UNIT 4 HW

This class allows you to practice preparing professional looking reports. Make sure all reports are typed and all graphs (unless otherwise noted) are computer generated and copied and pasted into your report. If you would like help with Word or Excel please don't hesitate to ask.

- 1. Read Chapter 4 from Statistical Sleuth and answer the conceptual problems at the end of the chapter. Note: You do not need to type these up and turn them in. The answers are at the very end of the chapter.
- 2. When wildfires ravage forests, the timber industry argues that logging the burned trees enhances forest recovery; the EPA argues the opposite. The 2002 Biscuit Fire in southwest Oregon provided a test case. Researchers selected 16 fire-affected plots in 2004, before any logging was done and counted tree seedlings along a randomly located transect pattern in each plot. They returned in 2005, after nine of the plots had been logged, and counted the tree seedlings along the same transects. The percent of seedlings lost from 2004 to 2005 is recorded in the table below for logged (L) and unlogged (U) plots:

Test the EPA's assertion (and thus the opposite of the logging industries assertion) that logging actually increases the percentage of seedlings lost from 2004 to 2005.

a. Perform a complete analysis using a rank sum test in SAS. (Logging data).

State the problem

We wish to test the claim that distribution percentage of seedlings lost from the unlogged plots is different than the distribution percentage of seedlings lost from the logged plots. In other words, we are testing the claim that logging affects the amount of seedlings lost.

 H_0 : Distribution_{logged} = Distribution_{unlogged}

 H_A : Distribution_{logged} \neq Distribution_{unlogged}

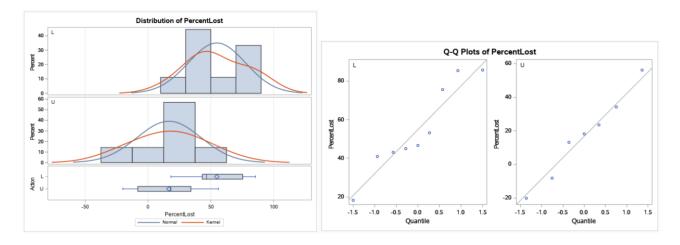
Assumptions

Normality – There is little evidence from the histograms and QQ plots to indicate the distribution is not normal for this data. Therefore, we will assume normal distribution.

Equal standard distribution – There is significant evidence to indicate the data has different standard distributions. Therefore, we will assume the standard distributions are not equal.

Independence – The plots were randomly assigned. Therefore, we will assume the observations are independent both between and within groups.

Decision – Although the data appears to be normally distributed, the variances are different and the sample sizes are too small for the t-tools. As a result, we will proceed with a rank sum test. The independence works well and the data is ordinal as well, which meets the requirements for a rank sum test.



Critical value: 2.131 Test statistic = 36 Z w/ CC: -2.4346 CI: [65.1, 10.8] P value: 0.0279

P value (exact): 0.0115



Decision

Reject the null hypothesis at significance level alpha =0.05

Conclusion

There is sufficient evidence at the alpha = .05 level of significance (p-value = .0115 two sided) to suggest that the distribution of seedlings lost from logging is different than the distribution of seedlings lost from unlogged plots.

Scope of Inference

A 95% confidence interval for this difference is [-65.1, -10.8] seedlings lost. Since the plots were randomly sampled, the results could be generalized to all plots of burnt forestry. The plots were drawn independently and from a continuous distribution, thus confirming the data is independent and non-paired.

b. Verify the p-value and confidence interval by running the rank sum test in R (using R function Wilcox.test). (You do not need to repeat the complete analysis ... simply cut and paste a screen shot of your code and the output.) You may use: https://www.r-bloggers.com/wilcoxon-mann-whitney-rank-sum-test-or-test-u/ for reference.

```
> logged <- c(45,53.1,40.8,75.5, 46.7, 85.4, 85.6, 18.2, 43.2)</pre>
> unlogged <- c(23.6,13.3,34.2,18.1,56.1,-8.1,-20.1)</pre>
> sum(rank(c(logged,unlogged))[1:9])
[1] 100
> sum(rank(c(logged,unlogged))[10:16])
[1] 36
>
> wilcox.test(logged, unlogged, conf.int = TRUE, correct = TRUE, exact =
 FALSE, alternative = "two.sided")
        Wilcoxon rank sum test with continuity correction
data: logged and unlogged
W = 55, p-value = 0.01491
alternative hypothesis: true location shift is not equal to 0
95 percent confidence interval:
 10.79996 65.10000
sample estimates:
difference in location
              33.39998
```

- 3. Conduct a Welch's two-sample t-test on the Education Data from HW 3 (untransformed). Perform a complete analysis using SAS to test the claim that the mean income of college educated people (16 years of education) is greater than the mean of those with a high school education only (12 years of education).
 - a. State the problem, address the assumptions. Be sure to support with your knowledge of theory (CLT) as well as with histograms, box plots, q-q plots, etc.

