

# Markov Chains, Carbon Cycling, and Stochastic Processes: A Mathematical Representation of Global Warming

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**Abstract:** *Global warming, the process by which carbon emissions enter the atmosphere and gradually trap reflected solar heat, has been at the forefront of socioeconomic discussion for the past decade. With that said, though many broadcasted disputes revolve around the implications of global warming on biodiversity, renewable resources, and the like, research on methods by which global warming can be mathematically simulated remain less publicized. Thus, carbon cycling—the engine of global warming—will be examined mathematically via Markov Chains with the intention of forecasting future atmospheric carbon concentrations.*

## Contents:

<b>1.0</b>	<b>Introduction</b>
<b>1.1</b>	Global Warming and Carbon Cycling
<b>1.2</b>	Markov Chains: Definitions, Facets, and Visualization
<b>1.3</b>	Markov Chain Properties
<b>2.0</b>	<b>The Global Carbon Cycle</b>
<b>2.1</b>	Mapping the Global Carbon Cycle
<b>2.2</b>	Transition Matrix Construction
<b>2.3</b>	Assumptions and Characteristics of Markov Chain Models
<b>3.0</b>	<b>Carbon Cycle Forecasting</b>
<b>3.1</b>	Carbon Forecasts and Trends
<b>3.2</b>	Refining Markov Chain Models: Stochastic Variation
<b>3.3</b>	Monte Carlo Simulation
<b>3.4</b>	Forecast Interpretation: Global Warming
<b>4.0</b>	<b>Conclusion</b>
<b>4.1</b>	Summary: Carbon Cycle Analysis
<b>4.2</b>	Further Research
<b>5.0</b>	<b>Bibliography</b>

# 1.0 Introduction

## *1.1 Global Warming and Carbon Cycling*

In society today, discussion of global warming is a fairly common in mass media as well as scientific research. Many are concerned about the repercussions that Earth's rising temperature, and as a result, the majority of publicized discussion is comprised of the socioeconomic and ecological implications of global warming overall. In turn, little to no dialogue on global warming from a stochastic, mathematical standpoint visibly takes place. Moreover, publicity rarely emphasizes the processes by which global warming occurs, overlooking the driving mechanisms of a process so many would like to assuage or reverse. Therefore, the following analysis will attempt to model global warming by forecasting the Earth's carbon cycle via Markov Chains. However, before attempting such an endeavor, it is crucial to first understand the nuances of the carbon cycle as well as its possible effects on global warming.

Global warming is a process by which the Earth's atmosphere retains solar heat energy through what is known as the Greenhouse Effect. At a superficial level, the Greenhouse Effect is the occurrence where solar energy (previously absorbed by the Earth) is released from the Earth's surface as Infra-Red (IR) Radiation. This release of IR radiation is the equivalent of the Earth "cooling down" [4]. Now, when radiatively active (greenhouse) gases exist in the Earth's atmosphere, they absorb this IR heat radiation. These gases—now sufficiently irradiated—subsequently heat the Earth to a "temperature above what is possible in the absence of an atmosphere." Common greenhouse gases include carbon dioxide and methane. Considering these molecules both contain carbon as their central atom, it is clear how the term "carbon emissions" arose [4]. With the basic functionality of global warming in mind, the driving mechanisms of greenhouse gas emissions into the atmosphere will be explored at length.

Now—due to the fact that primary greenhouse gases involve carbon—carbon cycling is often utilized when researching facets of global warming, particularly when attempting to determine human additions to atmospheric carbon levels. Extensive study on carbon cycling has been conducted and refined over multiple decades, and literature estimates regarding current carbon concentrations as well as carbon movement are widely considered reliable and consistent across time. Consequently, the carbon cycle is recognized as a globally connected, complex system (see Figure 4). Therefore, though an excess of data on carbon cycling is available, a mathematical inquiry via Markov Chains only requires current state carbon concentrations and carbon movement between individual states [6]. Thus, with the mechanisms of global warming in mind, attention will be turned to Markov Chains and their application to multifaceted stochastic processes.

### ***1.2 Markov Chains: Definition, Facets, and Visualization***

The stochastic (randomness) process known as a Markov Chain is one of many keystone achievements derived by mathematician Andrey Markov. Born in Russia in 1856, Andrey Markov grew up to be a major contributor to the fields of mathematics and applied statistics [1]. Technically speaking, a Markov Chain is a stochastic process that follows the Markov Property or Markov Assumption. Moreover, a stochastic process can be thought of as any process or simulation that incorporates a level of chance or randomness. For example, Figure 1 depicts the probability of making a free throw in basketball, given the shooter has a 70% shooting success rate. “M” represents missing the basket; “S” represents a successful free-throw.

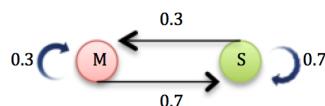


Figure 1.

Along with this, the Markov Property—also known as the memorylessness property—states that the conditional probability of future states (or, in this case, future carbon concentrations) is wholly independent of previous states. That is, a given state in the future can be determined solely by

information on the present state and the probability of present state concentrations moving to a new state, often within a given time period. The probability of one state moving to another is represented as a probability matrix, and is known as a transition matrix (represented as  $T$  in this analysis). For example, the transition matrix  $P$  for the basketball shooter in Figure 2 is:

$$\begin{array}{ccc} & \text{From:} & \\ & S & M & \text{To:} \\ P = & \begin{pmatrix} 0.7 & 0.7 \\ 0.3 & 0.3 \end{pmatrix} & \begin{matrix} S \\ M \end{matrix} \end{array}$$

**Figure 2.**

Where the first value in the matrix (0.7) corresponds to the probability of making a free throw after making a free throw previously (hence, from “S”, to “S”). Interpretations for all other values in the transition matrix above (and transition matrices in general) can be determined by the method aforementioned. The schematic set-up in Figure 1—deemed “*from, to*” notation—will be expounded on shortly. Its overlying purpose is to ensure every column of a transition matrix equals one, as the sum of probabilistic movement from a single state cannot exceed such a probability, for obvious reasons. However, the rows of a transition matrix may exceed a probability of one—as seen in Figure 2—because one state may be more easily reached.

