Taylor Series Discussion

Ch. 8.8 #31

Approximate the value of the given definite integral by using the first 4 nonzero terms of the integrand's Taylor series.

$$\int_0^{\sqrt{\pi}} \sin(x^2)dx$$

First, we find the taylor series...

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

$$\sin(x^2) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{(2n+1)!}$$

$$x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!}$$

Then we define the function and integrate each term.

```
antiD(x^2 ~ x)
```

```
## function (x, C = 0)
## 1/3 * x^3 + C
```

```
antiD(-x^6 / fact(3) ~ x)
```

```
## function (x, C = 0)
## -1/42 * x^7 + C
```

```
antiD(x^10 / fact(5) ~ x)
```

```
## function (x, C = 0)
## 1/1320 * x^11 + C
```

```r
antiD(-x^14 / fact(7) ~ x)
```

```
## function (x, C = 0)
## 0 * x^15 + C
```

$$C + \frac{x^3}{3} + \frac{x^7}{42} + \frac{x^{11}}{1320} + \frac{x^{15}}{75600}$$

Then, we evaluate it for the interval:

```r
f <- function(x) {x^3/3 - x^7/42 + x^11/1320 - x^15/75600}

f(sqrt(pi)) - f(0)
```

```
## [1] 0.8877069
```