Problem

We are testing to determine if the mean income attributed to those with 16-year education is equal to those with 12-year education.

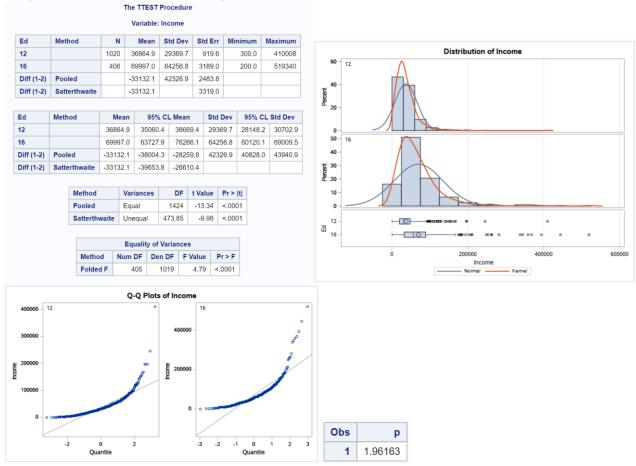
$$H_{0:}\mu_{16-years} = \mu_{12-years}$$
 $H_{A:}\mu_{16-years} \neq \mu_{12-yars}$

Assumptions

Normality – The distribution of both samples is assumed to be normal. The central limit theorem suggests that the normality of the data sample means increases as does the sample size.

Equal standard distribution – The boxplots and histograms clearly illustrate there is a difference in variance between the samples.

Decision – Based on the elements described above, the data must be transformed in order to properly compare the means. However, we'll proceed with a Welch's t-test for this exercise.



b. Show all 6 steps, including a thoughtful, thorough, yet non-technical conclusion. Include a confidence interval.

Calculations

Critical Value (two sided): 1.96163

Test Statistic: -9.98 P-value: <0.0001

CI: [-39,653.8, -26,610.4]

[draw]

Decision: Reject the null hypothesis

Conclusion

There is overwhelming evidence at the alpha =0.05 level of significance (p <0.0001 two sided) that the mean income in 2005 for people with 16 years of education is not the same as the mean income of people with 12 years of education. A 95% confidence interval for this factor is [-\$39,653.8, -\$26,610.4].

c. Include a scope of inference at the end. (You may copy and paste this from a previous HW if you like.)

Scope of Inference

Since the study was an observational study, we are not able to establish causality between education and income. There is no mention of the sampling method, we are unable to infer these results apply to those outside of this study.

d. Verify the Welch's t statistic and p-value with R (using R function t.test). Simply cut and paste your R code and output. You may use: http://rcompanion.org/rcompanion/d 02.html for reference.

Results are similar to those of SAS

```
> data <- read.csv(file = "C:/Users/javie/OneDrive/Documents/GitHub/SMU_</pre>
MSDS/Data_Sets/EducationData.csv")
> View(data)
> View(data)
> bartlett.test(Income2005 ~ Educ, data=data)
        Bartlett test of homogeneity of variances
data: Income2005 by Educ
Bartlett's K-squared = 406.28, df = 1, p-value < 2.2e-16
> t.test(Income2005 ~ Educ, data=data, var.equal=FALSE, conf.level=0.95)
        Welch Two Sample t-test
data: Income2005 by Educ
t = -9.9827, df = 473.85, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -39653.77 -26610.39
sample estimates:
mean in group 12 mean in group 16
        36864.90
                         69996.97
```

e. Would you prefer to run the log transformed analysis you ran in HW3, or do you feel this analysis is more appropriate? Why or Why not? (Make mention of the assumptions as well as the parameters that each test provides inference on. As you know, they are different.)

Transformed permutation

In order to transform the data, the data must be skewed and the larger sample must have a larger spread. If the data is not normally distributed, then a transformation may be used to correct it. Welch two sample t-test (unequal variance)

In order to use the two sample t-test, the data must be continuous, be normally distributed, variance be unequal, and the samples must be independent and randomly assigned.

