CMPU1018_CA_

Finished!

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Click on a question number to see how your answers were marked and, where available, full solutions.

Question Number	Score			
1	5	/	5	Review
2	5	/	5	Review
3	3	/	3	Review
4	4	/	4	Review
5	4	/	4	Review
6	2	/	2	Review
7	4	/	4	Review
8	2	/	2	Review
9	20	/	20	Review
Total	**49**	**/**	**49 (100%)**	

Question 1

The general method for solving an equation of the form $ax \equiv c \pmod{b}$ is called the **Extended Euclidean Algorithm**, which is procedure for finding $\gcd(a, b)$ along with the integers x, y such that

$$ax + by = \gcd(a, b).$$

a)

Remember that $ax \mod b$ will always a multiple of the $\gcd(a, b)$.

So it's possible to tell straight away that there are no solutions to the equation $3078x \equiv 1 \pmod{6156}$, because

1 is not a multiple of $\gcd(3078, 6156) =$ 3078 ✔ Expected answer: 3078 .

Your answer is correct. You were awarded **1** mark. ✔

You scored **1** mark for this part.

Score: 1/1 ✔

b)

The Extended Euclidean Algorithm can be used to solve $3078x \equiv 6 \pmod{1500}$ by reducing the problem via modular arithmetic. This is a similar method to the previous question, except this time we keep track of the quotients:

$3078 = 1500q_0 + 78$ where $q_0 =$ 2 ✔ Expected answer: 2

$1500 = 78q_1 + 18$ where $q_1 =$ 19 ✔ Expected answer: 19

$78 = 18q_2 + 6$ where $q_2 =$ 4 ✔ Expected answer: 4

$18 = 6 \times 3 + 0.$

Stop when you see a remainder of zero, because this means that the $\gcd(3078, 1500) = \gcd(6, 0) = 6$. Treat the numbers as symbols (no simplification) and use back-substitution to eliminate the remainders 18 and 78. The result is an expression of the $\gcd$ in the form $3078x + 1500y$.

$$
\begin{aligned}
6 &= 78 - 18q_2 && \text{substitute } 18 = 1500 - 78q_1 \\
&= 78 - (1500 - 78q_1)q_2 && \text{collect like terms} \\
&= 78(1 + q_1q_2) - 1500q_2 && \text{substitute } 78 = 3078 - 1500q_0 \\
&= (3078 - 1500q_0)(1 + q_1q_2) - 1500q_2 && \text{collect like terms} \\
&= 3078(1 + q_1q_2) - 1500(q_2 + q_0(1 + q_1q_2))
\end{aligned}
$$

Now, using the values of the quotients q_0, q_1, q_2 you computed earlier, this means:

$$6 = 3078 \times 77 - 1500 \times 158.$$

So the solution to $3078x \equiv 6 \pmod{1500}$ is $x =$ 77 ✔

Expected answer: 77

Gap 0

Your answer is correct. You were awarded **1** mark. ✔

Gap 1

Your answer is correct. You were awarded **1** mark. ✔

Gap 2

Your answer is correct. You were awarded **1** mark. ✔

Gap 3

Your answer is correct. You were awarded **1** mark. ✔

You scored **4** marks for this part.

Score: 4/4 ✔

Advice

This question shows the full Extended Euclidean Algorithm.

Part (a) shows a shortcut to determine if there are no solutions to a given equation. $3078x \pmod{6156}$ can only be equal to a mutliple of $\gcd(3078, 6156) = 3078$, and the multiples in modulo 6156 are 0 and 3078.

So $3078x \mod 6156 \neq 1$ because 1 is not a multiple of 3078.

Part (b) shows the full working for the Extended Euclidean Algorithm.

Question 2

This question will test your use of the Division Algorithm and the Extended Euclidean Algorithm.

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a)

Use the division algorithm to find the quotient q and the remainder r when dividing 11 by 8.

q = 1 ✔ Expected answer: 1 , r = 3 ✔ Expected answer: 3

q

Your answer is correct. You were awarded **1** mark. ✔

r

Your answer is correct. You were awarded **1** mark. ✔

You scored **2** marks for this part.

Score: 2/2 ✔

b)

The gcd of 20 and 29 is 1 ✔ Expected answer: 1 .

