Lesson 4 - Vectors in R^2 and R^3

PART A: R^2 **and** R^3

We will now be looking at representations of vectors in two-dimensional and three-dimensional space.

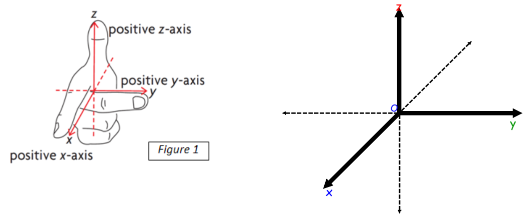
R^2 – refers to the coordinate system for two dimensions

R^3 – refers to the coordinate system for three dimensions

Note: R^n refers to the coordinate system for the nth dimension, where n is a positive integer value. Our studies will be limited to two and three dimensions, but just keep in mind that the universe is not so limited.

PART B: A "new" axis

We now introduce a "new" axis, the $z-axis$, which gives us the third dimension. As you can see below, we will be using the right-handed system of coordinates (figure 1), in which the positive direction for the x, y and z axes are denoted by the bold lines and labeled in figure 2.



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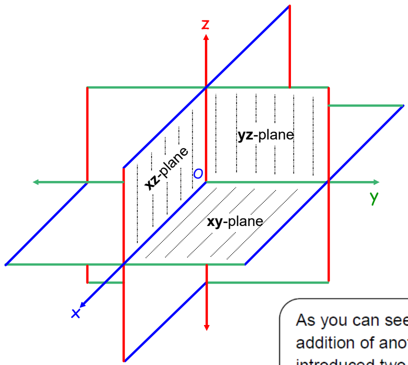

As you can see in Figure 3, with the addition of another axis, we have introduced two more planes, the $yz-plane$ and the $xz-plane$.

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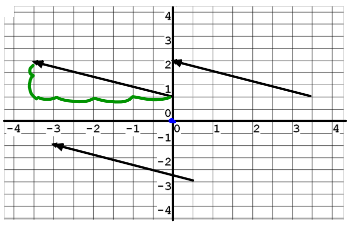
PART C: Differentiating between points and vectors

As you can see in Figure 4, all of the directed line segments represent the same vector, which starts at a certain point and moves 3.5 units to the left and 1.5 units up.

The notation we will use for this vector is:

$$\vec{v} = [-3.5, 1.5]$$

Each of the vectors in the sketch, are called **representations** of this vector.



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Note: we will distinguish between vector notation and the notation used for coordinates of points by using square brackets for vectors and round for coordinates of points.

A representation of the vector $\vec{v} = [a_1, a_2]$ in two dimensional space is any directed line segment, $\overline{AB}$, from the point $A = (x, y)$ to the point $B = (x + a_1, y + a_2)$.

Likewise a representation of the vector $\vec{v} = [a_1, a_2, a_3]$ in three dimensional space is any directed line segment, $\overline{AB}$, from the point $A = (x, y, z)$ to the point $B = (x + a_1, y + a_2, z + a_3)$.

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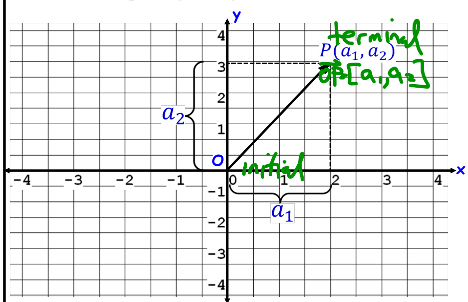
PART D: The position vector

The representation of the vector $\vec{v} = [a_1, a_2, a_3]$ that starts at the point $A = (0,0,0)$ and ends at the point $B = (a_1, a_2, a_3)$ is called the ***position vector*** of the point (a_1, a_2, a_3). So, when we talk about position vectors we are specifying the initial and final point of the vector.

