UNIVERSITY OF LONDON

BSc EXAMINATION 2024

For Internal Students of
Royal Holloway

DO NOT TURN OVER UNTIL TOLD TO BEGIN

CS2860: Algorithms and Complexity
CS2860R: Algorithms and Complexity – for FIRSTSIT/RESIT CANDIDATES

Time Allowed: **TWO hours**

Please answer **ALL** questions

Calculators are not permitted

1. (a) Describe the order of growth of the following functions in standard Θ notation as simply as possible. Sort the resulting Θ expressions by increasing speed of growth. [10 marks]

   i. $f(n) = 4 - 3n + 6n^2$

   ii. $f(n) = 5n + 2n \log n$

   iii. $f(n) = 5n^2 - 2 \log n + 3n \log n + 8$

   iv. $f(n) = 500 \log n + n$

   (b) Consider the two functions `fnA` and `fnB` given below. What are the running times of `fnA` and `fnB` as functions of n? You may use Big-Oh notation in your answer. Justify your answer with complete computations. (Don't try to explain what the functions compute.) [10 marks]

```
1       int fnA(int n) {
2        int sum=0;
3        int i=1;
4        while (i < n) {
5         sum += 3*i-2
6         i = 2*i
7        }
8        return sum;
9       }
10
```

```
1       int fnB(int n) {
2        int total=0;
3         for (int i=0; i<n; i++) {
4         total = total * fnA(2*i);
5        }
6        return total;
7       }
8
```

2. (a) State the heap property for binary min-heaps. [2 marks]

   (b) Which of the following two arrays is a binary min-heap?

   i. `A=[28, 31, 41, 57, 39, 64, 32, 61, 42, 50]`

   ii. `B=[3, 20, 13, 22, 46, 33, 19, 57, 31]` [2 marks]

   (c) Describe the makeheap algorithm from lectures and illustrate it on the array `C=[3,1,5,6,2,4,7]`. [6 marks]

3. This question is about linked lists. You may assume either single-linked lists or double-linked lists in your answer, as you prefer.

   (a) Show the structure of a linked list containing the values $2, 4, 3, 15, 9$ in this order. [3 marks]

   (b) Show the operations involved when an algorithm searches for the number 4 in the above linked list. Also state the worst-case running time (in Big-Oh notation) for searching for a number in a linked list with n items. [3 marks]

   (c) Show the operations involved when inserting the number 7 into the above linked list after 3 and before 15. Also state the running time (in Big-Oh notation) for inserting a number into a linked list with n items. [4 marks]

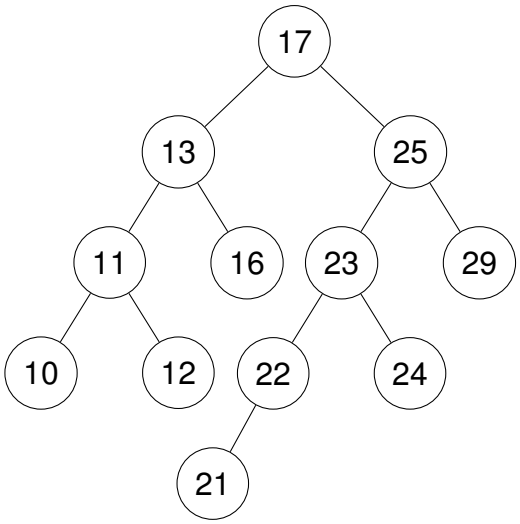

FIGURE 1: FIGURE FOR QUESTION 4.

