if i=j,

$$\frac{\partial \frac{e^{a_i}}{\sum_{k=0}^{N} e^{a_k}}}{\partial a_j} = \frac{\sum e^{a_k}.e^{a_j} - e^{a_i}.e^{a_j}}{\sum {e^{a_k}}^2}$$

from hypothesis 1 $\sum_{\mathbf{k=0}}^{\mathbf{N}} \mathbf{e}^{\mathbf{a_k}} = \mathbf{e}^{\mathbf{a_j}}$

$$\frac{\partial \frac{e^{a_i}}{\sum_{k=0}^{N} e^{a_k}}}{\partial a_j} = \frac{e^{a_j}}{\sum e^{a_k}} . \frac{\sum e^{a_k} - e^{a_i}}{\sum e^{a_k}}$$

$$\frac{\partial \frac{e^{a_i}}{\sum_{k=0}^{N} e^{a_k}}}{\partial a_j} = S_j * (1 - S_i)$$

if i!=j, from hypothesis 2 with condition i!=j

$$\frac{\partial \frac{e^{a_i}}{\sum_{k=0}^{N} e^{a_k}}}{\partial a_j} = \frac{\sum e^{a_k}.0 - e^{a_i}.e^{a_j}}{\sum {e^{a_k}}^2}$$

$$\frac{\partial \frac{e^{a_i}}{\sum_{k=0}^{N} e^{a_k}}}{\partial a_j} = -S_j.S_i$$