Game Theory

3.1 Describing a game
3.2 Static + complete
3.3 Dynamic + complete + perfect info
3.4 Repeated games
3.5 Bargaining + fair allocations
3.6 Games with incomplete + assymetric + imperfect info

Nash Equilibrium
No advantage in switching for either side

Introductory game theory =
study of rational behavior
in strategic settings
where players make flawless calculations
in determining the actions
that best serve their interests

General Information
Strategic game = game where the actions of one player influences the actions of the other player (Not pure skill or pure luck)
Noncooperative game theory = contracts / cooperation among competitors is not allowed (eg US business environment)

Rules of game theory
(1) Never play a dominated strategy
(2) If you have a dominant strategy, play it
(3) Eliminate strategies until all dominated are eliminated
(4) If dominant & iterated dominant don't exist, look for mutual best replies (5) In dynamic games: look forward, reason backwards

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3.1 Describing a Game

Underlying assumption - Players are rational

(1) You are aware of your preferences & constraints
(2) You act consistently given your preferences & constraints & behavior of others

Game theory characteristics

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(1) Strategy = players contingency plan or decision rule (optimal action in every possible scenario)

(2) Timing of play
(a) Static game = each player moves without knowing their opponents move
(b) Dynamic game = order of play is sequential

(3) Informtion Set = info each player has at each stage of the game
(a) Complete information = each player knows the payoff / objective of every other player
(b) Perfect information (dynamic games only) = each player knows the history of the other players in the game
(c) Common knowledge = Information known by everyone in the game, everyone knows everyone else knows
(d) Symmetric information = each player has the same set of information

Rules of the game
(1) Players
(2) Actions
(3) Timing of play
(4) Payoff function of each player
(5) Information available

3.2 Static Games + Complete Information

Everyone moves without knowing the others player's moves, but everyone knows the other players objectives.

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(1) Coordination games

Players benefit from cooperating, Communication before playing the game helps with the final outcome

Example 1: Driving on one side of the road It doesn't matter whether people choose left or right as long as they make the same choice.

Example 2: Pareto Coordination Game There is one outcome that would be best for everyone, but another where they will benefit more than the rest of the options. Both choosing the one correct box > both choosing a different box > choosing different boxes

Example 3: Dating game
Guy likes meat + red wine
Girl likes fish + white win
Good: Meat + red OR fish + white
Bad: Meat + white OR fish + red

Guy brings fish to make girl happy
Girl brings red wine to make guy happy
Dinner = ruined

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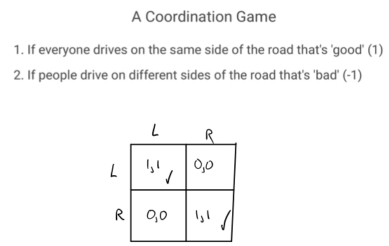

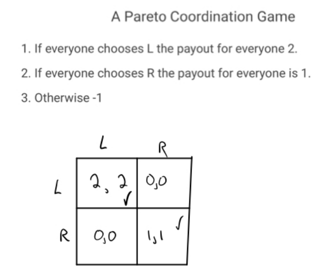

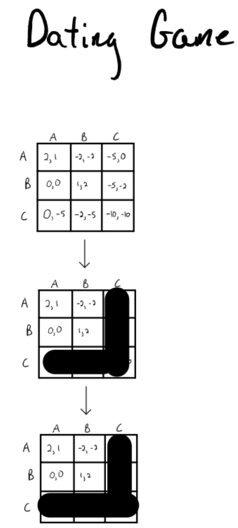

(2) competitive games

Example 1: Zero sum games (Matching pennies)

Example 2: Not zero sum (War - each side wants the other to be peaceful.)
From the perspective of A:
Best result = They are peaceful, we attack (+3)
2nd best result = We are both peaceful (+2)
3rd best result = They attack, we are peaceful but limited damage (+1)
4th best result = We both attack, full war (-1)

Example 3: Prisoners Dilemma
Similar to war from above, but now the "3rd best" option from above is going to be the worst for us. Not confessing is a bigger risk.

