University of Manitoba

Department of Statistics

STAT 4600: Computational Statistics

Assignment 5

Due: Thursday, March 15

Question 1:

Consider the integral

$$I = \int_{-1}^{1} (1 - x^2)|\sin(\pi x)|\, dx.$$

Our goal is to approximate this integral using different Monte Carlo schemes.

In each of the following cases, approximate the sampling distribution of the resulting Monte Carlo estimator by repeating the Monte Carlo experiment 500 times for a simulation size of $N = 1000$. Then, make a histogram of the 500 values thus obtained and obtain a 90% confidence interval for I based on the quantiles of the simulated sampling distributions. Make your comparisons based on this output.

(A) Briefly explain how I can be estimated by generating random samples from the uniform distribution on the interval $(-1, 1)$. Then, carry out the approximation using the approach you have proposed.

(B) Briefly explain how I can be estimated by generating random samples from the density

$$g(x) = \frac{3}{4}(1 - x^2), \qquad \text{for } x \in (-1, 1).$$

Also, explain how to sample from g using the inversion method. Then, carry out the approximation using the approach you have proposed.

(C) Briefly explain how I can be estimated by generating random samples from the density

$$h(x) = \frac{\pi}{4}|\sin(\pi x)|, \qquad \text{for } x \in (-1, 1).$$

Also, explain how to sample from h based on the Accep-Reject scheme. Then, carry out the approximation using the approach you have proposed.

(D) What are your conclusions? Which approach seems best and why?

Question 2:

What follows is based on the Poisson model for scores and estimates obtained from the EPL data as seen in Assignment 4. You will also want to use the function you created in that assignment to generate entire new seasons of the EPL and, perhaps, the function constructed in Assignment 2 that returns a summary of a season given the outcomes of all the games.

(A) Create an R function that, given the parameters `mu`, `Delta`, `alpha` and `beta` of the model and a number of seasons `N.seasons` to be simulated, will

1. simulate `N.seasons` different seasons,

2. for each simulated season, assign a rank to each team (calculated using the same rule as in Assignment 2),

3. return a table showing the number of times each team was awarded each rank and the average rank of each team (see below).

Then, use your function to simulate 100 seasons on the EPL and provide the appropriate summary of your simulation.

See Table 1 below for an example of the type of output that your function should produce. To see how to interpret this output note that, for instance, Chelsea finished at the top of the league on 21 occasions, finished second 38 times and never finished worst than in 7th place, which it did only once, over the 100 simulated seasons. Also, when averaging Chelsea's ranks over the 100 seasons, we get 2.58, the second best among all teams.

(B) At the end of a season, the bottom three teams in the league rankings are relegated from the EPL to a lower division of English professional soccer for the next season. Simultaneously, three new teams from lower divisions are promoted to the EPL. After the 2016/17 season, Hull City, Middlesbrough and Sunderland were relegated with, respectively, 37, 28 and 24 points.

Using the your results from part (A), how likely was it for each of these teams to be relegated? Also, among the teams that were not relegated, which team was the most likely to have been relegated?

(C) Finally, again relying on the table produced in part (A), how likely was Chelsea to end up League Champion?

Which team was most likely to finish first in the standings?

(D) Finally, there is a clear discrepancy between the top seven teams and the bottom 13 teams. The see this, and again using your previous output, calculate the approximate probability that each team finishes in the top 7.

This discrepancy is also indirectly visible from the average ranks. Explain how that is.

Table 1: Typical output obtained from using the function to simulate 100 seasons.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	AvgRank
AFC Bournemouth	0	0	0	0	0	0	2	12	10	19	12	8	11	6	3	6	3	5	3	0	11.84
Arsenal	2	13	10	22	27	18	6	0	1	1	0	0	0	0	0	0	0	0	0	0	4.50
Burnley	0	0	0	0	0	0	0	7	3	7	12	3	18	15	17	5	6	5	1	1	13.31
Chelsea	21	38	18	14	4	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	2.58
Crystal Palace	0	0	0	0	0	2	2	10	16	10	7	11	10	8	5	8	5	3	1	2	12.02
Everton	1	1	2	7	15	25	27	10	4	3	3	0	0	0	0	2	0	0	0	0	6.62
Hull City	0	0	0	0	0	0	0	0	0	0	0	2	0	2	4	6	11	18	26	31	18.33
Leicester City	0	0	0	0	0	0	5	10	13	14	7	10	12	7	8	4	5	2	2	1	11.86
Liverpool	3	6	24	20	16	16	13	1	0	1	0	0	0	0	0	0	0	0	0	0	4.52
Manchester City	7	19	24	27	15	5	2	1	0	0	0	0	0	0	0	0	0	0	0	0	3.52
Manchester United	1	5	9	8	20	27	25	3	2	0	0	0	0	0	0	0	0	0	0	0	5.49
Middlesbrough	0	0	0	0	0	0	1	0	0	2	2	3	3	5	12	17	14	15	15	11	16.59
Southampton	0	0	0	0	1	2	7	15	19	5	8	16	10	10	2	2	1	1	1	0	10.73
Stoke City	0	0	0	0	0	1	2	8	6	11	12	13	6	10	10	12	3	4	2	0	12.57
Sunderland	0	0	0	0	0	0	0	0	0	1	1	2	1	1	2	4	5	13	29	41	18.56
Swansea City	0	0	0	0	0	0	0	2	3	8	2	7	8	9	8	12	12	19	4	6	15.13
Tottenham Hotspur	65	18	13	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.58
Watford	0	0	0	0	0	0	0	2	0	2	4	7	2	8	11	11	23	11	13	6	15.99
West Bromwich Albion	0	0	0	0	0	0	2	14	19	8	17	7	7	8	9	4	2	2	1	0	11.39
West Ham United	0	0	0	0	0	0	5	5	4	8	13	11	12	11	9	7	10	2	2	1	12.87