# Multivariable Calculus

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**1. Find the equation of the regression line for the given points. Round any final values to the nearest hundredth, if necessary.**

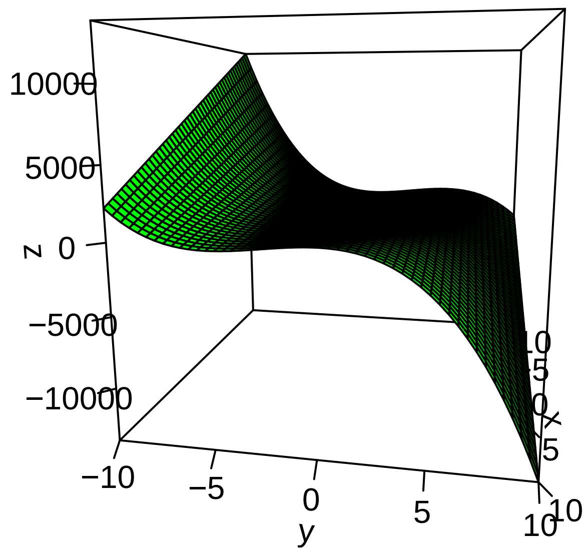
```
x <- c(5.6, 6.3, 7, 7.7, 8.4)
y <- c(8.8, 12.4, 14.8, 18.2, 20.8)

points <- cbind(x,y)

lm(y ~ x, data.frame(points))
```

```
##
## Call:
## lm(formula = y ~ x, data = data.frame(points))
##
## Coefficients:
## (Intercept)            x
##     -14.800        4.257
```

$$y = 4.26x - 14.8$$

**2. Find all local maxima, local minima, and saddle points for the function given below. Write your answer(s) in the form (x, y, z). Separate multiple points with a comma.**

```
f2 <- function(x,y) {24*x - 6*x*y^2 - 8*y^3}

x <- y <- (seq(-10, 10, length = 50))
z <- outer(x, y, f2)

persp(x,y,z, theta = 100, phi = 10, col = 'green', ticktype = 'detailed')
```




We take partial derivatives:

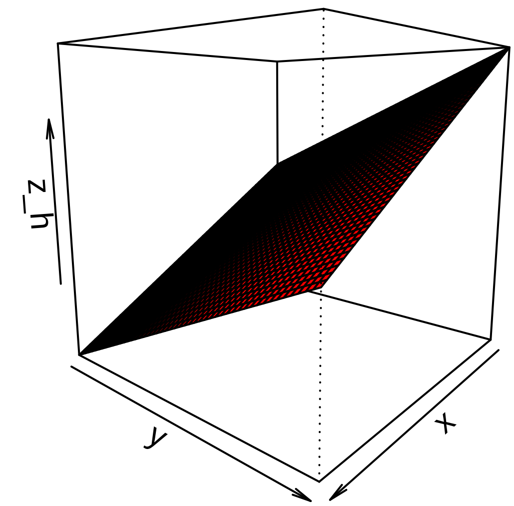
$$f_x(x,y) = -6y^2 + 24$$
$$f_y(x,y) = -24y^2 - 12xy$$

We set the first function to 0, yeilding $y = \pm 2$. Substituting both values into the second partial derivative yeilds $x = 4$ when $y = -2$ and $x = -4$ when $y = 2$.

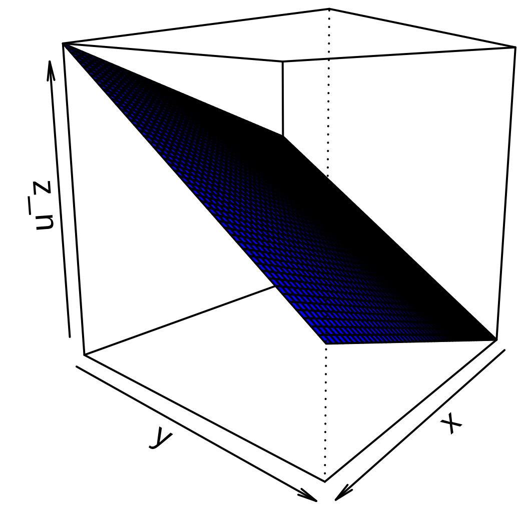
So we have two VIPs: $(4, -2, 64)$ and $(-4, 2, -64)$. It's unclear (analytically) what type of points they are. Visually, they appear to be saddle points.

