To: Dr. Wu
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Subject: Financial Engineering Project 1
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Memo: Optimal Portfolio Construction and the Efficient Frontier

I. Introduction

Portfolio theory, or postulates relating to the methods by which investors minimize the level of risk of their holdings, is founded in the nuances of asset allocation. Any and every security that carries risk will have a unique risk and return historically. By analyzing these trends, and considering these securities as an aggregate portfolio, one can weight these assets in a manner that minimizes the investor's odds of losing money for any given level of return desired. Economist Harry Markowitz introduced Modern Portfolio Theory (MPT) in his 1952 essay, for which he was later awarded a Nobel Prize in economics.

More specifically, the feasible set for a portfolio is defined as a set of every possible risk-return combination that can be obtained by weighting the securities of the portfolio differently. This set is continuous, and therefore infinite in nature. With that said, for the purposes of this project, considerations will be limited to the efficient frontier of a portfolio. The efficient frontier is a set of possible portfolios that minimize risk for any given level of return. Such a set is represented by a curve, and is a subset of the entire feasible set of the portfolio in general. Each point along the efficient frontier represents an optimal portfolio that minimizes the risk of an investor.

Thus, the portfolio in question for this project includes two large cap stocks (Visa (V), Merck & Co. (MRK)), small-cap stock (Tyler Technologies (TYL)), corporate bonds (LQD), and treasury bonds (TLT). The intentions of this project are to identify the efficient frontier of this portfolio (constrained and unconstrained) and the frontier of short term and long-term index data as well.

II. Findings

With regard to the five-stock portfolio efficient frontier created using weekly historical data from 2015, one can determine two key points. First and foremost, unconstrained efficient frontiers span a much larger range of possible expected returns, but carry with them a minimum level of risk much higher than that of a constrained efficient frontier, ceteris paribus (See Figures 1 and 2). Constrained efficient frontiers introduce a level of practicality not observed in its unconstrained counterpart, as it forces a minimum diversification of all optimal portfolios of at least 5% allocation per asset. Thus, diversification is an absolute necessity when considering portfolio optimization.

Furthermore—as depicted by the efficient frontiers of the short and long-term index portfolios (Figures 3,4,5, and 6)—short-term series are much more volatile with regard to their optimal returns than their long term counterparts. Indeed, the shorter the timeframe of the portfolio, the larger the range of optimal expected returns witnessed in the efficient frontier. Such a discovery is consistent with both Modern Portfolio Theory and the Law of Large Numbers. The longer the timeframe a portfolio exists, the less variation its expected returns will have overall (ceteris paribus), including its efficient frontier of optimal returns.

Lastly, it appears the 15-year index portfolio has a lower baseline optimal expected return than its 30-year counterpart. This occurrence runs counter to the trends aforementioned. However, it is likely that the Internet-tech bubble recession of 2000, the 2008 recession, and the flash crash of 2010 contributed to the lackluster returns of the 15-year index portfolio. Macroeconomic events have a profound impact on the performance of securities markets, and must be considered heavily in investment analysis.

III. Discussion

III. a. Methods

Understanding Constrained and Unconstrained Efficient Frontiers: Before explaining the methods of this project, it is important to understand the purpose of the analysis. Namely, it is crucial to recognize the distinctions between unconstrained and constrained efficient frontiers. An unconstrained efficient frontier allows the weight of any given security to exist unbounded; the stock can be sold short or long with any amount of weighting, as can be seen with the macro in *Q4* and *Q4 Unconstrained Diagram*. The macro aforementioned utilized solver to minimize portfolio risk given a certain level of return. More importantly, however, unconstrained efficient frontiers are impractical; no investor would consolidate holdings in the manner depicted in *Q4* (seen in Figure 1) as the portfolio is fully allocated as one stock, and not diversified.

Therefore, to prevent this occurrence, a constrained efficient frontier specifies minimum and maximum weights each security must have. This way, the portfolio will be comprised of at least five percent (but no more than fifty percent) of each security, diversifying the investments and mitigating future risk. These models were created with historical data, which cannot be expected to indicate future risk and return exactly. Thus, the need for diversification (i.e. a constrained efficient frontier) is necessary, and will be central to the analysis overall.

Data Manipulation: Weekly data for each of the aforementioned stocks and bonds in the portfolio was collected from Yahoo Finance. This data was comprised of the list of adjusted price of all of these securities (after dividends, if applicable) on a weekly basis. By exporting this data into Excel, the weekly return was found for each of these securities, and then averaged to find the expected weekly return (as seen in *Q1* and *Q2* of the data sheet). The risk (standard deviation), variance, and covariance matrix of these weekly returns were also calculated, and the risk-return relationship of each security is visualized in *Q3* of the datasheet.

