

1.

a. $2^{(\lg(n) + \lg(n))} / n$

$$2^{(2 * \log_2(n))} / n$$

$$2^{(\log_2(n^2))} / n$$

$$2^{(\log_2(n^2))} / n$$

$$n^2 / n$$

n

b. Yes because $2^{\lg(n)} = 2^{\log_2(n)} = n$. so for $n = 0$ and $c_1 = 1$, $c_1 * 2^{\lg(n)} = 2^{\lg(n)}$ and for $c_2 = 2$, $2^{\lg(n)} < c_2 * 2^{\lg(n)}$

c. Yes because $2^{(2^{\lg \lg n})} = 2^{(2^{\log_2(\log_2(n))})} = 2^{(\log_2(n))} = n$. so just like 1b $n = 0$ and $c_1 = 1$, $c_1 * 2^{\lg(n)} = 2^{\lg(n)}$ and for $c_2 = 2$, $2^{\lg(n)} < c_2 * 2^{\lg(n)}$

2. Loop invariant = $\text{prod}(i) + x(i) * y(i) = a * b$. It holds for i because x and y both change but change in ways that makes them still equal to $a * b$. It will hold for $i + 1$ as well because x will be divided by 2 and y will be multiplied by 2 so it will always $x(i) * y(i)$. x will eventually become 0 though and the loop will always conclude.

3. $\lg(n) \ll \ln(n) \ll \sqrt{n} \ll n \ll n \log n \ll n^2 \ll 2^n \ll n^{(1/n)} \ll n^{(1+1/\lg(n))} \ll (\lg(n))^{1000} \ll 2^{(\sqrt{\lg(n)})} \ll (\sqrt{2})^{(\lg(n))} \ll (\lg(n))^{(\sqrt{2})} \ll n^{(\sqrt{2})} \ll (1 + 1/n)^n \ll n^{(1/1000)} \ll H(n) \ll H(\sqrt{n}) \ll 2^{(H(n))} \ll H(2n) \ll F(n) \ll F(n/2) \ll \lg(F(n)) \ll F(\lg(n))$