Although both provided the same result (reject the null), I believe the transformation is more appropriate for this data since the two sample Welch relies on the CLT to kick in and normalize the data.

4.

a. Chapter 4, Problem 20 from the text. Show all work. "By hand" here means actually by hand. Simply take a picture of your work and include it in your pdf/doc file. Include your sorted, labeled, and ranked data; your calculations of the mean and standard deviation of the assumed distribution of the rank sum statistic under Ho; your calculation of the Z statistic with a continuity correction; your p-value, and conclusion. (No confidence interval necessary here.)

20. Trauma and Metabolic Expenditure. For the data in Exercise 18 in Chapter 3: (a) Determine the rank transformations for the data. (b) Calculate the rank-sum statistic by hand (taking the trauma patients to be group 1.) (c) Mimic the procedures used in Display 4.5 and Display 4.7 to compute the Z-statistic. (d) Find the one-sided p-value as the proportion of a standard normal distribution larger Ho: MI -M2 = 7 than the observed Z-statistic.

Metabolic Expenditures (kcal/kg/day)

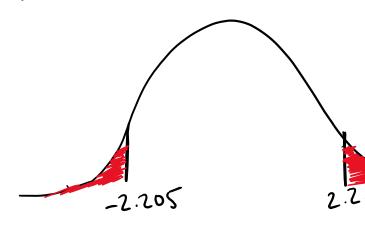
Nontrauma patients:	20.1	22.9	18.8	20.9	20.9	22.7	21.4	20.0
Trauma patients:	38.5	25.8	22.0	23.0	37.6	30.0	24.5	

Sort: 188, 20, 201, 2019, 2019, 21.4, 22, 22.7, 229, 23, 24.5, 25.8, 30, 37.6, 38.5

$$\frac{1}{R} = \frac{120}{15} = 8 \quad \text{Mean}(T) = n, R = 8 \times 8 = 64$$

$$S_{R} = \frac{1}{6} \cdot \frac{102161}{102161} = \frac{1179049520}{100161} = \frac{$$

$$Z = \frac{T - mean (T)}{5D(T)} = \frac{(38 - 64)}{11.790949520225} = -2.205081 | 052$$

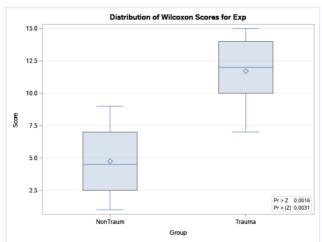


HA: NI - N2 7 0

b. Problem 21 from the text. Take a screen capture of the SAS output in addition to your response.

It appears the p values are close but not the same. The manual calculation produces p value 0.02745 and the SAS calculation produces p value 0.0105. SAS uses a 0.5 continuity correction, which was not accounted for in the manual calculation.

```
1 data TraumaExpenditures;
 2 input Group $ Exp;
 3 datalines;
 4 NonTrauma 20.1
 5 NonTrauma 22.9
 6 NonTrauma 18.8
 7 NonTrauma 20.9
 8 NonTrauma 20.9
 9 NonTrauma 22.7
10 NonTrauma 21.4
11 NonTrauma 20.0
12 Trauma 38.5
13 Trauma 25.8
14 Trauma 22.0
15 Trauma 23.0
16 Trauma 37.6
17 Trauma 30.0
18 Trauma 24.5
19 ;
20
21 proc ttest data=traumaexpenditures; *Welch's two sample t-test;
        class Group;
23
        var Exp;
24 run:
26 data criteval; *solve for critical value;
27 CriticalValue = quantile("T", 0.975, 13);
28
29 proc npar1way data=TraumaExpenditures wilcoxon; *Wilcoxon rank-sum test;
30
        class Group;
31
        var Exp;
        exact hl; *exact hodges-lehmann;
32
33 run;
              The NPAR1WAY Procedure
       Wilcoxon Scores (Rank Sums) for Variable Exp
              Classified by Variable Group
              Sum of Expected Std Dev
                                            Mean
              Scores
 Group
                      Under H0 Under H0
 NonTraum
           8
                38.0
                          64.0 8.633269
                                         4.750000
 Trauma
           7
                82.0
                          56.0 8.633269 11.714286
           Average scores were used for ties.
              Wilcoxon Two-Sample Test
         Statistic
                                  82.0000
         Normal Approximation
                                   2.9537
         One-Sided Pr > Z
                                   0.0016
         Two-Sided Pr > |Z|
                                   0.0031
         t Approximation
         One-Sided Pr > Z
                                   0.0105
         Two-Sided Pr > |Z|
         Z includes a continuity correction of 0.5.
                 Kruskal-Wallis Test
               Chi-Square
                             9.0698
               Pr > Chi-Square 0.0026
```

