

CMSC 409:
Artificial Intelligence
<http://www.people.vcu.edu/~mmanic/>

Virginia Commonwealth University,
Fall 2023,
Dr. Milos Manic
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CMSC 409: Artificial Intelligence
Session # 26,27

Topics for today

- Announcements
- Previous session review
- Fuzzy Logic
 - *Introduction, membership degree*
 - *Some of the key researchers*
- Fuzzy controller
 - *Zadeh min-max controller*
 - *Fuzzification*
 - *Fuzzy inference engine (rule table)*
 - *Defuzzification*
- Zadeh fuzzy controller
 - *Design example, inputs, outputs (singletons)*

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CMSC 409: Artificial Intelligence Announcements Session # 26,27

- Canvas
 - Prev. session slides updated
- TAs
 - Victor Cobilean <cobileanv@vcu.edu>, Harindra Sandun Mavikumbure mavikumbureh@vcu.edu;
 - TA office hours: Thursdays, 3:30 - 4:30pm (Zoom)
- Final exam
 - Dec. 12 (take home, 48hr open book exam) – **earlier preference?**
 - Prep examples posted
- MHRG & VR Lab visit: Dec. 5 or 7 (last two sessions)
- Course evaluations ([coming up soon](#)) open now
- Paper (optional)
 - The 4th draft (final submission) due Nov. 28
 - In addition to previous draft, it should contain a technique (or selection thereof), you plan on using to solve the selected problem (check out the class paper instructions for the 4th draft)
- Interest in PhD or MSc program?
- Subject line and signature
 - Please use [CMSC 409] Last_Name Question

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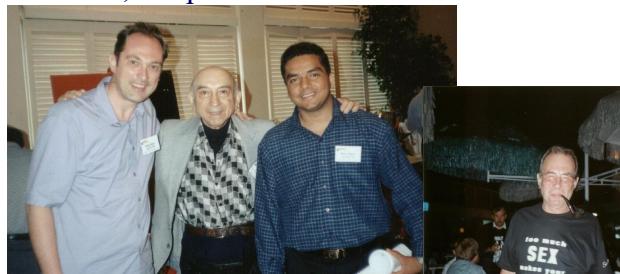
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Fuzzy systems

- Developed by Lofti Zadeh
- <http://www.cs.berkeley.edu/~zadeh/>
- Over 15 honorary doctorates
- His work was cited in over 30,000 publications



• Hans-Jurgen Zimmerman

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IJCNN 07

- James C. Bezdek, University of West Florida in Pensacola
 - Fuzzy c-means (FCM)
 - 2007 IEEE Frank Rosenblatt Award Recipient
 - http://www.ieee.org/portal/pages/about/awards/bios/2007_Bios/2007Rosenblatt-Bezdek.html

(Taken from IEEE Portal Pages)



“FCM is considered one of the most important discoveries in fuzzy pattern recognition and related areas and the clustering algorithm of choice for most practitioners in fuzzy exploratory data analysis. The original model has inspired many applications in related areas of pattern recognition and image processing.”



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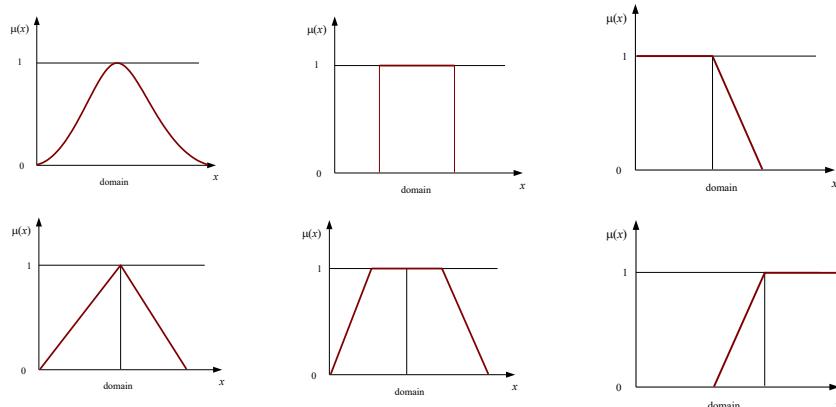
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Briefly on Fuzzy Logic (FL)

Data representation using fuzzy sets



‘Bell’ shaped, triangular, trapezoidal, shouldered fuzzy sets

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Fuzzy systems

- Inputs can be any value from 0 to 1.
- The basic fuzzy principle is similar to Boolean logic.
- Min and max operators are used instead of AND and OR, respectively.
- The NOT operator also becomes $1 - \#$.

$A \cap B \cap C \Rightarrow \min\{A, B, C\}$ – smallest value of A, B or C

$A \cup B \cup C \Rightarrow \max\{A, B, C\}$ – largest value of A, B or C

$$\overline{A} \Rightarrow 1 - A \quad \text{– one minus } A \quad \begin{matrix} \text{complement} \\ \text{Boolean} \end{matrix}$$

Boolean

Fuzzy

A \cap B	
0	0
0	1
1	0
1	1

A \cup B	
0	0
0	1
1	0
1	1

A \cap B	
0.2	0.3
0.2	0.8
0.7	0.3
0.7	0.8

conjunction

A \cup B	
0.2	0.3
0.2	0.8
0.7	0.3
0.7	0.8

disjunction

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■ Fuzzy controller

□ Zadeh min-max controller

■ Fuzzification

■ Fuzzy inference engine

■ Defuzzification

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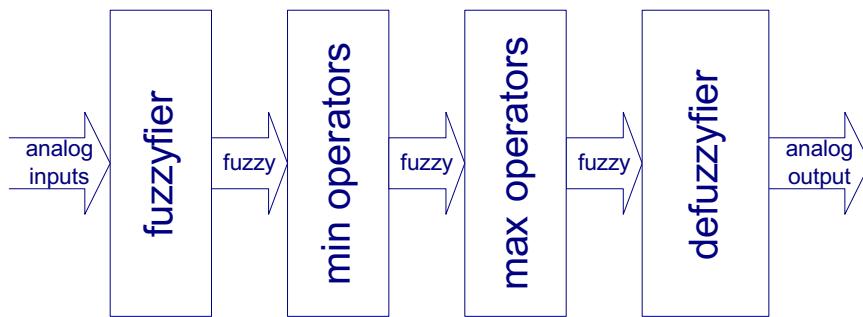
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Fuzzy Systems

