SRGroups

Self-replicating groups of regular rooted trees.

0.1

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Abstract

To do.

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Contents

Chapter 1

The package

??? is a package which does some interesting and cool things. To be continued...

1.1 Framework

Introduction... to do. Testing references: See `AutT` (1.2.2). See `RegularRootedTreeGroupDegree` (**??**). Why do function references work and attribute references don't?

1.1.1 IsRegularRootedTreeGroup (for IsPermGroup)

▷ `IsRegularRootedTreeGroup(`*arg*`)` (filter)

Returns: `true` or `false`

Groups acting on regular rooted trees are stored together with their degree (`RegularRootedTreeGroupDegree` (**??**)), depth (`RegularRootedTreeGroupDepth` (**??**)) and other attributes in this category.

Example

```
gap> G:=SymmetricGroup(3);
Sym( [ 1 .. 3 ] )
gap> IsRegularRootedTreeGroup(G);
false
gap> H:=RegularRootedTreeGroup(3,1,SymmetricGroup(3));
Sym( [ 1 .. 3 ] )
gap> IsRegularRootedTreeGroup(H);
true
```

1.1.2 RegularRootedTreeGroup (for IsInt, IsInt, IsPermGroup)

▷ `RegularRootedTreeGroup(`*k, n, G*`)` (operation)

Returns: the regular rooted tree group G as an object of the category `IsRegularRootedTreeGroup` (**??**), checking that G is indeed a subgroup of $\mathrm{Aut}(T_{k,n})$.

The arguments of this method are a degree $k \in \mathbb{N}_{\geq 2}$, a depth $n \in \mathbb{N}$ and a subgroup G of $\mathrm{Aut}(T_{k,n})$.

Example

```
to do
```

1.1.3 RegularRootedTreeGroupNC (for IsInt, IsInt, IsPermGroup)

▷ `RegularRootedTreeGroupNC(k, n, G)` (operation)

Returns: the regular rooted tree group G as an object of the category `IsRegularRootedTreeGroup` (**??**), without checking that G is indeed a subgroup of $\mathrm{Aut}(T_{k,n})$.

The arguments of this method are a degree $k \in \mathbb{N}_{\geq 2}$, a depth $n \in \mathbb{N}$ and a subgroup G of $\mathrm{Aut}(T_{k,n})$.

Example
```
to do
```

1.1.4 RegularRootedTreeGroupDegree (for IsRegularRootedTreeGroup)

▷ `RegularRootedTreeGroupDegree(G)` (attribute)

Returns: the degree k of the regular rooted tree that G is acting on.

The argument of this attribute is a regular rooted tree group $G \leq \mathrm{Aut}(T_{k,n})$ (`IsRegularRootedTreeGroup` (**??**)).

Example
```
to do
```

1.1.5 RegularRootedTreeGroupDepth (for IsRegularRootedTreeGroup)

▷ `RegularRootedTreeGroupDepth(G)` (attribute)

Returns: the depth n of the regular rooted tree that G is acting on.

The argument of this attribute is a regular rooted tree group $G \leq \mathrm{Aut}(T_{k,n})$ (`IsRegularRootedTreeGroup` (**??**)).

Example
```
to do
```

1.1.6 ParentGroup (for IsRegularRootedTreeGroup)

▷ `ParentGroup(G)` (attribute)

Returns: the regular rooted tree group that arises from G by restricting to $T_{k,n-1}$.

The argument of this attribute is a regular rooted tree group $G \leq \mathrm{Aut}(T_{k,n})$ (`IsRegularRootedTreeGroup` (**??**)).

Example
```
gap> G:=AutT(2,4);
<permutation group of size 32768 with 15 generators>
gap> ParentGroup(G)=AutT(2,3);
true
```

1.1.7 IsSelfReplicating (for IsRegularRootedTreeGroup)

▷ `IsSelfReplicating(G)` (property)

Returns: `true`, if G is self-replicating, and `false` otherwise.

The argument of this property is a regular rooted tree group $G \leq \mathrm{Aut}(T_{k,n})$ (`IsRegularRootedTreeGroup` (**??**)).

Example
```
gap> G:=AutT(2,2);
Group([ (1,2), (3,4), (1,3)(2,4) ])
gap> subgroups:=AllSubgroups(G);;
gap> Apply(subgroups,H->RegularRootedTreeGroup(2,2,H));
gap> for H in subgroups do Print(IsSelfReplicating(H),"\n"); od;
false
false
false
false
false
false
false
true
true
true
```

1.1.8 HasSufficientRigidAutomorphisms (for IsRegularRootedTreeGroup)

▷ `HasSufficientRigidAutomorphisms(`*G*`)` (property)

Returns: `true`, if G has sufficient rigid automorphisms, and `false` otherwise.

The argument of this property is a regular rooted tree group $G \leq \mathrm{Aut}(T_{k,n})$ (`IsRegularRootedTreeGroup` (**??**)).

Example
```
to do
```

1.1.9 RepresentativeWithSufficientRigidAutomorphisms (for IsRegularRootedTreeGroup)

▷ `RepresentativeWithSufficientRigidAutomorphisms(`*G*`)` (attribute)

Returns: a regular rooted tree group which is conjugate to G in $\mathrm{Aut}(T_{k,n})$ and which has sufficient rigid automorphisms, i.e. it satisfies `HasSufficientRigidAutomorphisms` (**??**). This returned group is G itself, if G already has sufficient rigid automorphisms. Furthermore, the returned group has the same parent group as G if the parent group of G has sufficient rigid automorphisms.