Deriving Efficient Frontiers for Five-Stock Portfolio: With an understanding of unconstrained and constrained efficient frontiers, the five-stock portfolio can now be analyzed for maximum returns and minimum variance. As seen in *Q4 Unconstrained Diagram* (which has weight constraints of -100% and 100% to ensure returns are reasonable), the maximum and minimum returns can be seen as the first and last point on Figure 1. Short sale is only permitted in the unconstrained efficient frontier. The macro that aggregates the data represented in these charts utilizes solver as well, but runs iterations so that an array of data is synthesized and placed into the spreadsheet. The same macro is again used in *Q5* to construct the constrained efficient frontier graph, as seen in Figure 2. Comparisons between these two figures will occur in the following section.

Manipulating and Understanding Historical Index Data Portfolio: After constructing the efficient frontiers of the five-security portfolio, attention was turned to an index portfolio. This portfolio, comprised of large cap (S&P 500), small cap (Russell 2000), corporate bond, and treasury bond indices, will be analyzed with regard to monthly returns in the past 3, 5, 15, and 30 years. In particular, the relationship between short-term and long-term series will be scrutinized in order to discern any poignant differences. The efficient frontiers for all of these indices will be constrained from a 5% minimum to a 50% maximum; the procedures by which these frontiers are derived is identical to those outlined for the five-security portfolio previously.

Comparison, Analysis, and Context: After collection of this data has occurred and comparisons between results have taken place, our aim is to tie the macroeconomic events of the past to these trends in our data with the ultimate goal of identifying overlying trends in the economy.

III. b. Analysis

Figure 1.

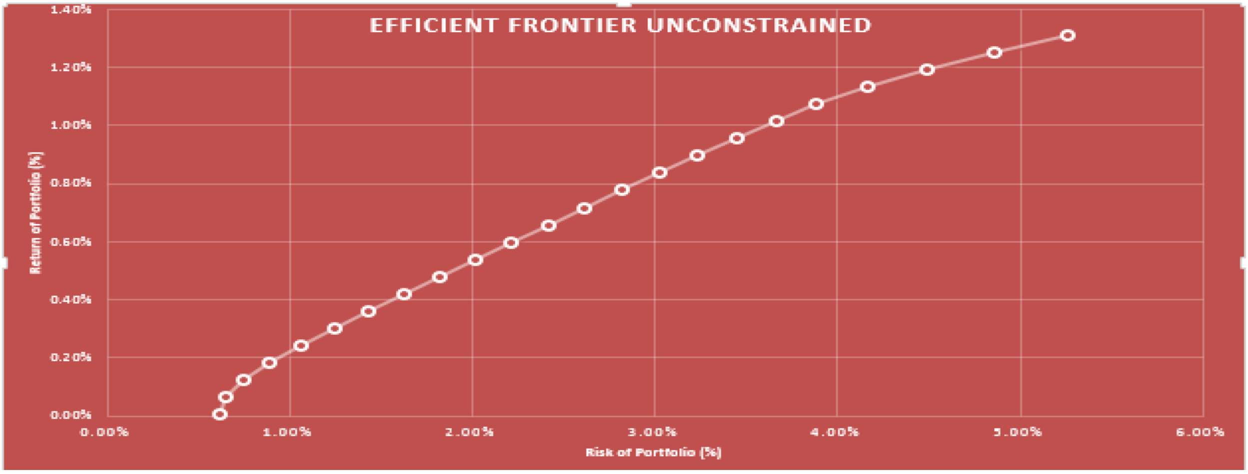
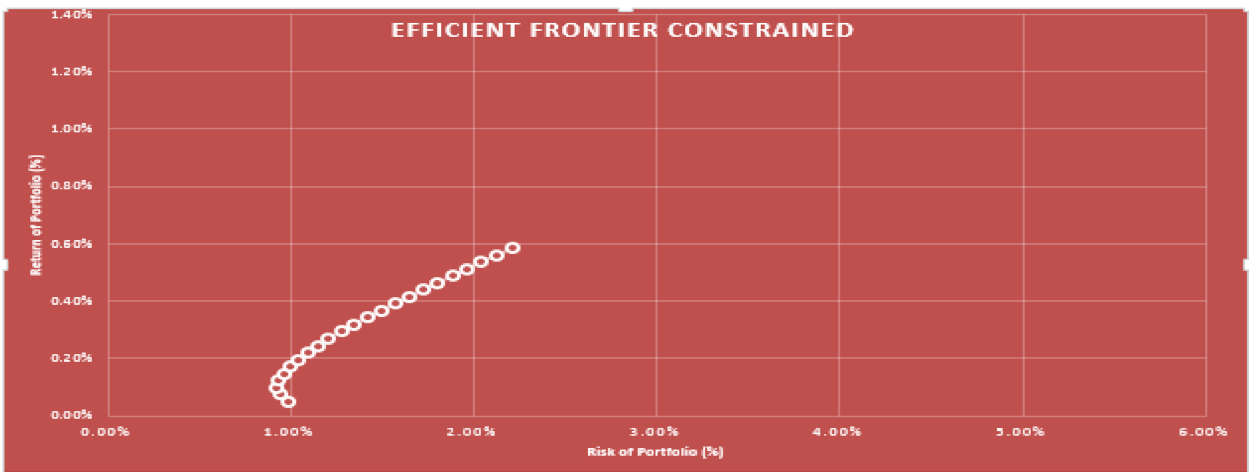


Figure 2.



