2021 MIDTERM 2 EXAM

1. A 7.0-kg block on a horizontal frictionless surface is attached to a light spring with the spring constant 1.2 kN/m. The block is initially at rest at its equilibrium position when a force of magnitude P acting parallel to the surface is applied to the block, as shown. When the block is 8.0 cm from the equilibrium position, it has a speed of 0.80 m/s. How much work is done on the block by the force P as the block moves the 8.0 cm?

2. The block shown is released from rest when the spring is stretched a distance d. If k = 50 N/m, m = 0.50 kg, d = 10 cm, and the coefficient of kinetic friction between the block and the horizontal surface is equal to 0.25, determine the speed of the block when it first passes through the position for which the spring is unstretched.

3. A ball is suspended by a string that is tied to a fixed point above a wooden block standing on end. The ball is pulled back as shown in the figure above and released. In trial A, the ball rebounds elastically from the block. In trial B, twosided tape causes the ball to stick to the block. In which case is the ball more likely to knock the block over?

4. A car of mass m traveling at speed v crashes into the rear of a truck of mass 2m that is at rest and in neutral at an intersection. If the collision is perfectly inelastic, what is the speed of the combined car and truck after the collision?

5. The only force acting on a 2.0-kg object moving along the x axis is shown. If the velocity vx is +2.0 m/s at t = 0, what is the velocity at t = 4.0 s?

6. A 6.0-kg object moving 2.0 m/s in the positive x direction has a one-dimensional elastic collision with a 4.0-kg object moving 3.0 m/s in the opposite direction. What is the total kinetic energy of the two-mass system after the collision?

7. Three particles are placed in the xy plane. A 30-g particle is located at (3, 4) m, and a 40-g particle is located at (−2, −2) m. Where must a 20-g particle be placed so that the center of mass of the three-particle system is at the origin?

8. Three objects of uniform density − a solid sphere, a solid cylinder, and a hollow cylinder − are placed at the top of an incline (see figure above). They are all released from rest at the same elevation and roll without slipping. Which object reaches the bottom first?

9. A wheel rotates about a fixed axis with an initial angular velocity of 20 rad/s. During a 5.0-s interval the angular velocity increases to 40 rad/s. Assume that the angular acceleration was constant during the 5.0-s interval. How many revolutions does the wheel turn through during the 5.0-s interval?

10. A wheel (radius = 0.25 m) is mounted on a frictionless, horizontal axis. The moment of inertia of the wheel about the axis is 0.040 kg · m 2 . A light cord wrapped around the wheel supports a 0.50-kg object as shown in the figure. The object is released from rest. What is the magnitude of the acceleration of the 0.50-kg object?.

11. An ice skater starts a spin with her arms stretched out to the sides. She balances on the tip of one skate to turn without friction. She then pulls her arms in so that her moment of inertia decreases by a factor of 2. In the process of her doing so, what happens to her kinetic energy?

12. Four identical particles (mass of each = 0.40 kg) are placed at the vertices of a rectangle (2.5 m × 4.0 m) and held in those positions by four light rods which form the sides of the rectangle. What is the moment of inertia of this rigid body about an axis that passes through the mid-points of the shorter sides and is parallel to the longer sides?

13. A 25 kg object is undergoing lightly damped harmonic oscillations. If the maximum displacement of the object from its equilibrium point drops to 1/3 its original value in 1.8 s, what is the value of the damping constant b?

14. A uniform meter stick is freely pivoted about the 0.20-m mark. If it is allowed to swing in a vertical plane with a small amplitude and friction, what is the frequency of its oscillations?

1.

$m = 7kg$

$k = 1.2\,kN/m$
$= 1200\,N/m$

$d_f = 8cm$
$= 0.08m$

$V_f = 0.8m/s$ $V_i = 0m/s$

Conservation of Energy:

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2}mv_i^2 + \frac{1}{2}kd_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kd_f^2$$

Work = Change in Energy:

$$0 = \frac{1}{2}\cdot 7\cdot(0.8)^2 + \frac{1}{2}(1200)(0.08)^2$$

$$W = 2.24J + 3.84J$$

$$W = 6.08J$$

$$W = 6.1J \quad \boxed{C}$$

2.