Block diagram of Zadeh fuzzy controller



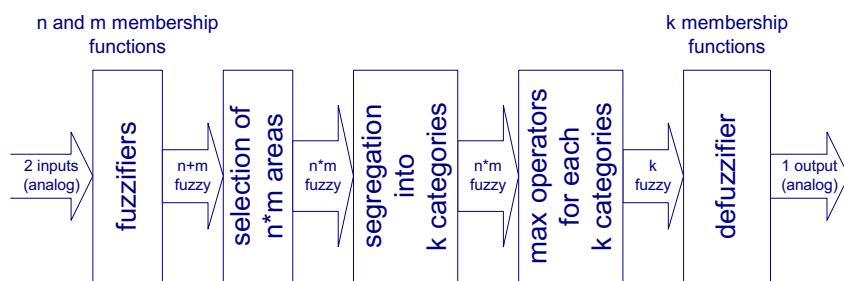
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Fuzzy Systems



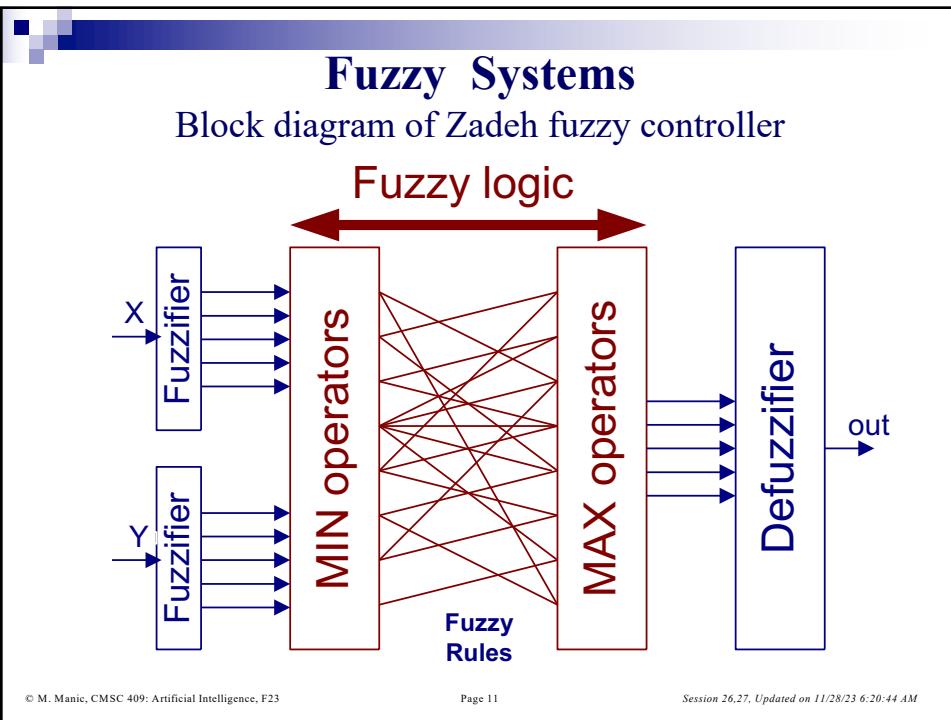
Block diagram for Zadeh fuzzy controller

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- *Fuzzy controller*
 - *Zadeh min-max controller*
- *Fuzzification*
- *Fuzzy inference engine*
- *Defuzzification*

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Fuzzy Systems

fuzzification

- There are three major types of membership functions
 - Gaussian, Triangular and Trapezoidal
- Some guidance on membership functions
 1. Each point of an input should belong to at least one membership function (preferable 2 or more)
 2. The sum of two overlapping functions should not be greater than 1.
 3. For higher accuracy, more membership functions can be used, but this can lead to system instability and will require a larger fuzzy table.

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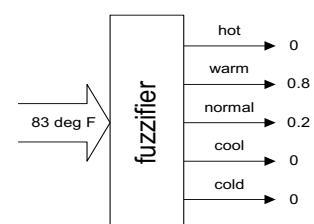
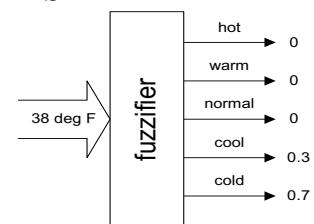
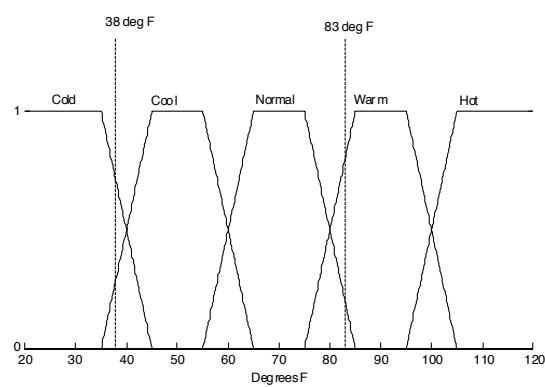
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Fuzzy Systems

fuzzification



fuzzification results

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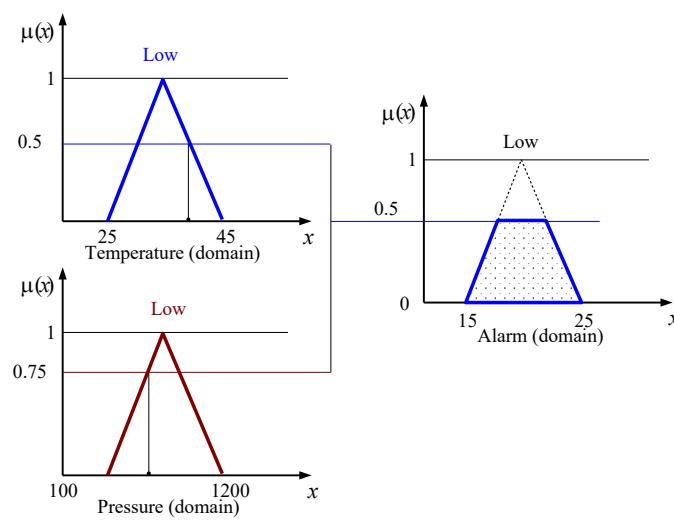
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- *Fuzzy controller*
 - Zadeh min-max controller
- *Fuzzification*
- *Fuzzy inference engine*
- *Defuzzification*

