

# Game Theory

## 3.1 Describing a game

### 3.2 Static + complete

### 3.3 Dynamic + complete + perfect info

### 3.4 Repeated games

### 3.5 Bargaining + fair allocations

### 3.6 Games with incomplete + asymmetric + imperfect info

#### Nash Equilibrium

No advantage in switching for either side

#### Introductory game theory =

study of rational behavior

in strategic settings

where players make flawless calculations

in determining the actions

that best serve their interests

#### General Information

Strategic game = game where the actions of one player influences the actions of the other player (Not pure skill or pure luck)

Noncooperative game theory = contracts / cooperation among competitors is not allowed (eg US business environment)

#### Rules of game theory

(1) Never play a dominated strategy

(2) If you have a dominant strategy, play it

(3) Eliminate strategies until all dominated are eliminated

(4) If dominant & iterated dominant don't exist, look for mutual best replies (5) In dynamic games: look forward, reason backwards

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## 3.1 Describing a Game

### Underlying assumption - Players are rational

(1) You are aware of your preferences & constraints

(2) You act consistently given your preferences & constraints & behavior of others

### Game theory characteristics

(1) Strategy = players contingency plan or decision rule (optimal action in every possible scenario)

(2) Timing of play

(a) Static game = each player moves without knowing their opponents move

(b) Dynamic game = order of play is sequential

(3) Information Set = info each player has at each stage of the game

(a) Complete information = each player knows the payoff / objective of every other player

(b) Perfect information (dynamic games only) = each player knows the history of the other players in the game

(c) Common knowledge = Information known by everyone in the game, everyone knows everyone else knows

(d) Symmetric information = each player has the same set of information

#### Rules of the game

(1) Players

(2) Actions

(3) Timing of play

(4) Payoff function of each player

(5) Information available

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## 3.2 Static Games + Complete Information

Everyone moves without knowing the others player's moves, but everyone knows the other players objectives.

### (1) Coordination games

Players benefit from cooperating, Communication before playing the game helps with the final outcome

**Example 1: Driving on one side of the road** It doesn't matter whether people choose left or right as long as they make the same choice.

**Example 2: Pareto Coordination Game** There is one outcome that would be best for everyone, but another where they will benefit more than the rest of the options. Both choosing the one correct box > both choosing a different box > choosing different boxes

#### Example 3: Dating game

Guy likes meat + red wine

Girl likes fish + white win

**Good:** Meat + red OR fish + white

**Bad:** Meat + white OR fish + red

Guy brings fish to make girl happy

Girl brings red wine to make guy happy

Dinner = ruined

A Coordination Game

1. If everyone drives on the same side of the road that's 'good' (1)
2. If people drive on different sides of the road that's 'bad' (-1)

	L	R
L	1, 1 ✓	0, 0
R	0, 0	1, 1 ✓

A Pareto Coordination Game

1. If everyone chooses L the payout for everyone 2.
2. If everyone chooses R the payout for everyone is 1.
3. Otherwise -1

	L	R
L	2, 2 ✓	0, 0
R	0, 0	1, 1 ✓

Dating Game

	A	B	C
A	2, 1	-2, -2	-5, 0
B	0, 0	1, 2	-5, -2
C	0, -5	-2, -5	-10, -10

	A	B	C
A	2, 1	-2, -2	
B	0, 0	1, 2	
C			

	A	B	C
A	2, 1	-2, -2	
B	0, 0	1, 2	
C			

### (2) competitive games

#### Example 1: Zero sum games (Matching pennies)

#### Example 2: Not zero sum (War - each side wants the other to be peaceful.)

From the perspective of A:

Best result = They are peaceful, we attack (+3)

2nd best result = We are both peaceful (+2)

3rd best result = They attack, we are peaceful but limited damage (+1)

4th best result = We both attack, full war (-1)

#### Example 3: Prisoners Dilemma

Similar to war from above, but now the "3rd best" option from above is going to be the worst for us. Not confessing is a bigger risk.