$k = 50N/m$ $d = 10cm$
$m = 0.50kg$ $= 0.1m$
$\mu_k = 0.25$

Step 1: Potential Energy

$$P_E = \frac{1}{2}kx^2$$

$$P_E = \frac{1}{2}(50)(0.1)^2$$

$$P_E = 0.25J$$

Step 2: Work done by frictional force

$$W = F\cdot d$$

$$W = \mu_k F_n \cdot d$$

$$W = \mu_k \cdot m \cdot g \cdot d$$

$$W = 0.25\cdot 0.5\cdot 9.8\cdot 0.1$$

$$W = 0.1225$$

Step 3: Change in Kinetic Energy:

$$K_E = P_E - W$$

$$\frac{1}{2}mv^2 = 0.25 - 0.1225$$

$$v = \sqrt{\frac{2(0.25-0.1225)}{0.5}}$$

$$*\, v = \sqrt{\frac{2(P_E - W)}{m}}$$

$$V = 0.7141428429\ m/s$$

$$V = 71.41428429\ cm/s$$

$$V = 71cm/s. \quad \boxed{C}$$

3.

Using Impulse:

Trial A:

$$I = mv_f - mv_i$$

$$I = m(-v) - m(v)$$

ball moves backwards after bouncing

$$I = -2mv$$

Trial B:

$$I = mv_f - mv_i$$

$$I = -mv$$

Trial A has a greater impulse, So it is more likely in trial A.

$\boxed{A)}$

4.

$m_{car1} = m$
$m_{car2} = 2m$

Using Conservation of momentum:

Completely inelastic collision:

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)V_f$$

$= 0$, car is initially at rest.

$$V_f = \frac{m_1v_{1i}}{m_1 + m_2}$$

$$V_f = \frac{mv}{m+2m}$$

$$V_f = \frac{1}{3}V \quad \boxed{C}$$

5. At $t=1$, $F(x)=-8N$

$\underline{A_1=-8N}$

from $t=1$ to $t=2$,

$F(x)=-8N$

$A_2=\frac{1}{2}(-8)(2-1)$

$\underline{A_2=-4N}$

from $t=2$ to $t=4$

$F(x)=16$

$A_3=\frac{1}{2}(16)(4-2)$

$\underline{A_3=16N}$

$A_T=-8-4+16$

$A_T=4N$

Using Change of momentum:

$m(V_f-V_i)=4$

$2(V_f-2)=4$ $\quad V_f=\frac{4}{m}+V_i$

$V_f=\frac{4}{2}+2$

$V_f=4\,m/s$ $\boxed{A)}$

6.

$m_1=6kg$ $m_2=4kg$

$V_1=2m/s$ $V_2=3m/s$

Using the Conservation of Kinetic Energy:

$K_{Total}=\frac{1}{2}m_1V_1^2+\frac{1}{2}m_2V_2^2$

$K_{Total}=\frac{1}{2}(6)(2)^2+\frac{1}{2}(4)(3)^2$

$K_{Total}=30J$ $\boxed{A)}$

7. $m_1=30g$ $m_2=40g$ $m_3=20g$

$r_1=(3,4)$ $r_2=(-2,-2)$ $r_3=?$

Using center of mass:

$$X_{cm}=\frac{m_1X_1+m_2X_2+m_3X_3}{m_1+m_2+m_3}$$

$$0=\frac{m_1X_1+m_2X_2+m_3X_3}{m_1+m_2+m_3}$$

$0=(30\cdot3)+(40\cdot-2)+(20X_3)$

$0=90-80+20X_3$

$-10=20X_3$

$\underline{X_3=-0.5}$

$$Y_{cm}=\frac{m_1Y_1+m_2Y_2+m_3Y_3}{m_1+m_2+m_3}$$

$$0=\frac{m_1Y_1+m_2Y_2+m_3Y_3}{m_1+m_2+m_3}$$

$0=(30\cdot4)+(40\cdot-2)+(20Y_3)$

$0=120-80+20Y_3$

$-40=20Y_3$

$\underline{Y_3=-2}$

$(-0.5,-2)$ $\boxed{E)}$

8.

