Problem Set #3 Solutions

1.

a) $y' = 12x^3 + 6x^2 - 2x + 4$

b) $y' = 36x^5 + 36x^3 - 12x^2 - 20x - 6$

c) $y' = \dfrac{-6x^2 + 28x + 12}{(2x^2 + 4)^2}$

d) y' = $50(10x - 2)^4 \cdot \left(3 \cdot x^2 - 1\right)^2 + 12(10x - 2)^5 \cdot \left(3 \cdot x^2 - 1\right) \cdot x$

OR when factored

$$32 \cdot \left(3 \cdot x^2 - 1\right) \cdot \left(135 \cdot x^2 - 12 \cdot x - 25\right) \cdot (5 \cdot x - 1)^4$$

OR when expanded

$$8100000x^8 - 1680000x^6 - 664000x^4 - 7200000x^7 + 3168000x^5 - 209152x^3 + 105600x^2 - 15616x + 800$$

e) y' = $\sec(\theta) \cdot \tan(\theta) \cdot \csc(\theta) - \sec(\theta) \cdot \csc(\theta) \cdot \cot(\theta)$

OR when simplified to cos

Y' = $\dfrac{\left(2 \cdot \cos(\theta)^2 - 1\right)}{\left[\cos(\theta)^2 \cdot \left(\cos(\theta)^2 - 1\right)\right]}$

f) y' = 1 + $\tan^2\theta$ **OR** y' = $\sec^2\theta$.∴

g) $y' = 5e^{5x+7}$

h) y' = $\dfrac{1}{3} \cdot \dfrac{\left[6 \cdot \dfrac{x}{(4 \cdot x + 2)} - 12 \cdot \dfrac{x^2}{(4 \cdot x + 2)^2}\right]}{x^2} \cdot (4 \cdot x + 2)$

which simplifies to

y' = $\dfrac{2}{x} \cdot \dfrac{(x + 1)}{(2 \cdot x + 1)}$

h) In this problem, it is useful to square both sides of the equation before carrying out the implicit differentiation.

$$y^6 = 2xy - 4xy^2$$

Differentiating, and solving for dy/dx, we get

$$\frac{dy}{dx} = \frac{y(1-2y)}{(3y^5 + 4xy - x)}$$

j) By the chain rule, we get

y' = $5 \cdot \left(x^2 + 4\right)^{\left(\frac{3}{2}\right)} \cdot x$

2. Taking the derivative, we get

$$f'(x) = 3ax^2 + 2bx + c$$

We now set this equation to zero and solve using the quadratic formula

$$x = \frac{-2b \pm \sqrt{4b^2 - 12ac}}{6a}$$

We see that the number of horizontal tangents is dependent on the number of real solutions.

a) If $4b^2 - 12ac > 0$
Then the quantity under the square root is positive and there are two roots (or horizontal tangents)

b) If $4b^2 - 12ac = 0$
Then the quantity under the square root is zero and there is a single root (-b/3a) and a single horizontal tangent.

c) If $4b^2 - 12ac < 0$

Then the quantity under the square root is negative, and we have a complex number which means there are no real roots (or horizontal tangents).

3. If $f(x) = x + 2\sin(x)$

Then $f'(x) = 1 + 2\cos(x)$

So, the derivative is simply a vertically shifted cosine function with a magnitude of 2. This being the case, we know that it will periodically cross the x-axis (y=0). Setting the derivative to zero, we get

$$\cos(x) = -\frac{1}{2} \qquad \text{and} \qquad \arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}, \frac{4\pi}{3}$$

And since this is a periodic function repeating every 2π radians, there are horizontal tangents at

$$x = \left\{ \begin{matrix} \dfrac{2\pi}{3} \pm 2n\pi \\ \dfrac{4\pi}{3} \pm 2n\pi \end{matrix} \right\} \quad \text{where n is an integer}$$

4. If $f(x) = \dfrac{x^3 + x}{x - 1}$

Then $f'(x) = \dfrac{2x^3 - 3x^2 - 1}{(x-1)^2}$

And so $f'(2) = 3$

Using the point slope form, the tangent line at the point (2,10) is

y-10=3(x-2) which in standard form is

y=3x+4

5. If $f(x) = \frac{6}{x+2}$

Then $f'(x) = -\frac{6}{(x+2)^2}$

So, $f'(1) = -\frac{2}{3}$

Using the point slope form and simplifying, the tangent line at (1,2) is

$$y = \frac{-2}{3}x + \frac{8}{3}$$

We know that the slope of the normal to the tangent is the negative reciprocal of the slope of the tangent (3/2 in this case) so using the point slope form and simplifying, the normal to the tangent is

$$y = \frac{3}{2}x + \frac{1}{2}$$