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1. Introduction & Review of Functions

Getting ready for MAT~~1320C, Fall 2022~~: MAT B20B FALL 2023

- ☐ Course Syllabus – policies
- ☐ Brightspace resources
- ☐ Textbook
- ☐ Mobius assignments (worth 10% overall) due almost every Saturday.
- ☐ Test 1 (20%) during lecture on Wednesday, October ~~5~~ 4
- ☐ Test 2 (20%) during lecture on Wednesday, November ~~16~~ 15
- ☐ Final Exam (50%), to be scheduled during Exam period (~~Dec. 9–22~~) 8-21

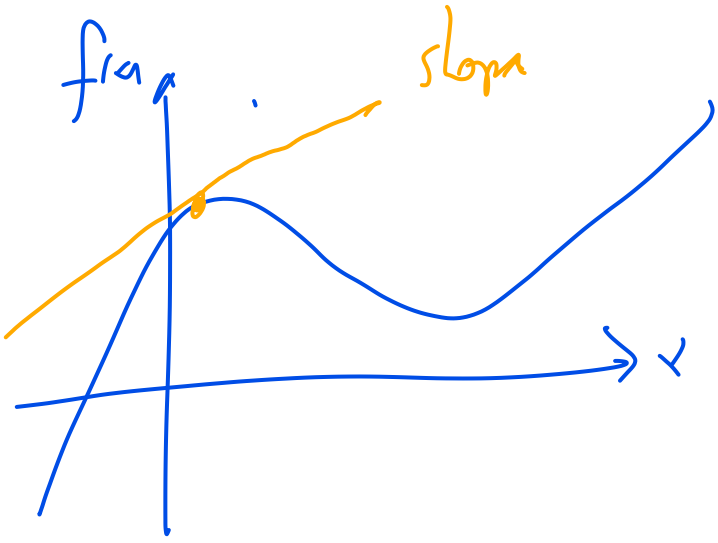
THE BIG PICTURE

Calculus was developed in the mid seventeenth century at about the same time by two scientists: Sir Isaac Newton and Gottfried Leibniz.

The idea was to develop general solutions to two problems:

Differential Calculus
(about 2/3 of MAT1320)

Goal: find the slope of a tangent at any point on a given function



Integral Calculus
(about 1/3 of MAT1320)

Goal: find the area bounded by a function and the x-axis over a given interval

BASIC ALGEBRAIC SKILLS

Keeping The Big Picture in mind, a lot of the problems you will be considering in this course will require you to be adept with some basic algebraic tricks and rules. You need to know how to

- ☐ simplify exponents
- ☐ add, subtract, multiply, divide, and simplify fractional expressions
- ☐ factor or expand algebraic expressions
- ☐ solve equations involving polynomials
- ☐ solve equations involving exponents or logarithms
- ☐ solve equations involving absolute value
- ☐ solve inequalities

We're going to quickly review functions, while highlighting some of the above skills.

FUNCTIONS AND CONCEPTS

Function: A **FUNCTION** f is a rule between two sets, called the **DOMAIN** and the **RANGE**, that assigns to each element x in the domain **exactly one** element, called $f(x)$, in the range.

Traditionally:

Example 1.1. area of circle depends on its radius

Graph: The **GRAPH** of $y = f(x)$ consists of all ordered pairs (coordinates) $(x, f(x))$ such that x belongs to the domain of f.

Vertical Line Test: Actually, any equation in 2 variables can be represented by some curve in the xy-plane, but not every 2-dimensional curve corresponds to the graph of a **function!** The graph of a function must pass the Vertical Line Test.

The most important function:

SYMMETRY AND PERIODICITY OF FUNCTIONS

Even Functions: If $f(-x) = f(x)$ for every x in the domain of f, then f is called an **EVEN** function.

Odd Functions: If $f(-x) = -f(x)$ for every x in the domain of f, then f is called an **ODD** function.

Periodic Functions: If there exists a positive constant p such that $f(x + p) = f(x)$ for every x in the domain of f, then f is called a **PERIODIC** function. The smallest such constant p is called the **period**.

INCREASING/DECREASING

Let I be some interval of the real number line.