$V_{cm}=\sqrt{\frac{2gh}{1+\frac{I}{mg^2}}}$ $\quad * \, c=\frac{I}{mg^2}$

$V_{cm}=\sqrt{\frac{2gh}{1+c}}$

Moments of inertia:

$I_{sphere}=\frac{2}{5}MR^2$

$I_{cylinder}=\frac{1}{2}MR^2$

$I_{Hollow}=\frac{1}{2}(R_1^2+R_2^2)$

$V_{sphere}=\sqrt{\frac{2gh}{1+2/5}}=\sqrt{\frac{2gh}{1.4}}$

$V_{cylinder}=\sqrt{\frac{2gh}{1+0.5}}=\sqrt{\frac{2gh}{1.5}}$

$V_{Hollow}=\sqrt{\frac{2gh}{1+0.5}}=\sqrt{\frac{2gh}{1.5}}$

$V_{sphere}>V_{cylinder}>V_{Hollow}$

Solid Sphere reaches bottom first.

$\boxed{A)}$

9.

Angular Motion

$a = \frac{(\omega_f - \omega_i)}{t}$

$a = \frac{(40-20)}{5}$

$a = 4 \, m/s^2$

$d = \omega t + \frac{1}{2} a t^2$

$d = 20(5) + \frac{1}{2}(4)(5)^2$

$d = 150$ rad

$d = \frac{150}{2\pi}$

$d = 23.87324146$

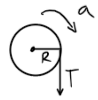
$d = 24$ Revolutions

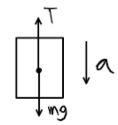
$\boxed{B)}$

10.

$r = 0.25 m$ $\quad M = 0.50 kg$

$I = 0.040 kg \cdot m^2$





$\Sigma T = I\alpha$

$RT = I\left(\frac{\alpha}{R}\right)$

$T = I\left(\frac{\alpha}{R^2}\right)$

$T = \frac{0.040 a}{0.25^2}$

$T = 0.64 a$

$\Sigma F = mg - T$

$mg - T = ma$

$0.5(9.81) - 0.64a = 0.5a$

$0.5(9.81) = 1.14a$

$a = \frac{4.905}{1.14}$

$a = 4.30 \, m/s^2$

$\boxed{C)}$

11.

$K_E = \frac{L^2}{2I}$ Decreases by 2

K_E is inversely proportional to I, therefore her kinetic energy doubles.

$\boxed{B)}$

12.

$m = 0.4 kg$

$I = mr^2$

Rectangle, so:

$I = 4mr^2$

$I = 4(0.4)\left(\frac{2.5}{2}\right)^2$

$I = 2.5 \, kg \cdot m^2$ $\boxed{C)}$

13.

$m = 25 kg$

$\frac{1}{3}$ its original value $\left(A = \frac{1}{3} A_0\right)$

$t = 1.8 s$

$A = A_0 e^{-\alpha t}$

$\frac{1}{3} A_0 = A_0 e^{-\alpha t}$

$e^{-\alpha t} = \frac{1}{3}$

$\ln e^{-\alpha t} = \ln \frac{1}{3}$

$-\alpha t = \ln \frac{1}{3}$

$a = \frac{\ln \frac{1}{3}}{-t}$

$a = \frac{\ln \frac{1}{3}}{-1.8}$

$a = 0.6103401604$

$a = \frac{b}{2m}$

$b = a 2m$

$b = 0.6103401604 \cdot 2(25)$

$b = 30.51700802$

$b = 31 \, kg/s$ $\boxed{A)}$

14.

$L = 0.20\text{-m}$

$$L_{eq} = \frac{I}{md}$$

Using the parallel axis theorem:

$$I = I_{cm} + md^2$$

$$I = \frac{mL^2}{12} + m(0.3L)^2$$

$$I = ML^2\left(\frac{1}{12} \cdot 0.09\right)$$

$$I = 0.173ML^2$$

$$L_{eq} = \frac{0.173ML^2}{M \cdot 0.3L}$$

$$L_{eq} = 0.578L$$

$$L_{eq} = 0.578 \cdot 1.0m$$

$$L_{eq} = 0.578\ m$$

$$T = 2\pi\sqrt{\frac{L_{eq}}{g}}$$

$$T = 2\pi\sqrt{\frac{0.578}{9.8}}$$

$$T = 1.53s$$

$$f = \frac{1}{T}$$

$$f = 0.66\ Hz \quad \boxed{B)}$$