Lesson 4 – Optimization in Economics

Rates of Change in Business and Economics

Business Functions

- The **demand function** or **price function**, $p(x)$, is the price per unit that the market place is willing to pay for a given product or service at a production level of x units.

- The **revenue function** is $R(x)=xp(x)$, where x is the number of units of a product or service sold at a price per unit of $p(x)$.

- The **cost function**, $C(x)$, is the total cost of producing x units of a product or service.

- The **profit function**, $P(x)$, is the profit from the sale of x units of a product or service. $P(x)= R(x) - C(x)$

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Derivatives of Business Functions

Economists use the word **marginal** to indicate the **derivative of a business function**.

$C'(x)$ or $\frac{dC}{dx}$ is the marginal cost function and refers to the instantaneous rate of change of total cost with respect to the number of items produced.

$R'(x)$ or $\frac{dR}{dx}$ is the marginal revenue function and refers to the instantaneous rate of change of total revenue with respect to the number of items sold.

$P'(x)$ or $\frac{dP}{dx}$ is the marginal profit function and refers to the instantaneous rate of change of total profit with respect to the number of items sold.

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Example 1: A company sells 1500 movie DVDs per month at $10 each. Market research has shown that sales will decrease by 125 DVDs per month for each $0.25 increase in price.

a) Determine the demand (or price) function. (Price per unit)
b) Determine the marginal revenue when sales are 1000 DVDs per month.
c) The cost of producing x DVDs is $C(x) = -0.004x^2 + 9.2x + 5000$. Determine the marginal cost when production is 1000 DVDs per month.
d) Determine the actual cost of producing the 1001st DVD.
e) Determine the profit and marginal profit from the monthly sales of 1000 DVDs.

Let n rep. the number of price increases.
Let x rep. the number of DVD's sold.

a) $p = 10 + 0.25n$
$x = 1500 - 125n$
$\frac{125n}{125} = \frac{1500 - x}{125}$

$p(x) = 10 + 0.25\left(\frac{1500-x}{125}\right)$
$p(x) = 10 + \frac{375 - 0.25x}{125}$
$p(x) = \frac{1250 + 375 - 0.25x}{125}$
$p(x) = \frac{1625 - 0.25x}{125}$
$\therefore p(x) = 13 - 0.002x$

b) $R(x) = x(13 - 0.002x)$
$= 13x - 0.002x^2$
$R'(x) = 13 - 0.004x$
$R'(1000) = 13 - 0.004(1000)$
$= 13 - 4$
$= \$9/\text{dvd}$

c) $C(x) = -0.004x^2 + 9.2x + 5000$
$C'(x) = -0.008x + 9.2$
$C'(1000) = -0.008(1000) + 9.2$
$= -8 + 9.2$
$= \$1.20/\text{dvd}$

d) $C(1001) - C(1000)$
$= [0.004(1001)^2 + 9.2(1001) + 5000]$
$- [-0.004(1000)^2 + 9.2(1000) + 5000]$
$= 10201.96 - 10200$
$= \$1.196$

e) $\text{Profit} = R - C$
$P(x) = (13x - 0.002x^2) - (-0.004x^2 + 9.2x + 5000)$
$P(x) = 0.002x^2 + 3.8x - 5000$
$P(1000) = 0.002(1000)^2 + 3.8(1000) - 5000$
$= \$800$

$P'(1000) = 0.004x + 3.8$
$= 0.004(1000) + 3.8$
$= \$7.80/\text{dvd}$

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Example 2
An ice cream shop sells 150 cookies 'n' cream cakes per month at a price of $40 each.
A customer survey indicates that for each $1 decrease in price, sales will increase by five cakes.

a) Determine a revenue function based on the number of price decreases. $\Rightarrow n$
b) Determine the marginal revenue for the revenue function developed in part a).
c) When is this marginal revenue function equal to zero? What is the total revenue at this time? How can the owners use this information?

n = number of price decreases

a) Revenue = (# of cakes sold) x (price per cake)

$\underbrace{\text{(\# of cakes sold)}}_{\text{sales}}$ $\underbrace{\text{(price per cake)}}_{\text{price}}$

$R(n) = (150 + 5n)(40 - n)$

b) Marginal Revenue is given by:

$R'(n) = (5)(40-n) + (150+5n)(-1)$
$= 200 - 5n - 150 - 5n$
$R'(n) = 50 - 10n$

c) set R' (n)= 0, solve for *n*, and then find *R(n)*

$0 = 50 - 10n$
$\frac{10n}{10} = \frac{50}{10}$
$n = 5 \leftarrow \$5$ decrease

$R(5) = (150 + 5(5))(40 - 5)$
$R(5) = (175)(35)$
$= \$6125$

Therefore, Marginal Revenue is zero when there is a $5 decrease in price. Decreasing the price any further would result in **more** *sales,* but **less** *revenue.*

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