If W is the waiting time till the x^{th} event in a Poisson process with average rate λ, then $W \sim GAM(x,\theta)$, where $\theta = \frac{1}{\lambda}$.

Suppose customers arrive at a shop according to a Poisson process at an average rate of 20 customers per hour. What is the probability that the shopkeeper will have to wait more than 15 minutes for the arrival of both of the next two customers?

Let Y = # customers per minute. Then Y ~ POI(1/3). Let W = # minutes till second customer. Then W ~ GAM(2,3), and

$$\mathrm{P}[W > 15] = 1 - \mathrm{P}[W \le 15] = 1 - \mathrm{P}\left[\tfrac{W}{3} \le 5\right] = 1 - F(5;2) \quad = 1 - 0.960 = 0.040$$

An important special case of the gamma distribution is the Chi-square distribution. A random variable Y is said to have a Chi-square distribution with *v* degrees of freedom if and only if Y~GAM(*v*/2, 2).

e.g. Suppose that the lifetime of a component follows an exponential distribution with an average life of 500 hours. If X denotes the time till failure of the component, then

$$P[X > x] = \int_x^{\infty} \frac{1}{500} e^{\frac{-t}{500}} dt = e^{\frac{-x}{500}}$$

What is the probability the component will last over 600 hours?

$$P[X > 600] = e^{\frac{-600}{500}} = e^{\frac{-6}{5}}$$

Now suppose that the component has been in use for 300 hours. What is the probability the component will last over 600 additional hours? (i.e., over 900 hours total, given that it has already lasted 300 hours).

$$P[X > 900 | X > 300] = P\left[\frac{(X > 900) \cap (X > 300)}{(X > 300)}\right]$$

$$= \frac{P[X > 900]}{P[X > 300]} = \frac{e^{\frac{-900}{500}}}{e^{\frac{-300}{500}}} = e^{\frac{-(900-300)}{500}} = e^{\frac{-6}{5}}$$

Note that this is the identical probability calculated above for a new component lasting over 600 hours! In other words, the component "forgot" that it was used. This property of the Exponential Distribution is referred to as the "no-memory" property, and makes the Exponential a good choice for modeling the lifetimes of electronic components that do not suffer physical wear and tear.