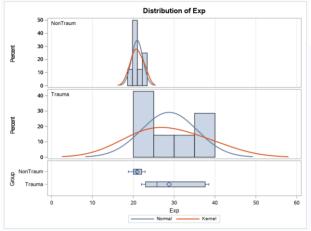


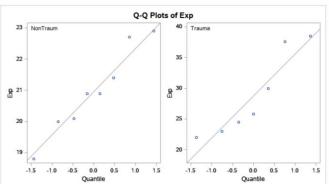
	The TTEST Procedure								
Variable: Exp									
Group Method N Mean Std Dev Std Err Minimum Maximum									
NonTraum		8	20.9625	1.3794	0.4877	18.8000	22.9000		
Trauma		7	28.7714	6.8354	2.5835	22.0000	38.5000		
Diff (1-2)	Pooled		-7.8089	4.7528	2.4598				
Diff (1-2)	Satterthwaite		-7.8089		2.6292				

Group	Method	Mean	95% CL Mean		Std Dev	95% CL	Std Dev
NonTraum		20.9625	19.8093	22.1157	1.3794	0.9120	2.8074
Trauma		28.7714	22.4498	35.0931	6.8354	4.4047	15.0520
Diff (1-2)	Pooled	-7.8089	-13.1230	-2.4949	4.7528	3.4455	7.6569
Diff (1-2)	Satterthwaite	-7.8089	-14.1398	-1.4781			

Method	Variances	DF	t Value	Pr > t
Pooled	Equal	13	-3.17	0.0073
Satterthwaite	Unequal	6.4282	-2.97	0.0230

Equality of Variances								
Method Num DF Den DF F Value Pr > F								
Folded F	6	7	24.56	0.0005				





Hodges-Lehmann Estimation							
Location Shift (Trauma - NonTraum) 5.3000							
Туре	95% Confid	dence Limits	Interval Midpoint	Asymptotic Standard Error			
Asymptotic (Moses)	1.9000	16.7000	9.3000	3.7756			
Exact	1.9000	16.7000	9.3000				

Obs	CriticalValue
1	2.16037

c. Write up a complete analysis using the information you have gained from A and B to test the claim that the distributions are different.

Problem – We are testing to determine if there is a difference in the mean difference of metabolic expenditure (kcal/kg/day) between trauma patients and non-trauma patients.

Assumptions – There is no evidence to suggest the data is not normally distributed. However, it appears there is a difference in variance, thus both samples do not have an equal standard deviation. The smaller sample size is associated w/ a larger standard deviation, which is when the t-test is least robust. As a result, a t-test is not appropriate in this case.

Test – We will proceed with a rank-sum test.

Hypothesis: The mean difference of metabolic expenditure (kcal/kg/day) of non-trauma patients equals the mean difference of metabolic expenditure (mkcal/kg/day) of trauma patients. Alternatively, the mean difference of metabolic expenditure (kcal/kg/day) of non-trauma patients does not equal the mean difference of metabolic expenditure (mkcal/kg/day) of trauma patients.

 H_0 : $\mu_{non-trauma} = \mu_{trauma}$ H_A : $\mu_{non-trauma} \neq \mu_{trauma}$

Critical Value: ±2.16037

t-statistic: 82 z-statistic: 2.9537 p-value: 0.0105 CI: [1.9, 16.7]

See graphics in previous answer for support.

Decision: reject the null hypothesis

Conclusion – There is substantial evidence to suggest that the mean difference between the trauma and non-trauma patients metabolic expenditure is different (p value 0.0105, alpha 0.05, two sided). As a result, we would reject the null hypothesis. A 95% confidence interval for this factor is [1.9, 16.7].

Scope of Inference – It is unclear about the randomness of the sample. Yet, it appears the patients were independent of each other. This was an observation study so we are unable to establish causality to the expenditure. Furthermore, without more information of the sampling, we are unable to infer these results outside of the study.

