CALC 2 Midterm PREP - Topics

Calculus II (University of Ottawa)

March 12, 2024 4:00 PM

$$x_{n+1} = x_n + \Delta x$$

$$y_{n+1} = y_n + f(x_n, y_n)\Delta x$$

$$\frac{dP}{dt} = kP \Rightarrow \text{Radioactive decay}$$

$\uparrow$ RGR

$$\frac{dm}{dt} = km$$

$$\Downarrow$$

$$m(t) = m_0 e^{kt}$$

$$m(t_{HL}) = \frac{1}{2} m_0 \Rightarrow e^{kt_{HL}} = \frac{1}{2}$$

$$k' = \frac{\ln(1/2)}{t_{HL}}$$

$$m_{HL}(t) = m_0 e^{k't}$$

$$= m_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{HL}}}$$

$$\frac{dT}{dt} = \pm k(T - T_{Amb})$$

$$\frac{dQ}{dt} = \frac{dS_{IN}}{dt} - \frac{dS_{out}}{dt}$$

$$\downarrow$$

$$\frac{dS_{IN}}{dt} = \left(C_{\text{salt en Run-off water}}\right) \cdot \left(V_{\text{water entering reservoir}}\right)$$

$$\downarrow$$

$$\frac{dS_{out}}{dt} = \left(C_{\substack{\text{Salt in}\\\text{reservoir}}}\right)\left(V_{\substack{\text{water used}\\\text{per day}}}\right)$$

$$\frac{dQ}{dt} = \frac{q_{s\text{-}OUT}}{V_{OUT}} \cdot V_{IN} - \frac{q_{s\text{-}IN}}{V_{Held}} \cdot V_{used/day}$$

$$\lim_{n\to\infty} \{a_n\} = L \iff \{a_n\} \text{ is Conv.}$$

$$\underset{\text{or } \infty}{\text{DNE}} \iff \text{Div.}$$

$$a_n \leq b_n \leq c_n$$

$$\lim_{n\to\infty} f(a_n) = f(L)$$

$$\{a_n\} \Rightarrow \{P_n\}$$

If $\{P_n\}$ converges to S,
then $\{a_n\}$ converges to S

Telescoping Series

Telescoping Series

↳ Where all but the 1st and last term remain

Geometric Series

↳ $\sum ar^n$

↳ If $|r|<1$, converge to $\frac{a}{1-r}$
else $(1 \geq |r|)$, div.

Initial limit test

$\sum_{1}^{\infty} a_n = L \rightarrow \lim_{n\to\infty} a_n = 0$

TFD

$\lim_{n\to\infty} a_n \neq 0 \rightarrow \sum_{1}^{\infty} a_n \Rightarrow$ Div.

Integral Test

If $f(n)$ is contin., decreasing, and pos$^+$ $\forall n \geq 1$,
then $\int_{0}^{\infty} f(n)\,dn \rightarrow$ Conv. OR div.

same conclusion can be made of the integral

PS

↳ $\sum^{\infty} 1$ (If $p>1 \rightarrow$ Conv.

$$\hookrightarrow \sum_{1}^{\infty} \frac{1}{n^p} \begin{cases} \text{If } p>1 \to \text{Conv.} \\ \text{If } p\le 1 \to \text{Div.} \end{cases}$$

$$\int_{n+1}^{\infty} f(x)\,dx \le R_n \le \int_{n}^{\infty} f(x)\,dx$$

Where $f(x) \Rightarrow a_x = a_n$

Pos+, decreasing, Continuous

$R_n = S - S_n$

Exact Sum

Partial sum

$$\underbrace{F(n) \le R_n}$$

From this we can figure out what N is required to give a certain R_n

$$S_n + \int_{n+1}^{\infty} f(x)\,dx \le S \le S_n + \int_{n}^{\infty} f(x)\,dx$$

$S_n+\int_{n+1} f(x)dx \leq S \leq S_n+\int_{n} f(x)dx$

LCT

Given a_n, b_n

If $\lim_{n\to\infty} \frac{a_n}{b_n} = M$

then $a_n \wedge b_n \Rightarrow$ Conv.

OR $a_n \wedge b_n \Rightarrow$ Div.

Abs. Conv.

If $\sum_{1}^{\infty} |a_n|$ es Conv.,

then a_n es Abs. Conv.

If Abs. Conv.

then Conv.

Con. Conv.

If $\sum_{1}^{\infty} |a_n| \Rightarrow$ Div $\wedge$ $\sum_{1}^{\infty} a_n \Rightarrow$ Conv.

If $\sum_{1} |a_n| \Rightarrow Div \wedge \sum_{1} a_n \Rightarrow Conv.$

then $a_n \Rightarrow$ Con. Conv.

RT

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = N$$

If $\begin{cases} N < 1, & a_n \text{ is Abs. Conv. } \therefore \text{Conv.} \\ N > 1, & a_n \text{ is Div.} \\ N = 1, & \text{more testing is required} \end{cases}$

NRT

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = Q$$

If $\begin{cases} Q < 1, & a_n \text{ is Abs. Conv. } \therefore \text{Conv.} \\ Q > 1, & a_n \text{ is Div.} \\ Q = 1, & \text{more testing is req.} \end{cases}$

[redacted]