For all homework in this class, you can use any written source, but you **must cite all your sources**. You can also talk with other students in this class, but you must cite them (list names for each problem you discussed with other students). Finally **all solutions must be written in your own words**.

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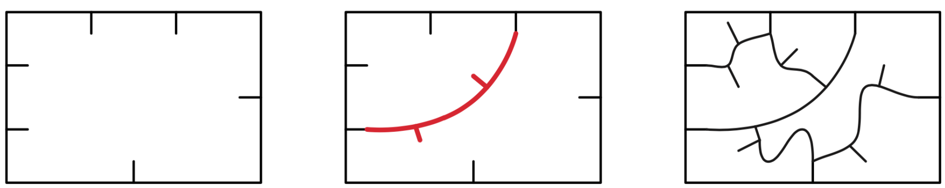
1. Herr Professor Doktor Georg von den Dschungel has a 23-node binary tree, in which every node is labeled with a unique letter of the German alphabet, which is just like the English alphabet with four extra letters: Ä, Ö, Ü, and ß. (Don't confuse these with A, O, U, and B!) Preorder and postorder traversals of the tree visit the nodes in the following order:

   - Preorder: **B K Ü E H L Z I Ö R C ß T S O A Ä D F M N U G**
   - Postorder: **H I Ö Z R L E C Ü S O T A ß K D M U G N F Ä B**

   (a) List the nodes in George's tree in the order visited by an inorder traversal.

   (b) Draw George's tree.

2. The Terminal Game is a two-person game played with pen and paper. The game begins by drawing a rectangle with n "terminals" protruding into the rectangles, for some positive integer n, as shown in the figure below. On a player's turn, she selects two terminals, draws a simple curve from one to the other without crossing any other curve (or itself), and finally draws a new terminal on each side of the curve. A player loses if it is her turn and no moves are possible, that is, if no two terminals may be connected without crossing at least one other curve.



The initial setup. The first turn. No more moves.

   Analyze this game, answering the following questions (and any more that you find the answers to): When is it better to play first, and when is it better to play second? Is there always a winning strategy? What is the fewest moves in which you can defeat your opponent? Prove your answers are correct.

3. (a) Recall that the ***Fibonacci numbers F_n*** are defined by the recurrence $F_n = F_{n-1} + F_{n-2}$ with base cases $F_0 = 0$ and $F_1 = 1$. The first several Fibonacci numbers are shown below:

F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}
0	1	1	2	3	5	8	13	21	34	55

   The Fibonacci sequence can be extended backward to negative indices by rearranging the defining recurrence: $F_n = F_{n+2} - F_{n+1}$. Here are the first several negative-index Fibonacci numbers:

F_{-10}	F_{-9}	F_{-8}	F_{-7}	F_{-6}	F_{-5}	F_{-4}	F_{-3}	F_{-2}	F_{-1}
-55	34	-21	13	-8	5	-3	2	-1	1

   Prove that $F_{-n} = (-1)^{n+1} F_n$ for each positive integer n.

   (b) Prove that every nonzero integer can be written in the form $\sum_i \pm 3^i$ where the exponents i are distinct non-negative integers. (For example: $42 = 3^4 - 3^3 - 3^2 - 3^1$, $25 = 3^3 - 3^1 + 3^0$, $17 = 3^3 - 3^2 - 3^0$.) Hint: When writing n, should 3^0 or -3^0 (or neither) be in the sum? Don't even *think* about using weak induction.