- i. State the problem.
- ii. State the assumptions you are making and why you are making them. Justify your decisions. Print out any histograms, q-q plots, box plots, etc. that you use in your justification.
- iii. Show all 6 steps of the hypothesis test for the rank sum test of the trauma data. Use the critical values, test statistics, p-values, etc. obtained above. Add a confidence interval from the Hodges-Lehmann procedure (from SAS).
- iv. Also include a scope of inference statement.
- 5. A study was performed to test a new treatment for autism in children. In order to test the new method, parents of children with autism were asked to volunteer for the study in which 9 parents volunteered their children for the study. The children were each asked to complete a 20 piece puzzle. The time it took to complete the task was recorded in seconds. The children then received a treatment (20 minutes of yoga) and were asked to complete a similar but different puzzle. The data from the study is below:

a. Calculate the statistic S for a signed rank test by hand showing the final table with the absolute differences, the signs, and the ranks. Also, show your calculation of the z-statistic (standardized S statistic).

	Child	Before	After	Dif.	ordered	rank		
	1	85	75	10	s l-`		$m_{20}(5) = n(n+1)/4$	
	2	70	50	20	10(-) 3	mean co) menon 1 1/2	
	3	40	50	-10	10	3	mean (S) = $n(n+1)/4$ SD(S) = $\left[\frac{n(n+1)(2n+1)}{24}\right]^{v_2}$	
	4	65	40	15	10	3	L 24 J	
	5	80	20	60	15	5.5		
	6	75	65	10	15	5.5	-5=41	
	7	55	40	15	20	7	S-mean(S)	
	8	20	25	-5	40	8	2 = SD(S)	
	9	70	30	40	60	9 ~	30007	
		(9 412),				DLS	$2 = \frac{S - mean(S)}{SD(S)}$ $= \frac{9(9+1)(2(9)+1)}{24}^{1/2}$ $= 25.32291452420119827$	
$Z = \frac{41 - 225}{253229145} = 0.7305$								

b. Verify your calculation in both SAS and R. Simply cut and paste your code and relevant output.

```
data Autism;
infile "/folders/myfolders/Autism.csv" firstobs=2 dlm=","; *calling in file storing data;
input Child $ Before After; *variables;
run;

data Autism1;
set Autism;
diff = before - after;
run;

proc univariate data=Autism1;
var diff;
run;
```

Tests for Location: Mu0=0							
Test	Statistic p Value						
Student's t	t	2.540341	Pr > t	0.0347			
Sign	М	2.5	Pr >= M	0.1797			
Signed Rank	s	18.5	Pr >= S	0.0313			

> Autism <- read.csv(file = "C:/Users/javie/OneDrive/Documents/Academics
/SMU/MSDS 6371/Unit_4/Autism.csv")</pre>

> View(Autism)

> wilcox.test(Autism\$Before, Autism\$After, paired = TRUE)

Wilcoxon signed rank test with continuity correction

```
data: Autism$Before and Autism$After V = 41, p-value = 0.03236 alternative hypothesis: true location shift is not equal to 0
```

```
Warning message:
In wilcox.test.default(Autism$Before, Autism$After, paired = TRUE) :
   cannot compute exact p-value with ties
> |
```

c. Conduct the six step hypothesis test using your calculations from above to test the claim that the yoga treatment was effective in reducing the time to finish the puzzle.

Hypothesis:

Participation in yoga (treatment) does not result in a difference in puzzle completion time. The alternative is that yoga does result in a difference in puzzle completion time.

```
H_0: \mu_{before\ yoga} = \mu_{after\ yoga} \qquad H_A: \mu_{before\ yoga} \neq \mu_{after\ yoga} Sign rank-sum test t-stat = 2.540341 alpha = 0.05 critical value = Z_{0.05} = \pm 1.645 p-value = 0.0313 Decision: reject the null hypothesis (p-value 0.0313, alpha 0.05, two sided)
```

d. Use SAS to conduct a six step hypothesis test using a paired t-test to test the claim that the yoga treatment was effective in reducing the time to finish the puzzle.

