Homework 3 – Sam Reeves

$ProblemSet1$

1. What is the rank of the matrix A?

```
A <- matrix(c( 1, 2, 3, 4,
              -1, 0, 1, 3,
               0, 1,-2, 1,
               5, 4,-2,-3), ncol = 4, byrow = TRUE)

pracma::rref(A)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0    0
## [2,]    0    1    0    0
## [3,]    0    0    1    0
## [4,]    0    0    0    1
```

The matrix A reduces down to the 4 dimensional Identity Matrix. Each column is a member of the set of dependent variables D. So, $r(A) = 4$. A has "full rank".

2. Given an mxn matrix where m > n, what can be the maximum rank? The minimum rank, assuming the matrix is non-zero?

If a matrix has nullity 0; it has no free variables, its rank is n, as in problem 1. Imagining another scenario, in which the set D has no members, the rank will be 0.

3. What is the rank of matrix B?

```
B <- matrix(c(1, 2, 1,
              3, 6, 3,
              2, 4, 2), ncol = 3, byrow = TRUE)

pracma::rref(B)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    1
## [2,]    0    0    0
## [3,]    0    0    0
```

$D = \{1, 3\}$ $\qquad$ $F = \{2\}$

$r(A) = 2$

$ProblemSet2$

Compute the eigenvalues and eigenvectors of the matrix A. You'll need to show your work. You'll need to write out the characteristic polynomial and show your solution.

```r
A = matrix(c(1, 2, 3,
             0, 4, 5,
             0, 0, 6), ncol = 3, byrow = TRUE)

pracma::rref(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1
```

The matrix A reduces to the 3 dimensional identity matrix I_3.

To find the eigenvalues λ of A, we must use the following formula:

$$\det(\lambda I_3 - A) = 0$$
$$or$$
$$\begin{vmatrix} \lambda - 1 & 0 & 0 \\ 0 & \lambda - 1 & 0 \\ 0 & 0 & \lambda - 1 \end{vmatrix} = 0$$

So, using the $Rule of Sarrus$, we can arrive at the characteristic polynomial by repeating the first $n-1$ columns of A in reduced row-echelon form in columns n through $2n-1$. Then we can add the sum of the three full length diagonal products, and substract the sum of the three opposite diagonal products.

Performing this action, we get:

$$((\lambda - 1)(\lambda - 1)(\lambda - 1) + 0 + 0) - (0 + 0 + 0) = 0$$
$$(\lambda - 1)^3 = 0$$
$$\lambda = 1$$

To compute the eigenspace, the set of vectors that satisfies $(\lambda I_n - A)\vec{v} = \vec{0}$, we can use the following:

$$E(A)_\lambda = N(\lambda I_n - A)$$
$$E(A)_\lambda = N(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix})$$
$$E(A)_\lambda = \left\{ \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = t \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, t \in \mathbb{R} \right\}$$