Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - **Representation: unsigned and signed**
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings
- Summary

Encoding Integers

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

```
short int x =  15213;
short int y = -15213;
```

- C `short` 2 bytes long

	Decimal	Hex	Binary
x	15213	**3B 6D**	**00111011 01101101**
y	-15213	**C4 93**	**11000100 10010011**

- **Sign Bit**
  - For 2's complement, most significant bit indicates sign
    - 0 for nonnegative
    - 1 for negative

$-1*2^{15}$

Calculating Two's complement

- Don't confuse two's complement of a number with Two's complement representation of a number.
- For positive numbers Two's complement representation of a number is itself in binary representation (if there is no overflow)
- For negative numbers to find the Two's complement representation
  1. Find the binary representation of absolute value of number
  2. Flip bits
  3. Add one

Binary representation of 5 is: 0 1 0 1

1's Complement of 5 is: 1 0 1 0

2's Complement of 5 is: (1's Complement + 1) i.e.

1 0 1 0 (1's Compliment)

\+ 1

1 0 1 1 (2's Complement i.e. -5)

Practice: Find decimal value in Unsigned and Two's complement representions

$\vec{x}$			
Hexadecimal	Binary	$B2U_4(\vec{x})$	$B2T_4(\vec{x})$
0xE	[1110]		
0x0	[0000]		
0x5	[0101]		
0x8	[1000]		
0xD	[1101]		
0xF	[1111]		

Practice: Find decimal value in Unsigned and Two's complement representions

$\vec{x}$			
Hexadecimal	Binary	$B2U_4(\vec{x})$	$B2T_4(\vec{x})$
0xE	[1110]	$2^3 + 2^2 + 2^1 = 14$	$-2^3 + 2^2 + 2^1 = -2$
0x0	[0000]	0	0
0x5	[0101]	$2^2 + 2^0 = 5$	$2^2 + 2^0 = 5$
0x8	[1000]	$2^3 = 8$	$-2^3 = -8$
0xD	[1101]	$2^3 + 2^2 + 2^0 = 13$	$-2^3 + 2^2 + 2^0 = -3$
0xF	[1111]	$2^3 + 2^2 + 2^1 + 2^0 = 15$	$-2^3 + 2^2 + 2^1 + 2^0 = -1$

1110
0001
+ 1
0010
$\to 2^1 = 2$
$\boxed{-2}$

1110
$0 \times 2^0 \to 0$
$1 \times 2^1 \to 2$
$1 \times 2^2 \to 4$
$1 \times 2^3 \to 8$
+
14

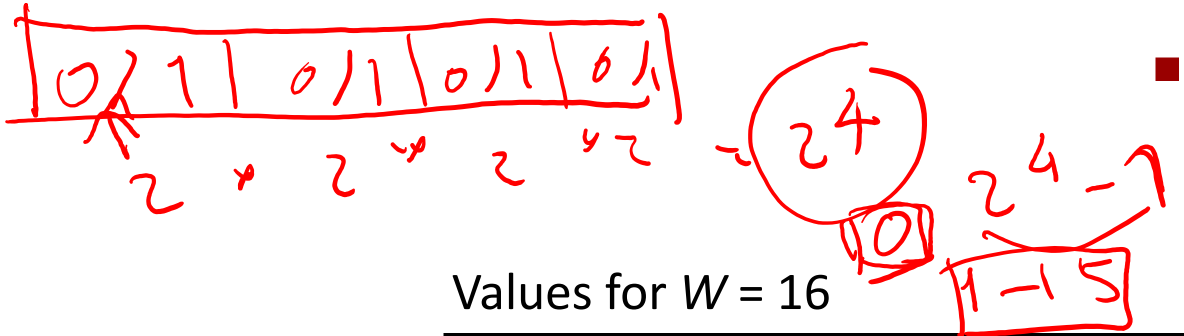
1110
$0 \times 2^0 \to 0$
$1 \times 2^1 \to 2$
$1 \times 2^2 \to 4$ } 6
$-1 \times 2^3 \to -8$
+
-2

