Scalar Transport Equation

The scalar transport equation is a fundamental expression in fluid mechanics and related fields that describes the transport of a scalar quantity ϕ (such as temperature, concentration, or another property) within a fluid medium. It is expressed as:

$$\frac{\partial(\rho\phi)}{\partial t} + \nabla \cdot (\rho\phi\mathbf{u}) = \nabla \cdot (\Gamma\nabla\phi) + S_\phi$$

Where:

- ρ: Fluid density.
- ϕ: Scalar quantity being transported.
- $\mathbf{u}$: Velocity vector of the fluid flow.
- Γ: Diffusion coefficient (depends on the type of ϕ).
- S_ϕ: Source term for ϕ, representing creation, destruction, or external addition.

Breakdown of Terms:

1. **Unsteady (time-dependent) term**: $\frac{\partial(\rho\phi)}{\partial t}$
   - Represents the rate of change of ϕ within a fluid element due to time variations.
2. **Convective term**: $\nabla \cdot (\rho\phi\mathbf{u})$
   - Describes the transport of ϕ due to the motion of the fluid (advection). It accounts for the flow carrying ϕ in and out of a region.
3. **Diffusive term**: $\nabla \cdot (\Gamma\nabla\phi)$
   - Models the rate of diffusion of ϕ caused by gradients in its concentration. Diffusion works to equalize differences in ϕ within the medium.
4. **Source term**: S_ϕ
   - Represents any generation or consumption of ϕ due to external factors, such as reactions, heat sources, or imposed conditions.

$$\frac{\partial(\rho\phi)}{\partial t} + \nabla \cdot (\rho\phi\mathbf{u}) = \nabla \cdot (\Gamma\nabla\phi) + S_\phi$$

Physical Interpretation:

The left-hand side of the equation represents the total change in ϕ in a control volume:

- The first term ($\frac{\partial(\rho\phi)}{\partial t}$) is the local, time-dependent change.
- The second term ($\nabla \cdot (\rho\phi\mathbf{u})$) is the change due to the flow of fluid carrying ϕ in and out of the control volume.

The right-hand side describes mechanisms that modify ϕ within the control volume:

- Diffusion smooths out gradients in ϕ.
- Sources and sinks add or remove ϕ.

This equation is widely used in simulations to model phenomena like heat transfer, mass transfer, and other scalar fields in fluids. It is typically solved using numerical methods in Computational Fluid Dynamics (CFD).