**3. A grocery store sells two brands of a product, the 'house' brand name and a 'name' brand. The manager estimates that if she sells the 'house' brand for $x$ dollars and the 'name' brand for $y$ dollars, she will be able to sell $81 - 21x + 17y$ units of the 'house' brand and $40 + 11x - 23y$ units of the 'name' brand.**

```
house <- function(x,y) {81 - 21*x + 17*y}
name <- function(x,y) {40 + 11*x - 23*y}

x <- y <- seq(0, 50, length = 50)

z_h <- outer(x,y,house)
z_n <- outer(x,y,name)

persp(x,y,z_h, theta = 130, phi = 10, col = 'red')
```




```r
persp(x,y,z_n, theta = 130, phi = 10, col = 'blue')
```

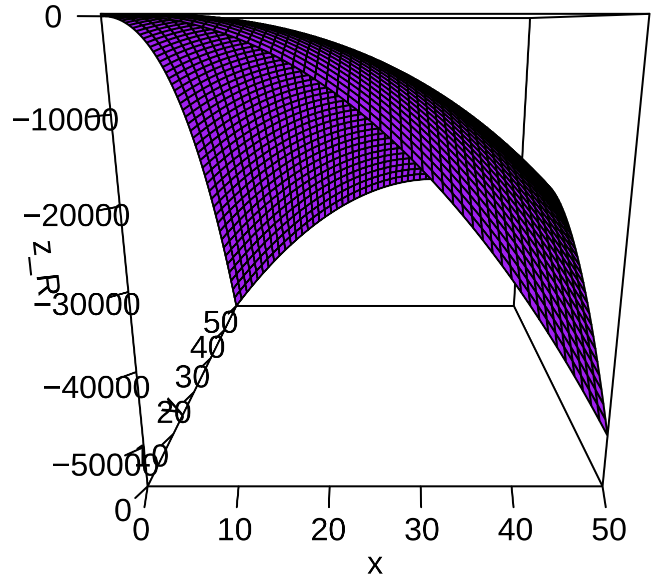



Step 1. Find the revenue function $R(x, y)$.

Let's add the functions together:

$$R(x, y) = R(x) + R(y)$$
$$x(81 - 21x + 17y) + y(40 + 11x - 23y)$$
$$R(x, y) = -21x^2 - 23y^2 + 28xy + 81x + 40y$$

```
R <- function(x,y) {-21*x^2 - 23*y^2 + 28*x*y + 81*x + 40*y}

z_R <- outer(x,y,R)
persp(x,y,z_R, col = 'purple', ticktype = 'detailed')
```




Step 2. What is the revenue if she sells the 'house' brand for \$2.30 and the 'name' brand for \$4.10?

```
R(2.3, 4.1)
```

```
## [1] 116.62
```

**4. A company has a plant in Los Angeles and a plant in Denver. The firm is committed to produce a total of 96 units of a product each week. The total weekly cost is given by $C(x, y) = \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700$, where $x$ is the number of units produced in Los Angeles and $y$ is the number of units produced in Denver. How many units should be produced in each plant to minimize the total weekly cost?**

We start with two equations:

$$f(x, y) = \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700$$
$$x + y = 96$$

Substitute the simple one into the long one:

$$x = -y + 96$$
$$f(x, y) = \frac{y^2}{3} - 14y + 2908$$

We find the minimum by taking the derivative of this function and setting it equal to zero.

$$f'(y) = \frac{2y}{3} - 14 = 0$$
$$y = 21$$

75 units should come from LA, 21 from Denver.

**5. Evaluate the double integral on the given region:**

$$\int\int e^{8x+3y} dA$$
$$R : 2 \leq x \leq 4 \text{and} 2 \leq y \leq 4$$

We can first evaluate the inner function on the interval:

$$\int_2^4 \int_2^4 e^{8x+3y} dydx$$
$$\int_2^4 \frac{1}{8} e^{3y+32} - e^{3y+16} dx$$

Then, evaluate the outer function on the same interval:

$$\frac{e^{44} - e^{38} - e^{28} + e^{22}}{24}$$