Numeric Ranges

- **Unsigned Values**
  - *UMin* = 0

    000…0
  - *UMax* = $2^w - 1$

    111…1

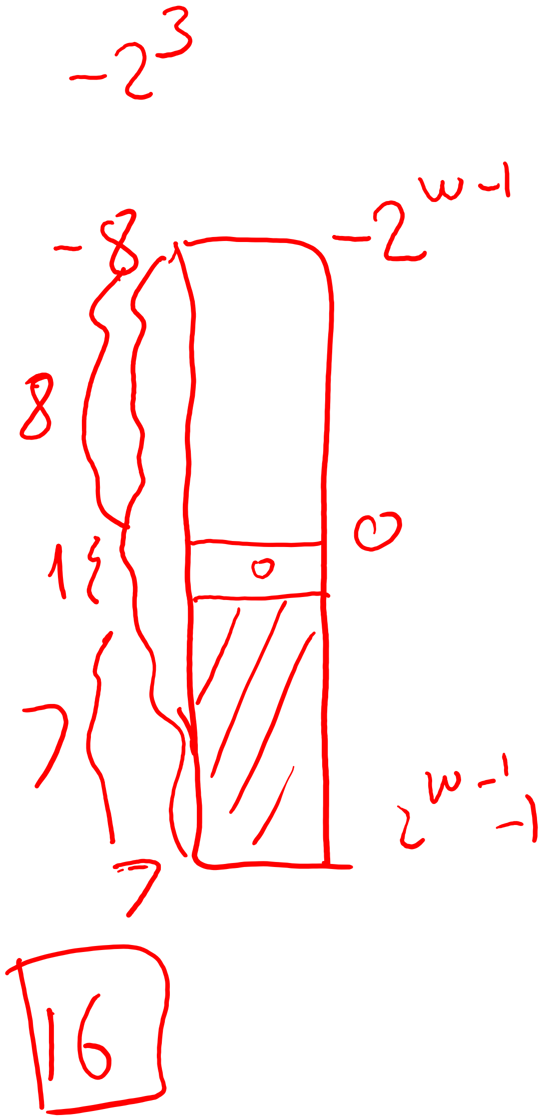


- **Two's Complement Values**
  - *TMin* = -2^{w-1}

    100…0
  - *TMax* = $2^{w-1} - 1$

    011…1
- **Other Values**
  - Minus 1

    111…1



Values for W = 16

	Decimal	Hex	Binary
`UMax`	65535	`FF FF`	`11111111 11111111`
`TMax`	32767	`7F FF`	`01111111 11111111`
`TMin`	-32768	`80 00`	`10000000 00000000`
`T-1`	-1	`FF FF`	`11111111 11111111`
`T0, U0`	0	`00 00`	`00000000 00000000`

Numeric Ranges

- **Unsigned Values**
  - *UMin* = 0

    000…0
  - *UMax* = $2^w - 1$

    111…1

- **Two's Complement Values**
  - *TMin* = -2^{w-1}

    100…0
  - *TMax* = $2^{w-1} - 1$

    011…1

- **Other Values**
  - Minus 1

    111…1

2^{16}

65536

BRU

0

65535

Values for *W* = 16

	Decimal	Hex	Binary
`UMax`	65535	`FF FF`	`11111111 11111111`
`TMax`	32767	`7F FF`	`01111111 11111111`
`TMin`	-32768	`80 00`	`10000000 00000000`
`T-1`	-1	`FF FF`	`11111111 11111111`
`T0, U0`	0	`00 00`	`00000000 00000000`

$1\times2^{14} + 1\times2^{13} + 1\times2^{12} \ldots$

$-1\times2^{15} + 0 + 0 + 0 \ldots$

Values for Different Word Sizes

	W			
	8	16	32	64
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

$2^{64}-1$

- **Observations**
  - $|TMin| \quad = \quad TMax + 1$
  - $UMax \quad = \quad 2 * TMax + 1$
- **C Programming**
  - #include <limits.h>
  - Declares constants, e.g.,
    - ULONG_MAX
    - LONG_MAX
    - LONG_MIN
  - Values platform specific

Unsigned & Signed Numeric Values

X	B2U(*X*)	B2T(*X*)
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	–8
1001	9	–7
1010	10	–6
1011	11	–5
1100	12	–4
1101	13	–3
1110	14	–2
1111	15	–1

- **Equivalence**
  - Same encodings for nonnegative values
- **Uniqueness**
  - Every bit pattern represents unique integer value (That would not be true if we were using 1’s complement, or sign and magnitude representation)
  - Each representable integer has unique bit encoding