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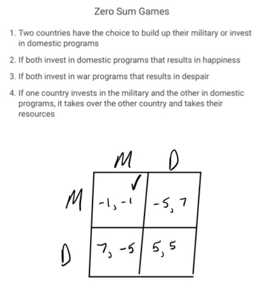

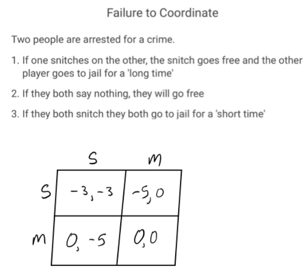

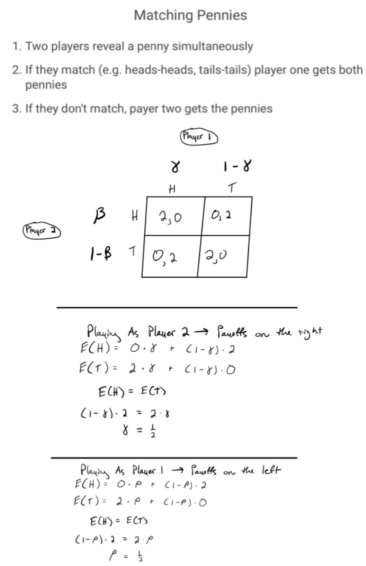

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(3) Static games 3 types of equilibrium

1st - Dominant Strategy Equilibrium
Strictly dominant strategy = choice that earns higher payoff than any other strategy
If both players have a strictly dominant strategy, the result will be a **dominant strategy equilibrium**
Example: Negative political ads

2nd - Iterated-Dominant Strategy Equilibrium
Example: Pig Push vs Not Players will elimate dominated strategies until they find a solution

3rd - Nash Equilibrium
The equilibrium where neither player would be better off making a change
Pure strategy (predictable) = decision rule that does not involve randomizing
Mixed strategy (unpredictable) = sometimes choose one, sometimes choose the other

Finding a mixed strategy Nash equilibrium to a game like matching pennies
(1) Define possible mixed strategies
(2) Identify each players best reply function (the best response to each possible move by the opponent)
(3) Mixed stratgy nash equilibrium = where the best replies of each player equal one another

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3.3 Dynamic Games + Perfect Information

Nodes
(1) Decision node = any point where a player is making a decision
(2) Initial node = Very first decision
(3) Terminal node = very bottom of the lowest branches

Threats

(1) False belief = you think the other player will do something if you don't give in, but they actually won't

(2) Credible threat = they actually will go through with what they're saying

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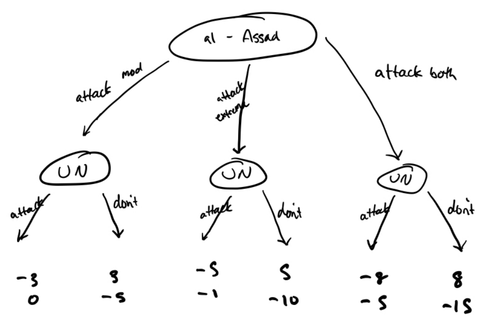
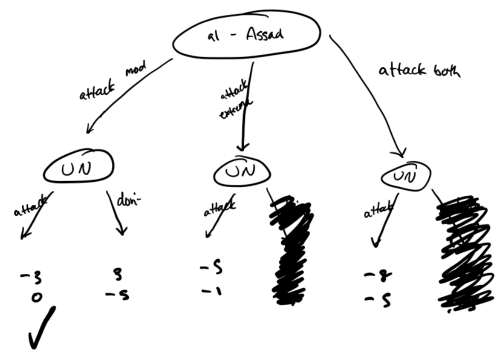
Consider Syria

