UNIVERSITY OF LONDON

BSc EXAMINATION 2023

For Internal Students of
Royal Holloway

DO NOT TURN OVER UNTIL TOLD TO BEGIN

CS2900: Multi-dimensional Data Processing
CS2900R: Multi-dimensional Data Processing – for FIRSTSIT/RESIT CANDIDATES

Time Allowed: **TWO hours**

Answer ALL questions
Calculators are NOT permitted

1. The following questions require short definitions of technical terms in the context of vectors and matrices. Component-wise definitions are sufficient. You may assume that $\underline{u}$ and $\underline{v}$ are real valued column vectors in N-dimensions (N is finite) and that $\mathbf{M}$ is a real valued $N \times N$ matrix.

   (a) List three properties of the addition of vectors that are true in any finite number of dimensions. [9 marks]

   (b) If $\underline{u} = \mathbf{M}\underline{v}$ then what is is this relationship on a component by component basis? [3 marks]

   (c) Define the following.

      i. The dot product operation for two vectors. [3 marks]

      ii. Tridiagonal matrix. [3 marks]

      iii. Orthogonal matrix. [3 marks]

      iv. Identity matrix. [3 marks]

2. (a) The following questions are with respect to Singular Value Decomposition (SVD).

   i. If $\mathbf{B}$ is a real valued matrix which has M rows and N columns what is the SVD of $\mathbf{B}$? Define all terms appropriately. [6 marks]

   ii. Explain briefly how SVD is employed to compute the pseudo-inverse of a matrix. [6 marks]

   (b) Show that if a matrix $\mathbf{M}$ is orthogonal and can be applied on a vector $\underline{u}$ then $|\mathbf{M}\underline{u}| = |\underline{u}|$. [6 marks]

   (c) The following questions are with respect to the ranking of web pages.

   i. Define what the diffusion matrix of an adjacency matrix is. [6 marks]

   ii. Why is the transpose of the diffusion matrix used in the diffusion algorithm? [6 marks]

   iii. Give two reasons why one cannot use the simple diffusion-based algorithm rather than the PageRank algorithm. [6 marks]

3. (a) The following is with respect to the projection of vectors onto embedded sub-spaces.

   i. Demonstrate that the unit vectors that lie in the same direction of $(1, -1, 1, 1)^\intercal$ and $(1, 1, -1, 1)^\intercal$ can form an orthonormal basis. [2 marks]

   ii. Compute the projection of the vector $(1\ 3\ -1\ 1)^\intercal$ onto the 2-dimensional plane that lies in a 4-dimensional space and passes through the origin and the points $(1, -1, 1, 1)$ and $(1, 1, -1, 1)$. The projected vector is to be represented in the original 4-dimensional space. Show all the necessary steps to compute this. [8 marks]

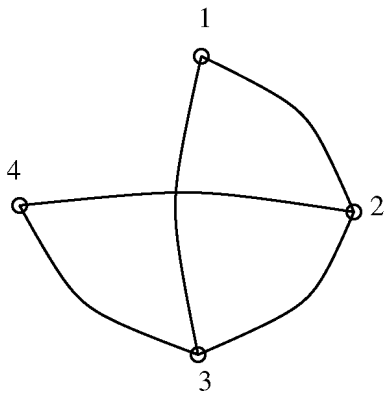
   (b) What is the rank of the following matrices? Demonstrate your reasoning.

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & 1 \\ -3 & 3 & -3 \end{pmatrix}, \quad \begin{pmatrix} 2 & 3 \\ 1 & 3 \\ 0 & 3 \end{pmatrix}$$

[6 marks]

   (c) The following is with respect to adjacency matrices of graphs.

   i. Define the adjacency matrix for the following graph (with vertex 1 represented by the first row and so on).



[3 marks]

   ii. Using the adjacency matrix find the only pair of separate vertices that have exactly two paths of length three between them. Show that there are no pair of separate vertices that have exactly three paths of length three between them. Show your reasoning. [5 marks]

4. The following data structure is defined in Python

```
G = { 1 : {1:2, 2:3}, 2 : {3:1}, 3 : {1:-1,3:1}}
```

(a) Demonstrate that this data structure can be used to define a matrix and write down what that matrix is. [6 marks]

(b) Demonstrate what mathematical operation the function `operation` (defined below) is carrying out if `G` is passed to it. What value will this function return if `G` is passed to it? [6 marks]

```
def operation(H):
    n = 0
    for i in H.keys():
        if i in H[i]:
            n += H[i][i]

    return n
```

(c) What would be the advantage of using a data structure like this in matrix operations? [4 marks]

END