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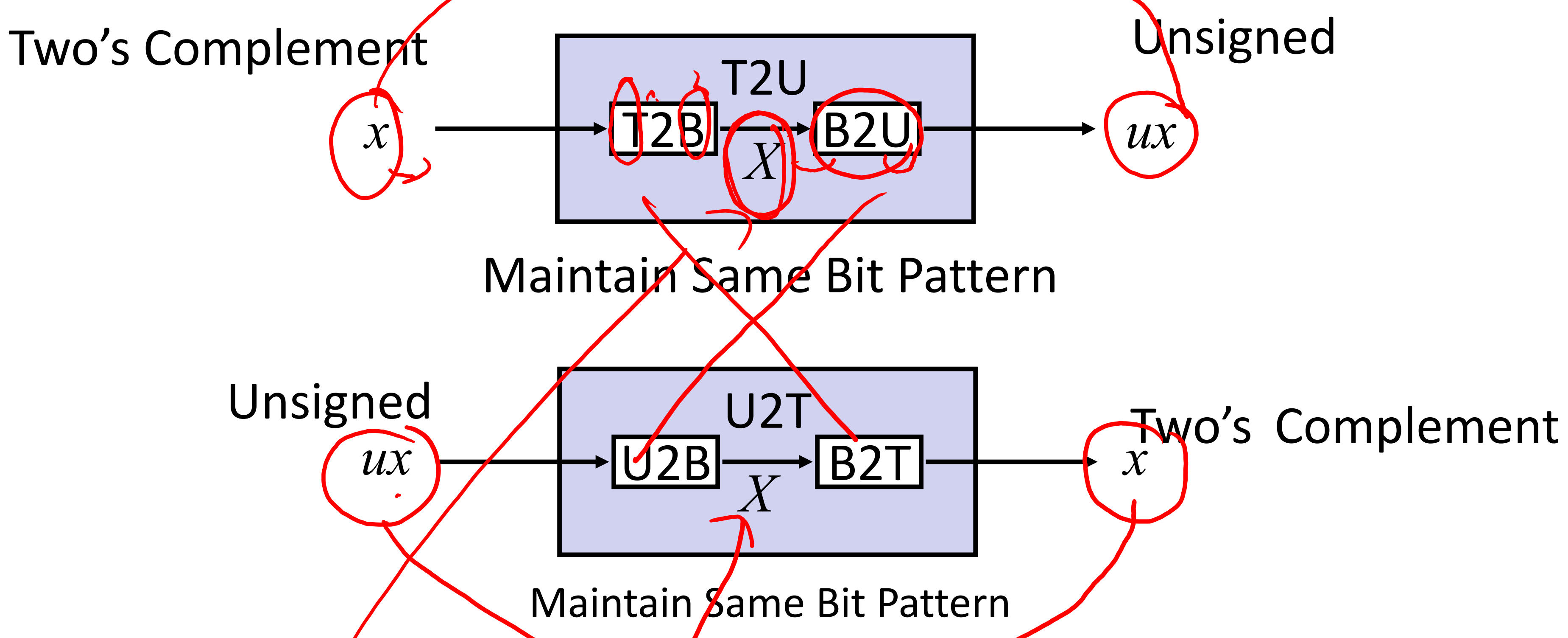
- **Bit-level manipulations**
- **Integers**
  - Representation: unsigned and signed
  - **Conversion, casting**
  - Expanding, truncating
  - Addition, multiplication
  - Summary
- **Representations in memory, strings**

Mapping (Casting) Between Signed & Unsigned



- **Mappings between unsigned and two's complement numbers:** **Keep bit representations and reinterpret**

Mapping Signed ↔ Unsigned

Bits	Signed		Unsigned
0000	0		0
0001	1		1
0010	2	T2U →	2
0011	3		3
0100	4		4
0101	5	← U2T	5
0110	6		6
0111	7		7
1000	-8		8
1001	-7		9
1010	-6		10
1011	-5		11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

Mapping Signed ↔ Unsigned

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0000	0	0
0001	1	1
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0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

Signed 0–7 $\longleftrightarrow$ Unsigned 0–7: =

Signed -8 – -1 $\longleftrightarrow$ Unsigned 8–15: +/- 16

$-8+2^4$

$-3+2^4$

Summary
Casting Signed ↔ Unsigned: Basic Rules

- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting 2^w

- Expression containing signed and unsigned int
  - `int` is cast to `unsigned`!!