4. Consider the binary search tree (BST) illustrated in Figure 1 above.

   (a) Define the *balance* property of AVL trees. Explain why the tree in the example does not have this property. [4 marks]

   (b) Show how the tree in Figure 1 can be "repaired" to have the balance property by the use of *rotations*. [4 marks]

   (c) What is the purpose of the balance property for AVL trees (e.g., what advantage does it bring compared to an unbalanced BST)? [4 marks]

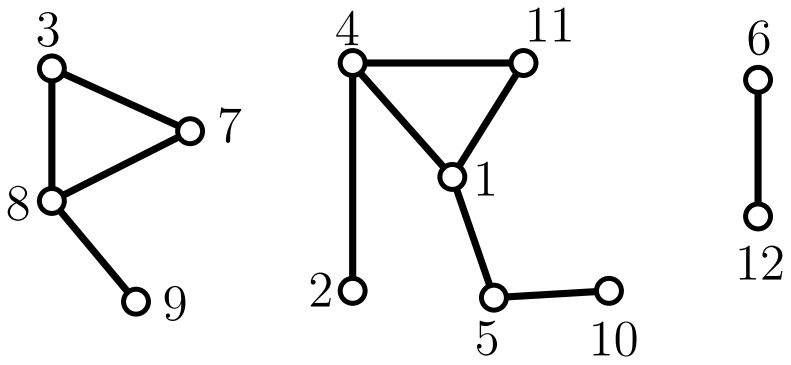

FIGURE 2: THE GRAPH G FOR QUESTION 5.

5. Give definitions of a connected graph and connectivity components. DFS is applied to find connectivity components in the graph G of Figure 2. How many connectivity components does G have? Assume that in the loops of DFS the vertices of G are considered in the natural order (i.e., when it has the option, it selects lower-numbered nodes before higher-numbered ones). Give the order in which the vertices of G are visited. [10 marks]

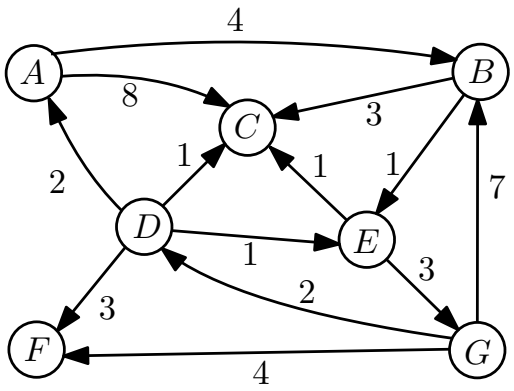

FIGURE 3: DIRECTED GRAPH FOR QUESTION 6.

6. (a) Describe Dijkstra's algorithm for finding single-source shortest paths in a graph, and illustrate it on the graph of Figure 3, by finding shortest paths to all destinations starting from vertex A. [7 marks]

   (b) Draw the resulting shortest paths tree. [2 marks]

   (c) Explain how to get an efficient implementation of Dijkstra's algorithm, i.e., an implementation that runs in time $O(m \log n)$ or better for a graph with m edges and n vertices. Justify your claim about the running time. [2 marks]

7. Consider the following problem: You have a large collection of music files (e.g., MP3 files), and for each file you know the artist name (as a string) and the song title (also as a string). You wish to find the name of the artist who has the largest number of songs featured in the collection.

   (a) Describe an algorithm for this problem. For full marks, the algorithm should be as efficient as possible. [6 marks]

   (b) State the running time of your algorithm, assuming that there are in total n songs, across t different artists. [4 marks]

8. Consider the following function `fn`.

```
1    int fn(n) {
2      if (n == 0) {
3       return 1;
4      }
5      if (n == 1) {
6       return 3;
7      }
8      return fn(n-1)+6*fn(n-2);
9    }
```

(a) Compute the value `fn(4)`. [3 marks]

(b) Illustrate all recursive calls made by the algorithm on input $n = 4$, i.e., when calling `fn(4)`, by drawing the recursion tree. [3 marks]

(c) What does `fn(n)` compute for $n \geq 0$? Justify your answer. [2 marks]

(d) Compute an estimate of the execution time as a function of n. Give your answer in Big-Oh notation and justify your answer. [3 marks]

(e) Describe how you can use the Memoisation technique to improve the running time of the function. [3 marks]

(f) Compute an estimate of the execution time as a function of n after using Memoisation. Give your answer in Big-Oh notation and justify your answer. [3 marks]

END