Homework 8

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1. A company buys 100 lightbulbs, each of which has an exponential lifetime of 1000 hours. What is the expected time for the first of these bulbs to burn out?

I accidentally chose this problem for the discussion board... The expected value is "just" mu / n.

```
n <- 100
mu <- 1000

mu / n
```

```
## [1] 10
```

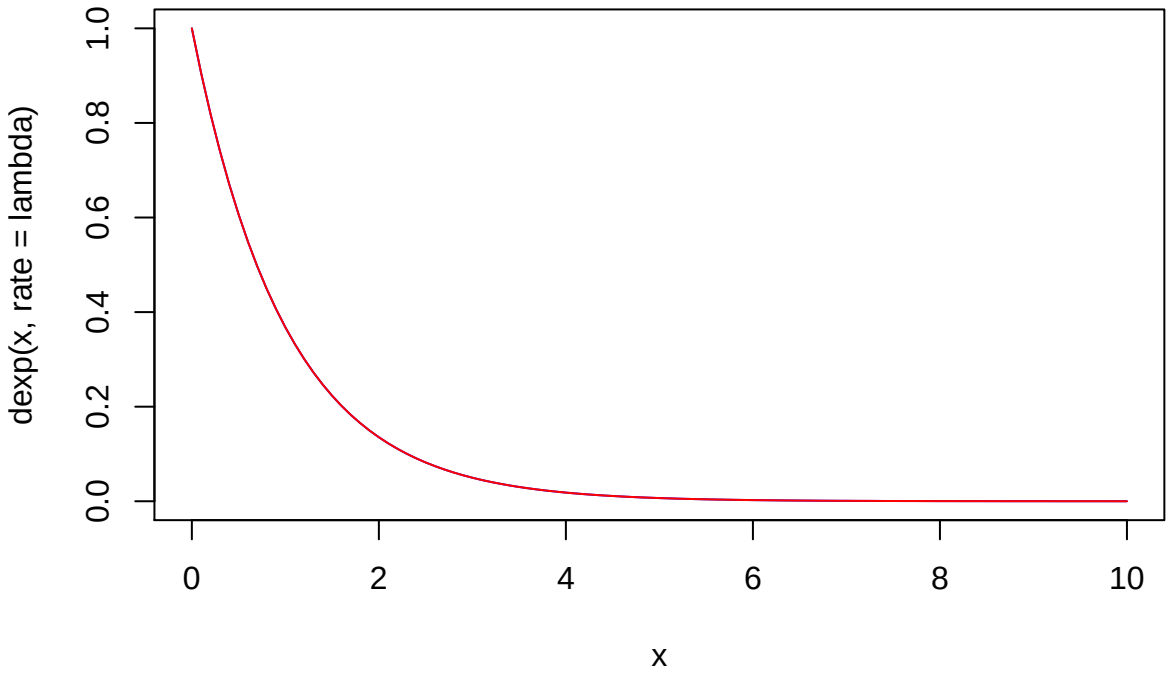
2. Assume that X_1 and X_2 are independent random variables, each having an exponential density with parameter λ. Show that $Z = X_1 - X_2$ has density

$f(z) = \frac{1}{2}\lambda e^{-\lambda|z|}$.

Because the two distributions have the same shape, the majority of values in Z will always be distributed around zero. As the lambda parameter of one X_1 approaches that of X_2, the distribution of Z will approach a uniform normal, and it will always be centered at zero.