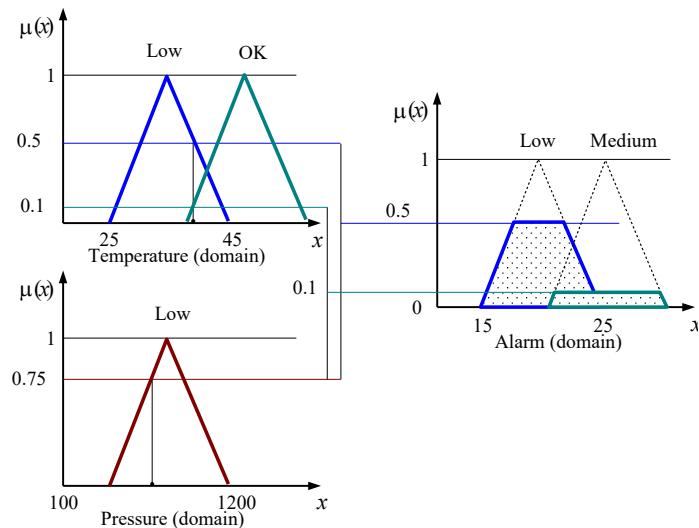
Briefly on Fuzzy Logic (FL)

Fuzzy sets connected by a Zadeh AND operator



Briefly on Fuzzy Logic (FL)

Zadeh min-max rule



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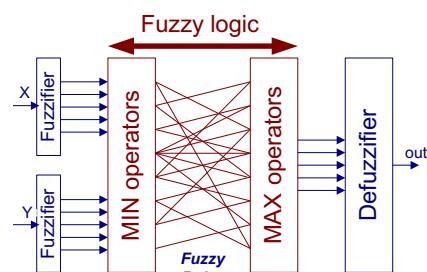
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Fuzzy Systems - Rule Evaluation - Zadeh fuzzy tables

Controller takes temperature reading from two inputs



	Input 2				
Input 1	→				
↓	cold	cool	normal	warm	hot
cold	A	A	A	B	A
cool	A	A	B	C	B
normal	A	B	C	C	C
warm	A	B	C	D	D
hot	B	C	D	E	E

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Fuzzy Systems - Rule Evaluation - Zadeh fuzzy tables

	Input 2				
Input 1	\rightarrow				
\downarrow	0 cold	0 cool	0.2 normal	0.8 warm	0 hot
0.7 cold	A	A	A	B	A
0.3 cool	A	A	B	C	B
0 normal	A	B	C	C	C
0 warm	A	B	C	D	D
0 hot	B	C	D	E	E
	Input 2				
Input 1	\rightarrow				
\downarrow	0 cold	0 cool	0.2 normal	0.8 warm	0 hot
0.7 cold	0	0	0.2 A	0.7 B	0
0.3 cool	0	0	0.2 B	0.3 C	0
0 normal	0	0	0	0	0
0 warm	0	0	0	0	0
0 hot	0	0	0	0	0

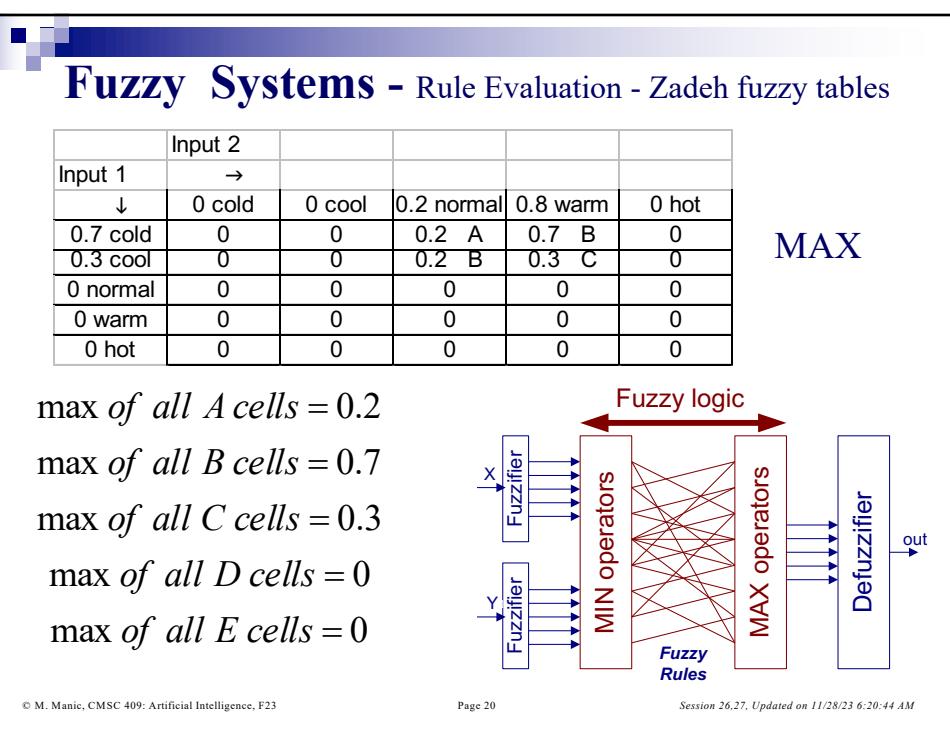
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- *Fuzzy controller*
 - *Zadeh min-max controller*
- *Fuzzification*
- *Fuzzy inference engine*
- *Defuzzification*

Fuzzy Systems - Defuzzification

- The equation to describe the defuzzification process.

- n – Number of membership functions
- z_k – Fuzzy output variables
- c_k – analog values from table

- Outputs:

- Zadeh

$$\text{Output} = \frac{\sum_{k=1}^n z_k c_k}{\sum_{k=1}^n z_k}$$

$$\text{Output} = \frac{0.2 * A + 0.7 * B + 0.3 * C}{0.2 + 0.7 + 0.3}$$

- Takagi-Sugeno

$$\text{Output} = \frac{0.2 * O3 + 0.7 * O4 + 0.2 * O8 + 0.3 * O9}{0.2 + 0.7 + 0.3 + 0.2}$$

Fuzzy Systems - Defuzzification

- The equation to describe the defuzzification process.
 - n – Number of membership functions
 - z_k – Fuzzy output variables
 - c_k – analog values from table

$$\text{Output} = \frac{\sum_{k=1}^n z_k c_k}{\sum_{k=1}^n z_k}$$

$$\text{Output} = \frac{0.2 * A + 0.7 * B + 0.3 * C}{0.2 + 0.7 + 0.3}$$

$$\text{Output} = \frac{0.2 * (-12) + 0.7 * (-3) + 0.3 * (8)}{0.2 + 0.7 + 0.3} = \frac{-2.1}{1.2} = 1.75$$

