Product Differentiation

What is product differentiation?

Law of one price = when every product is the same and when every customer is perfectly informed, each firm will charge the same price

According to Lancaster:
(1) Products in different markets when they serve different functions
(2) Differentiated products when same market (serve same function) but have different characteristics

According to Chamberlin:
(1) Objective characteristics: tangible, real differentiation
(2) Subjective characteristics: perceived product differentiation

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Sources of product differentiation:

(1) Physical differences in the products
(2) Differences in service quality by the seller
(3) Differences in geo location
(4) Differences in product image among customers
(5) Differences in quality

7.1 Types of Product Diff (Objective)

(1) Multichar product diff
(2) Horizontal product diff - only one char differs, people disagree over preference (eg color of car)
(3) Vertical product diff - only one char differs, everyone agrees (eg quality)

Solving the following problems:
(1) Determine the utility functions for choosing each of the firms
(2) Set the utility functions equal to one another
(3) Isolate the "choice desirability" variable. This is the quantity demanded for firm 1. Use this to set the profit equation.
(4) Take (1 - quantity demanded) for firm 2. Use this to set the profit function.
(5) Take the derivative for firm 1 and solve for price of firm 1.
(6) Take the derivative for firm 2 and solve for price of firm 2.
(7) Plug firm 2's price into the firm 1 profit derivative to solve for firm 1 profit max price.
(8) Optional: Take this price and plug it into firm 2's price equation. This will give the optimal price to set for both firm 1 and for firm 2.

7.2.1 Multichar

Best Response (US)

constant marg cost

$\Pi_1 = (a - q_1 - dq_2)(q_1) - cq_1$

Best Response (them)

constant marg cost

$\Pi = (a - q_2 - dq_1)(q_2) - cq_2$

$\frac{\partial \pi}{\partial q_1} = -1q_1 + a - q_1 - dq_2 - c$

$2q_1 = a - dq_2 - c$

$q_1 = \frac{a - dq_2 - c}{2}$

$\frac{\partial \pi}{\partial q} = -1q_0 + a - q_e - dq_1 - c$

$2q_2 = a - dq_1 - c$

$q_2 = \frac{a - dq_1 - c}{2}$

$\downarrow$

1) Plug in q_2 $\quad q_1 = \frac{a - d\left(\frac{a - dq_1 - c}{2}\right) - c}{2}$

2) Expand $\quad q_1 = \frac{1}{2}a - \frac{1}{4}da + \frac{1}{4}d^2q_1 + \frac{1}{4}dc - \frac{1}{2}c$

3) Isolate $\quad q_1 - \frac{1}{4}d^2q_1 = \frac{1}{2}a - \frac{1}{4}da + \frac{1}{4}dc - \frac{1}{2}c$

4) Factor $\quad q_1\left(1 - \frac{1}{4}d^2\right) = \frac{1}{2}a - \frac{1}{4}da + \frac{1}{4}dc - \frac{1}{2}c$

5) Simplify $\quad q_1 = \frac{\frac{1}{2}a - \frac{1}{4}da + \frac{1}{4}dc - \frac{1}{2}c}{\left(1 - \frac{1}{4}d^2\right)}$

7.2.2 Horizontal (Gas station, Salop model)

Horizontal product diff -- only one char differs, people disagree over preference (eg color of car)

V = free gas + free delivery
P = Our price
P_N = Their price
T = transportation costs
X = Distance to travel
D = Degree of substitutability

Best Response (US)

$\pi_1 = (a - q_1 - dq_2)(q_1) - cq_1$ constant marg cost

$\frac{\partial \pi}{\partial q_1} = -1q_1 + a - q_1 - dq_2 - c$

$2q_1 = a - dq_2 - c$

$q_1 = \frac{a - dq_2 - c}{2}$

Best Response (them)

$\pi = (a - q_2 - dq_1)(q_2) - cq_2$ constant marg cost

$\frac{\partial \pi}{\partial q} = -1q_0 + a - q_e - dq_1 - c$

$2q_2 = a - dq_1 - c$

$q_2 = \frac{a - dq_1 - c}{2}$

$\downarrow$

1) Plug in q_2 $\quad q_1 = \frac{a - d\left(\frac{a - dq_1 - c}{2}\right) - c}{2}$

2) Expand $\quad q_1 = \frac{1}{2}a - \frac{1}{4}da + \frac{1}{4}d^2q_1 + \frac{1}{4}dc - \frac{1}{2}c$

3) Isolate $\quad q_1 - \frac{1}{4}d^2q_1 = \frac{1}{2}a - \frac{1}{4}da + \frac{1}{4}dc - \frac{1}{2}c$

4) Factor $\quad q_1\left(1 - \frac{1}{4}d^2\right) = \frac{1}{2}a - \frac{1}{4}da + \frac{1}{4}dc - \frac{1}{2}c$

5) Simplify $\quad q_1 = \dfrac{\frac{1}{2}a - \frac{1}{4}da + \frac{1}{4}dc - \frac{1}{2}c}{(1 - \frac{1}{4}d^2)}$

In the scenario where product differentiation is due to geographic location, the price we should charge is our marginal cost + the cost the customer will need to pay to get to us.

