Lesson 5 – Operations with Algebraic Vectors in R^2

PART A: Vectors in R^2 expressed in terms of Unit Vectors

We can express the vector $\overrightarrow{OP}$ in terms of unit vectors $\hat{\imath}$ and $\hat{\jmath}$. The unit vectors $\hat{\imath}$ and $\hat{\jmath}$ have a magnitude of 1, and have their tails at the origin. The head of $\hat{\imath}$ is on the $x-axis$ at $(1,0)$ and the head of $\hat{\jmath}$ is on the $y-axis$ at $(0,1)$. In the notation for Cartesian vectors, $\hat{\imath} = [1,0]$ and $\hat{\jmath} = [0,1]$. The unit vectors $\hat{\imath}$ and $\hat{\jmath}$ are the building blocks for Cartesian vectors, and will be referred to as the *Standard Basis Vectors*.

> In two-dimensional space, there are two *standard basis* vectors:
>
> $\hat{\imath} = [1,0]$ $\hat{\jmath} = [0,1]$
>
> In three-dimensional space, there are three *standard basis* vectors
>
> $\hat{\imath} = [1,0]$ $\hat{\jmath} = [0,1]$ $\hat{k} = [0,0,1]$
>
> Standard basis vectors are unit vectors.

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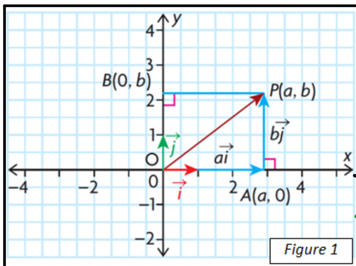

Figure 1

In figure 1, vector $\overrightarrow{OP}$ can now be written in terms of the standard basis vectors as follows:

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB}$
$= a\hat{\imath} + b\hat{\jmath}$

$\overrightarrow{OP} = 3\hat{\imath} + 2\hat{\jmath}$
$[3,2]$

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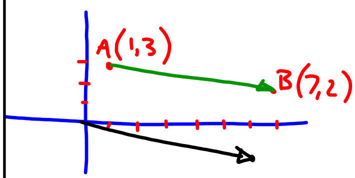
PART B: Magnitude of a Vector

To find the magnitude of a vector, use the formula for the distance between two points

> **Magnitudes in R^2**
>
> If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points, then $|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
>
> (Recall: the vector $\overrightarrow{AB} = [x_2 - x_1, y_2 - y_1]$ is the related position vector)

Example 1: Find the position vector and the magnitude of the vector $\overrightarrow{AB}$ with $A(1,3)$ and $B(7,2)$.



$\overrightarrow{AB} = [7-1, 2-3]$
$\overrightarrow{AB} = [6, -1]$

$|\overrightarrow{AB}| = \sqrt{6^2 + (-1)^2}$
$= \sqrt{36+1}$
$|\overrightarrow{AB}| = \sqrt{37}$

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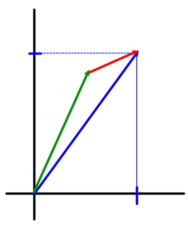
PART C: Adding vectors

To add two Cartesian vectors $\vec{u} = [u_1, u_2]$ and $\vec{v} = [v_1, v_2]$

$\vec{u} + \vec{v} = [u_1 + v_1, u_2 + v_2]$

Example 2: $\vec{u} = [2,1]$ and $\vec{v} = [3,5]$ determine $\vec{u} + \vec{v}$.



$\vec{u} + \vec{v} = [2+3, 1+5]$
$= [5, 6]$

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PART D: Subtracting vectors

To subtract two Cartesian vectors $\vec{u} = [u_1, u_2]$ and $\vec{v} = [v_1, v_2]$

$\vec{u} - \vec{v} = [u_1 - v_1, u_2 - v_2]$

Example 3: $\vec{u} = [2,1]$ and $\vec{v} = [3,5]$ determine $\vec{u} - \vec{v}$.

$\vec{u} - \vec{v} = [2-3, 1-5]$
$= [-1, -4]$

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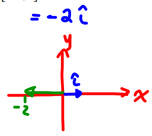
Example 4: Express each vector in terms of the standard basis vectors $\hat{\imath}$ and $\hat{\jmath}$.

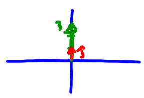
a) $[-2,0]$

$= -2\hat{\imath}$



b) $[0,3]$

$= 3\hat{\jmath}$



Example 5: Express each vector as a position vector.

a) $3\hat{\imath} + 2\hat{\jmath}$

$3\hat{\imath} + 2\hat{\jmath} = [3, 2]$

b) $4\hat{\jmath}$

$= [0, 4]$

Example 6: if $\vec{u} = [-3,5]$ and $\vec{v} = [1,4]$ determine

a) $-3\vec{u} + 4\vec{v}$

=-3[-3, 5] + 4[1, 4]
=[9, -15] + [4, 16]
=[9 + 4, -15 + 16]
=[13, 1]

b) $|-3\vec{v} - 2\vec{u}|$

$= |-3[1,4] - 2[-3,5]|$
$= |[-3,-12] + [6,-10]|$
$= |[3,-22]|$
$= \sqrt{3^2 + (-22)^2}$
$= \sqrt{493}$

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Example 7: If $\vec{a} = [5,-6]$, $\vec{b} = [-7,3]$, and $\vec{c} = [2,8]$, calculate $\left|\vec{a} - 3\vec{b} - \frac{1}{2}\vec{c}\right|$.

(Note: use standard basis vectors to represent the given vectors first.***)***

You don't have to, but it might be easier to use standard basis vectors

$\vec{a} = [5,-6]$ OR

$\vec{b} = [-7,3]$ OR

$\vec{c} = [2,8]$ OR

$$= \left| \underbrace{5\hat{i} - 6\hat{j}}_{\vec{a}} - 3\underbrace{(-7\hat{i} + 3\hat{j})}_{\vec{b}} - \frac{1}{2}\underbrace{(2\hat{i} + 8\hat{j})}_{\vec{c}} \right|$$

$$= \left| 5\hat{i} + 21\hat{i} - \hat{i} - 6\hat{j} - 9\hat{j} - 4\hat{j} \right|$$

$$= \left| 25\hat{i} - 19\hat{j} \right|$$

$$= \left| [25,-19] \right|$$

$$= \sqrt{25^2 + (-19)^2}$$

$$= \sqrt{986}$$

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