Binomial X~BIN(n,p)

X counts the number of successes in n iid Bernoulli trials, where the probability of success for each trial is p.

$$\mathrm{P}\left[X = x\right] = \binom{n}{x} p^{x}\left(1-p\right)^{n-x} \qquad X = 0,1,2,\ldots$$

$$\text{mean} = \mu_x = np \qquad \text{variance} = \sigma_x^2 = np\left(1-p\right)$$

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Hypergeometric X~HYP(n,M,N)

X counts the number of successes in a sample of size n, where the sampling is without replacement, and there are M successes out of N total items at the start of the experiment.

$$\mathrm{P}\left[X = x\right] = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} \qquad \max\left(0, n-N+M\right) \le X \le \min\left(n, M\right)$$

$$\text{mean} = \mu_x = n\frac{M}{N} \qquad \text{variance} = \sigma_x^2 = n\frac{M}{N}\left(1-\frac{M}{N}\right)\left(\frac{N-n}{N-1}\right)$$

****

Negative Binomial X~NB(r,p)

X counts the number of failures until r successes are observed. Trials are iid Bernoulli, and p is the probability of success for each trial.

$$\mathrm{P}\left[X = x\right] = \binom{x+r-1}{r-1} p^{r}\left(1-p\right)^{x} \qquad \mathbf{X = 0,\ 1,\ 2,\ \ldots}$$

$$\text{mean} = \mu_x = \frac{r\left(1-p\right)}{p} \qquad \text{variance} = \sigma_x^2 = \frac{r\left(1-p\right)}{p^2}$$

If Y = # trials to achieve r successes, then $Y = X + r$. Then $\mu_y = \frac{1}{p}$ and $\sigma_x^2 = \frac{r\left(1-p\right)}{p^2}$.

Poisson X~POI(λ)

X is the number of occurrences of an event in a given amount of time (or some other measure such as area or volume) when the average rate of occurrences per unit time (area, volume) is λ.

$$\mathrm{P}\left[X = x\right] = \frac{e^{-\lambda}\lambda^{x}}{x!} \qquad X = 0, 1, 2, \ldots$$

$$\text{mean} = \mu_x = \lambda \qquad \text{variance} = \sigma_x^2 = \lambda$$

****

Exponential X~EXP(λ)

X is the waiting time between successive occurrences in a POI(λ) process.

$$f\left(x\right) = \lambda e^{-\lambda x} \qquad 0 \le x \qquad F\left(x\right) = \mathrm{P}\left[X \le x\right] = 1 - e^{-\lambda x}$$

$$\text{mean} = \mu_x = \frac{1}{\lambda} \qquad \text{variance} = \sigma_x^2 = \frac{1}{\lambda^2}$$

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Uniform X~UNIF(a,b)

$$f\left(x\right) = \frac{1}{b-a} \qquad a \le x \le b \qquad F\left(x\right) = \mathrm{P}\left[X \le x\right] = \frac{x-a}{b-a}$$

$$\text{mean} = \mu_x = \frac{a+b}{2} \qquad \text{variance} = \sigma_x^2 = \frac{\left(a-b\right)^2}{12}$$

****

Normal $X \sim \mathrm{N}\left(\mu, \sigma^2\right)$

$$f\left(x\right) = \frac{1}{\sqrt{2\pi}\ \sigma} e^{\frac{-\left(x-\mu\right)^2}{\sigma^2}} \qquad -\infty < x < \infty$$

****

Gamma X ~ GAM(α,β)

X is the waiting time until the k^{th} occurrence in a POI(λ) process.

$$\text{mean} = \mu_x = \alpha\beta \qquad \text{variance} = \sigma_x^2 = \alpha\beta^2 \qquad \alpha = k,\ \beta = \frac{1}{\lambda}$$