Output data for *singleton* encoding

A	-12
B	-3
C	8
D	20
E	35

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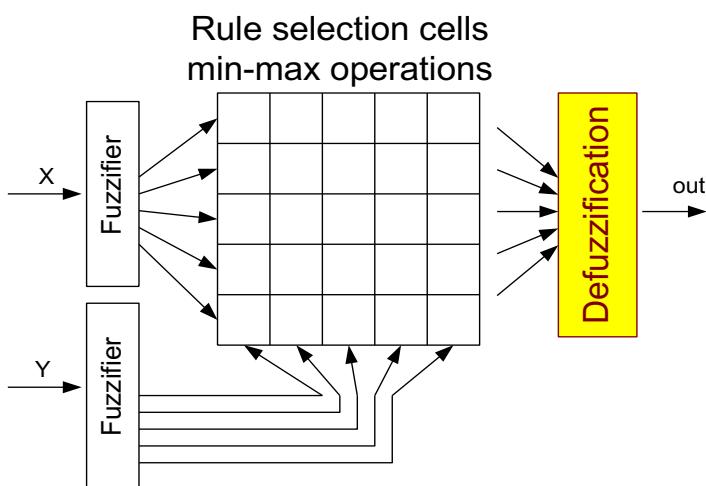
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Fuzzy Systems

Block diagram of Zadeh fuzzy controller

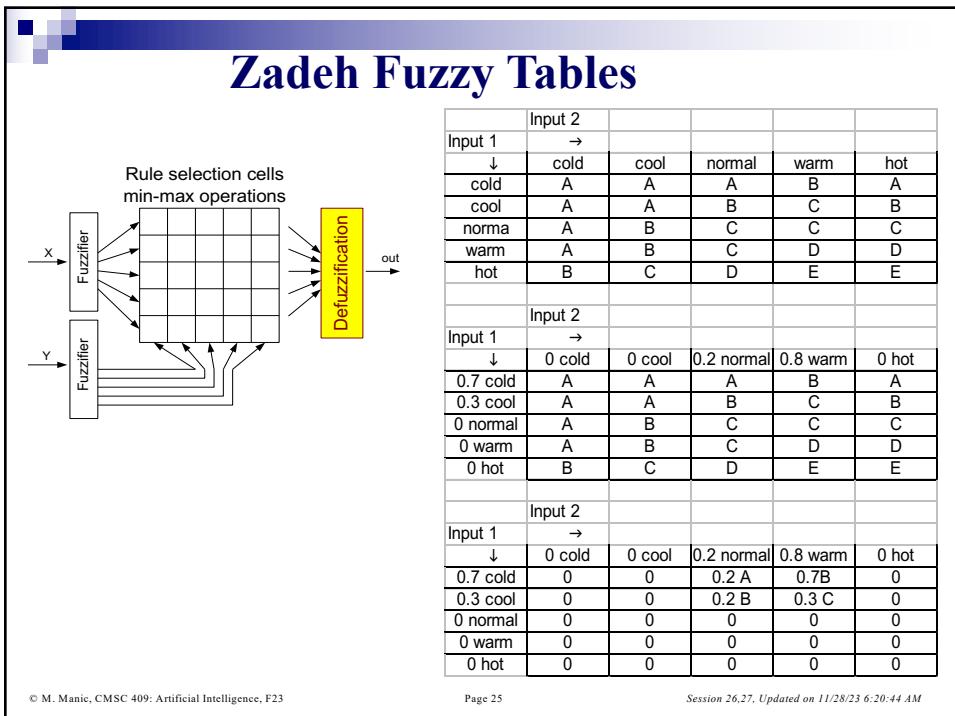


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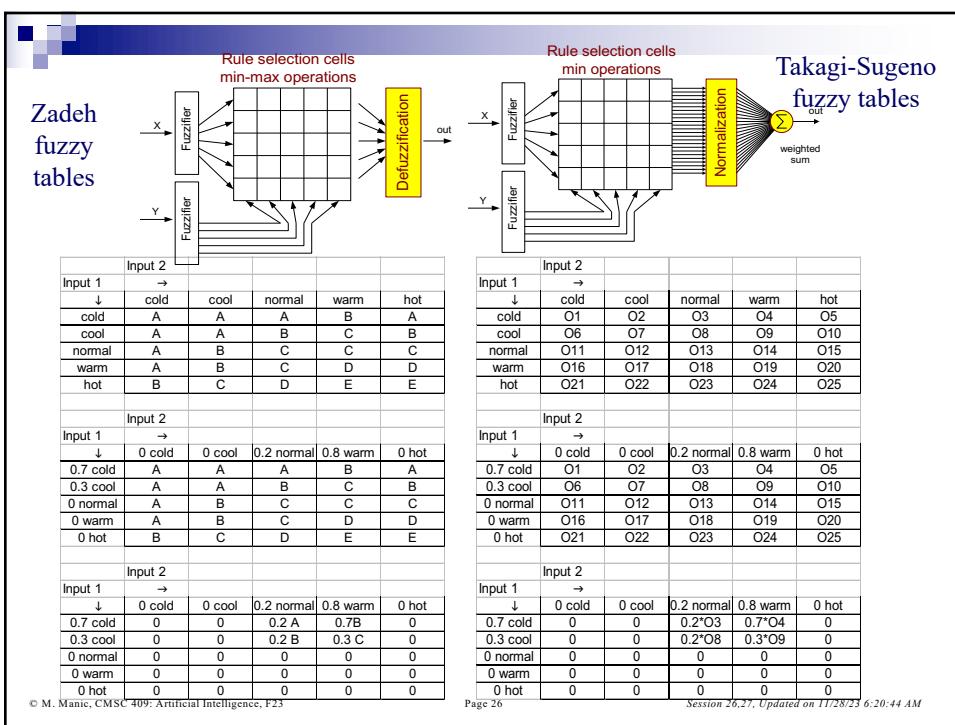