Due to the probabilistic nature of Markov Chains, the Markov property is central to interpreting Markov Chains analytically as well as conceptually. Since probabilistic transfer in the future only depends on current state parameters, the transition matrix  $T$  will subsequently remain constant (though more complex research is done with dynamic transition matrices, they will not be addressed by this analysis). Mathematically, finding state values (represented as a matrix/vector  $\mathbf{x}$ ) one period of time in the future can be represented as

$$\mathbf{x}_2 = T\mathbf{x}_1$$

With this in mind, it follows that

$$\mathbf{x}_3 = T\mathbf{x}_2 = TT\mathbf{x}_1$$

Thus, in general

$$\mathbf{x}_{k+1} = T^k \mathbf{x}_1$$

Here,  $k$  represents an interval measurement (e.g. year 1, free-throw 3, day 4). It is important to note here that transition matrices ( $T$ ) will always be square  $n \times n$  matrices and state matrices (also considered vectors) ( $\mathbf{x}$ ) will always be of dimension  $n \times 1$ . Thus, the multiplication of  $T$  and  $\mathbf{x}$  will always be possible. A general representation of a transition matrix  $T$  can be seen below. All values residing in transition matrix  $T$  are probability vectors for different states.

$$\begin{array}{ccccccc} & & & \text{From:} & & & \text{To:} \\ & 1 & 2 & \dots & n & & \\ \left( \begin{array}{ccccc} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{array} \right) & & & & & 1 & 2 \\ & & & & & & \vdots \\ & & & & & & n \end{array}$$

**Figure 3.**

### 1.3 Markov Chain Properties

A state in a Markov chain is said to be accessible if it is possible (i.e. nonzero probability) to transition to that state from its current state at some point in time. Often, accessibility is formally represented as  $a \rightarrow b$ , where  $a$  is the current state and  $b$  is a possible future state. Similarly, two states are considered to communicate with one another if  $a \rightarrow b$  and  $b \rightarrow a$ , which is logically written as  $a \leftrightarrow b$ . If two or more states communicate with one another, then they are deemed a communicating class. Now, if an entire transition matrix is a single communicating class, then the matrix is considered to be irreducible, as every state can eventually be reached from any other [1].

Moreover, a transition matrix is considered to be periodic if the probability of returning to any state  $a$  may only occur in multiples of  $k$  interval (typically time interval) steps. An excellent analogy to visualize periodicity is the transition matrix for a stoplight. In order to attain a state of “red light,” from a state of “green light.” Attainment of “red light” will only occur after 2 time steps, due to the existence of the “yellow light” state. Thus, the transition matrix for a stoplight would be considered periodic. By contrast, an aperiodic transition matrix states that for any given state, recurrence to that state or any other state will occur irregularly [6].

Also, transience is another characteristic of transition matrices, and is described as when—given a starting state of  $a$ —there is a non-zero probability of not returning to state  $a$ . If the previous statement is not satisfied, then the matrix is deemed recurrent, and will be able to return to its starting state with 100% probability. Mean recurrence time, in time steps  $k$ , can either be finite or infinite, and is calculated by summing the probability of attaining a particular state in  $n$  time intervals. Formally, this theorem is displayed as

$$\sum_{n=1}^{\infty} n \cdot f_{ii}^{(k)}$$

where

$$f_{ii}^{(n)} = P(R_i = k)$$

or, the probability of returning to state  $i$  in  $k$  time intervals, where  $k$  is any integer. Let  $R_i$  signify the event of returning to state  $i$ , the given starting state.

Lastly, a transition matrix is ergodic if it is both aperiodic and has a finite mean recurrence time (known as having positive recurrence). The property of ergodicity is particularly important for manipulating transition matrices to determine eigenvalues and eigenvectors, but is not a particularly useful property unless intended for use in more advanced processes that are unnecessary for this exploration [1]. Thus, with the essential facets and characteristics of Markov Chains available, enough information is now known to successfully map the global carbon cycle.

## 2.0 The Global Carbon Cycle

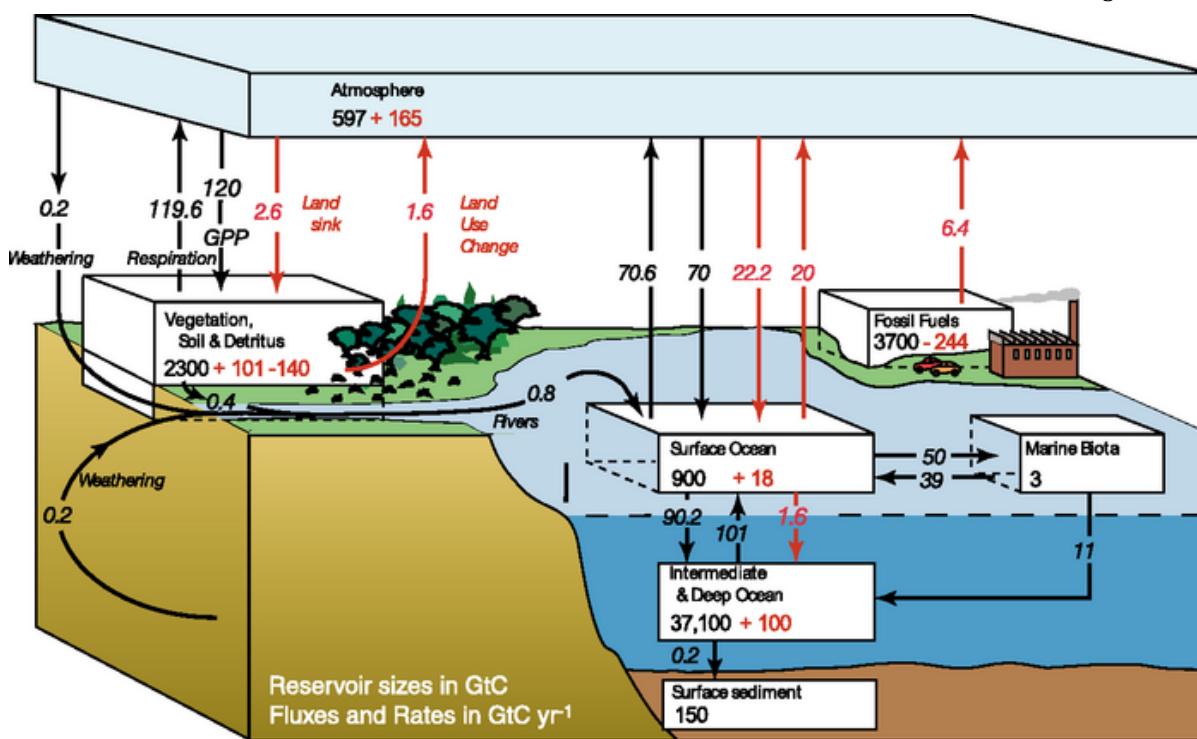
### 2.1 Mapping the Global Carbon Cycle

Before any global warming forecasts can occur, the world's carbon cycle must be mapped.

In order to do this, the global carbon cycle must be understood to the extent that numerical values for carbon concentration and transfer between states can be discerned. Fortunately, ecologists have researched this topic for decades; as depicted below in Figure 4, carbon states—the locations of carbon on Earth—are comprised of the atmosphere, surface ocean, deep ocean, sediment (Earth's crust), rivers and lakes, marine biota (marine life), fossil fuels, terrestrial life, and detritus [3]. Detritus is considered to be any carbon byproduct by a living (or once living) organism, including dead organisms of all kinds. Between these carbon states, carbon transfers by a multitude of mechanisms, including dissolution, sequestration, consumption, and respiration, to name a few.

Figure 4 depicts many—but not all—of the states and transfer mechanisms of the carbon cycle [6]. Its purpose is to assist the reader in visualizing the carbon cycle, as its carbon concentration values are incomplete.

Figure 4.



Fortunately, complete and recent carbon concentrations for different states as well as annual carbon transfer between states are available and corroborated by extensive research. Using carbon cycle data provided by [2], carbon concentrations are summarized in the following matrix.

