UNIVERSITY OF LONDON

BSc EXAMINATION 2022

For Internal Students of
Royal Holloway

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CS2900: Multi-dimensional Data Processing
CS2900R: Multi-dimensional Data Processing – for FIRSTSIT/RESIT CANDIDATES

Time Allowed: **TWO hours**

Answer ALL questions
Calculators are NOT permitted

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1. The following questions require short definitions of technical terms in the context of vectors and matrices. Component-wise definitions are sufficient. You may assume that $\underline{u}$ is a real valued vector in N-dimensions (N is finite) and that $\mathbf{M}$, $\mathbf{A}$ and $\mathbf{B}$ are real valued $N \times N$ matrices.

   (a) List three properties of the dot product of two vectors that are true in any finite number of dimensions. [9 marks]

   (b) If $\mathbf{M} = \mathbf{AB}$ then what is M_{ij} in terms of $\mathbf{A}$ and $\mathbf{B}$? [3 marks]

   (c) Define the following.

      i. The scalar product operation for a vector. [3 marks]

      ii. Diagonal matrix. [3 marks]

      iii. Symmetric matrix. [3 marks]

      iv. Upper diagonal matrix. [3 marks]

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2. (a) The real-valued vectors $\underline{u}$ and $\underline{v}$ have the same number of dimensions.

   i. What are the necessary steps to compute $\underline{y}$, the projection of $\underline{u}$ onto $\underline{v}$? [9 marks]

   ii. How do you compute the vector $\underline{w}$ that is orthogonal to $\underline{y}$ and whose sum with $\underline{y}$ is $\underline{v}$? [3 marks]

(b) The following questions regard algorithms for ranking the importance of web pages.

   i. Explain the necessary steps in the PageRank algorithm. A pseudo-code description is sufficient. [8 marks]

   ii. Give two reasons why one cannot use the simple diffusion-based algorithm rather than the PageRank algorithm. [6 marks]

(c) Do all matrices have an inverse? Explain your reasoning with a specific example. [6 marks]

(d) Given two rectangular matrices $\mathbf{C}$ and $\mathbf{D}$ what one must check to determine if $\mathbf{CD}$ exists? If $\mathbf{CD}$ exists, what does it corresponds to in terms of linear transformations? [4 marks]

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3. (a) Compute the projection of the vector $(1 \ \ 0 \ \ 0 \ \ 1)^{\intercal}$ onto the 2-dimensional plane that lies in a 4-dimensional space and passes through the origin and the points $(1, -1, 0, 1)$ and $(1, 1, 1, 0)$. Do this using basis vectors for the plane. The projected vector is to be represented in the original 4-dimensional space. Show all the necessary steps to compute this. [8 marks]

(b) A graph has the following adjacency matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

i. Draw the equivalent directed graph, labelling the vertices 1 to 4. [3 marks]

ii. Using the adjacency matrix list the one ordered pair of nodes that have two paths of length two between them. Show your reasoning. [4 marks]

iii. Compute the corresponding diffusion matrix for this graph. [3 marks]

iv. This graph corresponds to set of web pages and their links and one is starting from the page corresponding to vertex 1. Assuming random selection of web links from a page is it more likely that after two iterations one will be at vertex 1 or vertex 3? Explain your reasoning. [6 marks]

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4. (a) Using the singular value decomposition of a matrix $\mathbf{X} = \mathbf{U}\mathbf{D}\mathbf{V}^\intercal$

   i. derive an expression for $\mathbf{X}\mathbf{X}^\intercal$ in terms of the components of the above decomposition. [3 marks]

   ii. Using the previous derived expression in 4(a)(i) derive an expression for $(\mathbf{X}\mathbf{X}^\intercal)^{2n}$. [3 marks]

   (b) On the next page a Python class `specialMatrix` is defined. This class is used to define a matrix with a specific structure.

   i. What is the structure of the matrix? [2 marks]

   ii. What does the function `func1` do? [3 marks]

   iii. What is the advantage of using this object for storing this type of matrix over a standard Numpy matrix? [2 marks]

   iv. Write a function `trace` that can be added to `specialMatrix` which returns the trace of the matrix. It is not necessary to write down the rest of the `specialMatrix` code. [3 marks]

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```
import numpy as np
import sys

class specialMatrix:
    """
    input :-
        M is a Numpy vector
    """
    def __init__(self,M):
        self.MSize = np.size(M)
        self.a = np.copy(M)
    """
    input :- integers i,j
    output (i,j) entry of matrix
    """
    def getEntry(self,i,j):
        if i<0 or j<0 or i>=self.MSize or j>=self.MSize:
            sys.exit("getEntry - indices out of range")
        if i==j:
            return(self.a[i])
        else:
            return(0)
    """
    input :- Numpy array A
    """
    def func1(self,A):
        s = np.shape(A)
        if s[0] != self.MSize or s[1] != self.MSize:
            sys.exit("specialMatrix - matrix is not correct 
               dimensions!")
        X = np.zeros_like(A)
        for i in range(self.MSize):
            for j in range(self.MSize):
                X[i,j] = self.a[i] * A[i,j]
        return(X)
```

END

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