Signed vs. Unsigned in C

- **Constants**
  - Literals by default are considered to be signed integers
  - Unsigned if have "U" as suffix

```
0U, 4294967259U
```

- **Casting**
  - Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

  - Implicit casting also occurs via assignments and procedure calls

```
tx = ux;
uy = ty;
```

Question

What will be printed out?

```
void main()
{
    int a= 0;
    unsigned int b = 0;

    if((b-1)<10000)
    {
        printf("b-1 is smaller than 10000");
    }
    else
        printf("b-1 is bigger than 10000");

}
```

$0 - 1 = -1$

$\underbrace{0u - 1}_{u} = \underline{umAx} > \underline{10000}$

Casting Surprises

- **Expression Evaluation**
  - If there is a mix of unsigned and signed in single expression, ***signed values implicitly cast to unsigned***
  - Including comparison operations **<, >, ==, <=, >=**
  - Examples for $W = 32$: **TMIN = -2,147,483,648 ,** **TMAX = 2,147,483,647**

Constant$_1$	Constant$_2$	Relation	Evaluation	
0	0U	==	unsigned	1
-1	0	<	signed	1
-1	0U	<	unsigned	0*
2147483647	-2147483648	>	signed	1
2147483647U	-2147483648	>	unsigned	0*

0u==0u

-1<0 √

-1<0u

umax<0u

2147483647

$2^{31}-1$

0111 1111 1111 1111 1111 1111 1111 1111
31 15 0

-2147483648

2^{31}

1000 0000 0000 0000 0000 0000 0000 0000
31 15 0

Practice

- Consider the following code that attempts to sum the elements of an array a, where the number of elements is given by parameter length
- When run with argument length equal to 0, this code should return 0.0. Instead it encounters a memory error. Why?

```
/* WARNING: This is buggy code */
float sum_elements(float a[], unsigned length) {
    int i;
    float result = 0;

    for (i = 0; i <= length-1; i++)
        result += a[i];
    return result;
}
```

0

int

0u - 1

UMAX

i < length

https://onlinegdb.com/Bk3WcxKvS

Practice

- Consider the following code that attempts to sum the elements of an array a, where the number of elements is given by parameter length
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    int i;
    float result = 0;

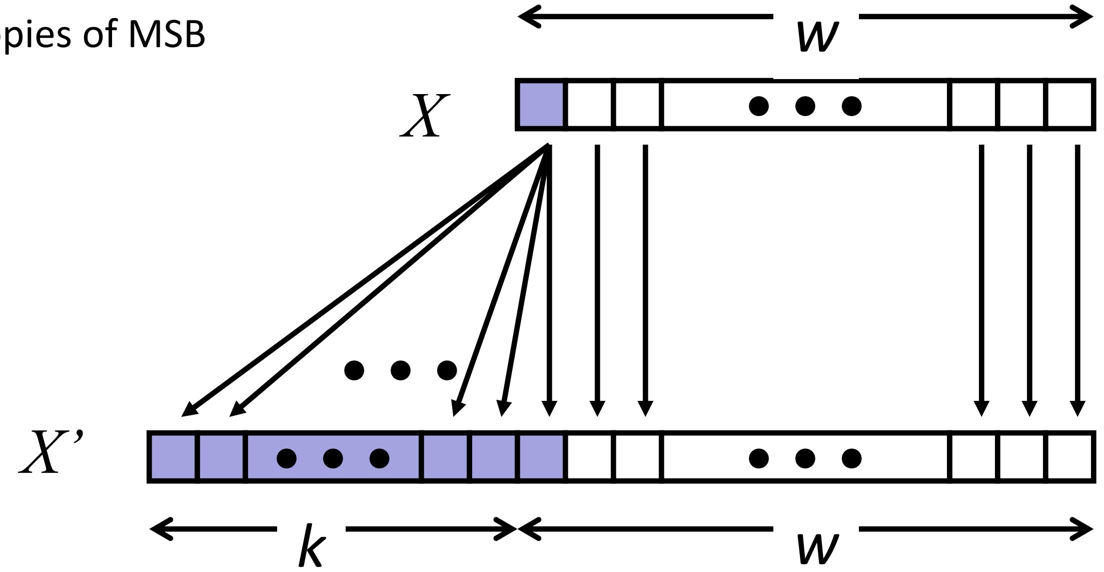
    for (i = 0; i <= length-1; i++)
        result += a[i];
    return result;
}
```

length -1 is Umax. It is always >= any other unsigned int

i will cast to unsigned

Sign Extension

- **Task:**
  - Given w-bit signed integer x
  - Convert it to $w+k$-bit integer with same value (e.g. int to long int casting)
- **Rule:**
  - Make k copies of sign bit:
  - $X' = \underbrace{x_{w-1},\ldots, x_{w-1}}_{k \text{ copies of MSB}}, x_{w-1}, x_{w-2},\ldots, x_0$



Sign Extension Example

```
short int x =  15213;
int       ix = (int) x;
short int y = -15213;
int       iy = (int) y;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
y	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

