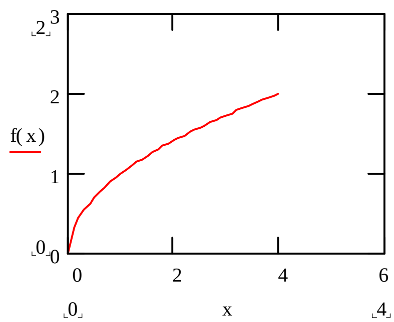
Problem Set #3 Solutions

1.

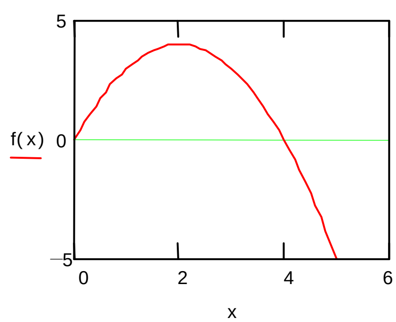
a) The graph is looks like



To find the volume as we rotate the function around the x-axis, we consider the volume to be made up of disks with radius f(x). Then, by integrating the area of the disks from 0 to 4 with respect to x, we get the volume.

$$\int_0^4 \pi (\sqrt{x})^2 \, dx = 25.1$$

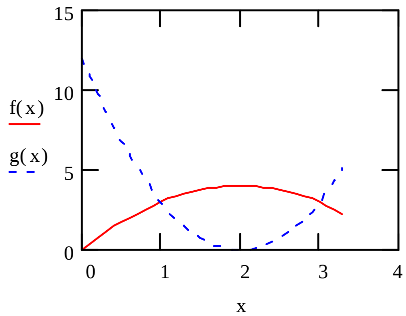
b) The graph looks like



Setting the function equal to zero and solving for x, we find that the function crosses the x-axis at x=0 and x=4. Once again, we consider the function to be the radius of a circle and integrate the area of the disks from 0 to 4.

$$\int_0^4 \pi (4x - x^2)^2 \, dx = 107.2$$

c) The graph looks like



When we rotate around the x-axis, we will have a washer-like cross section in most places. To get the area of the washers we use the functions as areas and take the top minus the bottom.

$$\int_1^3 \left(\pi(f(x))^2 - \pi(g(x))^2\right)dx$$

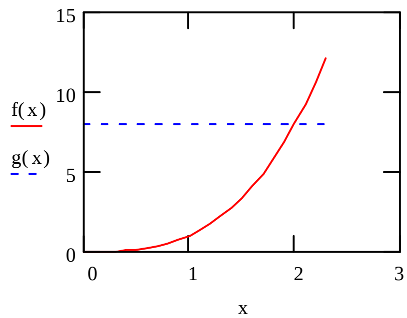
$$= \pi \int_1^3 \left((4x - x^2)^2 - (3(x-2)^2)^2\right)dx = 107.2$$

2.

See 1 a) for the graph. To find the volume, we use the shell method. To use the shell method, we calculate the area of the cylinder (2πrh) at each x and integrate. X will be our "r" and the value of the function will be our "h."

$$\int_0^4 2\pi \cdot x \cdot \sqrt{x} \cdot dx = 80.4$$

b) The graph looks like



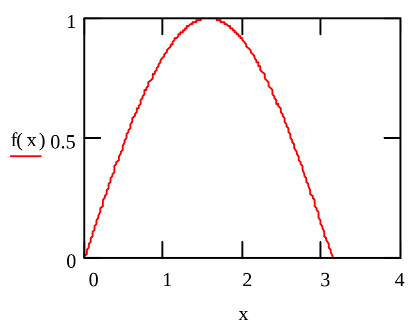
Using the shell method again, our height becomes 4-f(x) and our radius is still x. We see by solving

$$x^3 = 8$$

for x, x=2. Therefore, we integrate from 0 to 2.

$$\int_0^2 2\pi \cdot (8 - x^3) \cdot x \cdot dx = 60.3$$

d) The graph looks like



Using the shell method again, we get

$$\int_0^{\pi} 2\pi \cdot x \cdot Sin(x)dx = 19.7$$

3.

a) $$\int \frac{x^2}{\sqrt{1-x^2}}\,dx = \frac{\arcsin(x)}{2} - \frac{x\cdot\sqrt{1-x^2}}{2} + C$$

b) $$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2} + C$$

c) $$\int \frac{25}{(x-4)(2x+1)}\,dx = \frac{25}{9}\cdot\ln\left(\frac{x-4}{2x+1}\right) + C$$

d) $$\int \frac{6x^2-4}{x^2(x-2)}\,dx = \ln[x\cdot(x-2)^5] - \frac{2}{x} + C$$

e) $$\int \frac{4e^x}{e^{2x}-4}\,dx = \ln\left(\frac{e^x-2}{e^x 2}\right) + C$$

4. Once we have the following expression

$$\int \frac{\cos(\theta)}{1-\sin^2(\theta)}\,d\theta$$

We can use the following substitution for the denominator

$$\sin^2(\theta) = \frac{1}{2}(1-\cos(2\theta))$$

The integral becomes

$$\int \frac{2\cos\theta}{1+\cos(2\theta)}\,d\theta$$

Decomposing into partial fractions and completing the integral, we see that

$$\int \sec(\theta)\,d\theta = \frac{1}{2}\left(\frac{\sin(\theta)+1}{\sin(\theta)-1}\right) + C$$