18. Min/Max Values

Lec 17 mini review.

 \diamond Midpoint Rule: $M_n = \sum_{i=1}^n f(\overline{x_i}) \Delta x$ $(\overline{x_i} = \frac{1}{2}(x_{i-1} + x_i))$

 \diamond Trapezoidal Rule: $T_n = \frac{\Delta x}{2} \left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n) \right]$

 \diamond Simpson's Rule (*n* even):

$$S_n = \frac{\Delta x}{3} \left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n) \right]$$

Error Bounds:

• If $|f''(x)| \le K$ for all $a \le x \le b$, then $\left| T_n - \int_a^b f(x) dx \right| \le \frac{K(b-a)^3}{12n^2}$ and $\left| M_n - \int_a^b f(x) dx \right| \le \frac{K(b-a)^3}{24n^2}$

• If $|f^{(4)}| \le K$ for all $a \le x \le b$, then $\left| S_n - \int_a^b f(x) dx \right| \le \frac{K(b-a)^5}{180n^4}$

MAX/MIN VALUES

A function y = f(x) has...

...an Absolute/Global Maximum at x = c if ...a x = c if

...an ABSOLUTE/GLOBAL MINIMUM AT

The value f(c) is called the...

GLOBAL MAXIMUM VALUE of f.

GLOBAL MINIMUM VALUE of f.

- \diamond The max/min values of f are called ${\bf EXTREME\ VALUES}$.
- \diamond A function f can attain global max/min values at many numbers.

^{*} These notes are solely for the personal use of students registered in MAT1320. ©EJM All rights reserved.

♦ Some functions do not have max and/or min values.	
Local vs. Global Extreme Values	
A function $y = f(x)$ has	
a LOCAL MAXIMUM AT $x=c$ if	a LOCAL MINIMUM AT $x=c$ if
 Some local max/min are also absolute ext 	treme points.
⋄ Not every local max/min is an absolute n	nax/min.

 \diamond Not every absolute max/min is a local max/min.

EXTREME VALUE THEOREM

Theorem 18.1. (Extreme Value Theorem)

If y = f(x) is continuous on the closed interval [a, b], then, restricted to the interval [a, b], f has an absolute maximum and an absolute minimum on [a, b].

Note. If f is not continuous on [a, b], then the Extreme Value Theorem is not applicable. Even if f is continuous, if the interval is not closed, then the Extreme Value Theorem is not applicable.

CRITICAL NUMBERS AND HOW TO FIND EXTREME VALUES

The Extreme Value Theorem tells us of a situation in which absolute extreme values must exist, but it does not tell us how to find them. For that, we need Fermat's Theorem.

Theorem 18.2. (Fermat's Theorem)

A number x is called a **CRITICAL NUMBER** of a function f(x) if either

• x is in the domain of f and f'(x) = 0 (type 1)

• or x is in the domain of f but x is **not** in the domain of f'. (type 2)

Graphically, these two types of critical numbers correspond to numbers \boldsymbol{x} such that

• f(x) has a horizontal tangent at x (type 1)

• f(x) has a corner, jump discontinuity, or vertical tangent line at x (type 2)

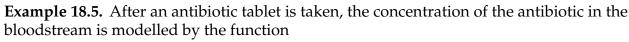
Example 18.3. Find all the critical numbers of $h(x) = x^{2/3}(x-2)^2$.

The Closed Interval Method

Follow these steps to find the absolute extrema of a <u>continuous</u> function f(x) on a closed interval [a,b]

- **1.** Find the critical numbers of f.
- **2.** For each critical number c such that $c \in [a, b]$, compute its value f(c).
- **3.** For the endpoints x = a and x = b of [a, b], compute the values f(a) and f(b).
- **4.** The absolute maximum value of f on [a, b] is largest value computed in steps 2 and 3.
- **5.** The absolute minimum value of f on [a, b] is smallest value computed in steps 2 and 3.