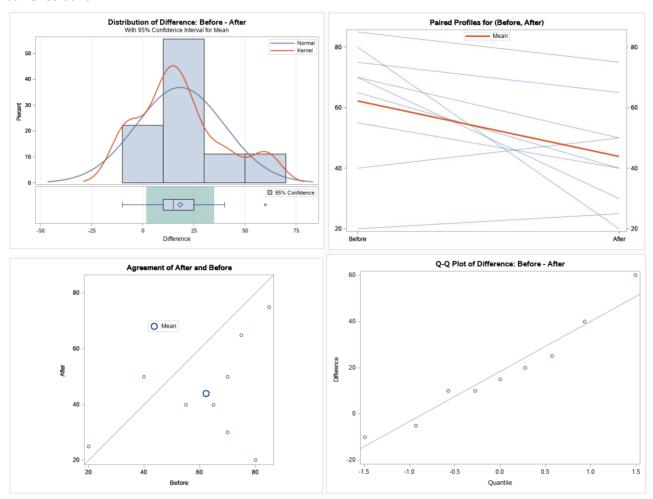
Hypothesis:

Participation in yoga (treatment) does not result in a difference in puzzle completion time. The alternative is that yoga does result in a difference in puzzle completion time.

```
H_0: \mu_{before\ yoga} = \mu_{after\ yoga} \qquad H_A: \mu_{before\ yoga} \neq \mu_{after\ yoga} Paired t-test t-stat = 2.54 alpha = 0.05 critical value = t_{0.05,\ 8} = \pm 1.86 p-value = 0.0347 CI = (95%) [1.6912, 34.9755] decision: reject the null hypothesis
```

Conclusion: There is sufficient evidence to suggest the treatment mean doesn't equal the mean before treatment, therefore we would reject the null hypothesis. The confidence interval is [1.6912, 34.9755] with a factor of 95%.





e. Verify your calculations in R. Simply cut and paste your code and relevant output.

The results are similar to those in the SAS output.

f. Use your data from above to construct a "complete analysis" of the test that you feel is most appropriate to test the claim that the yoga treatment was effective in reducing the time to finish the puzzle. This is simply formatting your results. You should be able to cut and paste most of the work from above. Problem – We will test the claim that yoga (treatment) has an effect on the puzzle completion time of children with autism.

Assumptions:

Based on the data above, it appears there is not sufficient evidence to indicate the differences do not come from normal distributions. Since the same subjects were tested before and after, the data is not independent. Instead, it is paired. Based on these assumptions, we will proceed with a paired t-test

Hypothesis:

Participation in yoga (treatment) does not result in a difference in puzzle completion time. The alternative is that yoga does result in a difference in puzzle completion time.

```
H_0: \mu_{before\ yoga} = \mu_{after\ yoga} \qquad H_A: \mu_{before\ yoga} \neq \mu_{after\ yoga} Paired t-test t-stat = 2.54 alpha = 0.05 critical value = t_{0.05,\ 8} = \pm 1.86 p-value = 0.0347 CI = (95%) [1.6912, 34.9755] decision: reject the null hypothesis
```

Conclusion: There is sufficient evidence to suggest the treatment mean doesn't equal the mean before treatment, therefore we would reject the null hypothesis. The confidence interval is [1.6912, 34.9755] with a factor of 95%. In other words, the treatment did have an effect on the completion times of the children puzzles.

Scope of Inference: Since the study was an experiment and all of the participants were assigned the treatment, we can infer causality. However, the subjects were self-selcted since they were asked to volunteer, which means that we can infer these results apply to subjects outside of this study. In other words, the yoga (treatment) did result in a difference in completion time within the subjects but random sampling would be required in order to be able to generalize the results.

BONUS (1 pt on 20 pt scale, 5pts on 100 point scale, etc.) This one is challenging and involves hard core SAS coding! Using our permutation test SAS code that we have used in prior HWs, do the following:

- a. Build the permutation distribution for the rank sum statistic for the Trauma data used above. Use 5000 permutations. Use SAS to fit / overlay a normal curve to the resulting histogram. Compare the mean and standard deviation of this normal curve that was fit to the permutation / randomization distribution to the mu and sigma you found in earlier in the homework.
- b. Compare the one-sided p-value found in this permutation distribution with the one found in prior questions.

HINT: Don't mind the highlight; the whole thing is the hint. You will need to work code similar to what is to the right into the permutation test SAS code we used before (in place of Proc ttest). You will also have to do some research on how to get your hands on the sum of the ranks statistic (a good start is to print the outnpar data set!).

```
ODS OUTPUT WilcoxonTest = outnpar;
PROC NPARIWAY DATA=learn WILCOXON;
CLASS group;
VAR score;
EXACT;
RUN;
PROC PRINT DATA=outnpar;
RUN;
```