- Converting from smaller to larger integer data type
- C automatically performs sign extension

Summary:
Expanding, Truncating: Basic Rules

- **Expanding (e.g., short int to int)**
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected result

- **Truncating (e.g., unsigned to unsigned short)**
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod
    - For small numbers yields expected behavior

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  - **Addition, negation, multiplication**
- Representations in memory, strings
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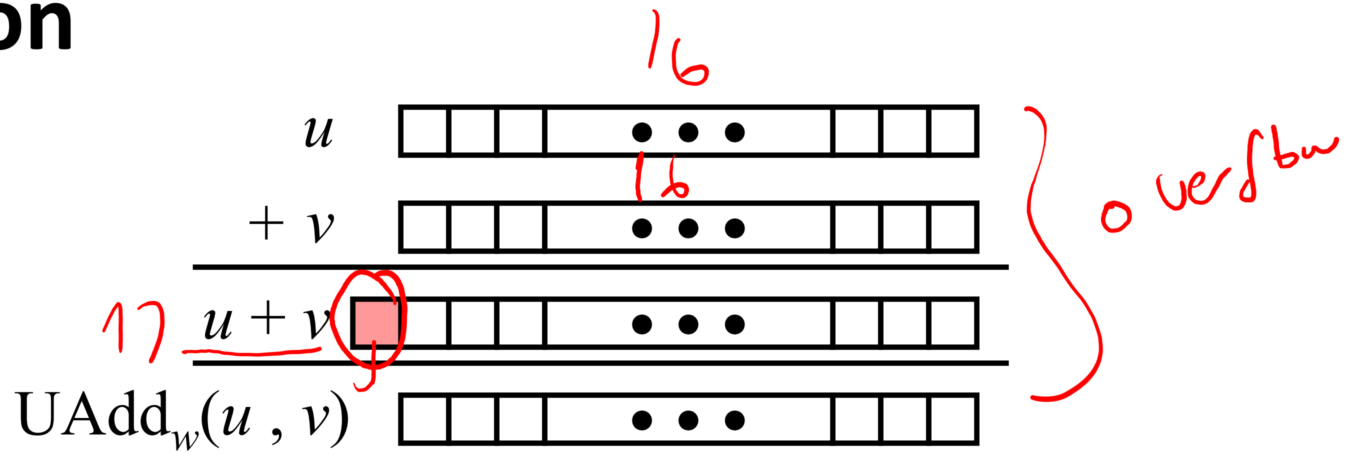
Unsigned Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



- **Standard Addition Function**
  - Ignores carry output
- **Implements Modular Arithmetic**

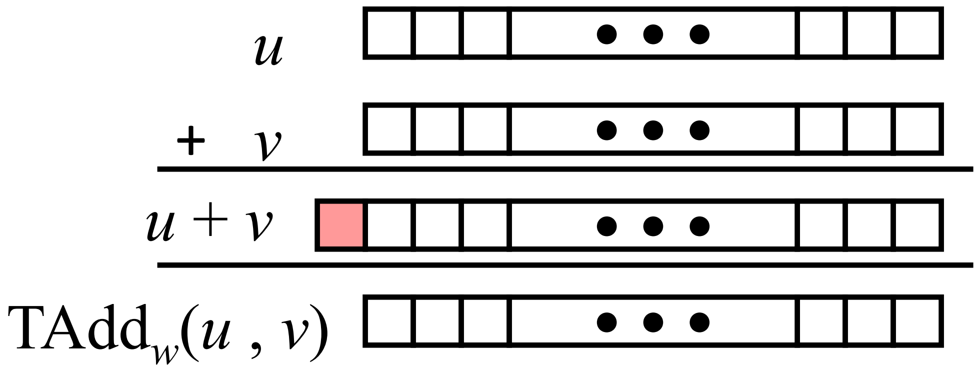
$s \quad = \quad \text{UAdd}_w(u, v) \quad = \quad u + v \mod 2^w$