Your answer is correct. You were awarded **1** mark. ✔

You scored **1** mark for this part.

Score: 1/1 ✔

c)

$gcd(20, 29) =$ -13 ✔ Expected answer: -13 x$20+$ 9 ✔ Expected answer: 9 x29

1

Your answer is correct. You were awarded **1** mark. ✔

2

Your answer is correct. You were awarded **1** mark. ✔

You scored **2** marks for this part.

Score: 2/2 ✔

Question 3

Perform the following calculations in $\mathbb{Z}_2, \ \mathbb{Z}_9, \ \mathbb{Z}_{10}$.

	$\mathbb{Z}_2$	$\mathbb{Z}_9$	$\mathbb{Z}_{10}$
$1+3$	0 ✔ Expected answer: 0	4 ✔ Expected answer: 4	4 ✔ Expected answer: 4
2×3	0 ✔ Expected answer: 0	6 ✔ Expected answer: 6	6 ✔ Expected answer: 6
$6\times(6+6)$	0 ✔ Expected answer: 0	0 ✔ Expected answer: 0	2 ✔ Expected answer: 2
7×9	1 ✔ Expected answer: 1	0 ✔ Expected answer: 0	3 ✔ Expected answer: 3
$(6+7)\times(7+7)$	0 ✔ Expected answer: 0	2 ✔ Expected answer: 2	2 ✔ Expected answer: 2

Gap 0

Your answer is correct. You were awarded **0.2** marks. ✔

Gap 1

Your answer is correct. You were awarded **0.2** marks. ✔

Gap 2

Your answer is correct. You were awarded **0.2** marks. ✔

Gap 3

Your answer is correct. You were awarded **0.2** marks. ✔

Gap 4

Your answer is correct. You were awarded **0.2** marks. ✔

Gap 5

Your answer is correct. You were awarded **0.2** marks. ✔

Gap 6

Your answer is correct. You were awarded **0.2** marks. ✔

Gap 7

Your answer is correct. You were awarded **0.2** marks. ✔

Gap 8

Your answer is correct. You were awarded **0.2** marks. ✔

Gap 9

Your answer is correct. You were awarded **0.2** marks. ✔

Gap 10

Your answer is correct. You were awarded **0.2** marks. ✔

Gap 11

Your answer is correct. You were awarded **0.2** marks. ✔

Gap 12

Your answer is correct. You were awarded **0.2** marks.

Gap 13

Your answer is correct. You were awarded **0.2** marks.

Gap 14

Your answer is correct. You were awarded **0.2** marks.

You scored **3** marks for this part.

Score: 3/3

Advice

In the the last part, working out $(6+7) \times (7+7) \bmod X$, it is sometimes easier to work out $(6+7) \bmod X$ and $(7+7) \bmod X$ separately, giving two numbers in the range $[0 \ldots X-1]$, and then to multiply them together.

For example, working $\bmod 9$ we have:

$$6+7 \equiv 4 \bmod 9,$$
$$7+7 \equiv 5 \bmod 9.$$

$$\begin{aligned}(6+7) \times (7+7) &\equiv 4 \times 5 \bmod 9 \\ &\equiv 2 \bmod 9\end{aligned}$$

Question 4

Multiplication in modular arithmetic looks different to regular multiplication. Here is an example for multiplication in modulo 7.

$\times$	**0**	**1**	**2**	**3**	**4**	**5**	**6**
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

Use this multiplication table to answer following questions.

a)

The number x such that $5x \equiv 1 \pmod 7$ is called the **inverse** of 5 in $\mathbb{Z}_7$. Find the inverse of 5 in $\mathbb{Z}_7$.

3

Expected answer: 3

$5 \times 3 = 1 \pmod 7$. You were awarded **1** mark.

You scored **1** mark for this part.

Score: 1/1

b)

Solve $5x \equiv 2 \pmod 7$ for x.

6

Expected answer: 6

$5 \times 6 = 2 \pmod 7$. You were awarded **1** mark.

You scored **1** mark for this part.

Score: 1/1

c)

Solve $5x \equiv 6 \pmod 7$ for x.

4

Expected answer: 4

$5 \times 4 = 6 \pmod 7$. You were awarded **1** mark.

You scored **1** mark for this part.

Score: 1/1

d)

Solve $5x + 2 \equiv 3 \pmod 7$ for x.

