Cyclic Sum and Difference Sets

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Details About Our Approach

There are three main aspects when it comes to the methods of work that we used.

1. Computation of cyclic sum sets using a program: This allowed us to turn the unknown space of cyclic sums into charted territory that we were able to analyze better.
2. Possibility analysis and elimination: By discovering all of the mathematical possibilities within discovering cyclic sum sets, we were able to solve a few problems involving verification of certain cyclic sum rules.
3. Mathematical proofing: Although a smaller part of our work, we attempted as much as possible to work out proofs with fundamental rules of summation and algebra, but we were left with much questioning with the unique nature of trying to mathematically derive rules for this kind of series.

Part 1: Problem #2 Verify that (1) is the only CSS of order 0, (1, 2) is the only CSS of order 1, and (1, 2, 4) is the only CSS

- The problem represents a fundamental basis upon which one's understanding of cyclic sum sets can be built.
- An order of 0 indicates a set of 1 term.
- Recall that only positive integers can be used and the numbers have to be in a sequential order.
- **"1" is the only number that can be counted sequentially up in the set of positive integers that also equals it's last term, itself, which is "1."**

Part 2: Problem #2 - Verification of (1, 2) being the only CSS of order 1.

- Four combinations are created by the 2 terms of this order of CSS.
- One of these combinations is subtracted because two of these sums are cumulatively equal.
- The previous statement means that the third sum has to be equal to 3.
- **This leaves the only two sequentially counting numbers whose sum is 3, which is 1 and 2.**

Problem #8 Characterizing CSS that are also CDS

- One of the problems we focused on in this project is Problem #8 because it is interesting to explore the overlap between these two sets of tuples
- To characterize these two sets we first need to find them so I wrote a few python programs that finds all CSS or CDS
- After that we would need to check which tuples overlap between the two sets and try to determine why

Finding CSS

- Using a few python programs I was able to find these CSS of different orders
- I did this by searching every combination of numbers from 1 to n(n+1)/2 + 1 (using Problem 6) and checked if that set of numbers fit the properties of a CSS
- As the number of terms in the tuple increased, the time the computer took to search did as well
- I stopped at Order 6 and it is not shown here because there are no CSS of Order 6

Order 0: (1)
Order 1: (1, 2)
Order 2: (1, 2, 4)
Order 3:
(1, 2, 6, 4)
(1, 3, 2, 7)
Order 4:
(1, 3, 10, 2, 5)
Order 5:
(1, 2, 5, 4, 6, 13)
(1, 2, 7, 4, 12, 5)
(1, 3, 2, 7, 8, 10)
(1, 3, 6, 2, 5, 14)
(1, 7, 3, 2, 4, 14)

Finding CDS

- Using a few python programs I was able to find these CDS of different orders.
- I did this by searching every combination of numbers from 1 to n^2 + n + 1 (using Problem 7) and checked if that set of numbers fit the properties of a CDS
- As the number of terms in the tuple increased the time the computer took to search did as well
- I stopped at Order 4 because that is when it became clear that there was no direct overlap between CSS and CDS greater than Order 2

Order 0: [1]
Order 1: [1, 2]
Order 2: [1, 2, 4]
Order 3:
[1, 2, 4, 10]
[1, 2, 5, 7]
[1, 2, 6, 12]
[1, 2, 9, 11]
[1, 3, 4, 8]
[1, 3, 6, 7]
[1, 3, 10, 11]
[1, 4, 5, 12]
[1, 5, 6, 8]
[1, 7, 9, 10]
[1, 7, 11, 12]
[1, 8, 9, 12]
Order 4:
[1, 2, 5, 15, 17]
[1, 2, 7, 9, 19]
[1, 3, 8, 9, 12]
[1, 3, 13, 16, 17]
[1, 4, 5, 10, 12]
[1, 6, 7, 10, 20]
[1, 11, 13, 18, 19]
[1, 11, 14, 15, 20]

Overlaps between CSS and CDS

- As it turns out that only CSS that are CDS are the sets (1), (1,2), and (1,2,4)
- This is due to that fact that CSS are related to the equation $N = n^2 + n + 1$ while CDS sets are related to $S = s^2 - s + 1$
- Basically CSS of Order 3 have to add up to: $9 + 3 + 1 = 13$ while CDS of Order 3

Problem #9 Is there a correspondence in the amount of Sets of the same Order?

- Another problem we focused on in this project is Problem #9 because it is interesting to explore the correspondence between these two sets of tuples
- With the Sets compiled we can compare the amount of each Set of the same order and see if there are trends or even a direct correspondence
- After that we would need to try to understand why there is or isn’t a correspondence for different Orders.

Comparing CSS to CDS

Order 0: (1)
Order 1: (1, 2)
Order 2: (1, 2, 4)
Order 3:
(1, 2, 6, 4)
(1, 3, 2, 7)
Order 4:
(1, 3, 10, 2, 5)
Order 5:
(1, 2, 5, 4, 6, 13)
(1, 2, 7, 4, 12, 5)
(1, 3, 2, 7, 8, 10)
(1, 3, 6, 2, 5, 14)
(1, 7, 3, 2, 4, 14)

- On the left CSS and on the right CDS
- When the Order is greater than 2 there seems to be no correspondence between the amount of tuples that are Sets.
- This is especially true when you consider that to the right is all of the CDS in normal form, if the non-normal form CDS were included there would be many more CDS than CSS for every Order.

Order 0: [1]
Order 1: [1, 2]
Order 2: [1, 2, 4]
Order 3:
[1, 2, 4, 10]
[1, 2, 5, 7]
[1, 2, 6, 12]
[1, 2, 9, 11]
[1, 3, 4, 8]
[1, 3, 6, 7]
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[1, 6, 7, 10, 20]
[1, 11, 13, 18, 19]
[1, 11, 14, 15, 20]

Future Work

The future of this work to improve its content would likely start with the research of further information relating to cyclic summation notation as a different form of summation, as it seems it likely has a fundamental part in mathematically acquiring proof for much of what was performed.