Horizontal example - Salop Model (circle)

We get to choose advertising costs and our own price

V = free gas + free delivery
P = Our price
P_N = Their price
T = transportation costs
X = Distance to travel
A = Advertising
C = Marginal cost

Salop Model - we get to choose our ad costs and our own price

① $U = v - tx - p + A$

Cost: C, bA^2

② Firm 1 Utility = Firm 2 Utility

$v - tx - p_1 + A = v - t(\frac{1}{n} - x) - p_2 + A_2$

circle

③ Demand for firm 1 $= x = \dfrac{-p + \frac{t}{n} + P_2 - A_2}{2t}$

Firm 1

① $\pi_1 = (2x)(p - c) - bA^2$

2 sides of circle; demand from above; price side; ad costs up front

② $\dfrac{\partial \pi_1}{\partial p} \longrightarrow$ A's cancel and P's cancel

$P = C + \frac{t}{n}$

③ $\dfrac{\partial \pi_1}{\partial A} \longrightarrow A = \dfrac{\frac{1}{t}(p - c)}{2b}$

④ Plug P into A $\longrightarrow A = \frac{2b}{n}$

7.2.3 Vertical, circle

Vertical product diff - only one char differs, everyone agrees (eg quality)

base utility · quality you care about · price

$U_1 = V + zL_i - P_1$ ← TU with features

$U_2 = V - P_2$ ← Basic TU

} Utility Functions

$U_1 = U_2$

$V + zL_i - P_1 = V - P_2$

$L_i = \frac{P_1 - P_2}{z}$ → High L_i wants high quality

} Isolate desire for quality

Profit Equations

Firm 2 (Low Qual)	Firm 1 (High Qual)
$\pi_2 = \left(\frac{P_1 - P_2}{z}\right)(P_2 - c)$ $\frac{\partial \pi_2}{\partial P_2} = \frac{-1}{z}(P_2 - c) + \frac{P_1 - P_2}{z}$ $P_2 = \frac{C + P_1}{2}$	$\pi_1 = \left(1 - \frac{P_1 - P_2}{z}\right)(P_1 - c)$ $\frac{\partial \pi_1}{\partial P_1} = \frac{-1}{z}(P_1 - c) + \frac{z - P_1 + P_2}{z} = 0$ $P_1 = \frac{C + P_2 + z}{2}$
Firm 2 $P_2 = \frac{C + \left(\frac{C + P_2 + z}{2}\right)}{2}$ $P_2 = C + \frac{1}{3}z$	Firm 1 $P_1 = \frac{C + C + \frac{1}{3}z + z}{2}$ $P_1 = C + \frac{2}{3}z$

Best price for P_2 (low qual)

Best price for P_1 (high qual)

} Profit Max

Vertical example - Hotelling

Salop Model - we get to choose our ad costs and our own price

① $U = V - tx - p + A$
Cost: C, bA^2

② Firm 1 Utility = Firm 2 Utility

$V - tx - p_1 + A = V - t(\frac{1}{n} - x) - p_2 + A_2$

↑ circle

③ Demand for firm 1 $= x = \frac{-p + \frac{t}{n} + P_2 - A_2}{2t}$

Firm 1

① $\pi_1 = (2x)(p - c) - bA^2$

2 sides of circle　demand from above　price side　ad costs up front

② $\frac{\partial \Pi_1}{\partial p} \longrightarrow$ A's cancel and P's cancel

$$P = C + \frac{t}{n}$$

③ $\frac{\partial \Pi_1}{\partial A} \longrightarrow A = \frac{\frac{1}{t}(p-c)}{2b}$

④ Plug P into A $\longrightarrow A = \frac{2b}{n}$

R&D Example (Think cost leadership)

$$V - P - tx = V - P_N - t(1-x)$$
$$-tx + t(1-x) = -P_N + P$$
$$-tx + t - tx = -P_N + P$$
$$-2tx = -P_N + P - T$$
$$x = \frac{-P_N + P - T}{-2t}$$

$$\Pi_1 = \left(\frac{-P_N + P - T}{-2t}\right)\left(p - \frac{c}{b}\right) - b^2$$

$$\frac{\partial \Pi_1}{\partial p} = \left(\frac{1}{-2t}\right)\left(p - \frac{c}{b}\right) + \left(\frac{-P_N + P - T}{-2t}\right)$$

$$\frac{P - \frac{c}{b}}{2t} = \frac{P_N - P + T}{2T}$$

$$P = T + \frac{c}{b}$$

$$\frac{\partial \Pi}{\partial b} = \frac{c}{b^2}\left(\frac{-P_N + P - T}{-2t}\right) - 2b$$

$$2b = \frac{c}{b^2}\left(\frac{-P_N + P - T}{-2t}\right)$$

$$2b^3 = c\left(\frac{-P_N + P - T}{-2t}\right)$$

$$b = \frac{c}{2}\left(\frac{-P_N + P - T}{-2t}\right)^{\frac{1}{3}}$$

$$b = \left(\frac{c}{4}\right)^{1/3}$$

$$P = T + \frac{c}{b}$$

$$P = T + \left(c^{2/3}\right)\left(4^{1/3}\right)$$