ArsDigita University

Month 2: Discrete Mathematics - Professor Shai Simonson

Examination 3 – 100 points

Show all work for partial credit. You may use two hours for this exam. After one hour, raise your hand if you feel that the time constraint will be too tight.

Name: _____

1.	**/30**
2.	**/20**
3.	**/15**
4.	**/15**
5.	**/20**
Total:	**/100**

1. **Recurrence Equations (30 points)**

   a. Solve the following recurrence equation:

   $a_n = 2a_{n-1} - a_{n-2}$, $a_0 = 2$, $a_1 = 1$.

   b. The scheme function below calculates the sum of a list of elements.

```
(define (sumup alist)
    (cond   ((null? alist) 0)
            ((null? (cdr alist)) (car alist))
            (else (+ (car alist) (cadr alist) (sumup (cddr alist))))))
```

Write a recurrence equation for the time complexity of the algorithm, and solve it showing a closed form answer. Assume that each car and cdr can be done in one step.

2. Recurrence Equations (20 points)

a. What is the order of complexity of each of the following recurrence equations?

$T(n) = 5T(n/2) + n^2 \quad T(1) = 1$

$T(n) = 4T(n/2) + n^2 \quad T(1) = 1$

$T(n) = 6T(n/6) + n \quad T(1) = 1$

$T(n) = T(2n/3) + n^2 \quad T(1) = 1$

$T(n) = T(2n/3) + 6 \quad T(1) = 1$

b. A bank pays 8 percent interest per year and you deposit \$1000. After the *k*th year, you withdraw *k* dollars. Write a linear non-homogeneous recurrence equation for the balance in your account after *n* years.

3. Counting (15 points)

You choose three cards in order from a set of cards numbered 1 to 10.

a. How many possible outcomes are there? Explain.

b. How many of these are in sorted order from highest to lowest. Explain.

4. Counting (15 points)

How many ways are there to choose three different numbers each between one and a hundred so that their sum is even? Explain.

5. Counting (20 points)

Emma's pizza shop has 21 different toppings, three kinds of sauce, and three kinds of cheese. Assume that someone can order any combination of sauces and cheeses (including none), but must order exactly two different toppings.

a. How many different pizzas can one create? Explain.

b. If three people are sharing a pizza, and each can order their third as in (a), then how many different pizzas can they order? Explain carefully.