3

Expected answer: 3

$5 \times 3 + 2 = 1 \pmod 7$. You were awarded **1** mark.

You scored **1** mark for this part.

Score: 1/1

Advice

This question introduces the concept of multiplication in modular arithmetic. All the answers can be found by looking up the appropriate row and column in the given multiplication table.

Aside: It is easy to find the inverse of a when working in $\mathbb{Z}_p$ for some prime number p. By Fermat's Little Theorem $a^p \equiv a \pmod{p}$, so if a is not a multiple of p then $a^{-1} \equiv a^{p-2} \pmod{p}$.

Question 5

a)

Express the following as a single power of 19:

$$19^{11} \times 19^4$$

Enter the power 15 15 ✔

Expected answer: 15 15

Your answer is numerically correct. You were awarded **1** mark. ✔

You scored **1** mark for this part.

Score: 1/1 ✔

b)

Express the following as a single power of 2:

$$(2^2)^9$$

Enter the power 18 18 ✔

Expected answer: 18 18

Your answer is numerically correct. You were awarded **1** mark. ✔

You scored **1** mark for this part.

Score: 1/1 ✔

c)

Express the following as a power of a single number. The power should be a prime.

$$3^{11} \times 19^{11}$$

Enter the number 57 57 Expected answer: 57 57 and the power 11 11 Expected answer: 11 11

Gap 0

Your answer is numerically correct. You were awarded **1** mark.

Gap 1

Your answer is numerically correct. You were awarded **1** mark.

You scored **2** marks for this part.

Score: 2/2

Advice

(a) Add the powers: $11 + 4 = 15$ (first index law).

(b) Multiply the powers $2 \times 9 = 18$ (second index law).

(c) Multiply 3 and 19 to get 57, with the power unchanged at 11. [This is the only way for the power to be a prime.]

Question 6

Solve the following equation for x.

Input your answer as a fraction or an integer as appropriate and not as a decimal.

$$\log_3(x+20) - \log_3(x+4) = 2$$

$x =$ -2 -2 Expected answer: -2 -2

Your answer is numerically correct. You were awarded **2** marks.

You scored **2** marks for this part.

Score: 2/2

Advice

We use the following two rules for logs :

1. $\log_a(\frac{x}{y}) = \log_a(x) - \log_a(y)$

2. $a^x = y \iff \log_a y = x$

Using rule 1 we get

$$\log_3(x+20) - \log_3(x+4) = \log_3\left(\frac{x+20}{x+4}\right)$$

So the equation to solve becomes:

$$\log_3\left(\frac{x+20}{x+4}\right) = 2$$

and using rule 2 this gives:

$$\begin{aligned} \frac{x+20}{x+4} &= 3^2 \Rightarrow \\ x+20 &= 3^2(x+4) = 9(x+4) \Rightarrow \\ 8x &= 20 - 4\times 9 = -16 \Rightarrow \\ x &= -2 \end{aligned}$$

We should check that this solution gives positive values for $x+20$ and $x+4$ as otherwise the logs are not defined.

Substituting this value for x into $\log_3(x+20)$ we get $\log_3(18)$ so OK.

For $\log_3(x+4)$ we get on substituting for x, $\log_3(2)$ so OK.

Hence the value we found for x is a solution to the original equation.

Question 7

Let $A = \{8, 5, 19\}$, let $B = \{45, 85\}$, let $C = \{5, 8, 21\}$ and let $D = \{85, 21\}$.

List the elements of the following sets.

Input sets in the form `set(a,b,c,d)` .

For example `set(1,2,3)` gives the set $\{1, 2, 3\}$.

Element (a, b) of a Cartesian product is entered, and represented, as $[a, b]$.

For example `set([1,1],[1,2],[2,3])` gives the set $\{[1,1], [1,2], [2,3]\}$.

The empty set is input as `set()` .

a)

$A \times B =$

set([8,45],[5,45],[19,45],[8,85],[5,85],[19,85]) $\{[8,45]\,,[5,45]\,,[19,45]\,,[8,85]\,,[5,85]\,,[19,85]\}$

Expected answer: set([8, 45],[8, 85],[5, 45],[5, 85],[19, 45],[19, 85])

$\{[8,45]\,,[8,85]\,,[5,45]\,,[5,85]\,,[19,45]\,,[19,85]\}$

Your answer is numerically correct. You were awarded **1** mark.