Estimates are represented in petagrams (gigatons).

$$\begin{array}{ll}
 \begin{matrix} A & 720 \\ SO & 670 \\ DO & 36,730 \\ S & 75,000,000 \\ R & 60 \\ MO & 1,000 \\ F & 4,130 \\ T & 800 \\ DE & 1,200 \end{matrix} & = \mathbf{x} \\
 \end{array}
 \quad
 \begin{array}{l}
 *A = \text{Atmosphere} \\
 SO = \text{Surface Ocean} \\
 DO = \text{Deep Ocean} \\
 S = \text{Sediment} \\
 R = \text{Rivers/Lakes} \\
 MO = \text{Marine Biota} \\
 F = \text{Fossil Fuels} \\
 T = \text{Terrestrial Life} \\
 DE = \text{Detritus}
 \end{array}$$

*\*Please reference the legend above throughout the remainder of this analysis*

As depicted above, the carbon concentration for each state is represented by a different row in matrix  $\mathbf{x}$ . Almost unintentionally, the present state matrix  $\mathbf{x}$  has been created for the year 2014. Though Falkowski's research was not conducted in 2014, his work is the most recent estimate of carbon concentrations, and will be used accordingly [3].

## 2.2 Transition Matrix Construction

Analogously, transition matrix  $T$  must now be created in order to successfully conduct Markov Chain calculations. To assist intuitive understanding, transition matrix  $T$  will be comprised of carbon transfers between states. From a conceptual standpoint, these values can be thought of as the carbon a given state will lose to other states over the course of the year, or as the carbon retained by a state. As a result, carbon cycle matrices will utilize "*from, to*" reference syntax to maximize readability (see  $T_1$ ). That is, the carbon emitted by car exhaust cars can be thought of as "*from* fossil fuels, *to atmosphere*" and will exist as a raw value (in gigatons). Therefore, to determine carbon transfer from a probability standpoint, one must alter the statement above so that it determines the percentage of fossil fuel carbon (or carbon of any state) will enter the atmosphere (a different state).

These values are discerned by dividing carbon transfer amounts by the total carbon existing in the state in which the transferring carbon resides. A concrete example using the above fossil fuel analogy is shown below.

$$\frac{F \text{ Carbon Transferred to } A}{\text{Total } F \text{ Carbon}} = \text{Carbon transfer percentage, } F \text{ to } A.$$

Now able to transfer raw carbon transfer values into percentages, two transition matrices— $T_1$  and  $T_2$ —will be created;  $T_1$  will include all carbon transfers that do not involve human impact, and  $T_2$  will include all carbon transfers present today. Essentially, the Markov Chain with transition matrix  $T_1$  will forecast the global carbon cycle as if humans ceased to detrimentally impact the environment and reverted back to hunter-gatherer societies (or at least zero carbon footprint societies) [2]. By contrast, the Markov Chain involving  $T_2$  forecasts future carbon concentrations as if humans continue to emit carbon emissions at the level they do today.

Accordingly, the construction of transition matrix  $T_1$  occurs as follows. Based on extensive transfer information provided by [2], one can deduce that roughly 511.6 petagrams of atmospheric carbon remain in the atmosphere over the course of a year. Hence, using the process outlined in the paragraphs above, one notes that.

$$\frac{A \text{ Carbon Kept}}{\text{Total } A \text{ Carbon}} = \frac{511.6}{720} = 0.7105$$

The atmosphere retains about 71.05 percent of its carbon in any given year. Using the “from, to” reference syntax, 0.7105 translates to the  $a_{11}$  value for transition matrix  $T_1$ . It is important to note here that atmospheric carbon retained in a year is calculated via deduction (as is carbon retainment for any state) since the sum of raw value transfer and retainment must equal a state’s beginning carbon concentration; correspondingly, percentage transfer and retainment must sum to one. With this in mind, carbon transfer percentages must be determined before retainment.

Therefore, by repeating the process above for all carbon transfers (and deductions for retainment), transition matrix  $T_1$  is found below (“from, to” notation represented in bold). Numbers are rounded to four significant figures.

$$T_1 = \begin{pmatrix} & \text{From:} & & & & & & & \text{To:} \\ \text{A} & \text{SO} & \text{DO} & \text{S} & \text{R} & \text{MO} & \text{F} & \text{T} & \text{DE} \\ \left( \begin{array}{ccccccc} 0.7105 & 0.1054 & 0 & 0 & 0 & 0 & 0 & 0.1095 & 0 \\ 0.1026 & 0.6754 & 2.75e^{-3} & 0 & 0.0133 & 0.039 & 0 & 0 & 0 \\ 0 & 0.1346 & 0.9972 & 0 & 0 & 0.011 & 0 & 0 & 0 \\ 0 & 0 & 5.445e^{-6} & 0.9999 & 0 & 0 & 0 & 0 & 0 \\ 2.778e^{-4} & 0 & 0 & 2.667e^{-20} & 0.9867 & 0 & 0 & 0 & 3.333e^{-4} \\ 0 & 0.0846 & 0 & 0 & 0 & 0.95 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6.667e^{-9} & 0 & 0 & 1 & 0 & 4.167e^{-9} \\ 0.1867 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8401 & 0.1933 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0504 & 0.8064 & 0 \end{array} \right) \end{pmatrix} \begin{matrix} \text{A} \\ \text{SO} \\ \text{DO} \\ \text{S} \\ \text{R} \\ \text{MO} \\ \text{F} \\ \text{T} \\ \text{DE} \end{matrix}$$

In interpreting  $T_1$ , in a given year, 10.54 percent of surface ocean carbon is released into the atmosphere, 18.67 percent of atmospheric carbon is absorbed by terrestrial life via photosynthesis, and so on. Transition matrix  $T_2$  can be found using the same process outlined above, except with data that incorporates human interaction [2]. By adding human impact on the current carbon cycle matrix  $T_1$ , transition matrix  $T_2$  takes the following form.

$$T_2 = \begin{pmatrix} & \text{From:} & & & & & & & \text{To:} \\ \text{A} & \text{SO} & \text{DO} & \text{S} & \text{R} & \text{MO} & \text{F} & \text{T} & \text{DE} \\ \left( \begin{array}{ccccccc} 0.8014 & 0.1352 & 0 & 6.667e^{-7} & 0 & 0 & 0.0586 & 0.1515 & 0 \\ 0.1281 & 0.6526 & 2.75e^{-3} & 0 & 0.0133 & 0.189 & 0 & 0 & 0 \\ 0 & 0.1376 & 0.9972 & 0 & 0 & 0.011 & 0 & 0 & 0 \\ 0 & 0 & 5.445e^{-6} & 0.9999 & 0 & 0 & 0 & 0 & 0 \\ 2.778e^{-4} & 0 & 0 & 2.667e^{-9} & 0.9867 & 0 & 0 & 0 & 3.333e^{-4} \\ 0 & 0.0746 & 0 & 0 & 0 & 0.80 & 0 & 0 & 0 \\ 0.0667 & 0 & 0 & 6.667e^{-8} & 0 & 0 & 0.9414 & 0 & 4.167e^{-9} \\ 0.1867 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6081 & 0.0891 \\ 0.0036 & 0 & 0 & 6.667e^{-8} & 0 & 0 & 0 & 0.2404 & 0.8064 \end{array} \right) \end{matrix} \begin{matrix} \text{A} \\ \text{SO} \\ \text{DO} \\ \text{S} \\ \text{R} \\ \text{MO} \\ \text{F} \\ \text{T} \\ \text{DE} \end{matrix}$$

Now that both transition matrices  $T_1$  and  $T_2$ —as well as present state matrix  $\mathbf{x}$ —have been calculated using global carbon cycle data, concentration forecasting may begin after the assumptions of the Markov Chain models are identified. Global warming interpretations may also be made, though at a superficial level.

