PROBLEM SET 6. DUE WEDNESDAY, 13 SEPTEMBER

Reading. *Quick Calculus*, pp. 151–157; 171–190.
Supplementary reading. Simmons, Chapter 6.

1. Compute the following integrals by substitution.
   (a) $\int (1+\frac{1}{x})^2 \frac{1}{x^2}\ dx$
   (b) $\int \tan(x)\ dx = \int \frac{\sin(x)}{\cos(x)}\ dx$
   (c) $\int \cos(x)\cos(\sin(x))\ dx$
2. Compute the following two trigonometric integrals.
   (a) Remember that $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$. Use this to compute
   $$\int \sin^2(x)\ dx.$$
   (b) Now use the fact that $\cos^2(x) = 1 - \sin^2(x)$ to compute
   $$\int \cos^2(x)\ dx.$$
3. Use right-hand Riemann sums (in this case, the same as upper Riemann sums) to show that the area under the graph of $y = x^3$ from $x = 0$ to $x = b$ is $\frac{b^4}{4}$.
4. Each of the following functions has one arch above the x-axis. Find the area of the region under that arch.
   (a) $f(x) = 9 - x^2$
   (b) $f(x) = x^3 - 9x$
   (c) $f(x) = 4x - x^3$
5. Evaluate the following definite integrals using the Fundamental Theorem of Calculus. (This is their *algebraic area.*)
   (a) $\int_0^{2\pi} \sin(x)\ dx$
   (b) $\int_{-3}^{2} x^4 + 2x^3 - 5x^2 - 6x\ dx$
   (c) $\int_0^{\frac{3\pi}{2}} \cos(x)\ dx$
6. Compute the geometric area of the following functions on the corresponding intervals. These are the same functions and intervals as in the previous problem. Note the difference between geometric and algebraic area!
   (a) $f(x) = \sin(x)$ on $[0, 2\pi]$
   (b) $f(x) = x^4 + 2x^3 - 5x^2 - 6x = x(x-2)(x+1)(x+3)$ on $[-3, 2]$
   (c) $f(x) = \cos(x)$ on $[0, \frac{3\pi}{2}]$