Week 13

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```r
library(mosaicCalc)
```

1. Use integration by substitution to solve the integral below.

$$\int 4e^{-7x}dx$$

$$u = -7x$$
$$\frac{du}{dx} = -7$$
$$du = -7dx$$
$$dx = \frac{du}{-7}$$

$$\int 4e^{u}\frac{du}{-7}$$
$$-\frac{4}{7}\int e^{u}du = -\frac{4}{7}e^{u}$$

$$-\frac{4}{7}e^{-7x} + C$$

2. Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of $\frac{dN}{dt} = -\frac{3150}{t^4} - 220$ bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a function $N(t)$ to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimeter.

$$\frac{dN}{dt} = -\frac{3150}{t^4} - 220$$
$$N(1) = 6530$$

$$\int \frac{dN}{dt} = \int -\frac{3150}{t^4} - 220$$

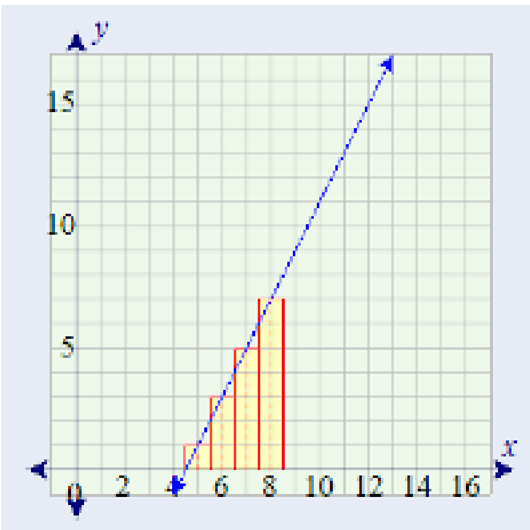
```r
antiD(-3150 * t^-4 - 220 ~ t)
```

```
## function (t, C = 0)
## 1050 * t^-3 - 220 * t + C
```

$$N(t) = \frac{3150}{3t^3} - 220t + C$$
$$N(1) = 1050 - 220t + C$$
$$C = 5700$$

$$N(t) = \frac{1050}{t^3} - 220t + 5700$$

3. Find the total area of the red rectangles in the figure below, where the equation of the line is $f(x) = 2x - 9$.



Assuming the blocks fill the space from $x = 4.5$ to $x = 8.5$:

```
f_x <- function(x) {y = 2*x - 9}
integrate(f_x, 4.5, 8.5)
```

```
## 16 with absolute error < 1.8e-13
```

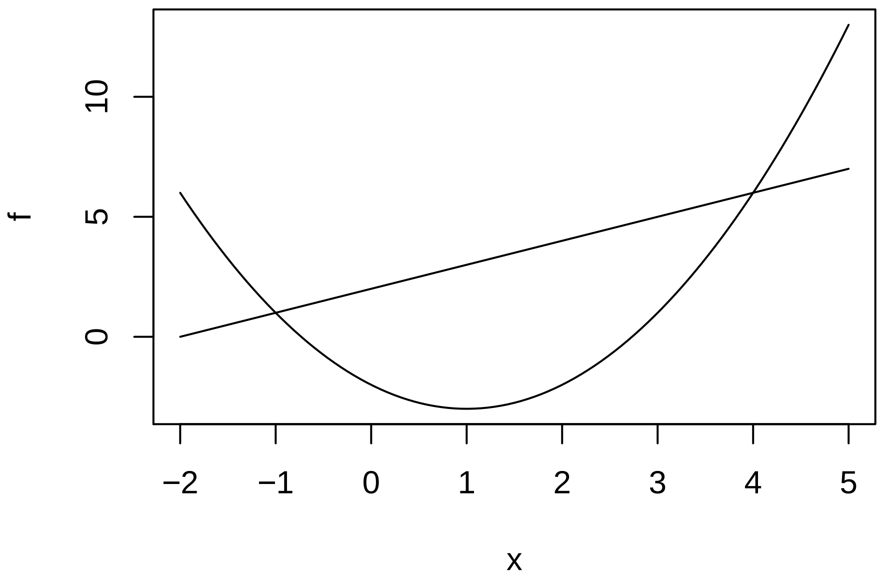
Or more simply: $A = \frac{4 \times 8}{2} = 16$

4. Find the are of the region bounded by the graphs of the given equations:

$$y = x^2 - 2x - 2$$
$$y = x + 2$$

```
f <- function(x) {y = x^2 - (2*x) -2}
g <- function(x) {y = x + 2}

plot(f, -2, 5)
plot(g, -2, 5, add = TRUE)
```




```
f1 <- integrate(f, -1, 4)
g1 <- integrate(g, -1, 4)

abs(f1$value - g1$value)
```

```
## [1] 20.83333
```

5. A beauty supply store expects to sell 110 flat irons during the next year. It costs $3.75 to store on flat iron for one year. There is a fixed cost of $8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

$$110 = \text{orders} \times \text{lot size}$$
$$\text{cost} = \$8.25 \times \text{orders} + \$3.75 \times \text{average inventory}$$
$$\text{average inventory} = \frac{110}{\text{orders} \times 2}$$
$$\text{cost} = \$8.25 \times \text{orders} + \frac{\$206.25}{\text{orders}}$$

We find the minimum with a derivative:

```
cost <- expression(8.25*orders + 206.25/orders)
D(cost, "orders")
```

```
## 8.25 - 206.25/orders^2
```

$$\text{orders} = \sqrt{\frac{\$206.25}{\$8.25}} = 5$$
$$\text{lot size} = 22$$

6. Use integration by parts to solve the integral below:

$$\int ln(9x)x^6 dx$$

$$\int udv = uv - \int vdu$$

```
u <- expression(log(9*x))
(du <- D(u, "x"))
```

```
## 9/(9 * x)
```

```
(v <- antiD(x^6 ~ x))
```

```
## function (x, C = 0)
## 1/7 * x^7 + C
```

$$ln(9x)\frac{x^7}{7} - \int \frac{x^6}{7} dx$$

```
antiD(x^6/7 ~ x)
```

```
## function (x, C = 0)
## 1/49 * x^7 + C
```

$$ln(9x)\frac{x^7}{7} - \frac{x^7}{49} + C$$

7. Determine whether $f(x)$ is a probability density function on the interval $[1, e^6]$. If not, determine the value of the definite integral:

$$f(x) = \frac{1}{6x}$$

```
f_x <- function(x) {y = 1 / (6*x)}
integrate(f_x, 1, exp(6))
```

```
## 1 with absolute error < 9.3e-05
```

$f(x)$ is a probability density function on this interval.