The argument of this attribute is a regular rooted tree group $G \leq \mathrm{Aut}(T_{k,n})$ (`IsRegularRootedTreeGroup` (**??**)), which is self-replicating (`IsSelfReplicating` (**??**)).

Example
```
to do
```

1.1.10 MaximalExtension (for IsRegularRootedTreeGroup)

▷ `MaximalExtension(`*G*`)` (attribute)

Returns: the regular rooted tree group $M(G) \leq \mathrm{Aut}(T_{k,n})$ which is the unique maximal self-replicating extension of G to $T_{k,n+1}$.

The argument of this attribute is a regular rooted tree group $G \leq \mathrm{Aut}(T_{k,n})$ (`IsRegularRootedTreeGroup` (**??**)), which is self-replicating (`IsSelfReplicating` (**??**)) and has sufficient rigid automorphisms (`HasSufficientRigidAutomorphisms` (**??**)).

Example
```
to do
```

1.1.11 ConjugacyClassRepsSelfReplicatingGroupsWithProjection (for IsRegularRootedTreeGroup)

▷ `ConjugacyClassRepsSelfReplicatingGroupsWithProjection(`*G*`)` (attribute)

Returns: a list $\mathrm{Aut}(T_{k,n+1}$-conjugacy class representatives of regular rooted tree groups which are self-replicating, have sufficient rigid automorphisms and whose parent group is *G*.

The argument of this attribute is a regular rooted tree group $G \leq \mathrm{Aut}(T_{k,n})$ (`IsRegularRootedTreeGroup` (**??**)), which is self-replicating (`IsSelfReplicating` (**??**)) and has sufficient rigid automorphisms (`HasSufficientRigidAutomorphisms` (**??**)).

Example
```
to do
```

1.1.12 ConjugacyClassRepsSelfReplicatingGroupsWithConjugateProjection (for IsRegularRootedTreeGroup)

▷ `ConjugacyClassRepsSelfReplicatingGroupsWithConjugateProjection(`*G*`)` (attribute)

Returns: a list $\mathrm{Aut}(T_{k,n+1}$-conjugacy class representatives of regular rooted tree groups which are self-replicating, have sufficient rigid automorphisms and whose parent group is conjugate to *G*.

The argument of this attribute is a regular rooted tree group $G \leq \mathrm{Aut}(T_{k,n})$ (`IsRegularRootedTreeGroup` (**??**)), which is self-replicating (`IsSelfReplicating` (**??**)) and has sufficient rigid automorphisms (`HasSufficientRigidAutomorphisms` (**??**)).

Example
```
to do
```

1.2 Auxiliary methods

This section explains the methods of this package.

1.2.1 RemoveConjugates

▷ `RemoveConjugates(`*G, subgroups*`)` (function)

Returns: n/a. This method removes *G*-conjugates from the mutable list *subgroups*.

The arguments of this method are a group *G* and a mutable list *subgroups* of subgroups of *G*.

Example
```
gap> G:=SymmetricGroup(3);
Sym( [ 1 .. 3 ] )
gap> subgroups:=[Group((1,2)),Group((2,3))];
[ Group([ (1,2) ]), Group([ (2,3) ]) ]
gap> RemoveConjugates(G,subgroups);
gap> subgroups;
[ Group([ (1,2) ]) ]
```

1.2.2 AutT

▷ `AutT(`*k, n*`)` (function)

Returns: the regular rooted tree group $\mathrm{Aut}(T_{k,n})$ (`IsRegularRootedTreeGroup` (**??**)) as a permutation group of the k^n leaves of $T_{k,n}$, generated as an iterated wreath product.

The arguments of this method are a degree $k \in \mathbb{N}_{\geq 2}$ and a depth $n \in \mathbb{N}$.

Example
```
gap> G:=AutT(2,2);
Group([ (1,2), (3,4), (1,3)(2,4) ])
gap> RegularRootedTreeGroupDegree(G);
2
gap> RegularRootedTreeGroupDepth(G);
2
```

1.2.3 BelowAction

▷ `BelowAction(`*`k, n, aut, i`*`)` (function)

Returns: the automorphism of $\mathrm{Aut}(T_{k,n})$ that arises from *aut* by restricting to the subtree below the *i*-th vertex at depth 1.

The arguments of this method are a degree *k* $\in \mathbb{N}_{\geq 2}$, a depth *n* $\in \mathbb{N}_{\eth\nvDash}$, an automorphism *aut* $\in \mathrm{Aut}(T_{k,n})$ and an index *i* $\in$`[1..k]`.

Example
```
gap> G:=AutT(2,2);
Group([ (1,2), (3,4), (1,3)(2,4) ])
gap> a:=Random(G);
(1,3,2,4)
gap> BelowAction(2,2,a,1);
()
gap> BelowAction(2,2,a,2);
(1,2)
```

Chapter 2

The library

2.1 Methods

2.1.1 bar (for IsObject)

▷ `bar(`*arg*`)` (filter)

Returns: `true` or `false`

foo

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