<u>Five-Stock Portfolio Efficient Frontier and Analysis</u>: When observing the unconstrained and constrained efficient frontier graphs of the five-stock portfolio (Figures 1 and 2, respectively), one notices several differences. First and foremost, the unconstrained efficient frontier (which allows for short sale) is able to achieve higher and lower expected returns than its constrained counterpart. The unconstrained frontier spans returns from 0% to ~1.3% while the constrained frontier can only achieve returns of just over 0% to around .0.60%. This wide range of returns, however, is also associated with a much higher potential risk for the unconstrained portfolio. Optimal risk levels for the unconstrained portfolio span from ~0.60% to ~5.3% while the constrained frontier risk is confined between ~0.9% and ~1.3%.

The unconstrained frontier clearly offers a higher potential for return, but its allocation is both dangerous and unrealistic. Conversely, the constrained efficient frontier diversifies its assets and mitigates much of the excess risk seen in unconstrained frontier portfolios, but at the cost of reduced optimal returns. Even so, the constrained efficient frontier is more useful practically as it diversifies the portfolio, mitigating future risk.

<u>Index Data Portfolio Efficient Frontier and Analysis</u>: A slight alteration from the five-stock portfolio, the index portfolio emulates the behavior of all of the securities in the market it relates to. Therefore, by observing the trends in a portfolio comprised of stock and bond indices, one can observe the nature of the securities market as a whole. By optimizing this portfolio, one can determine the lowest risk for any given level of return. The following data analyzes the 3, 5, 15, and 30-year monthly returns of four indices specified previously.

Using Table 1 and Figures 3, 4, 5, and 6, we can compare the relationship of risk and return for each time period by observing the constrained efficient frontiers of each index portfolio. Moreover, by observing the *Optimized* sheets attached to each one of these index portfolios, one can determine which sectors of the market are performing the best. **Note:** Table 1's risk levels are all minimized for each level of return; even data that says "Maximum Risk" are the minimum amount of risk for the maximum expected return.

Table 1.

Index Year Range	*Min. Exp. Return*	*Minimum Risk*	*Max. Exp. Return*	*Maximum Risk*
3 Year	0.25%	1.03%	1.03%	2.98%
5 Year	0.39%	0.87%	0.89%	3.39%
15 Year	0.46%	1.23%	0.62%	4.41%
30 Year	0.61%	1.35%	0.86%	4.24%

Figure 3.

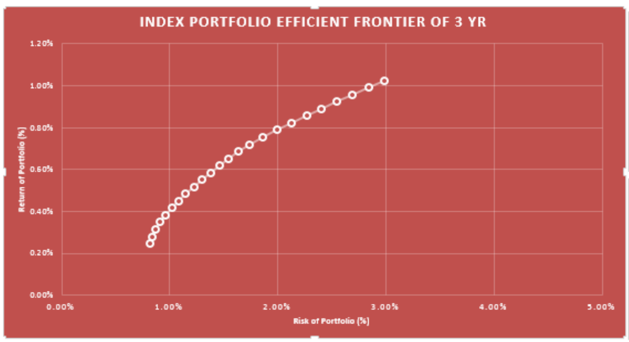


Figure 4.

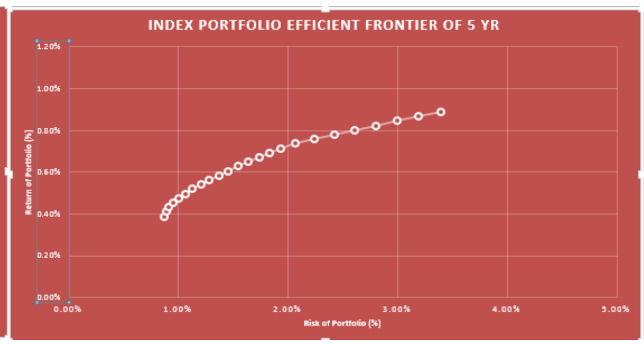


Figure 5.