1. Write down the players in the Syria conflict, make sure to consider
    1. Bashar al-Assad
    2. "Extreme" Rebels
    3. "Moderate" Rebels
    4. "UN" (for lack of a better description) Forces
2. Now consider the available actions of al-Assad and UN forces, explain (with a tree) why it was in al-Assad's interest to attack "Moderate" rebels, but not "Extreme " rebels





My Solution (Not 'the' Solution)

First Realization:

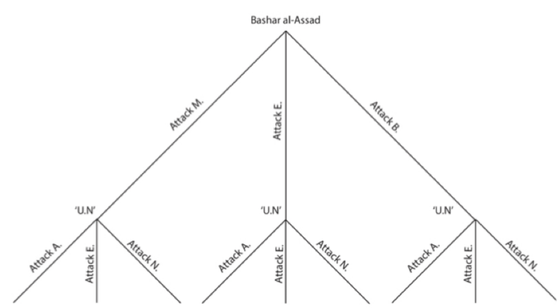
- Moderate and extreme rebels only have payouts, not actions. Under all conditions, they attack al-Assad.

Second Realization:

- 'Getting involved' is costly for the 'UN'. We know this because we *did not* (militarily) get involved until ISIL became a threat.

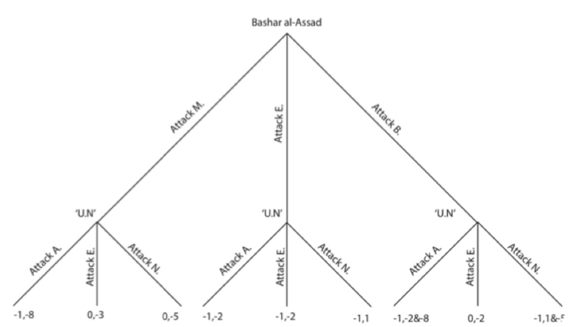
Third Realization:

- If the rebels defeat al-Assad, the stronger of the two rebel forces will take over the government.



Assumptions

1. Bashar al-Assad only cares about staying in power. The combined rebel forces *could* defeat him without outside intervention. For tractability, we will assume they *will* defeat him without outside intervention.
2. al-Assad has three choices, attack moderates, attack extremist or attack both.
3. The 'UN' can attack al-Assad, the extremists or no one. They will never attack the moderates as they are allies.



Payouts

1. al-Assad will get a -1 payout if he looses power, 0 if he stays in power
2. The UN will get a +1 payout if the moderate rebels take over, 0 if al-Assad stays in power (as it is the status quo) and -5 if the extremists take power. If they get involved, they will expend -3 in effort.

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Probabilities With Attacking Both

The U.N.'s action depends how likely they think the extremist will take over:

$$-2 = (1 - \mu) - 5\mu$$

Solve: $\mu = \frac{1}{2}$

If the UN thinks there is a 50% chance or more of the extremist taking over, they will attack the extremist.

Al-Assad has a payout of 0 for sure when attacking the moderates, a -1 payout for sure when attacking the extremists and a payout between 0 and -1 (depending on μ) when attacking both.

3.4 Repeated games

Finite repeated games
Tit for tat = we cooperate with our competitor because it helps us out in the end

Infinite repeated games (super games)
Trigger strategy = they cooperated with us last time, we will this time, they will next time, etc
To solve, we have to decide whether the PV of all future returns from cooperating is greater than competing

π = **5mil** (payoff from cooperating each period)
PV = **10mil** (payoff for competing in first period, 0 afterwards)
β = Discount factor (.9 would be someone who cares about the future a alot)

$Value(\pi, \infty) = \frac{\pi}{1-\beta} = PV$

$Value(\pi, \infty) = \frac{5}{1-\beta} \geq 10$
So in this example, we should only cooperate if our discount value is $\geq$ than .5

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