You scored **1** mark for this part.

Score: 1/1 ✔

b)

$(B \cap D) \times (A \cap C) =$ set([85,8],[85,5]) $\{[85, 8], [85, 5]\}$ ✔

Expected answer: set([85, 8],[85, 5]) $\{[85, 8], [85, 5]\}$

Your answer is numerically correct. You were awarded **1** mark. ✔

You scored **1** mark for this part.

Score: 1/1 ✔

c)

$[(A - C) \cup (C - A)] \times [(B - D) \cup (D - B)] =$

set([19,45],[19,21],[21,45],[21,21]) $\{[19, 45], [19, 21], [21, 45], [21, 21]\}$ ✔

Expected answer: set([19, 45],[19, 21],[21, 45],[21, 21]) $\{[19, 45], [19, 21], [21, 45], [21, 21]\}$

Your answer is numerically correct. You were awarded **1** mark. ✔

You scored **1** mark for this part.

Score: 1/1 ✔

d)

$(A \times D) \cap (C \times B) =$ set([8,85],[5,85]) $\{[8, 85], [5, 85]\}$ ✔

Expected answer: set([8, 85],[5, 85]) $\{[8, 85], [5, 85]\}$

Your answer is numerically correct. You were awarded **1** mark. ✔

You scored **1** mark for this part.

Score: 1/1 ✔

Advice

a)

$A \times B$ is the set of all pairs (a, b), where $a \in A$ and $b \in B$.

b)

$B \cap D$ is the set of all elements present in both B and D, i.e. $\{85\}$.

$A \cap C$ is the set of all elements present in both A and C, i.e. $\{8, 5\}$.

$(B \cap D) \times (A \cap C)$ is the set of pairs of all pairs (x, y), where $x \in B \cap D$ and $y \in A \cap C$.

c)

$(A \cap C) \times (A \cap C) \times (C \cap D)$ is the set of all triples (x, y, z), where $x \in A \cap C$, $y \in A \cap C$ and $z \in C \cap D$. Note that x and y do not have to be different.

d)

$A - C$ is the set of all elements present in A but not in C, i.e. $\{19\}$.

$C - A$ is the set of all elements present in C but not in A, i.e. $\{21\}$.

$(A - C) \cup (C - A)$ is the set of all elements which are either in $A - C$, or in $C - A$, so $(A - C) \cup (C - A) = \{19, 21\}$.

e)

$(A \times D)$ is the set of all pairs of elements (a, d), with $a \in A$ and $d \in D$, i.e. $\{[8, 85], [8, 21], [5, 85], [5, 21], [19, 85], [19, 21]\}$.

$C \times B)$ is the set of all pairs of elements (c, b), with $c \in C$ and $b \in B$, i.e. $\{[5, 45], [5, 85], [8, 45], [8, 85], [21, 45], [21, 85]\}$.

$(A \times D) \cap (C \times B)$ is the set of all pairs present in both of the previous sets.

f)

$C \cap D$ is the set of all elements in both C and in D, so $C \cap D = \{21\}$.

$C - D$ is the set of all elements in C and not in D, so $C - D = \{5, 8\}$.

$(C \cap D) \times (C - D)$ is the set of all pairs of elements (x, y), where x is in $C \cap D$ and y is in $C - D$, so $C \cap D) \times (C - D) = \{[21, 5], [21, 8]\}$.

Similarly, $(D - C) \times (C \cap D) = \{[85, 21]\}$.

Finally, $[(C \cap D) \times (C - D)] \cup [(D - C) \times (C \cap D)]$ is the set of all pairs present in either of the above sets, i.e. $\{[21, 5], [21, 8], [85, 21]\}$.

Question 8

The sum of the first 3 terms of an arithmetic progression is 14 and the $9th$ term of the same series is 12.

Calculate the value of the common difference. d = 1.0 *Round your answer to 1 decimal place.*

Expected answer: 1.0

Calculate the value of the first term of the series. a = 3.6 *Round your answer to 1 decimal place.*

Expected answer: 3.6

Gap 0

Your answer is correct. You were awarded **1** mark.

Gap 1

Your answer is correct. You were awarded **1** mark.

You scored **2** marks for this part.