A function $y = f(x)$ is called...

- **INCREASING** on the interval I if

- **DECREASING** on the interval I if

Linear Functions:

Horizontal Line.

Vertical Line.

Example 1.2. What is the slope of the line passing through $(4, 1)$ and $(\sqrt{3}, -1)$? Find the equation of this line.

ALGEBRAIC SKILL **Rationalize the denominator** of the slope you found above:

CATALOGUE OF IMPORTANT FUNCTIONS: POLYNOMIALS

Polynomials:

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degree 0: **(Constant Functions)**

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degree 1: **(Linear Functions)**

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degree 2: **(Quadratic Functions)**

Exercise 1.3. The height $h(t)$ (in metres) of a ball t seconds after it has been thrown up from an initial height of h_0 m above the ground, with an initial velocity of v_0 m/s, is given by the equation

$$h(t) = -4.9t^2 + v_0 t + h_0$$

If the ball is dropped from an initial height of 49 m, sketch the graph of ball's height as a function of time. How many seconds does it take for the ball to reach the ground?

ALGEBRAIC SKILL **Solving a nonlinear equation, factoring**:

How long until the ball reaches the ground if the ball is thrown upward with an initial velocity of 8 m/s, from an initial height of 1.6 m above the ground?

The Cubic Equation for finding the roots of $f(x) = ax^3 + bx^2 + cx + d$:

Let

$\Delta = 18abcd - 4b^3d + b^2c^2 - 4ac^3 - 27a^2d^2$

$\Delta_0 = b^2 - 3ac$

$\Delta_1 = 2b^3 - 9abc + 27a^2d$

$C = \sqrt[3]{\dfrac{\Delta_1 \pm \sqrt{{\Delta_1}^2 - 4{\Delta_0}^3}}{2}}$

$\zeta = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

Then the roots of $f(x) = ax^3 + bx^2 + cx + d$ are given by

$$x_k = -\frac{1}{3a}\left(b + \zeta^k C + \frac{\Delta_0}{\zeta^k C}\right) \quad \text{for } k = 0, 1, 2$$

In fact, if $\Delta > 0$, then f has 3 distinct real roots.
If $\Delta = 0$, then f has a multiple root and all its roots are real.
If $\Delta < 0$, then f has 1 real root, and 2 imaginary roots (which are complex conjugates of each other).

$\star\star\star$ Don't worry! You do not *need* to know this! $\star\star\star$

Exercise 1.4. Find the root(s) of $f(x) = x^3 + x^2 + \frac{5}{4}x + 3$.

hint: $(x + \frac{3}{2})$ is a factor; use long division.

ALGEBRAIC SKILL **Long division of Polynomials**:

Rational Functions:

Example 1.5. Find the domain of $g(x) = \dfrac{-x^2 - 4x + 5}{x^2 - 1}$. Does the graph of g have any holes or vertical asymptotes?

ALGEBRAIC SKILL **Solving a nonlinear equation**

Find all x-intercepts of $g(x) = \dfrac{-x^2 - 4x + 5}{x^2 - 1}$

CATALOGUE OF IMPORTANT FUNCTIONS: ROOT FUNCTIONS

Root Functions:

Algebraic Functions:

Example 1.6. Find the domain of $g(x) = \dfrac{x + \sqrt{2 + x^2}}{\sqrt{3 - \frac{2}{x}}}$

ALGEBRAIC SKILL **Solving nonlinear inequalities**

ALGEBRAIC SKILL **Solving a nonlinear equation**

Example 1.7. Find the point(s) of intersection of $f(x) = x$ and $g(x) = \sqrt{x+6}$ and sketch their graphs.

Example 1.8. Find the domain of $f(x) = \sqrt{1-x^2}$.

Example 1.9. What is the domain of $G(x) = \sqrt{x^2}$? What does its graph look like?

ABSOLUTE VALUE — A PIECEWISE-DEFINED FUNCTION

Absolute Value:

Exercise 1.10. Sketch the graph of $g(x) = |x^2 - 5|$. For what values of x is $g(x) = 1$?

ALGEBRAIC SKILL **Solving an equation involving absolute value**