ArsDigitaUniversity

Month2:DiscreteMathematics   -ProfessorShaiSimonson

Examination1 –100points

Showallworkforpartialcredit.Youmayusetwohoursforthisexam.Afterone hour,raiseyourhandifyoufeelthatthetimeconstraintwill   betootight.

Name:_____

1.   /30

2.   /10

3.   /25

4.   /10

5.   /25

Total: /100

1. (30 points) Applications of Logic in Computer Architecture

A very common circuit in computers is called a multiplexer. (A multiplexer is like a switch on a railroad, except that it helps route electrical signals rather than trains. For example, sometimes the data going to memory will come from an arithmetic calculation, and sometimes from an input buffer. A 2-value multiplexer controls which data gets sent to memory.)

A 2-value multiplexer has three inputs, two for data and one for control. The output is equal to the first data input when the control input equals zero, and the output equals the second data input when the control is one.

    a. Draw a truth table for a 2-value multiplexer.

    b. Write the CNF and DNF formulas for this circuit.

    c. If a computer program converted one of the above formulas above (your choice) into one with just *nor* gates, and made no attempt to minimize the result, how many gates would be in the result of each? Explain your answer.

    d. Given four data values, how many total inputs are in the 4-value multiplexer?

2. (10 points) Boolean Algebra

Prove or disprove the following equalities about the XOR operator:

   a. $a \oplus (b \oplus c) = (a \oplus b) \oplus c$

   b. $a \oplus (b \oplus c) = (a \oplus b) \oplus (a \oplus c)$

3. (25 points) Proofs by Induction

   a. Prove by induction that the sum of any six consecutive squares leaves a remainder of seven when divided by 12.

   b. What's wrong with the proof by induction below?

   Any non-zero number raised to the nth power equals one. The proof is by induction on n. For $n=0$, the theorem is trivial. By induction, since $n-1$ and $n-2$ are both less than n, $a^n = (a^{n-1} a^{n-1})/a^{n-2} = (1 \times 1)/1$.

4. (10 points) Inclusion/Exclusion Theorem

How many numbers between 7500 and 7800 are divisible by 5 but not divisible by 7? Explain how you got your answer. Make a picture if necessary.

5. (25 points) Logic and Theorem Proving

Turn the following sentences into logic, and use resolution to prove the theorem from the hypotheses.

Hypotheses:

A. All students at ADU work hard.
B. There is a student at ADU who plays hard.
C. If a student plays hard, the student has fun.
D. If a student works hard and has fun, the student is happy.

Theorem:

Some student at ADU is happy.