Hyperharmonic Centrality

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Introduction

Network analysis as a study analyzes connections between entities to understand underlying information about the relationships between their nodes. Our research focuses primarily on the harmonic centrality measure in network analysis--a global measure of the importance of a node within a network based on its distance from other nodes--hypergraphs--a type of network that extends the notion of simple graph pairwise connections to simultaneous connections between 3+ entities--and generalizing the harmonic centrality measure to hypergraphs.

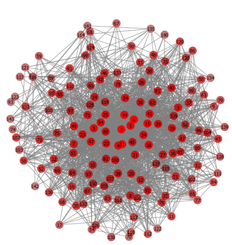
Methods

Firstly, we defined a walk in a hypergraph to be the same as a walk in its unweighted projected graph. We proved this in two scenarios: travelling within a hyperedge and travelling between adjacent hyperedges. Travelling between nodes in the same hyperedge has a cost of 1 and travelling between adjacent edges has a cost of 2 in both the hypergraph and the clique. Since all walks can be described with these two rules, every walk that exists in the hyperedge exists and is the same in the clique. We then implemented our design of the projected graph with Julia. We converted the hypergraph into an unweighted projected graph and then performed harmonic centrality on its adjacency matrix using the standard harmonic centrality equation for simple graphs. We then compared this result to the harmonic centrality obtained from using the weighted projected graph rather than the unweighted, using the Enron email dataset with nodes representing email addresses and hyperedges connecting sender and recipient(s) to model our concepts.

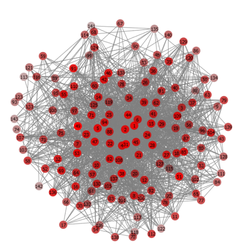
Data Statistics

- Nodes - 143
- Edges - 1800
- Hyperedges - 10551

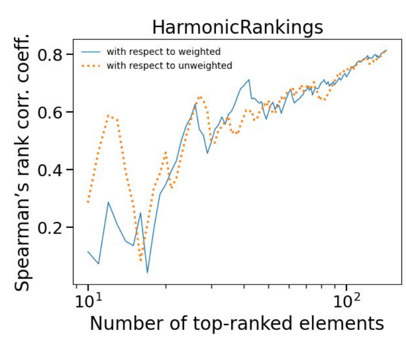
Results



weighted graph projection



unweighted graph projection



node-by-node comparison

Color intensity is used to indicate higher node centrality

Discussion

We see there is a noticeable difference between the centralities induced by the projections. The plot indicated the greatest difference in centrality within approximately the first 20 nodes, with this difference decreasing as node number increases. We also use L_2 normalization (0-1 scale) to measure the "distance" between the two datasets, finding our L_2 measure = 0.11, indicating a considerable difference between them. Our unweighted graph approach is a versatile tool that can be used to generalize any path based graph centrality, extending our understanding of analyzing higher-order networks.

Our research will be presented at a symposium on August 6th with no plans to continue into the following academic year.

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References

- J. Gao, Q. Zhao, W. Ren, A. Swami, R. Ramanathan and A. Bar-Noy, "Dynamic Shortest Path Algorithms for Hypergraphs," in *IEEE/ACM Transactions on Networking*, vol. 23, no. 6, pp. 1805-1817, Dec. 2015, doi: 10.1109/TNET.2014.2343914.
- Rochat, Y. (2009). *Closeness centrality extended to unconnected graphs: The harmonic centrality index* (No. CONF).

Code Repository

- https://github.com/SamRod33/HyperharmonicCentrality