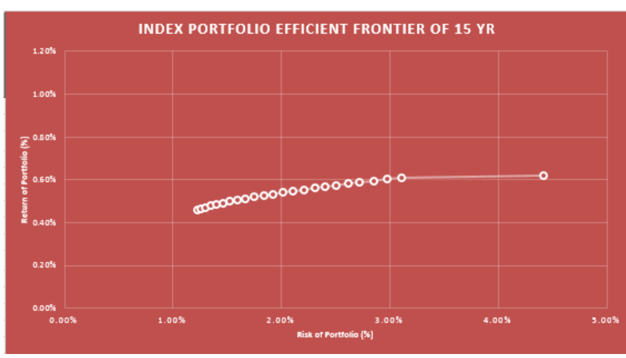
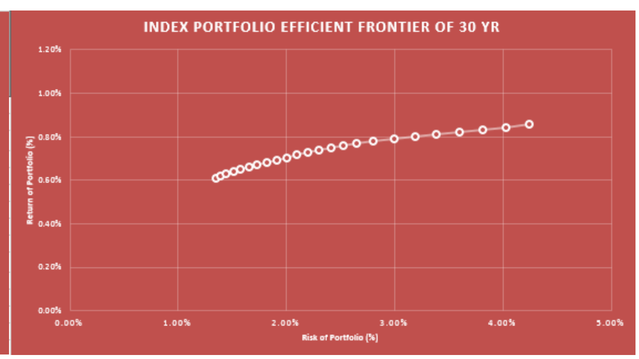


Figure 6.



When observing the efficient frontier of the 3-year index, the lowest possible monthly return (0.25%) is correlated to a minimum of 0.82% risk; conversely, the highest possible expected return of 1.03% correlates to an expected minimum risk of 2.98%. As can be discerned from Figure 3, this efficient frontier is able to achieve a relatively wide range of returns, depicted by the steep slope.

Now, the efficient frontier of the 5-year index exhibits slightly different trends. The lowest possible monthly return (0.39%) is correlated to a minimum of 0.87% risk; conversely, the highest possible expected return of 0.89% corresponds to a minimum risk of 3.39%. Figure 4, illustrates these characteristics, hinting that the five year index portfolio is less volatile in its performance, sports a higher baseline average return, but may require a higher minimum risk than the three year index. These facets of the efficient frontier are evident from the flatter slope of Figure 4 compared to Figure 3.

Transitioning to a long-term series, the efficient frontier of the 15-year index starkly contrasts those of its 3 and 5 year counterparts. The lowest possible monthly return (0.46%) is correlated to a minimum of 1.23% risk. At the same time, the highest possible expected return of 0.62% corresponds to a minimum risk of 4.41%. Figure 5, the efficient frontier of this data, is almost completely flat. Long-term expected monthly returns are consistent at thing point, and the nearly non-existent slope hints that an increased long-term return would require taking on an unbelievable large amount of risk. The implications of these long-term trends will be contextualized shortly.

Lastly, the 30-year index (Figure 6) maintains that the lowest possible monthly return (0.61%) is correlated to a minimum of 1.35% risk, while the highest possible expected return of 0.86% corresponds to a minimum risk of 4.24%. Similar to the efficient frontier of the 15-year index, the 30-year frontier also has a very small slope. With that said, the baseline minimum expected returns of the 30 year efficient frontier surpass those of the 15-year index by over 30% (0.61% versus 0.42%). In fact, for any given level of risk, the 30-year index portfolio will generate a higher return than the 15-year portfolio.

III. c. Limitations

There are several drawbacks to the current portfolio model. First and foremost, our raw data only comprises one year of historical prices and does not provide enough information to analyze the securities comprehensively. In reality, the situations would surely be much more complex. Also, the analytical macros utilized only worked with the solver function; there are definitely more complex and advanced methods that could yield better results. Lastly, macroeconomic events are one of the most important contributors to the return on stocks and securities, and need to be incorporated into the model in some way. After more practice, our efficient frontier models could surely become more reliable.

IV. Conclusions

Throughout this examination, the overlying purpose of efficient frontier analysis was to glean insight from the interplay between optimal portfolios (minimum variance, maximum returns) and the exogenous factors at play. In particular, understanding the distinction between unconstrained and constrained efficient frontiers is crucial to portfolio optimization. Unconstrained efficient frontiers tend to allocate heavily to a few securities while possibly short selling the others. These practices undoubtedly allow for increased possible returns, but also introduce a large amount of potential risk. Therefore, a constrained efficient frontier analysis is a more beneficial investing strategy.

Moreover, when understanding the nature of market returns via index-based portfolios, it is important to also consider macroeconomic events as well a diversification. As seen with the 15-year index returns, macroeconomic events can heavily impact the optimal returns of a portfolio. These trends will run counter to the mathematical trends of investing seen in a controlled setting (i.e. a market sheltered from macroeconomic events). Thus, before investing in anything, an extensive analysis of market trends should occur so that macroeconomic influence can be accounted for in investing models.

V. References

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