### ***2.3 Assumptions and Characteristics of Markov Chain Models***

Before Markov Chains can be utilized for forecasting and contextualized for interpretation, it is crucial to understand the limitations (and assumptions therein) of the Markov Chain models as well as the characteristics of different Markov Chains. To begin, the Markov Chain models above fall short of modeling carbon flow fluctuations for several reasons. Primarily, these Markov Chains assume the Earth's carbon is fixed (i.e. no additions or losses), and indeed it is not. Carbon additions via particulates from space contribute a minuscule but consistent amount to Earth's carbon level annually [4]. At the same time, Markov Chains assume that carbon transfers are fixed from year-to-year; while transfer percentages are indeed homologous on an annual basis, they are by no means constant, for obvious reasons [6]. Branching off of the previous pitfall, due to the fixed percentages of Markov Chain transition matrices, carbon concentration forecasts will not account for extraneous factors (e.g. supply and demand for fossil fuels, carbon emission regulations, etc.). All the same, with these assumptions in mind, Markov Chains account for complex carbon cycle fluctuations particularly well, due to the memorylessness property being so conducive for modeling probabilistic transfer [1].

Moving forward, the characteristics of the “no human impact” (NHI) Markov chain (derived from  $T_1$ ) differ subtly from those of its “human impact” HI counterpart (derived from  $T_2$ ). These differences are visible from the transition matrices of each chain. For example, only the HI chain is irreducible, as the NHI matrix does not include F in its communication class. Fossil fuels are not utilized in a NHI world, and therefore state F is known as an absorbing state (no carbon can escape said state). Along the same lines, the HI Chain is considered recurrent, as carbon can always return to the state that it started in. However, due to the absorbing F state in the NHI Markov Chain, the NHI Chain is deemed transient; the odds of not returning to a starting state are greater than zero due

to the risk of being absorbed by the fossil fuel state. With that said, both Markov Chains due exhibit similarities. Both matrices are aperiodic—as recurrence to any given state occurs irregularly. With these characteristics and assumptions in mind, forecasting may begin.

## 3.0 Carbon Cycle Forecasting

### 3.1 Carbon Forecasts and Trends

Forecasting carbon concentrations becomes fairly simple once the method of forecast is known and understood. Currently, both transition matrices  $T_1$  and  $T_2$ , as well as present state vector  $\mathbf{x}$ , are known. Therefore, in order to estimate carbon concentrations for 2015 (one year into the future), transition matrices are applied once to  $\mathbf{x}$ . As a result, the number of times a transition matrix and  $\mathbf{x}$  is multiplied—also known as the power  $k$  a transition matrix is taken to (see page 5)—is equivalent to the number of years into the future carbon concentrations are forecasted for. When the NHI and HI Markov Chain models forecast 2015 carbon concentrations, the mathematical process is as follows (in petagrams).

$$\text{2015: NHI Carbon} = T_1 \mathbf{x} = \begin{pmatrix} 669.80 \\ 667.15 \\ 36730 \\ 74,999,999 \\ 59.80 \\ 1006.7 \\ 4130.5 \\ 1038.4 \\ 1008.0 \end{pmatrix} \quad \text{HI Carbon} = T_2 \mathbf{x} = \begin{pmatrix} 1080.81 \\ 820.25 \\ 36732 \\ 74,999,945 \\ 59.99 \\ 849.98 \\ 3898.0 \\ 641.4 \\ 1287.6 \end{pmatrix} \begin{matrix} \text{A} \\ \text{SO} \\ \text{DO} \\ \text{S} \\ \text{R} \\ \text{MO} \\ \text{F} \\ \text{T} \\ \text{DE} \end{matrix}$$

When observing the 2016 carbon concentrations above, one can note rather large fluctuations in annual carbon concentration for many states. These large fluctuations are due to another pitfall of Markov Chains and their application to the global carbon cycle. Since Markov Chain systems tend to approach equilibrium, they are more likely to change fastest at the beginning forecasting times as opposed to later times. Consequently, carbon concentrations likely become more and more accurate further into the future. However, eventually too much time will have

passed—and too many changes to the carbon cycle will have occurred—to consider carbon concentration forecasts to be accurate. Thus, the ideal forecasting time will be placed at 2066, 51 years into the future. Correspondingly, transition matrices will be raised to the  $k = 51$  power to determine 2066 carbon concentrations for NHI and HI systems, shown below.

$$\text{2066: } \begin{aligned} \text{NHI Carbon} &= T_1^{(51)} \mathbf{x} = \begin{pmatrix} 749.31 \\ 690.08 \\ 36801 \\ 74,999,985 \\ 43.40 \\ 1155.68 \\ 4155.40 \\ 1306.49 \\ 343.94 \end{pmatrix} & \text{HI Carbon} &= T_2^{(51)} \mathbf{x} = \begin{pmatrix} 1975.12 \\ 1336.56 \\ 40823.56 \\ 74,997,250 \\ 83.06 \\ 504.44 \\ 426.74 \\ 859.45 \\ 2300.50 \end{pmatrix} \end{aligned}$$

A	1975.12
SO	1336.56
DO	40823.56
S	74,997,250
R	83.06
MO	504.44
F	426.74
T	859.45
DE	2300.50

Though the above forecasts (and any between 30 and 70 years) are likely to be among the most accurate for reasons aforementioned, it is still important to notice the trends present in carbon cycle forecasts. By graphing yearly carbon concentration estimates from 2015 onward, one can accomplish exactly this, as illustrated by Figures 5, 6, and 7. By observing the graphical trends of the global carbon system under different parameters (NHI and HI)—as well as noting concentration differences 50 years into the future, one can contextualize the implications of human impact on the environment, as will be discussed in Section 3.4.

Figure 5.