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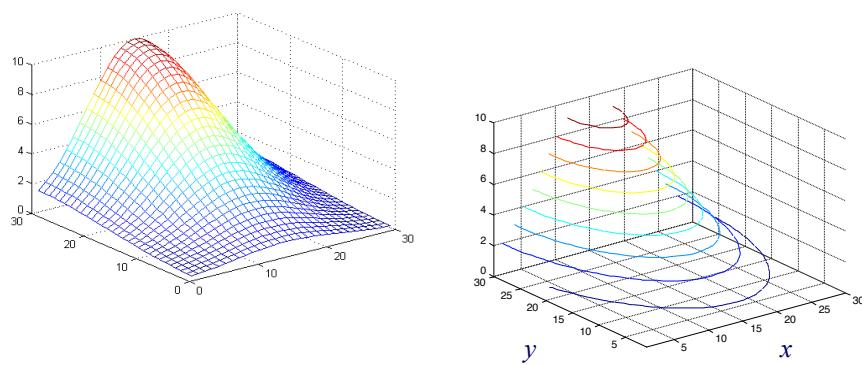
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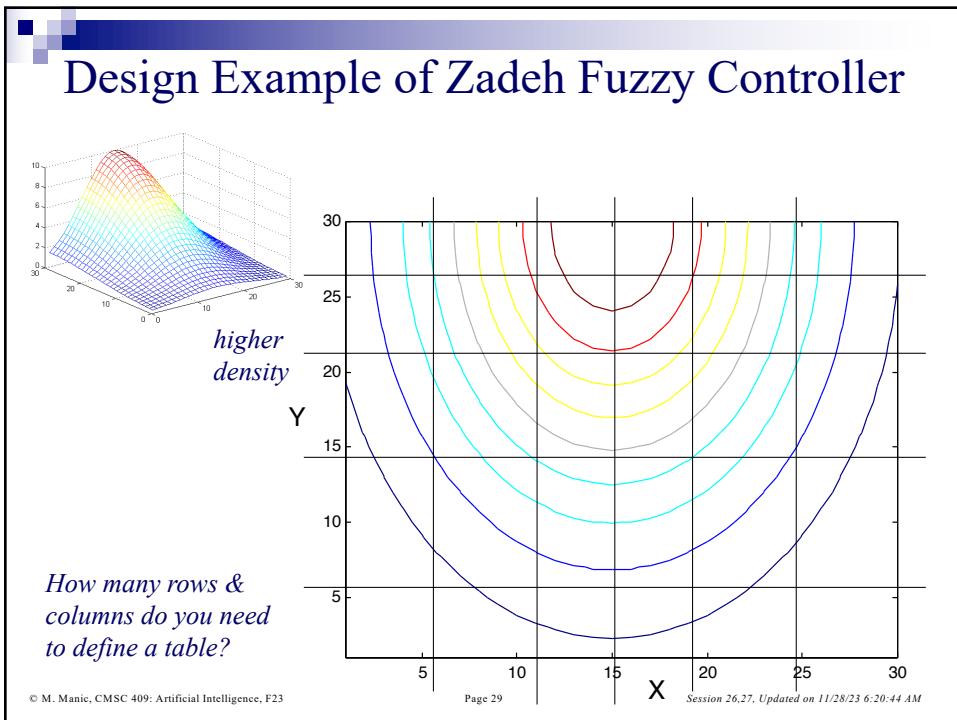
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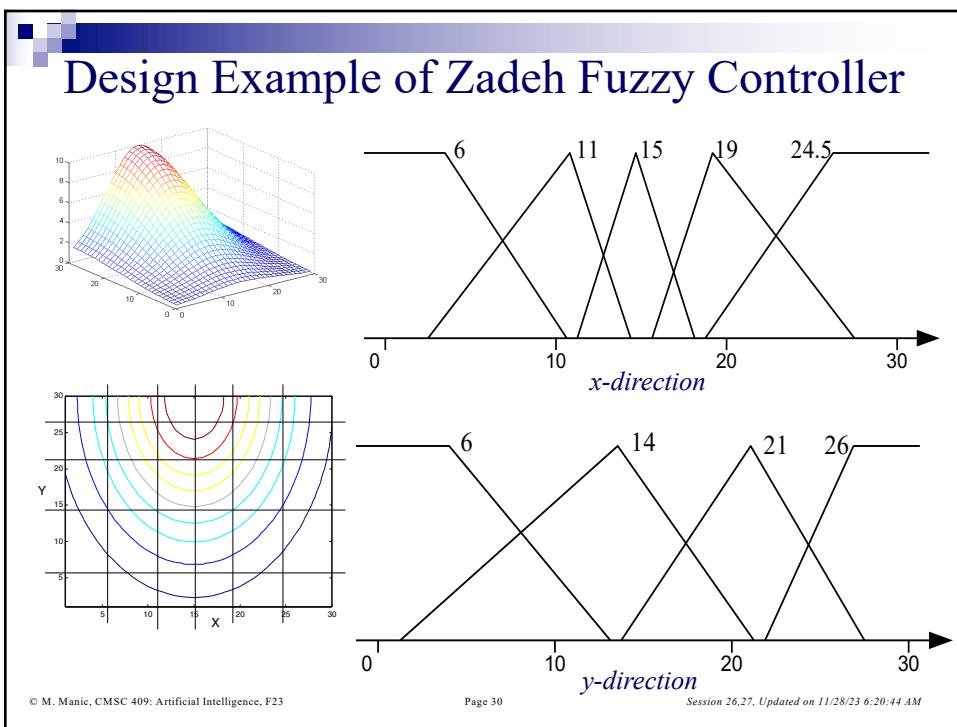
- *Zadeh fuzzy controller*
 - *design example*
 - *inputs, outputs (singletons)*
- *Fuzzy systems, various implementations*