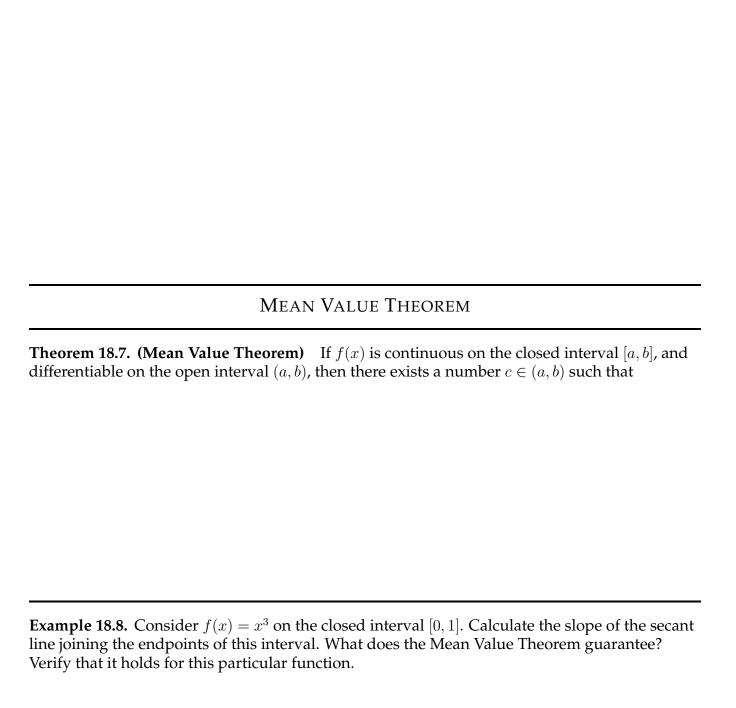
Example 18.4. Find the absolute/global extreme points of $h(x) = x^{2/3}(x-2)^2$ on the closed interval [-1, 1] (the function from Ex. 18.3)

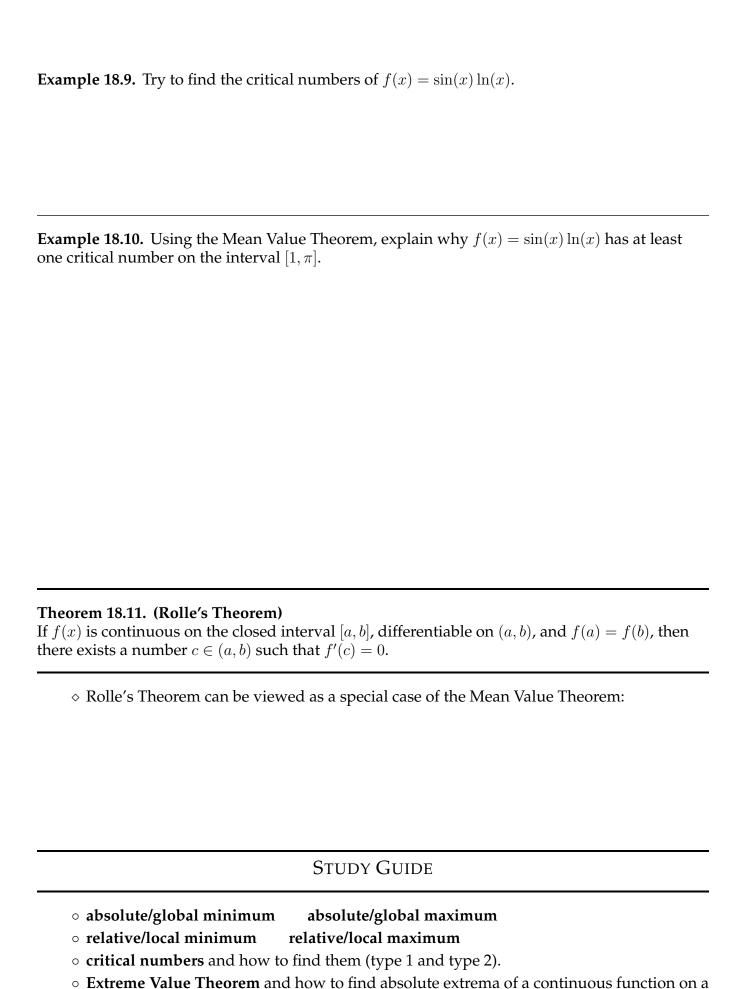


$$C(t) = 8(e^{-0.4t} - e^{-0.6t})$$

where the time t is measured in hours and C is measured in $\mu g/mL$. What is the maximum concentration of the antibiotic during the first 12 hours?

Example 18.6. Find the absolute maximum and absolute minimum of $f(x) = \sin(x)\cos(x)$ on the interval $[-\pi, \pi]$.





closed interval