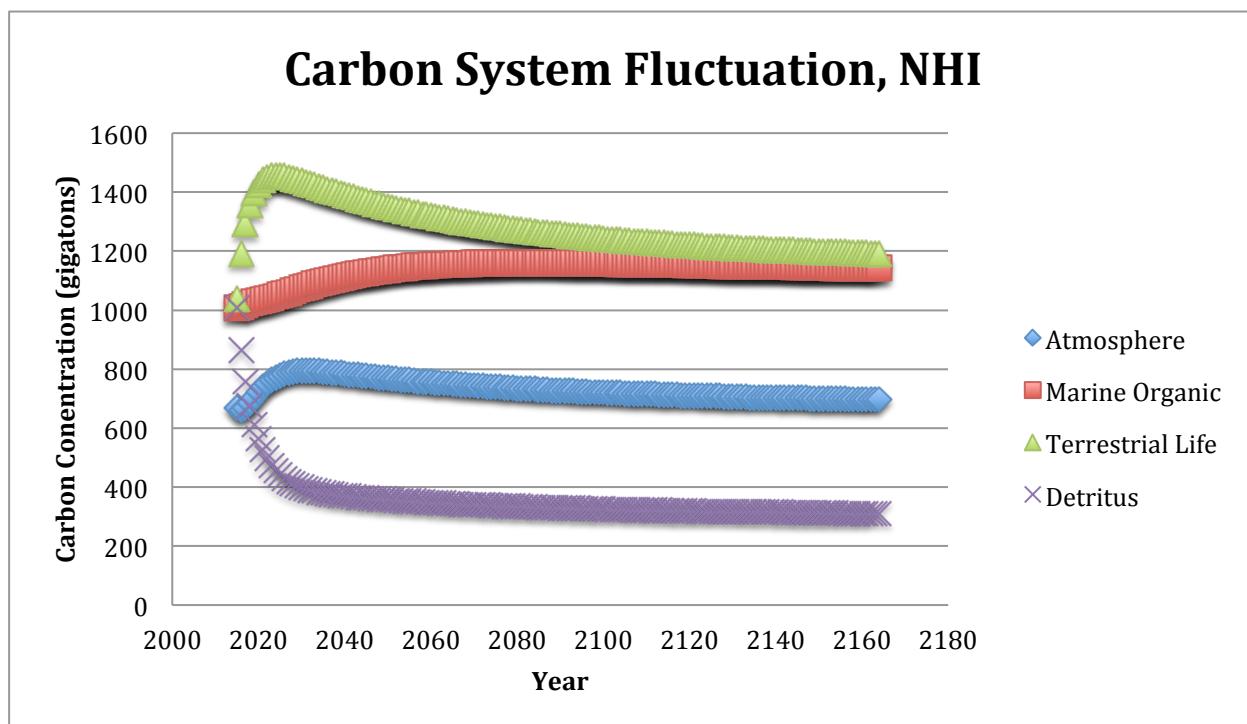


Figure 6.

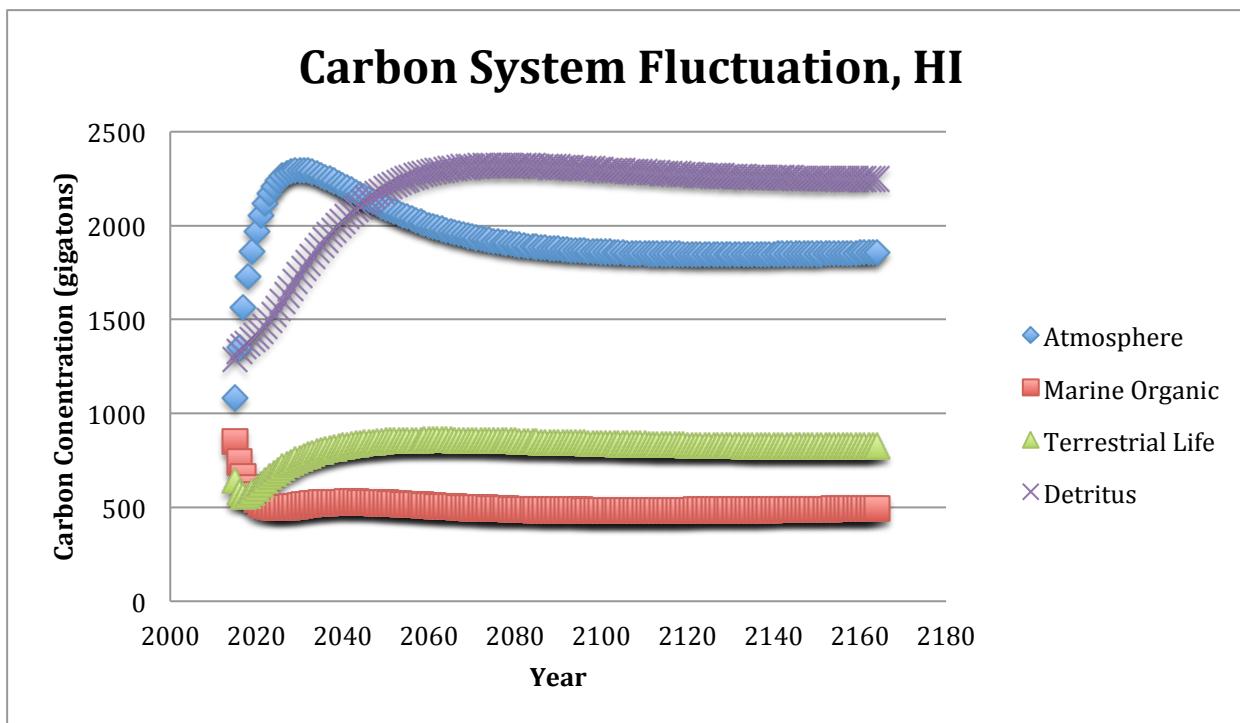
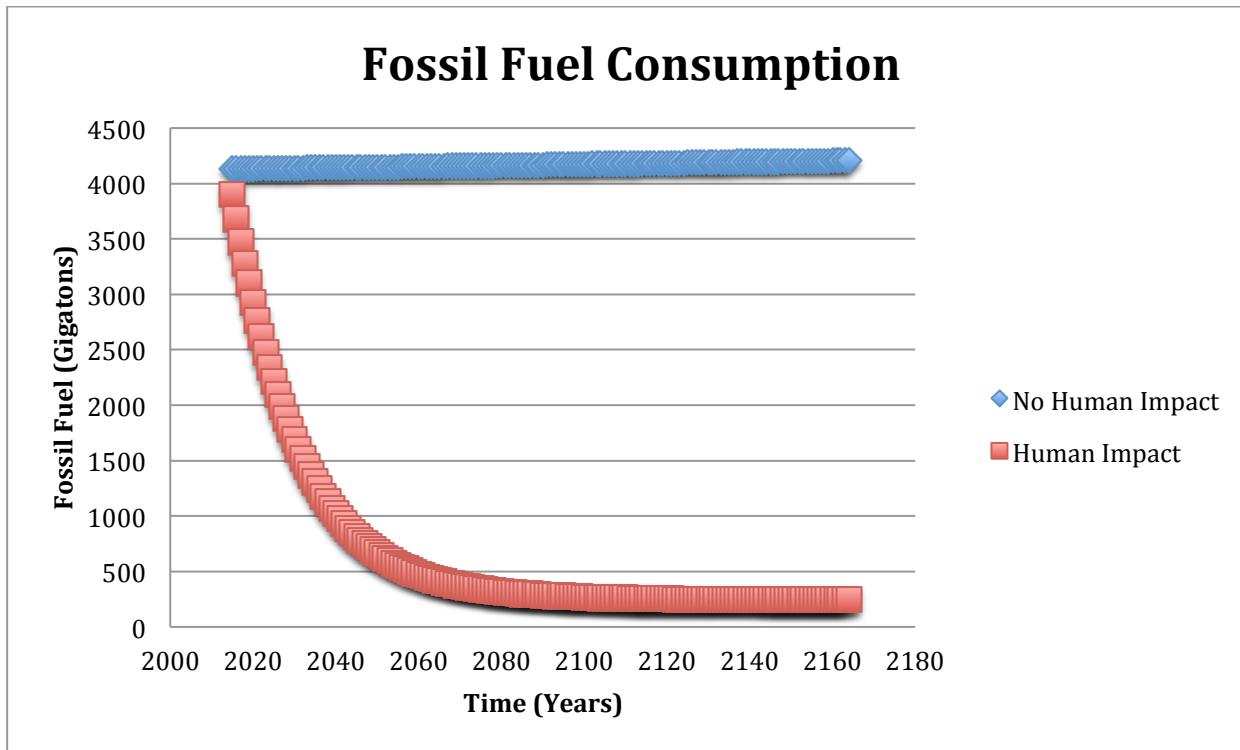