Design Example of Zadeh Fuzzy Controller

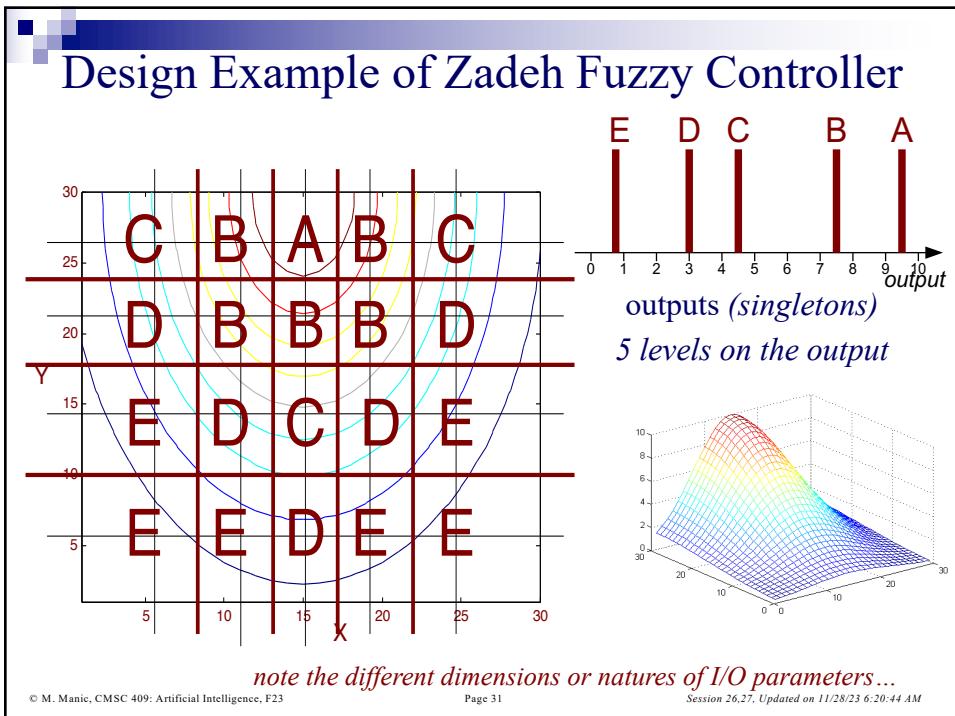




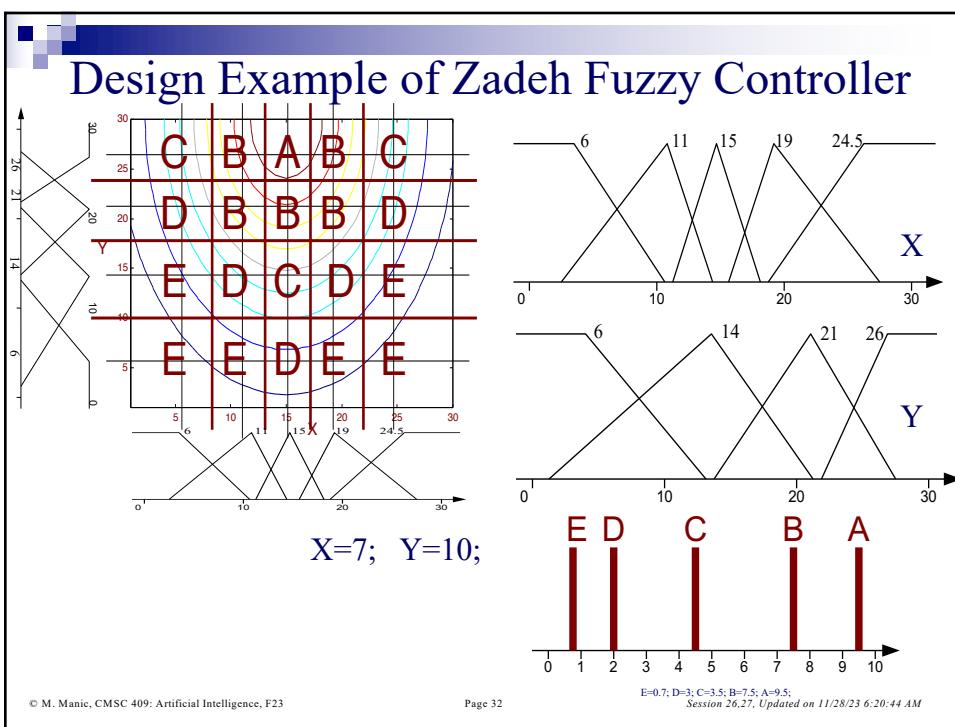
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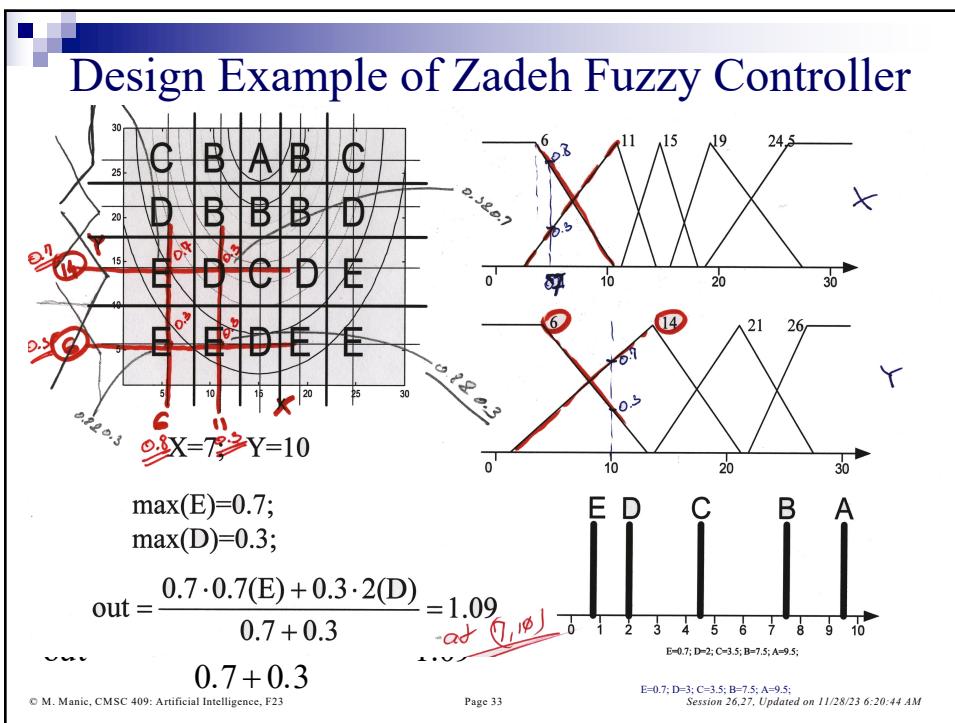
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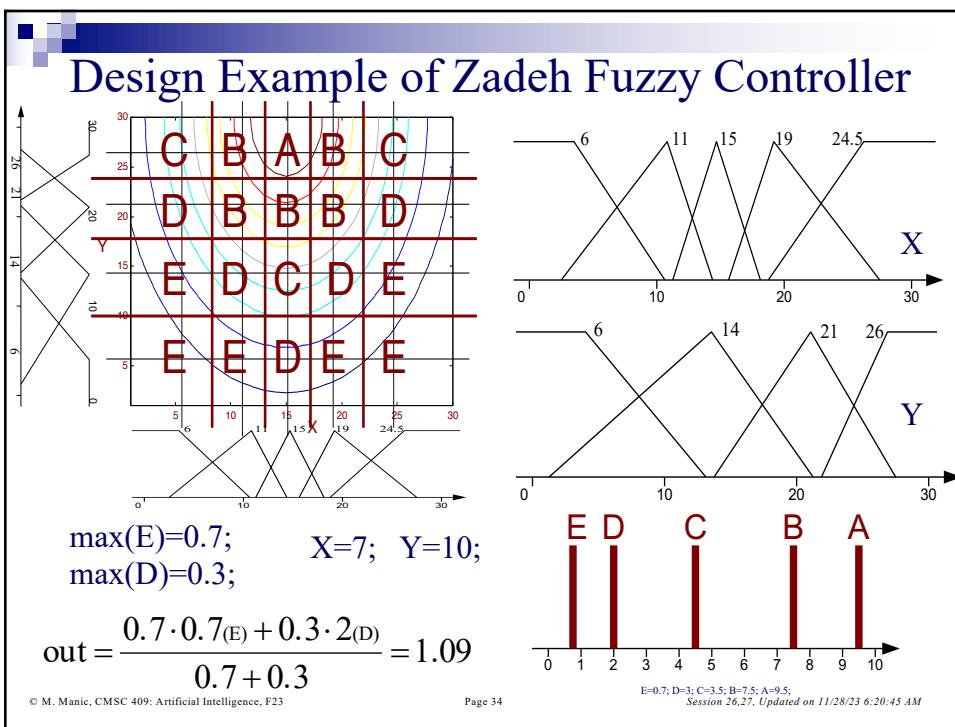
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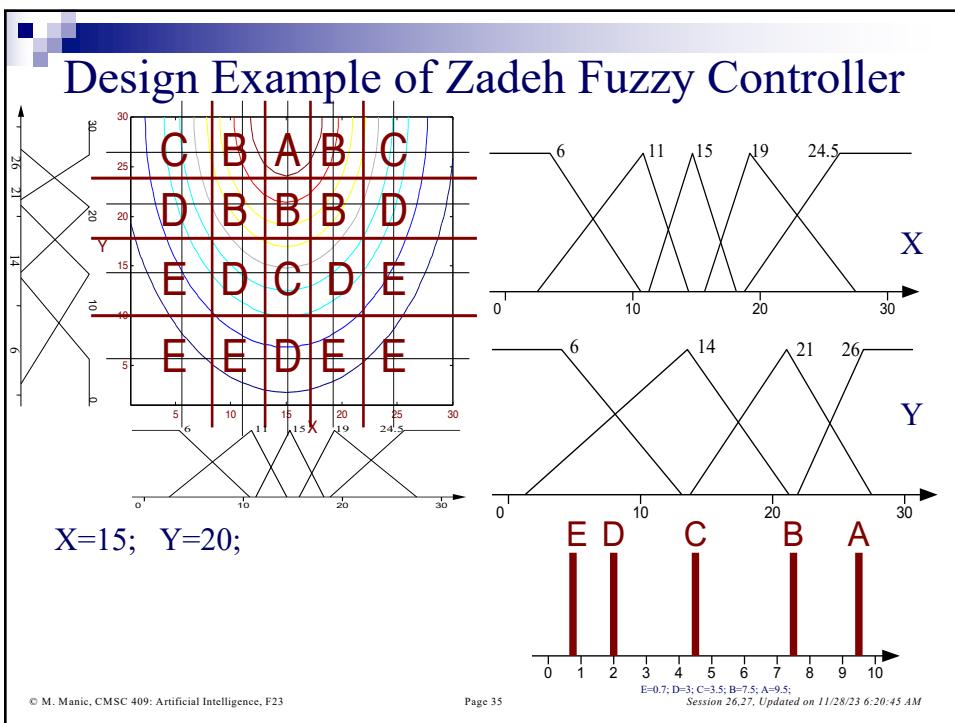