#### Zero Sum Games

- Two countries have the choice to build up their military or invest in domestic programs
- If both invest in domestic programs that results in happiness
- If both invest in war programs that results in despair
- If one country invests in the military and the other in domestic programs, it takes over the other country and takes their resources

	M	D
M	-1, -1	-5, 7
D	7, -5	5, 5

#### Failure to Coordinate

Two people are arrested for a crime.

- If one snitches on the other, the snitch goes free and the other player goes to jail for a 'long time'
- If they both say nothing, they will go free
- If they both snitch they both go to jail for a 'short time'

	S	M
S	-3, -3	-5, 0
M	0, -5	0, 0

#### Matching Pennies

- Two players reveal a penny simultaneously
- If they match (e.g. heads-heads, tails-tails) player one gets both pennies
- If they don't match, player two gets the pennies

		H	T
Player 1	H	2, 0	0, 2
Player 2	T	0, 2	2, 0

Playing As Player 2 → Payoffs on the right

$$E(H) = 0 \cdot x + (1-x) \cdot 2$$

$$E(T) = 2 \cdot x + (1-x) \cdot 0$$

$$E(H) = E(T)$$

$$(1-x) \cdot 2 = 2 \cdot x$$

$$x = \frac{1}{2}$$

Playing As Player 1 → Payoffs on the left

$$E(H) = 0 \cdot p + (1-p) \cdot 2$$

$$E(T) = 2 \cdot p + (1-p) \cdot 0$$

$$E(H) = E(T)$$

$$(1-p) \cdot 2 = 2 \cdot p$$

$$p = \frac{1}{2}$$

## (3) Static games 3 types of equilibrium

### 1st - Dominant Strategy Equilibrium

Strictly dominant strategy = choice that earns higher payoff than any other strategy

If both players have a strictly dominant strategy, the result will be a **dominant strategy equilibrium**

Example: Negative political ads

### 2nd - Iterated-Dominant Strategy Equilibrium

Example: Pig Push vs Not Players will eliminate dominated strategies until they find a solution

### 3rd - Nash Equilibrium

The equilibrium where neither player would be better off making a change

Pure strategy (predictable) = decision rule that does not involve randomizing

Mixed strategy (unpredictable) = sometimes choose one, sometimes choose the other

#### Finding a mixed strategy Nash equilibrium to a game like matching pennies

- Define possible mixed strategies
- Identify each player's best reply function (the best response to each possible move by the opponent)
- Mixed strategy Nash equilibrium = where the best replies of each player equal one another

## 3.3 Dynamic Games + Perfect Information

### Nodes

- Decision node = a point where a player is making a decision
- Initial node = Very first decision
- Terminal node = very bottom of the lowest branches

### Threats

- False belief = you think the other player will do something if you don't give in, but they actually won't
- Credible threat = they actually will go through with what they're saying

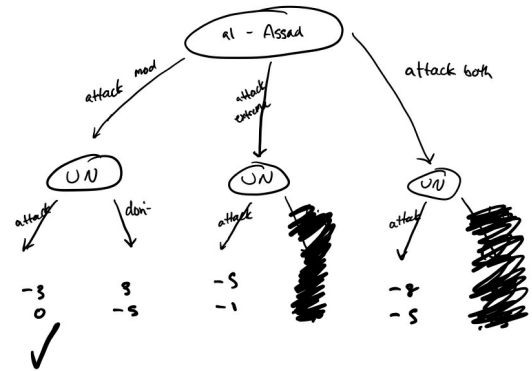
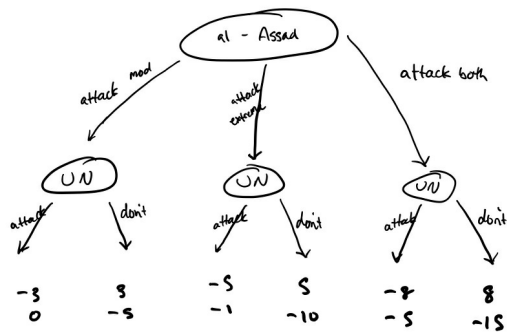
#### Consider Syria

