UNIVERSITY OF LONDON

BSc EXAMINATION 2022

For Internal Students of
Royal Holloway

DO NOT TURN OVER UNTIL TOLD TO BEGIN

CS2860: Algorithms and Complexity
CS2860R: Algorithms and Complexity – FOR FIRST SITS/RESIT CANDIDATES

Time Allowed: **TWO hours**

Please answer **ALL** questions.

1. (a) Describe the order of growth of the following functions in standard Θ notation as simply as possible. Sort the resulting Θ expressions by increasing speed of growth. [10 marks]

   i. $f(n) = 5(\log n)^3 + 3n^2 + 5$

   ii. $f(n) = 3n + 4n \log n + 42$

   iii. $f(n) = 15n^2 \log n - 3 \log n + \frac{7n-2}{\log n}$

   iv. $f(n) = 47 \log n - 25$

   (b) Explain what is the meaning of c and n_0 in the definition of Big-Oh notation. Given $4n^2 - 2n = O(n^4)$, find the minimum possible value of c for $n_0 = 1$ and for $n_0 = 10$. [6 marks]

2. Write a recursive function to compute the heights of all nodes in a given binary tree. The height of a leaf node is 1, and the height of the root is the number of nodes in the longest path from the root to a leaf.

   The tree nodes have the structure:

```
1   class Node {
2     int height;
3     Node left;
4     Node right;
5   }
```

   Your function should correctly fill in the `height` field for every node in the tree. [10 marks]

3. Both parts of the question relate to the tree in Figure 1 below.

   (a) Perform the following operations on the binary search tree (BST) in the given order, and show the shape of the tree after each one: Delete 20. Delete 43. Insert 26. Delete 32. Show the ordinary binary search tree operations; do not rebalance the tree after insertions or deletions. [4 marks]

   (b) Consider the tree in Figure 1 again. The tree can also be seen as an AVL tree into which we just inserted 43. Describe the rotation(s) that need to be performed to rebalance the tree. [8 marks]

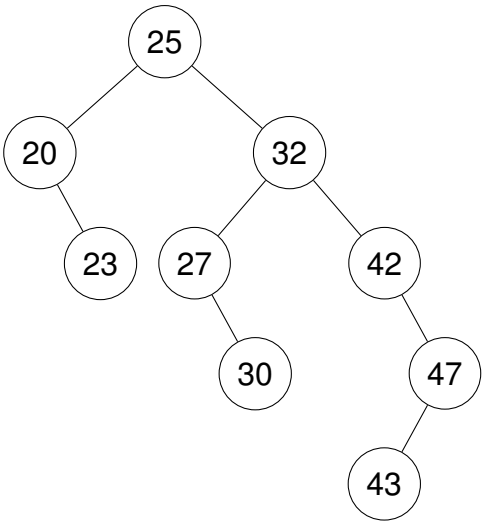

FIGURE 1: BST FOR QUESTION 3.

4. (a) State the heap property for binary min-heaps. [2 marks]

   (b) Which of the following two arrays is a binary min-heap?

   i. `A=[1, 3, 18, 25, 4, 48, 27, 41, 31, 49]`

   ii. `B=[4, 23, 28, 32, 92, 27, 96, 97, 67, 53]` [2 marks]

   (c) Consider the following array `C = [10, 19, 24, 43, 41, 31, 92, 75, 47, 62]` representing a binary min-heap.

   i. Draw the tree representation of the binary min-heap. [2 marks]

   ii. Describe the algorithm from lectures for deleting the minimum from binary min-heap and illustrate it on the heap `C`. [3 marks]

   iii. Describe the algorithm from lectures for inserting a new element to a binary min-heap and illustrate it by inserting the number 25 in the heap `C` (i.e., the original min-heap `C`, not the outcome of 4(c)ii). [3 marks]

5. What is the difference between the `HashMap` and `TreeMap` implementations of the Map abstract data type? Give an advantage and a disadvantage of each. [8 marks]

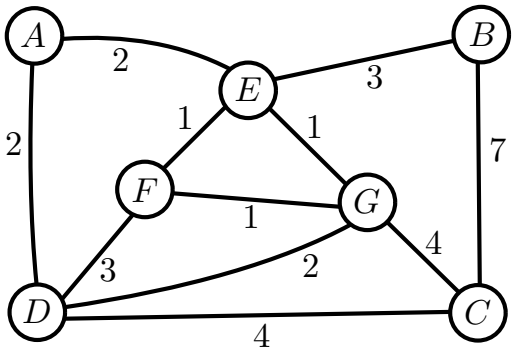

FIGURE 2: DIRECTED GRAPH FOR QUESTION 6.

6. (a) Illustrate Kruskal's algorithm on the graph in Figure 2. In addition show how the Union-Find data structure changes throughout the computation. [5 marks]

   (b) Draw the minimum spanning tree found in the part (a). [2 marks]

   (c) By examining the choices made in your computation, determine how many minimum spanning trees (MSTs) the graph in Figure 2 has. [4 marks]

7. Consider the following function `fn`.

```
 1    int fn(n) {
 2      if (n == 0) {
 3       return 1;
 4      }
 5      int sum = 0;
 6      for (int i=0; i<n; i++) {
 7       sum += fn(i);
 8      }
 9      return sum;
10    }
```

   (a) Compute the value `fn(5)`. [4 marks]

   (b) Illustrate all recursive calls made by the algorithm on input $n = 4$, i.e., when calling `fn(4)`, by drawing the recursion tree. [4 marks]

   (c) What does `fn(n)` compute for $n \geq 1$. You do not need to justify your answer. [3 marks]

   (d) Compute an estimate of the execution time as a function of n. Give your answer in Big-Oh notation and justify your answer. [3 marks]

   (e) Describe how you can use the Memoisation technique to improve the running time of the function. [2 marks]

   (f) Compute an estimate of the execution time as a function of n after using Memoisation. Give your answer in Big-Oh notation and justify your answer. [4 marks]

8. Suggest an algorithm that helps you select all possible holiday destinations within a given budget. Its input is a list of possible connections between cities, where each connection is given as a combination of a starting location, a destination location, and a cost of a ticket (For example, one such connection could be “a train from London to Manchester, £30”). Propose an algorithm that, given a starting location and overall budget, finds all destinations that you can reach with given budget. [11 marks]

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