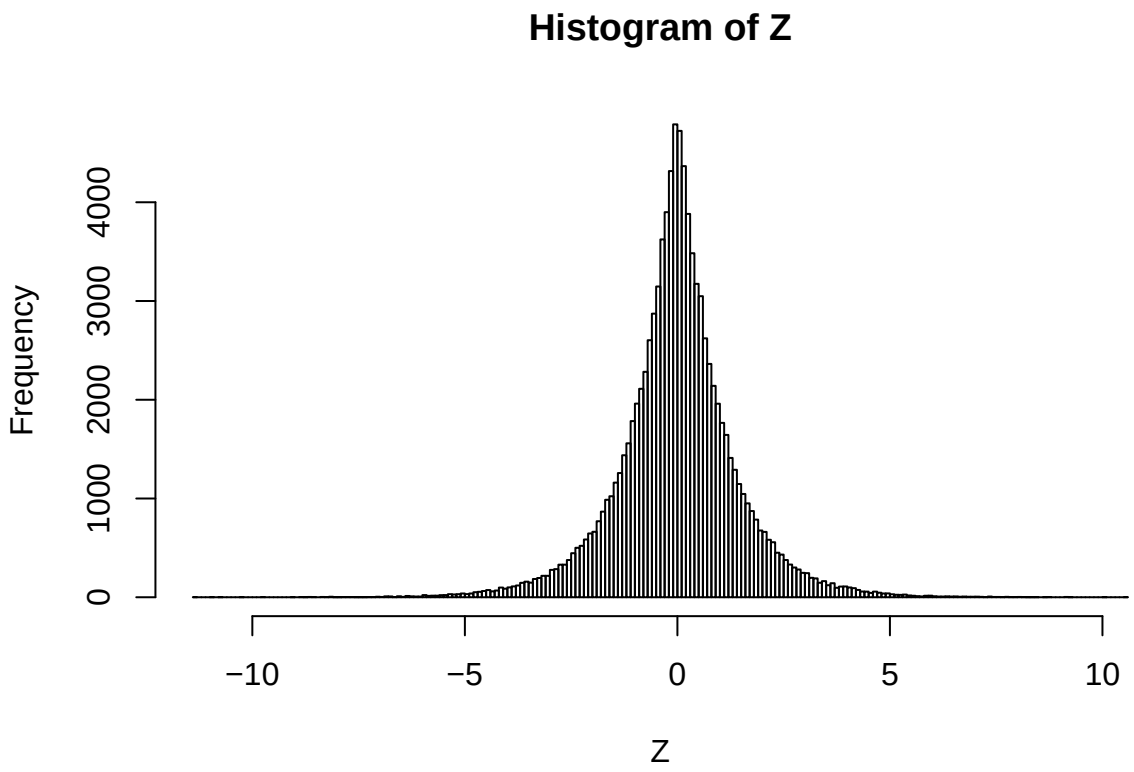
```
n <- 100000
lambda <- 1
X_1 <- rexp(n, lambda)
X_2 <- rexp(n, lambda)

curve(dexp(x, rate = lambda), from=0, to=10, col='blue')
curve(dexp(x, rate = lambda), from=0, to=10, col='red', add = TRUE)
```



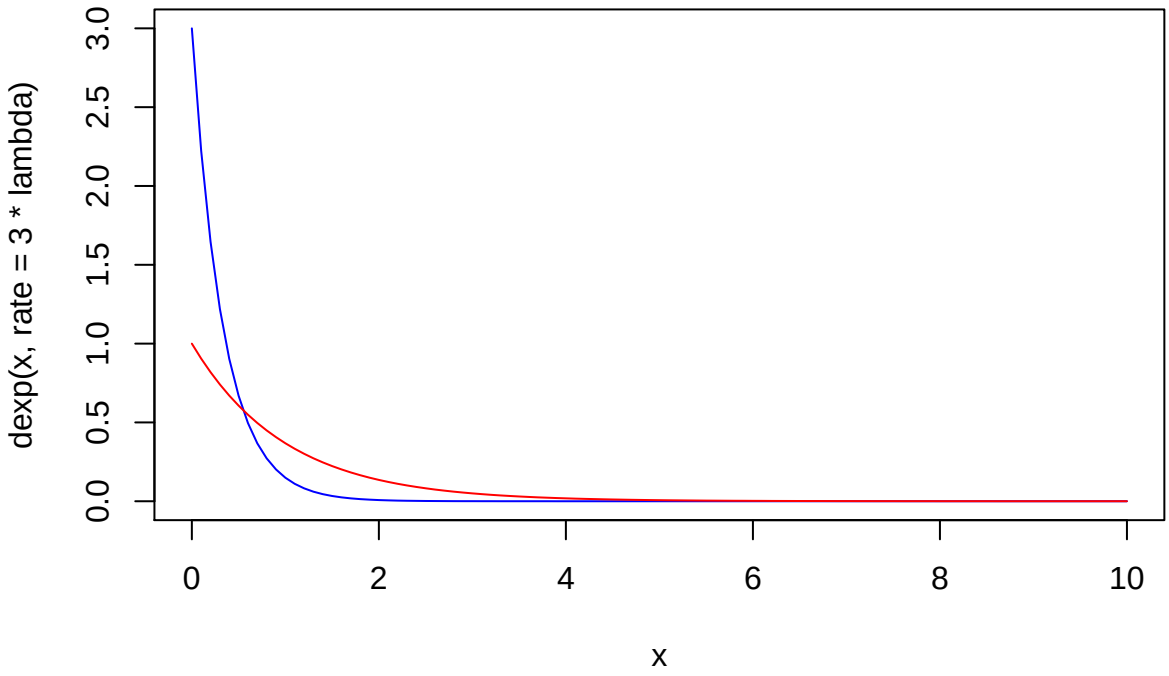

```
Z <- X_1 - X_2

hist(Z, breaks = 200)
```



