Problem Set #11 Solutions

1.

a) The augmented matrix for the series of equations is

$$\begin{pmatrix} 3 & 1 & 1 & 6 \\ 1 & -1 & -1 & -2 \\ 0 & 4 & 1 & 3 \end{pmatrix}$$

After performing Gaussian elimination and solving, we find that

$$\begin{aligned} x &= 1 \\ y &= 0 \\ z &= 3 \end{aligned}$$

b) The augmented matrix for the series of equations is

$$\begin{pmatrix} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 3 \\ 2 & 1 & 4 & 1 \end{pmatrix}$$

After performing Gaussian elimination and solving, we find that

$$\begin{aligned} x &= \frac{14}{9} \\ y &= \frac{11}{27} \\ z &= -\frac{17}{27} \end{aligned}$$

2. Starting with the original matrix and beginning the Gaussian elimination

$$\begin{pmatrix} 1 & 2 & 4 \\ -1 & -3 & -2 \\ 0 & 1 & c \end{pmatrix} = \begin{pmatrix} 1 & 2 & 4 \\ 0 & -1 & 2 \\ 0 & 1 & c \end{pmatrix} = \begin{pmatrix} 1 & 2 & 4 \\ 0 & -1 & 2 \\ 0 & 0 & c+2 \end{pmatrix}$$

So,

$$c \neq -2$$

3. Performing the Gausssian elimination on the augmented matrix, we get

$$\begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & c & 2 \\ 0 & 0 & -5 & -1 \end{pmatrix}$$

So, there is always a single solution regardless of the choice of c.

4.

a)

$$\begin{pmatrix} 3 & 1 & 2 \\ -1 & 0 & 1 \\ 0 & 1 & 6 \end{pmatrix}^{-1} = \begin{pmatrix} -1 & -4 & 1 \\ 6 & 18 & -5 \\ -1 & -3 & 1 \end{pmatrix}$$

b)

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 4 & -2 \\ 1 & 3 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 10 & 1 & -8 \\ -1 & 0 & 1 \\ -7 & -1 & 6 \end{pmatrix}$$

5. The inverse of

$$\begin{pmatrix} a & b & b \\ a & a & b \\ a & a & a \end{pmatrix}$$

is

$$\frac{1}{\left(a^3 - 2\cdot b\cdot a^2 + a\cdot b^2\right)}\cdot\begin{bmatrix} (-b+a)\cdot a & 0 & -b\cdot(-b+a) \\ -(-b+a)\cdot a & (-b+a)\cdot a & 0 \\ 0 & -(-b+a)\cdot a & (-b+a)\cdot a \end{bmatrix}$$

There is no inverse for the matrix if the denominator of the coefficient is zero

$$a^3 - 2ba^2 + ab^2 \neq 0$$
$$a(a-b)^2 \neq 0$$
$$a \neq 0, b$$

6. The inverse of

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

is

$$\frac{1}{(a \cdot d - b \cdot c)} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

As long as

$$ad - bc \neq 0$$

The inverse of the matrix exists

7. $A\left(\frac{1}{2}(v+w)\right) = \frac{1}{2}A(v+w)$ **BUT** $A(v+w) = Av + Aw$

$= \frac{1}{2}Av + \frac{1}{2}Aw$ $\qquad = b + b$

$= \frac{1}{2}b + \frac{1}{2}b$ $\qquad = 2b$

$A\left(\frac{1}{2}(v+w)\right) = b$ $\qquad A(v+w) \neq b$

8. If we reverse the order of the multiplication AP, instead of seeing the rows permuted, we see that the columns are permuted instead.

Example: PA

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} d & e & f \\ a & b & c \\ g & h & i \end{pmatrix}$$

AP

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} b & a & c \\ e & d & f \\ h & g & i \end{pmatrix}$$