PROBLEM SETS 1 & 2. DUE THURSDAY 7 SEPTEMBER

Problem Set 1. Problems from Lecture 1.

1. Given a quadratic equation of the from $ax^2 + bx + c = 0$, we can solve for x using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Using the above formula, solve the equation $4x^2 - 5x - 6 = 0$ for x.

In the above equation, $a = 4, b = -5$, and $c = -6$. Therefore, the solutions are given by

$$\begin{aligned} x &= \frac{5 \pm \sqrt{(-5)^2 - 4 * 4 * (-6)}}{2 * 4} \\ &= \frac{5 \pm \sqrt{(25 - (-96))}}{8} \\ &= \frac{5 \pm \sqrt{121}}{8} \\ &= \frac{5 \pm 11}{8} \\ &= \{-\frac{3}{4}, 2\} \end{aligned}$$

These answers can be checked by resubstitution into the original quadratic.

2. Find $f(x)$ if $f(x+1) = x^2 - 5x + 3$.

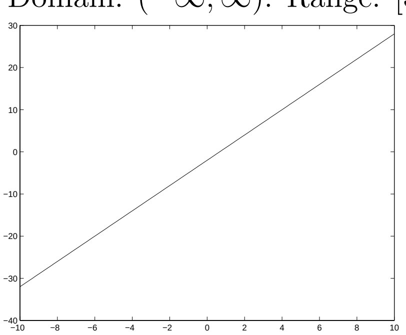
We can find $f(x)$ by substituting $x - 1$ for x in the formula for $f(x+1)$:

$$f(x) = f((x-1)+1) = (x-1)^2 - 5(x-1) + 3 = x^2 - 7x + 9$$

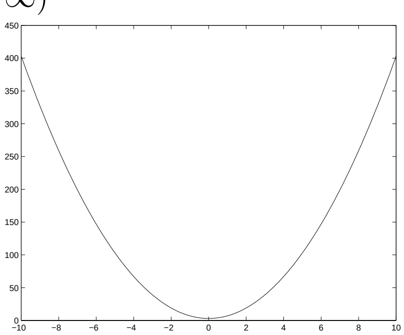
3. **Graph the following functions, and give their domain and range.**

(a) $y = 3x - 2$**.**
Domain: $(-\infty, \infty)$. Range: $(-\infty, \infty)$

(b) $y = 4x^2 + 3$.
Domain: $(-\infty, \infty)$. Range: $[3, \infty)$



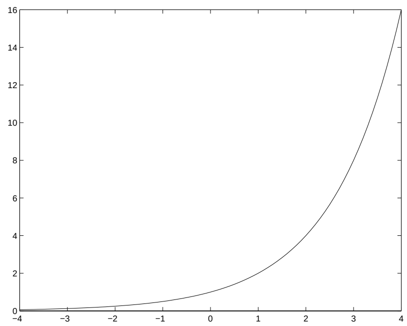
$y = 3x - 2$



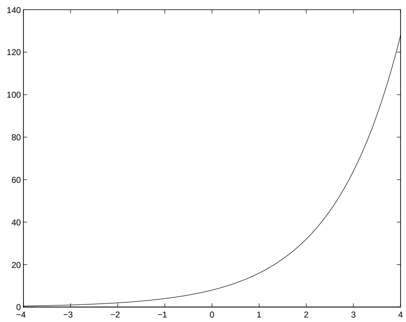
$y = 4x^2 + 3$

4. **Graph the following functions, and give their domain and range.**
(a) $y = 2^x$**.**
Domain: $(-\infty, \infty)$. Range: $(0, \infty)$

(b) $y = 2^{x+3}$.
Domain: $(-\infty, \infty)$. Range: $(0, \infty)$



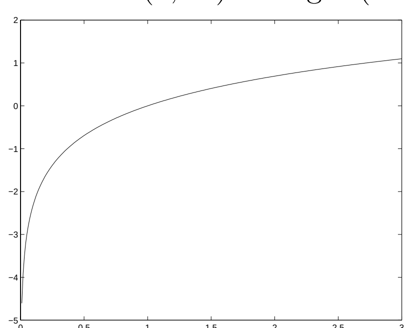
$y = 2^x$



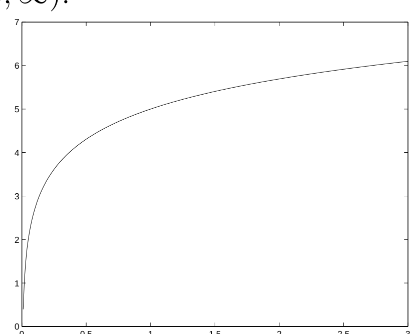
$2^x + 3$

5. **Graph the following functions, and give their domain and range.**
(a) $y = log_2(x)$**.**
Domain: $(0, \infty)$. Range: $(-\infty, \infty)$.

(b) $y = log_2(x) + 5$**.**
Domain: $(0, \infty)$. Range: $(-\infty, \infty)$.



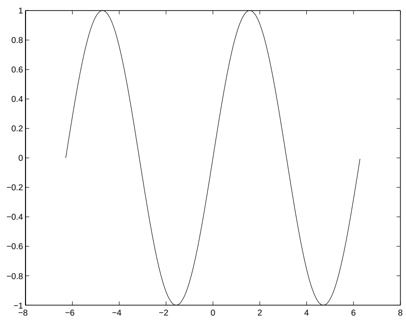
$y = log_2(x)$



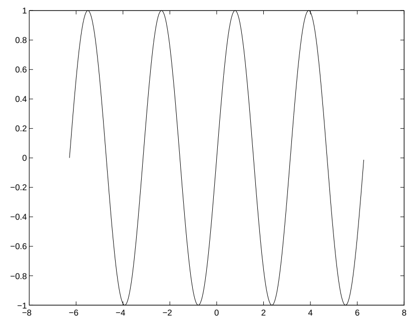
$log_2(x) + 5$

6. **Graph the following functions, and give their domain and range.**
(a) $y = \sin(x)$**.**
Domain: $(-\infty, \infty)$. Range: $[-1, 1]$.