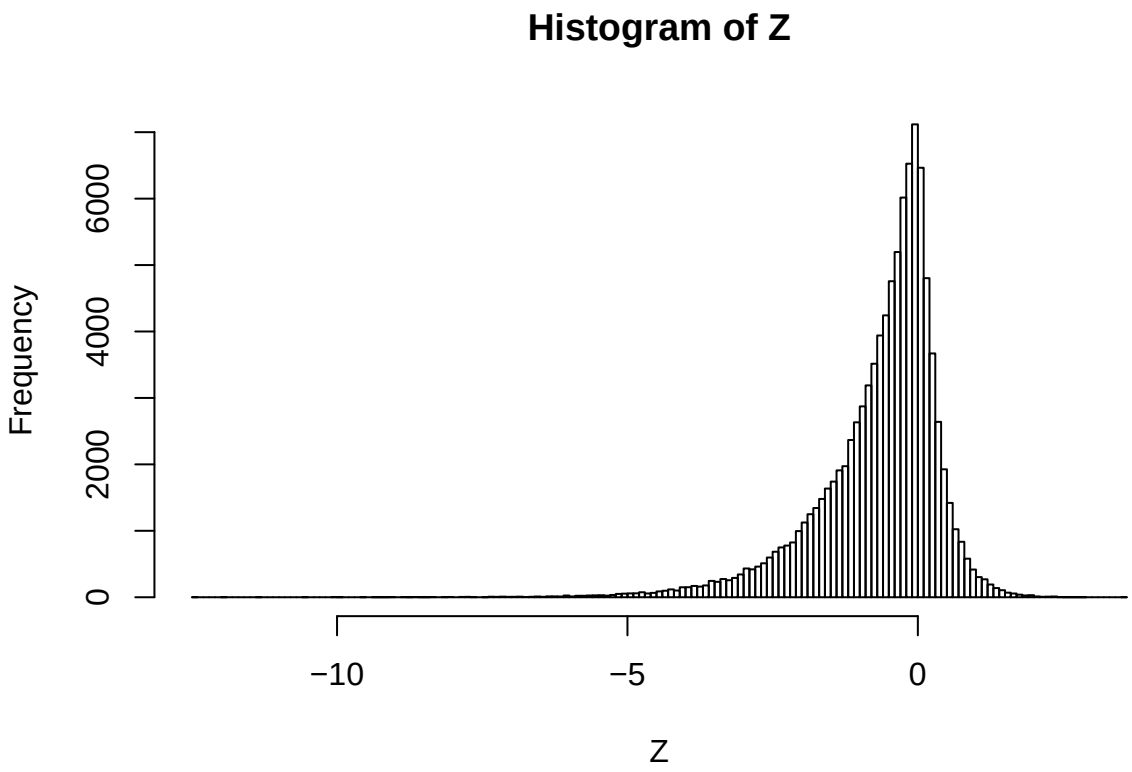

```
n <- 100000
lambda <- 1
X_1 <- rexp(n, 3*lambda)
X_2 <- rexp(n, lambda)

curve(dexp(x, rate = 3*lambda), from=0, to=10, col='blue')
curve(dexp(x, rate = lambda), from=0, to=10, col='red', add = TRUE)
```




```
Z <- X_1 - X_2

hist(Z, breaks = 200)
```



3. Let X be a continuous random variable with mean $\mu = 10$ and variance $\sigma^2 = \frac{100}{3}$. Using Chaebyshev's Inequality, find an upper bound for the following probabilities:

$$Chebyshev's\ \ Inequality$$

$$P(|X - \mu| \geq \epsilon) \leq \frac{V(X)}{\epsilon^2}$$

```
variance <- 100/3

cheb <- function(epsilon, var = variance) {
  upperBound <- var / epsilon^2
  if (upperBound > 1) {
    upperBound <- 1
  }

  return(upperBound)
}
```

(a) $P(|X - \mu| \geq 2)$

```
cheb(2)
```

```
## [1] 1
```

(b) $P(|X - \mu| \geq 5)$

```
cheb(5)
```

```
## [1] 1
```

(c) $P(|X - \mu| \geq 9)$

```
cheb(9)
```

```
## [1] 0.4115226
```

(d) $P(|X - \mu| \geq 20)$

```
cheb(20)
```

```
## [1] 0.08333333
```