Two's Complement Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



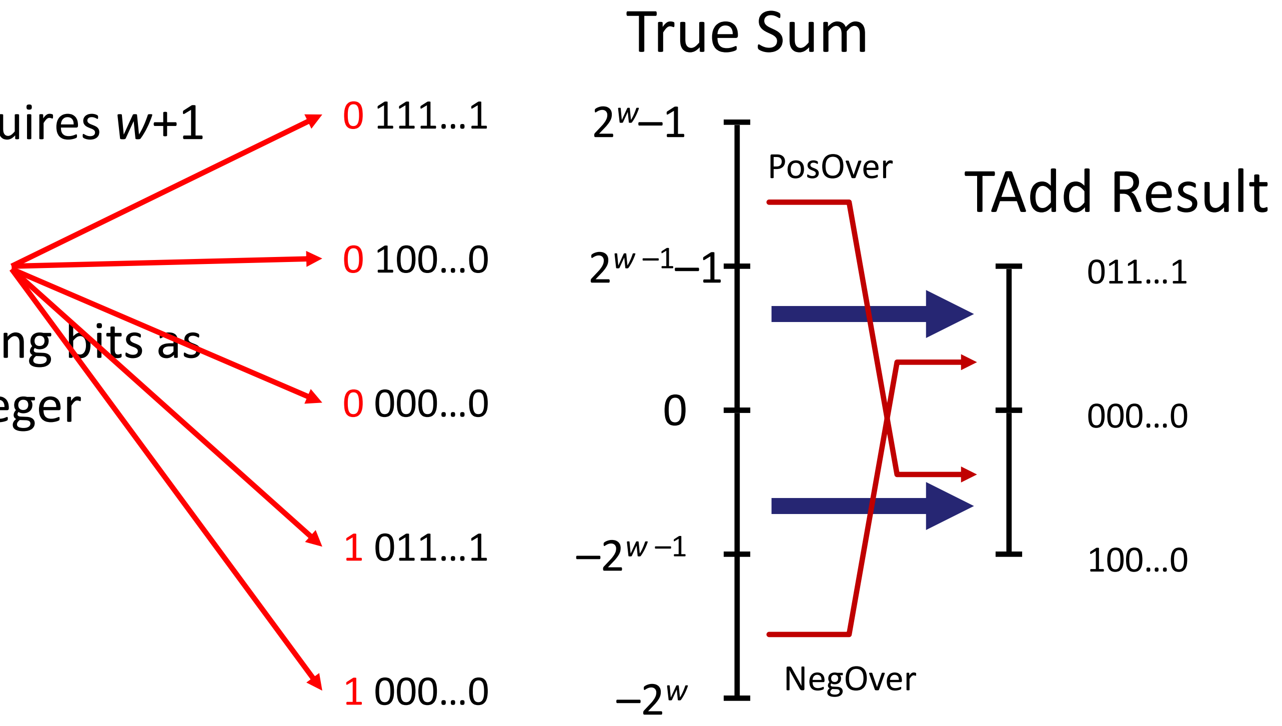
- **TAdd and UAdd have Identical Bit-Level Behavior**
  - Signed vs. unsigned addition in C:

```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v
```

  - Will give `s == t`

TAdd Overflow

- **Functionality**
  - True sum requires w+1 bits
  - Drop off MSB
  - Treat remaining bits as 2's comp. integer



Multiplication

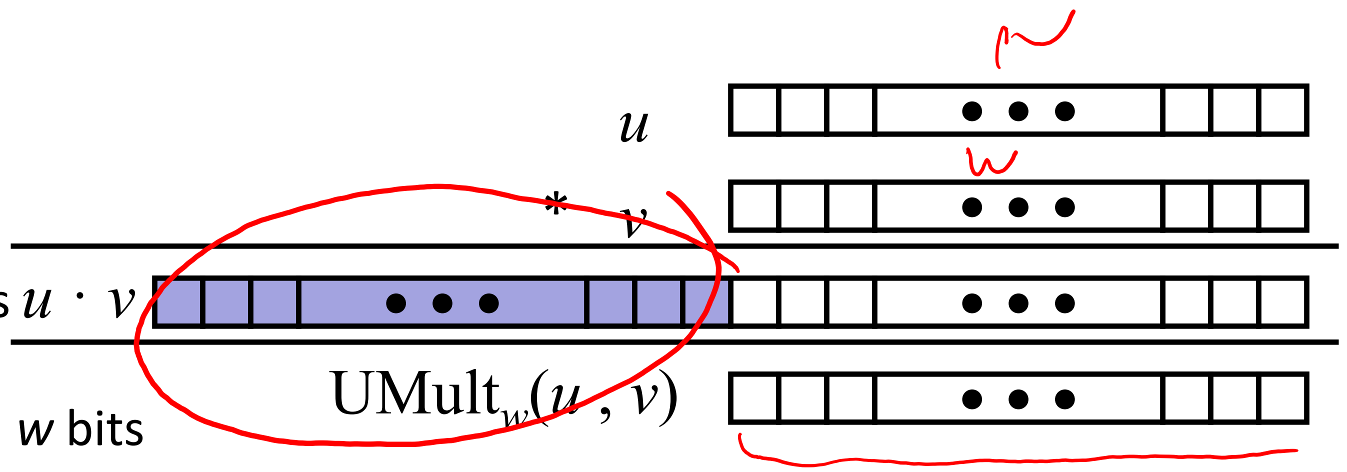
- **Goal: Computing Product of *w*-bit numbers *x*, *y***
  - Either signed or unsigned
- **But, exact results can be bigger than *w* bits**
- **So, maintaining exact results…**
  - would need to keep expanding word size with each product computed
  - is done in software, if needed
    - e.g., by “arbitrary precision” arithmetic packages
    - C has arbitrary precision library gmp.h (GNU Multiprecision Library)
    - Java has BigNumber class and subclases