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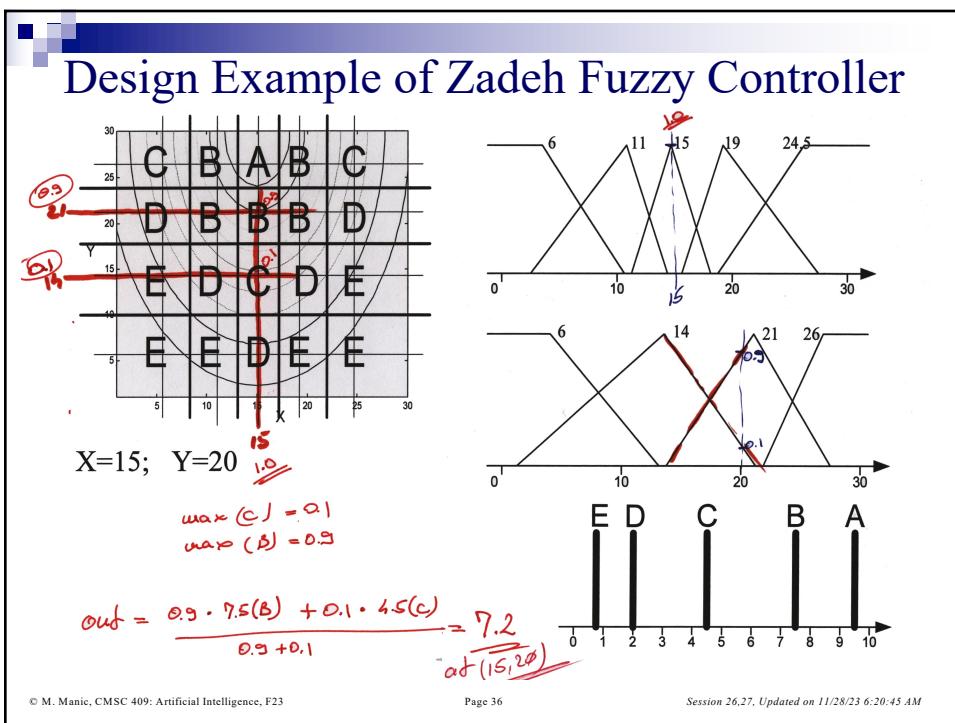
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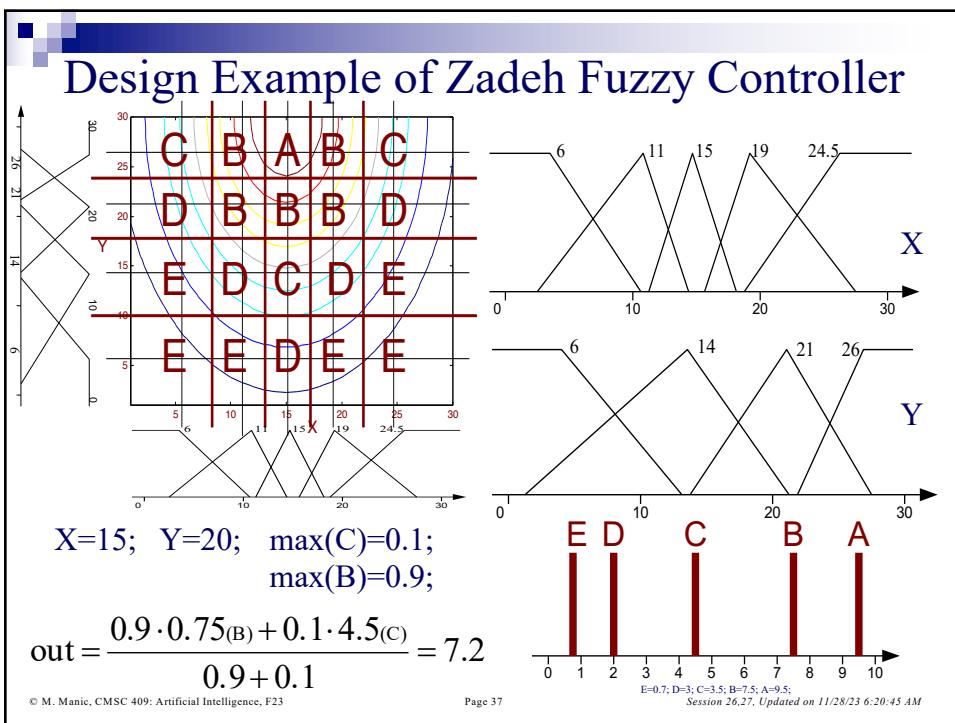
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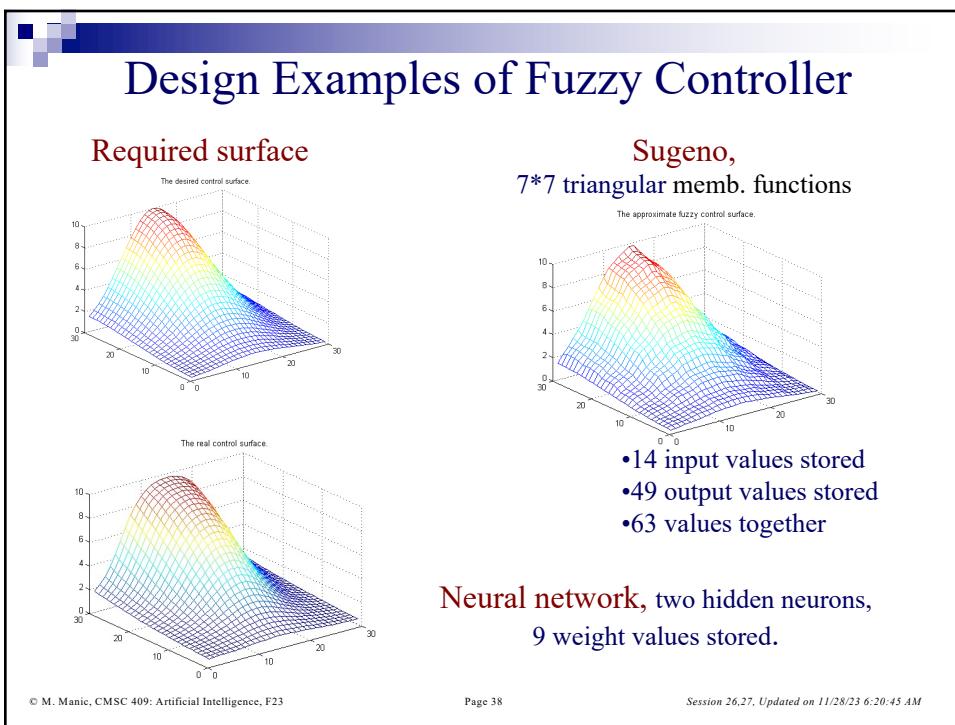
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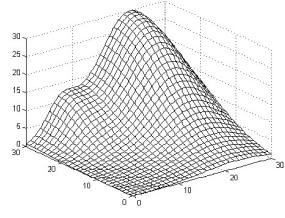


