ArsDigita University
Month 8: Theory of Computation
Professor Shai Simonson

Assignment 4

1. Context Free or Not

Determine and prove whether each of the following languages is Context Free or not.

a. $\{1^k 0^i 1^i 0^j 1^j 0^k \mid i,j,k > 0\}$.
b. {w#x | w is a substring of x, where w, x are in {0, 1}*}.
c. $\{0^i 1^j 0^j 1^i \mid , i,j > 0\}$.
d. **Extra Credit:** The complement of $\{(0^n 1^n)^m \mid m, n > 0\}$.

2. Decision Algorithms

Describe algorithms to decide the problems below.

a. Does a given Deterministic Pushdown Automaton generate (0+1)*?
b. Given a CFG and a string z in its language, does the string have 2 distinct derivation trees? (Note: your algorithm does not test whether or not the grammar is ambiguous! For that you would have to test every string.)
c. Text: 4.12: Show that the problem of testing whether a CFG generates some string in 1* is decidable.
d. **Extra Credit:** Text 4.13: Show that the problem of testing whether a CFG generates all strings in 1* is decidable.

3. Closure Problems for CFL's

a. Explain why the intersection of a regular language and a CFL must be a CFL (i.e. CFL's are closed under intersection with regular sets). You should illustrate your argument by constructing the machine that generates L intersected with R, where $L = 0^n1^n$ and R = (0+1)*110(0+1)*.
b. Show that the intersection of a regular language and a CFL is *not* necessarily regular (though it must be a CFL - see the previous problem).
c. Let L be some regular set in which all strings happen to have length equal to a multiple of three. Let Twist3(L) be the set of all strings in L where every three symbols are reversed. For example if L = {aag, cttgta, ttggagagc, ...} then Twist3(L) = {gaa, ttcatg, gttgagcga, ...}. Explain why Twist3(L) is a CFL. You should illustrate your argument by constructing the machine that generates Twist3(L), where L = (0+1)*110(0+1)*.
d. **Extra Credit:** Is Twist3(L) regular if L is regular?

4. Parsing and the CYK Decision Algorithm

a. Exhibit the table you get by doing the CYK algorithm on the strings 00000 and 000000 for the grammar below.

S → AB | BC A → BA | 0
B → CC | 1 C → AB | 0

b. Write a NPDA that accepts exactly what the grammar above generates.

5. Extra Credit: Chomsky-3 Normal Form

A grammar is in C3NF if every production is of the form A→BCD or A→b.

a. How many production steps does a C3NF grammar use to generate a string of length n? Explain.
b. If a C3NF grammar has n non-terminals, then how long does a string have to be, for it to be bound by the pumping lemma? Explain.
c. Can every CNF grammar be put into C3NF? Explain why or why not?.

6. Turing Machine Basics

a. Text 3.1 a.
b. Text 3.1 c.
c. Text 3.2 a.
d. Text 3.2 d.

7. Turing Machine Design

a. Design a TM that accepts the language of odd integers written in binary.
b. Design a TM program that accepts the language a#b#c, where a,b,c are in {0,1}*, and a+b=c, where a,b and c are interpreted as positive binary integers.
c. Design a TM that enumerates the language of odd integers written in binary.
d. Think about how tedious it would be to design a TM that enumerates all primes in binary.

8. Turing Recognition and Turning Decidability.

a. Text 3.14 a.
b. Text 3.14 d.
c. Text 3.15 a.
d. Text 3.15 c.

9. Extra Credit: A Problem that is Easier than it Seems

Show that if every subset of a set is a CFL, then the set must be regular.