Unsigned Multiplication in C

Operands: *w* bits

True Product: 2**w* bits $u \cdot v$

Discard *w* bits: remain *w* bits



- **Standard Multiplication Function**
  - Ignores high order *w* bits
- **Implements Modular Arithmetic**

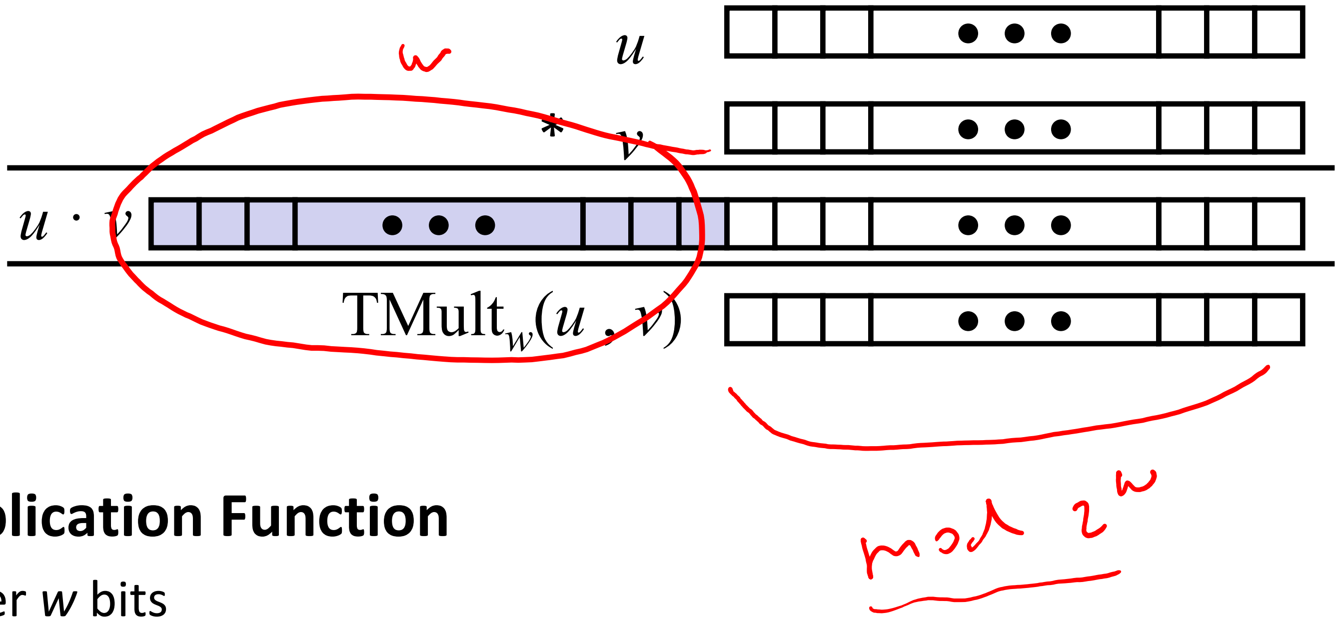
  $\text{UMult}_w(u, v) \;=\; u \cdot v \mod 2^w$

Signed Multiplication in C

Operands: *w* bits

True Product: 2**w* bits

Discard *w* bits: *w* bits



- **Standard Multiplication Function**
  - Ignores high order *w* bits
  - Some of which are different for signed vs. unsigned multiplication
  - Lower bits are the same

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Arithmetic: Basic Rules

- **Addition:**
  - Unsigned/signed: Normal addition followed by truncate, same operation on bit level
  - Unsigned: addition mod 2^w
    - Mathematical addition + possible subtraction of 2^w
  - Signed: modified addition mod 2^w (result in proper range)
    - Mathematical addition + possible addition or subtraction of 2^w