Figure 5 shows vector $\overline{OP}$, which in component form is represented by $[a_1, a_2]$. This is a vector with its tail at $O(0,0)$, and its head at $P(a_1, a_2)$.

a_1 is the $x - component$, and
a_2 is the $y - component$ of $\overline{OP}$



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PART E: Generating a vector given initial and final points

Given two points, $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$, the vector with the representation $\overline{AB}$ is:

$$\overline{AB} = [b_1 - a_1, b_2 - a_2, b_3 - a_3]$$

Direction is *very* important here! The vector above, *starts* at A and *ends* at B. Be sure to subtract the *initial* point from the *terminal* point.

Example 1: Give the vector for each of the following:

a) The vector from $(1, -3, -5)$ to $(2, -7, 0)$. A B

$\overrightarrow{AB}$ = [2 -1, - 7 + 3, 0 + 5] = [1, -4, 5]

b) The vector from $(2, -7, 0)$ to $(1, -3, -5)$. C D

$\overrightarrow{CD}$ = [1 - 2, -3+7, -5 - 0] = [-1, 4, -5]

c) The position vector for $(-90, 4)$

$\overrightarrow{OP}$ = [-90 - 0, 4 - 0] = [-90, 4]

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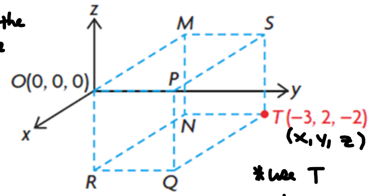
Example 2: Using the diagram below, complete the following:

a) Determine the coordinates of P, Q, R, M, N and S.

P( )
Q( )
R( )
M( )
N( )
S( )

Note
on an axis, the values of the other axes are 0



*use T as the reference point

b) Determine the position vector for T.

$\vec{OT} = [-3-0, 2-0, -2-0]$
$= [-3, 2, -2]$

c) Draw the vector $\overrightarrow{OT}$.

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Example 3: Using the diagram below, complete the following:

a) Write an equation for the $xy-plane$.

$z = 0$

b) Write an equation for the $plane$ containing the points $P, M, Q,$ and T.

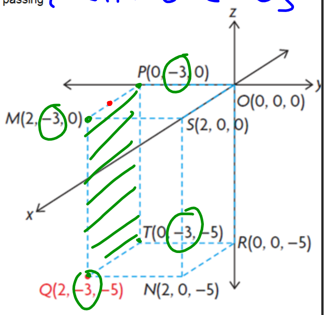
$y = -3$ (on the xz-plane)

c) Write a mathematical description of the set of points in rectangle $PMQT$.

any pt on rect. PMQT can be described by:
$(x, -3, z)$, $\{x \in \mathbb{R} \mid 0 \leq x \leq 2\}$
$\{z \in \mathbb{R} \mid -5 \leq z \leq 0\}$

d) What is the equation of the plane parallel to the $xy-plane$ passing through $R(0, 0, -5)$?

$z = -5$



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PART F: Summary

Key Idea

- In R^2 or R^3 the location of every point is unique. As a result, every vector drawn with its tail at the origin and its head at a point is also unique. These type of vectors are called position vectors.

Need to Know

- In R^2, $P(a, b)$ is a point that is a units from $O(0, 0)$ along the x-axis and b units parallel to the y-axis.
- The position vector $\overrightarrow{OP}$ has its tail located at $(0, 0)$ and its head at $P(a, b)$. $\overrightarrow{OP} = (a, b)$.
- In R^3, $P(a, b, c)$ is a point that is a units from $O(0, 0, 0)$ along the x-axis, b units parallel to the y-axis, and c units parallel to the z-axis. The position vector $\overrightarrow{OP}$ has its tail located at $(0, 0, 0)$ and its head at $P(a, b, c)$. $\overrightarrow{OP} = (a, b, c)$.
- In R^3, the three mutually perpendicular axes form a *right-handed* system.

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