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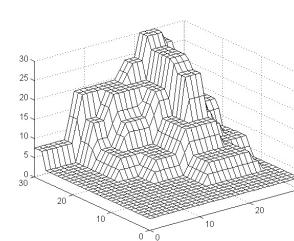


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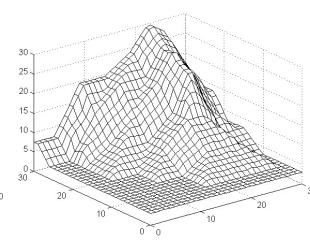
Various Implementations of Fuzzy Systems



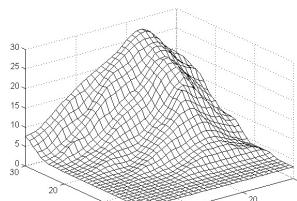
Required
control surface



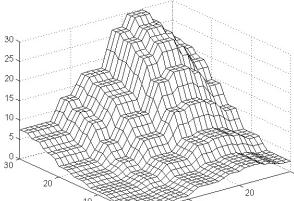
Zadeh,
trapezoidal memb. functions



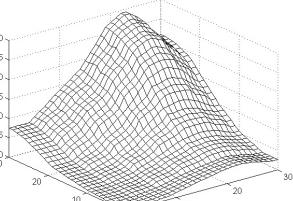
Zadeh,
triangular memb. functions



Zadeh,
Gaussian memb. functions



Takagi-Sugeno,
trapezoidal memb. functions



Takagi-Sugeno,
triangular memb. functions

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