- **Multiplication:**
  - Unsigned/signed: Normal multiplication followed by truncate, same operation on bit level
  - Unsigned: multiplication mod 2^w
  - Signed: modified multiplication mod 2^w (result in proper range)

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Machine Words

- **Any given computer has a "Word Size"**
  - Nominal size of integer-valued data
    - and of addresses
  - Until recently, most machines used 32 bits (4 bytes) as word size
    - Limits addresses to 4GB (2^{32} bytes)
  - Increasingly, machines have 64-bit word size
    - Potentially, could have 18 EB (exabytes) of addressable memory
    - That's 18.4 X 10^{18}
  - Machines still support multiple data formats
    - Fractions or multiples of word size
    - Always integral number of bytes

2^{32}

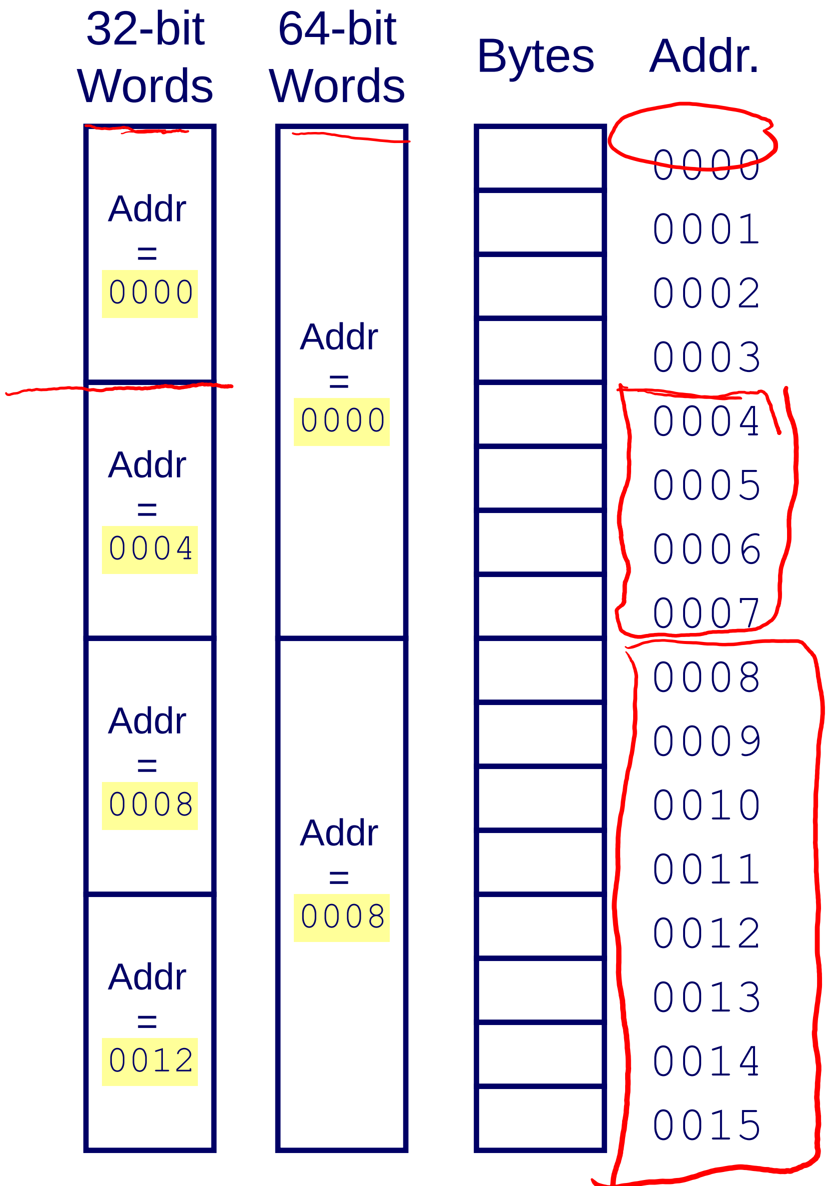
2^{32} bytes

4 GB

2^{64} bytes

Word-Oriented Memory Organization

- **Addresses Specify Byte Locations**
  - Address of first byte in word
  - Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Representing Strings

- **Strings in C**
  - Represented by array of characters
  - Each character encoded in ASCII format
    - Standard 7-bit encoding of character set
    - Character "0" has code 0x30
      - Digit *i* has code 0x30+*i*
  - String should be null-terminated
    - Final character = 0
- **Compatibility**
  - Byte ordering not an issue

```
char S[6] = "18213";
```