Figure 7.



Fossil fuels are emphasized (Figure 7) because of their strong connection to global warming.

Contextual discussion of fossil fuels and Figures 5-7 will occur Section 3.4 as well.

### **3.2 Refining Markov Chain Models: Stochastic Variation**

Of the assumptions that must be made of the NHI and HI Markov chain models (to account for the shortcomings of the models), one of particular annoyance is the constant transference property. That is, the percentage transfer between states is constant. However, simulation methods can be utilized to correct for this shortcoming in Markov Chain models. By adding a stochastic disturbance term to percentage transfer values, one can account for annual variation in carbon movement. Now, this stochastic variation is normally distributed, and does not account for definitive changes from original distribution parameters, but all the same is a notable improvement to fixed transfer percentage models. Carbon concentration forecasts can now be thought of probabilistic distributions instead of definitive values.

Mathematically, incorporating a stochastic disturbance term can be conducted in the following way. For non-pivot positions in transition matrices, the percentage transfer can be thought of as the transfer parameter ( $\mu$ ) with an error term ( $\varepsilon_{ni}$ ) attached to it. The distribution of error terms will be normal, with a mean of zero and a standard deviation of 10% of  $\mu$ . The reasoning behind the error term standard deviation assignment is the thought that, in general, larger percentage transfers will vary more extensively. Every carbon transfer will exhibit a different level of variation, so allowing variation to exist as a function of  $\mu$  is an excellent way to synthesize error distributions.

Next, in order to add an element of randomness to pivot positions (carbon retainment percentages), one can subtract the sum of all carbon transfer percentages for a carbon state (matrix column) from one. Such an action determines the value assigned to both the parameter transfer value and the variation exhibited by a given year ( $\mu + \varepsilon_\phi$ ) simultaneously. Although these two factors of pivot position transfer cannot be untangled, it is of little consequence; for any given state, transfer and retainment percentages will vary normally and sum to one. Thus, the following

transition matrix with the given properties encapsulates the nuances of the Monte Carlo data generation process (DGP) for NHI and HI carbon cycling models.

$$T_i = \begin{pmatrix} & & & \text{From:} & & & & & \text{To:} \\ \text{A} & \text{SO} & \text{DO} & \text{S} & \text{R} & \text{MO} & \text{F} & \text{T} & \text{DE} \\ & & & & & & & & \\ \mu + \varepsilon_\varphi & \mu + \varepsilon_{SOi} & \mu + \varepsilon_{DOI} & \mu + \varepsilon_{Si} & \mu + \varepsilon_{Ri} & \mu + \varepsilon_{MOi} & \mu + \varepsilon_{Fi} & \mu + \varepsilon_{Ti} & \mu + \varepsilon_{DEi} \\ \mu + \varepsilon_{Ai} & \mu + \varepsilon_\varphi & \mu + \varepsilon_{DOI} & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mu + \varepsilon_{Ai} & \mu + \varepsilon_{SOi} & \mu + \varepsilon_\varphi & \mu + \varepsilon_{Si} & & & & & \\ & & \mu + \varepsilon_{DOI} & \mu + \varepsilon_\varphi & \mu + \varepsilon_{Ri} & & & & \\ & & & \mu + \varepsilon_{Si} & \mu + \varepsilon_\varphi & \mu + \varepsilon_{MOi} & & & \\ & & & & \mu + \varepsilon_{Ri} & \mu + \varepsilon_\varphi & \mu + \varepsilon_{Fi} & & \\ & & & & & \mu + \varepsilon_{MOi} & \mu + \varepsilon_\varphi & \mu + \varepsilon_{Ti} & \\ \vdots & \\ \mu + \varepsilon_{Ai} & \mu + \varepsilon_{SOi} & \mu + \varepsilon_{DOI} & \mu + \varepsilon_{Si} & \mu + \varepsilon_{Ri} & \mu + \varepsilon_{MOi} & \mu + \varepsilon_{Fi} & \mu + \varepsilon_{Ti} & \mu + \varepsilon_\varphi \end{pmatrix} \begin{array}{c} \text{A} \\ \text{SO} \\ \text{DO} \\ \text{S} \\ \text{R} \\ \text{MO} \\ \text{F} \\ \text{T} \\ \text{DE} \end{array}$$

$\mu$  = Parameter for probabilistic state transfer. (Conditional on row/column identification)

$i = A, SO, \dots, DE$  (row reference for  $\varepsilon_{ni}$ )

$\varepsilon_{ni} : N(\mu, \sigma)$ ,  $\sigma = 0.10 \cdot \mu$

$\mu + \varepsilon_\varphi = 1 - \sum \mu_{ni} + \varepsilon_{ni}$   $n = A, SO, \dots, DE$  (columns)

