Topics in Algebra — Feedback Exercise 4

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Question (1)

By Theorem 38.11 in Fraleigh, we know that a subgroup of a free abelian group is also a free abelian group. Furthermore, it has a rank less than or equal to 3.

For example, if G is a free abelian group, then it could be of the form $G \cong \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$ - by the Fundamental Theorem of Finitely Generated Abelian Groups. Consequently, we have a rank 3 subgroup: $H \cong \mathbb{Z} \times \mathbb{Z} \times 2\mathbb{Z}$, generated by $\{(1, 0, 0), (0, 1, 0), (0, 0, 2)\}$. We have a rank 2 subgroup, $H \cong \mathbb{Z} \times \mathbb{Z}$, generated by $\{(1, 0), (0, 1)\}$. We have a rank 1 subgroup, $H \cong \mathbb{Z}$, generated by $\{1\}$. Finally, we have a rank 0 subgroup, the trivial group.

Question (2)

a) Need to read Theorem 39.12.

b) Part 2