- Write down the players in the Syria conflict, make sure to consider
  - Bashar al-Assad
  - "Extreme" Rebels
  - "Moderate" Rebels

3. Moderate rebels

4. "UN" (for lack of a better description) Forces

2. Now consider the available actions of al-Assad and UN forces, explain (with a tree) why it was in al-Assad's interest to attack "Moderate" rebels, but not "Extreme" rebels



### My Solution (Not the Solution)

First Realization:

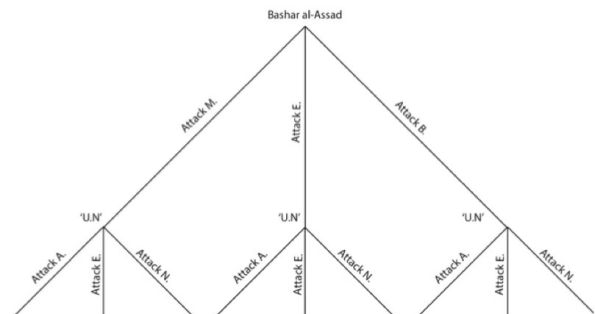
- Moderate and extreme rebels only have payouts, not actions. Under all conditions, they attack al-Assad.

Second Realization:

- 'Getting involved' is costly for the 'UN'. We know this because we *did not* (militarily) get involved until ISIL became a threat.

Third Realization:

- If the rebels defeat al-Assad, the stronger of the two rebel forces will take over the government.

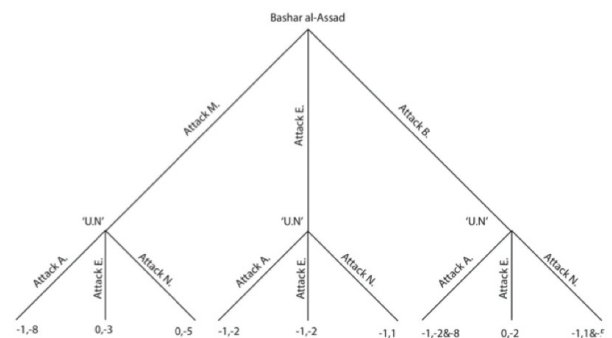


### Assumptions

- Bashar al-Assad only cares about staying in power. The combined rebel forces *could* defeat him without outside intervention. For tractability, we will assume they *will* defeat him without outside intervention.
- al-Assad has three choices, attack moderates, attack extremist or attack both.
- The 'UN' can attack al-Assad, the extremists or no one. They will never attack the moderates as they are allies.

### Payouts

- al-Assad will get a -1 payout if he loses power, 0 if he stays in power
- The UN will get a +1 payout if the moderate rebels take over, 0 if al-Assad stays in power (as it is the status quo) and -5 if the extremists take power. If they get involved, they will expend -3 in effort.



## Probabilities With Attacking Both

The U.N.'s action depends how likely they think the extremist will take over:

$$-2 = (1 - \mu) - 5\mu$$

Solve:  $\mu = \frac{1}{2}$

*If the UN thinks there is a 50% chance or more of the extremist taking over, they will attack the extremist.*

Al-Assad has a payout of 0 for sure when attacking the moderates, a -1 payout for sure when attacking the extremists and a payout between 0 and -1 (depending on  $\mu$ ) when attacking both.

## 3.4 Repeated games

### Finite repeated games

Tit for tat = we cooperate with our competitor because it helps us out in the end

### Infinite repeated games (super games)

Trigger strategy = they cooperated with us last time, we will this time, they will next time, etc

To solve, we have to decide whether the PV of all future returns from cooperating is greater than competing

$\pi$  = **5mil** (payoff from cooperating each period)

$PV$  = **10mil** (payoff for competing in first period, 0 afterwards)

$\beta$  = Discount factor (.9 would be someone who cares about the future a lot)

$$Value(\pi, \infty) = \frac{\pi}{1-\beta} = PV$$

$$Value(\pi, \infty) = \frac{5}{1-\beta} \geq 10$$

So in this example, we should only cooperate if our discount value is  $\geq$  than .5