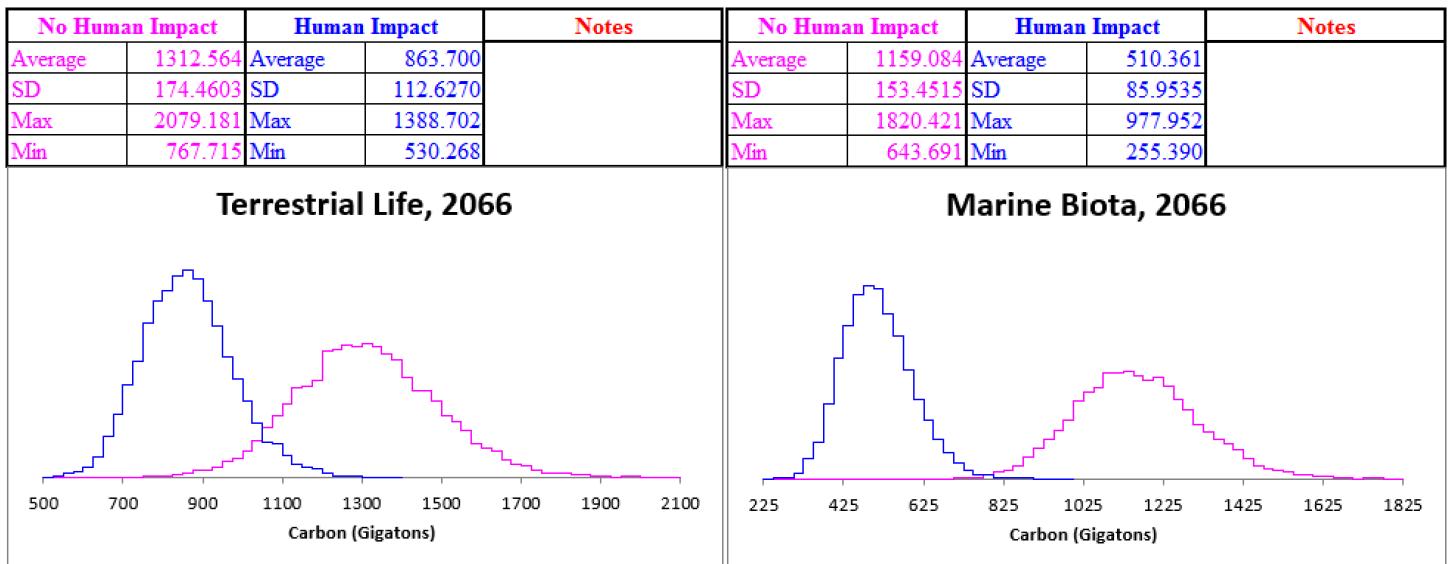
With this generalized stochastic matrix in mind, the NHI and HI transition matrices— $T_1$  and  $T_2$ —can now be converted to live data subject to randomness incorporated by the parameters of the data generation process aforementioned. To give an example, 18.26% of atmospheric (A) carbon is transferred to terrestrial life (T) via (primarily) photosynthesis. However, it is known intuitively that this percentage transfer will not be constant from year to year (though it is likely to remain similar from year to year). To mathematically depict this variation, percentage transfer for A to T in a given year is no longer known definitively as 18.26%, but as some value that exhibits a distribution of  $N(0.1826, \sigma)$ , where  $\sigma = 0.10 \cdot 0.1826$ . As will be explained at length shortly, NHI and HI Markov Chain benefit greatly from the inclusion of this variation aforementioned.

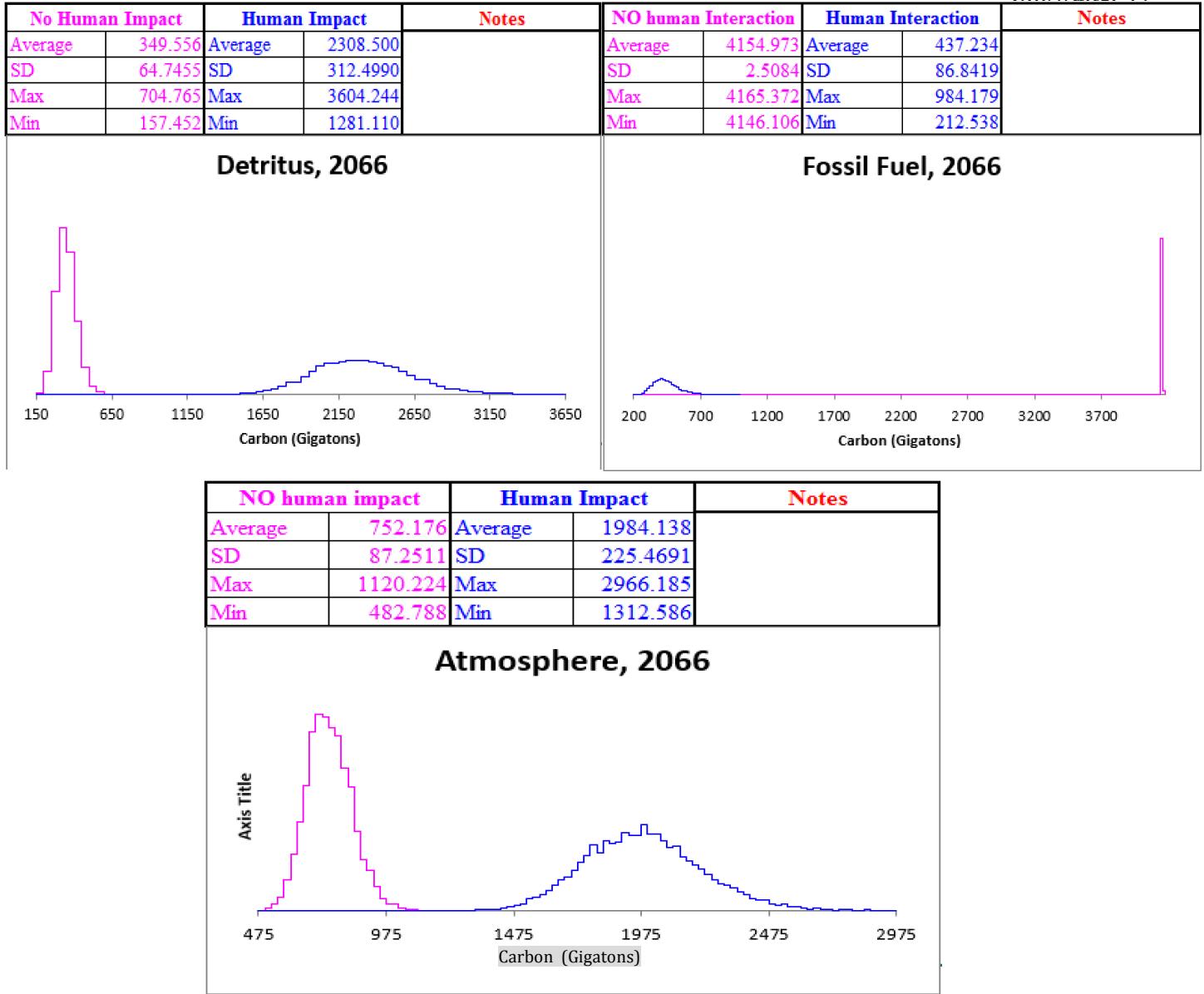
### 3.4 Monte Carlo Simulation

As explored in detail by [5], Markov Chains are particularly useful when analyzed in coordination with Monte Carlo simulations. A Monte Carlo (MC) simulation is a process (often a program) that pulls a sample from a population (given certain DGP parameters) and examines the center and spread of said sampling distribution. MC simulations are excellent tools to use when a phenomena exhibits variation at some level, as it does not extrapolate for definitive values, but forecasts the probability that some parameter (say, carbon concentration) will exist between a certain concentration range at some future time.

More directly, once stochastic disturbance is incorporated into NHI and HI carbon cycle models, future percentage transfers become random variables. By “refreshing” the data (re-sampling from the DGP), a different future is realized. Monte Carlo simulations become quite powerful here, as for each carbon transfer, percentages will vary every year as well as with every new sample taken. Investigating this system analytically is nigh impossible; it is for this reason precisely that MC simulations are so commonly used today. Repeatedly sampling a DGP-characteristic population and discerning results can construct reliable forecasts in lieu of analytical methods [5].