(b) $y = \sin(2x)$**.**
Domain: $(-\infty, \infty)$. Range: $[-1, 1]$.



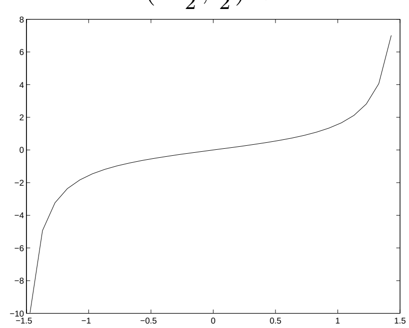
$y = \sin(x)$



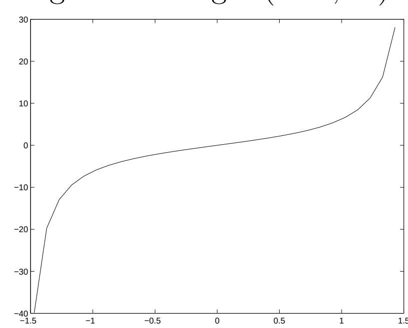
$y = \sin(2x)$

7. **Graph the following functions, and give their domain and range.**
   (a) $y = \tan(x)$.
   Domain: $(-\frac{\pi}{2}, \frac{\pi}{2}) + n\pi$ for all integers n. Range: $(-\infty, \infty)$.

   (b) $y = 4\tan(x)$.
   Domain: $(-\frac{\pi}{2}, \frac{\pi}{2}) + n\pi$ for all integers n. Range: $(-\infty, \infty)$.



$y = \tan(x)$



$y = 4\tan(x)$

8. **Simplify the following expressions.**
   (a) $\log_{10}(\frac{x+y}{z})$**.**

   $\log_{10}(\frac{x+y}{z}) = \log_{10}(x+y) - \log_{10} z$

   (b) $25^{\log_{25}(x+y)+\log_5(\frac{x}{y})}$**.**

$$\begin{aligned}
25^{\log_{25}(x+y)+\log_5(\frac{x}{y})} &= 25^{\log_{25}(x+y)}25^{\log_5(\frac{x}{y})} \\
&= (x+y)(5^2)^{\log_5(\frac{x}{y})} \\
&= (x+y)(5^{\log_5(\frac{x}{y})})^2 \\
&= (x+y)(\frac{x}{y})^2
\end{aligned}$$

9. **Make the following computations using right triangles.**
   (a) **For** $\theta = \frac{\pi}{4} = 45°$**, compute** $\sin(\theta)$**,** $\cos(\theta)$**, and** $\tan(\theta)$**.**

   One triangle with angle $\theta = \frac{\pi}{4}$ has "adjacent", "opposite" and "hypotenuse" sides of lengths 1, 1 and $\sqrt{2}$, respectively. Therefore:

$$\begin{aligned}
\sin(\theta) &= \frac{\textit{opposite}}{\textit{hypotenuse}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\
\cos(\theta) &= \frac{\textit{adjacent}}{\textit{hypotenuse}} = \frac{2}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\
\tan(\theta) &= \frac{\textit{opposite}}{\textit{adjacent}} = \frac{1}{1} = 1
\end{aligned}$$

(b) **For** $\theta = \frac{\pi}{6} = 30°$**, compute** $\sec(\theta)$**,** $\csc(\theta)$**, and** $\cot(\theta)$**.**
One triangle with angle $\theta = \frac{\pi}{6}$ has "adjacent", "opposite" and "hypotenuse" sides of lengths 2, 1 and $\sqrt{5}$, respectively. Therefore:

$$\begin{aligned}
\sec(\theta) &= \frac{1}{\cos(\theta)} = \frac{\textit{hypotenuse}}{\textit{adjacent}} = \frac{\sqrt{5}}{2} \\
\csc(\theta) &= \frac{1}{\sin(\theta)} = \frac{\textit{hypotenuse}}{\textit{opposite}} = \sqrt{5} \\
\cot(\theta) &= \frac{1}{\tan(\theta)} = \frac{\textit{adjacent}}{\textit{opposite}} = 2
\end{aligned}$$

10. **Simplify the following expressions (i.e. write them in terms of elementary trig functions** $\sin(\phi)$**,** $\sin(\theta)$**, etc.).**
(a) $\sin(\theta + \phi)$**.**
$\sin(\theta + \phi) = \sin(\theta)\cos(\phi) + \cos(\theta)\sin(\phi)$. This is a basic trigonometric identity.
(b) $\cos(3\theta)$**.** Using a formula similar to that used in (a), above, we write:

$$\begin{aligned}
\cos(3\theta) &= \cos(\theta + 2\theta) \\
&= \cos(\theta)\cos(2\theta) - \sin(\theta)\sin(2\theta)
\end{aligned}$$

Using the standard formulas for $\cos(2\theta)$ and $\sin(2\theta)$, this can be further simplified to:

$$\begin{aligned}
\cos(3\theta) &= \cos(\theta)(1 - 2\sin^2(\theta)) - \sin(\theta) * 2\sin(\theta)\cos(\theta) \\
&= \cos(\theta) - 2\sin^2(\theta)\cos(\theta) - 2\sin^2(\theta)\cos(\theta) \\
&= \cos(\theta)(1 - 4\sin^2(\theta))
\end{aligned}$$