Score: 2/2

Advice

Recall the formula for the sum of the first n terms of an arithmetic progression is $S_n = \frac{n}{2}(2a + (n-1)d)$.

The sum of the first 3 terms of an arithmetic progression is 14

$\frac{3}{2}(2a + 2d) = 14$ equation (i)

The formula for nth term of an arithmetic progression is $T_n = a + (n-1)d$.

The $9th$ term of the same series is 12

$a + 8d = 12$ equation (ii)

Here we have two simultaneous equations. We can eliminate the a term.

$3a + 3d = 14$ equation (i)

$3a + 24d = 36$ equation (ii)*3

$-21d = -22$

$d = \frac{-22}{-21}$

$d = \frac{22}{21}$

Using this result and equation (ii) we can find the value for a

$a + \frac{176}{21} = 12$

$a = 12 - \frac{176}{21}$

$a = 12 - \frac{176}{21}$

Question 9

In the following question you are asked to construct a truth table for:

$$((q \wedge \neg p) \to (\neg p \to q)) \wedge (\neg q \vee p).$$

Enter T if true, else enter F.

Complete the following truth table:

p	q	$q \wedge \neg p$	$\neg p \to q$	$(q \wedge \neg p) \to (\neg p \to q)$	$\neg q \vee p$	$((q \wedge \neg p) \to (\neg p \to q)) \wedge (\neg q \vee p)$
T	T	f Expected answer: F	t Expected answer: T	t Expected answer: T	t Expected answer: T	t Expected answer: T
T	F	f Expected answer: F	t Expected answer: T	t Expected answer: T	t Expected answer: T	t Expected answer: T
F	T	t Expected answer: T	t Expected answer: T	t Expected answer: T	f Expected answer: F	f Expected answer: F
F	F	f Expected answer: F	f Expected answer: F	t Expected answer: T	t Expected answer: T	t Expected answer: T

Gap 0

Your answer is correct. You were awarded **1** mark.

Gap 1

Your answer is correct. You were awarded **1** mark.

Gap 2

Your answer is correct. You were awarded **1** mark.

Gap 3

Your answer is correct. You were awarded **1** mark.

Gap 4

Your answer is correct. You were awarded **1** mark.

Gap 5

Your answer is correct. You were awarded **1** mark.

Gap 6

Your answer is correct. You were awarded **1** mark.

Gap 7

Your answer is correct. You were awarded **1** mark.

Gap 8

Your answer is correct. You were awarded **1** mark.

Gap 9

Your answer is correct. You were awarded **1** mark.

Gap 10

Your answer is correct. You were awarded **1** mark.

Gap 11

Your answer is correct. You were awarded **1** mark.

Gap 12

Your answer is correct. You were awarded **1** mark.

Gap 13

Your answer is correct. You were awarded **1** mark.

Gap 14

Your answer is correct. You were awarded **1** mark.

Gap 15

Your answer is correct. You were awarded **1** mark.

Gap 16

Your answer is correct. You were awarded **1** mark.

Gap 17

Your answer is correct. You were awarded **1** mark.

Gap 18

Your answer is correct. You were awarded **1** mark.

Gap 19

Your answer is correct. You were awarded **1** mark.

You scored **20** marks for this part.

Score: 20/20

Advice

First we find the truth table for $q \wedge \neg p$:

p	q	$q \wedge \neg p$
T	T	F
T	F	F
F	T	T
F	F	F

Then the truth table for $\neg p \to q$:

p	q	$\neg p \to q$
T	T	T
T	F	T
F	T	T
F	F	F

Putting these together to find $(q \wedge \neg p) \to (\neg p \to q)$:

p	q	$q \wedge \neg p$	$\neg p \to q$	$(q \wedge \neg p) \to (\neg p \to q)$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	F	F	T

Next we find the truth table for $\neg q \vee p$:

p	q	$\neg q \vee p$
T	T	T
T	F	T
F	T	F
F	F	T

Putting this all together to obtain the truth table we want:

p	q	$(q \wedge \neg p) \to (\neg p \to q)$	$\neg q \vee p$	$((q \wedge \neg p) \to (\neg p \to q)) \wedge (\neg q \vee p)$
T	T	T	T	T
T	F	T	T	T
F	T	T	F	F
F	F	T	T	T