Consequently, Monte Carlo simulation of both the NHI and HI stochastic Markov Chains yield the following distributions for 2066 carbon concentration, given a certain carbon state. For more information on the DGP of these Monte Carlo distributions, please refer to Section 3.2.





When comparing the definitive 2066 carbon concentration values to the centers of the sampling distributions above, it becomes rather clear that definitive carbon concentration values are not unbiased estimators of carbon concentration. Indeed, some carbon concentration values (e.g. detritus = 1287.6 versus sampling a distribution center of detritus = 2308.5 petagrams). These results are not necessarily cause for concern; variation in these sampling distributions is quite large, and therefore the center of these sampling distributions is not as important as the confidence intervals that can be made with this data [5]. A confidence interval of a carbon state can be found to be  $\bar{X} \pm 2(s)$ , where  $\bar{X}$  is the distribution center and s is the standard deviation (95% confidence).

### ***3.4 Forecast Interpretation: Global Warming***

As a preface to the following contextual interpretation, it is crucial to keep in mind that although the global carbon cycle can be mapped with Markov Chains, and concentration range forecasts can be made, quantitative interpretation of global warming is impossible. Even if future carbon concentrations were known exactly, they would act only as a contributor to global warming (granted, the major contributor) [4]. Unfortunately, global warming is an even more complex phenomenon than what is included in this analysis of the carbon cycle. The nitrogen cycle—as well as changes in human actions (e.g. deforestation rates, environmental regulations)—is connected to the carbon cycle and global warming. Such a relationship between variables confounds data. Even so, a general, qualitative analysis of the above forecasts can be conducted.

To begin, the HI carbon cycle model depicts fossil fuel concentration to be diminishing rapidly, while the NHI model witnesses a small increase in fossil fuels over time. The Monte Carlo simulations above corroborate these general claims as well. In turn, as fossil fuel concentrations decrease, the price of oil will skyrocket with increased demand, international tensions will correspondingly increase, and carbon emissions will continue to enter the atmosphere at the alarming rate witnessed in the past decade [3]. The discovery of a sustainable alternative energy could solve all of the aforementioned issues, though no major innovations have occurred thus far.

Moreover, terrestrial life continues to increase over time in the HI model (with regard to carbon levels), but detritus increases accordingly. A build-up of detritus is often associated with inefficiency in an ecosystem; this build-up can often signify an environmental imbalance and the risk of species loss [2]. The same concept applies to marine biota, which actually decreases substantially in the HI carbon cycle model. One possibility for the sharp decline in marine biota carbon levels could be overexploitation of marine life via over fishing and the like [3].

Lastly, atmospheric carbon concentration increases in both the NHI and HI carbon cycle models, but in different amounts and likely for different reasons. If today's society could exist with a zero carbon footprint and maintain its production and innovative abilities, many species populations would grow substantially, especially as previously destroyed habitats re-grew. Thus, larger terrestrial and marine life populations will contribute more to atmospheric carbon via respiration. However, the NHI model assumes carbon will not leave the system, and in reality it is likely that even with more living carbon, atmospheric carbon levels would decrease rather than remain constant. All the same, the cause of the slight increase to atmospheric carbon in the NHI model is the increased terrestrial and marine life.

By contrast, the atmospheric carbon concentrations of the HI model depict an atmospheric concentration that more than doubles in fifty years time. Such an occurrence is highly correlated with a respective increase in the average temperature of the Earth. Though little information is included in this analysis of other contributors to global warming—and the Markov Chain models of the carbon cycle are far from perfect—the significance of the carbon cycle analysis is, in the author's opinion, compelling enough to implicate current human practices with the destruction of Earth's climate and environment. Actions must be taken to assuage the emission of carbon via artificial means and, in turn, avoid a global environmental crisis.

## 4.0 Conclusion

### *Summary: Carbon Cycle Analysis*

The Earth's carbon cycle and global warming is one of many complex probabilistic processes propitious for Markov Chain modeling. Markov Chains and their marriage to Monte Carlos simulations is an excellent example of an instance where analytical calculations are enriched by simulative inquiry. When witnessed in reality, carbon cycle transfer mechanisms are as immense as they are complex. Consumption, respiration, defecation, death, and reproduction are all ways a single organism can transfer carbon [4]. By taking a top-down approach to carbon cycle estimates, one eliminates the need for massive amounts of data and computational processing. Categorizing carbon into major states and conducting transfer estimates has yielded great improvements in modeling global carbon fluctuations [2]. By harnessing these estimates, and—in a similar manner to carbon state estimation processes—generalizing carbon movement as probabilistic transfers vis-à-vis Markov Chains, valuable forecasting can be conducted.

Moreover, by corroborating (or, in this case, refuting) forecast data with Monte Carlo simulations, carbon cycle research garners an element of dynamism. The complexity of global warming as a system cannot be understated; this research endeavor seeks only to infer basic qualitative assertions about global warming with its results, not to quantify the carbon cycle's effect on global warming as much as forecast the carbon cycle itself. With that said, human impact on the carbon cycle and global warming—if only from a quantitative standpoint—stands as a stark indicator of the detriment mankind has wrought on the Earth. By observing the unambiguous differences between the NHI and HI Markov Chain forecasts, it is clear that in order to preserve any level of climate, environment, and species stability, stringent regulations regarding pollution, deforestation, carbon emissions, and the like must be drafted and aggressively enforced.

The author would like to emphasize that the purpose of this carbon-forecasting endeavor is neither encapsulated by an appeal for policy change nor an environmental admonition. All the same, these considerations are not, and cannot, be removed from this research entirely. In any discipline, to ignore the social implications of one's work is to forsake those impacted by said implications. Today, many ignore such sentiments, and, in the case of global warming, are forsaking themselves as well as rest of the world to an uncertain future ecologically, economically, and socially.

### ***Further Research***

As hinted at in Forecast Interpretation, global warming is a notably complex system. In turn, perhaps the most readily available research extension would be to map the global nitrogen cycle and analyze how these systems interact with one another as well as examine their tandem impact on greenhouse gas contributions to the atmosphere. Second to carbon, the nitrogen cycle contributes most heavily to greenhouse gas emission into the atmosphere [3]. Along with this, Monte Carlo simulations would map the variation among the combination of the carbon and nitrogen cycles, adding perspective to cycle interactions and additions to the atmosphere.

When noting the global carbon cycle specifically, efforts could be made to determine ergodicity of transition matrices  $T_1$  and  $T_2$ , perhaps identifying any eigenvalues and eigenspaces in the process. Correspondingly, literature data for carbon cycle transfers on an annual basis (if available) could be analyzed to determine any trends in transition matrix changes. In turn, this variation in transition matrix values could be modeled more accurately by adding stochastic trends, rather than a random error term. In doing so, carbon cycle fluctuation forecasts would simultaneously become more accurate and precise, as would be witnessed through Monte Carlo simulation. On the other hand, however, mapping fluctuation patterns is likely to be complex analytically, unless annual carbon transfer trends depict predictable patterns. Regardless, a plethora of extensions exist off of the baseline carbon cycle forecasting made in this analysis.

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