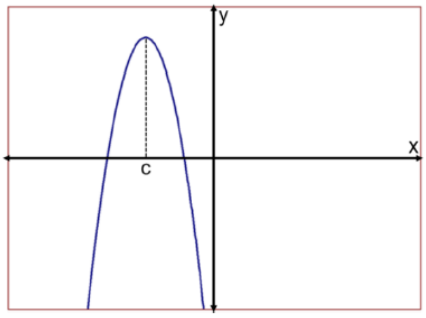
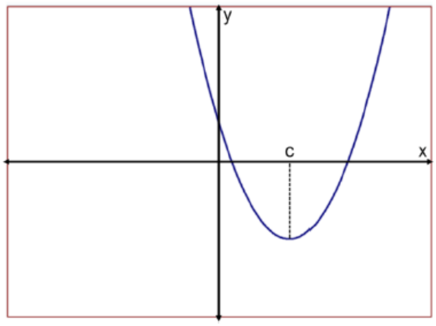
Lesson 2 - Maxima, Minima and Critical Points

PART A: Terminology



Local Maximum – a point is a local max if the *y*-coordinates of all the points in the vicinity are less than the *y*-coordinate of the point

If $f'(x)$ changes from positive to zero to negative as x increases from $x < c$ to $x > c$ then $[c, f(c)]$ is a local max and c is a local maximum value.



Local Minimum – a point is a local minimum if the *y*-coordinates of all the points in the vicinity are greater than the y-coordinates of the point.

If $f'(x)$ changes from negative to zero to positive as x increases from $x < c$ to $x > c$ then $[c, f(c)]$ is a local min and c is a local minimum value.

Note: Local maximum and minimum values of a function are also called local extreme values, local extrema, or turning points.

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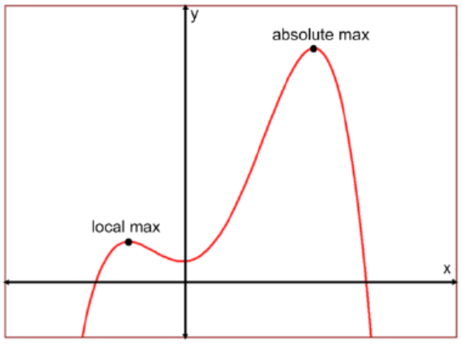
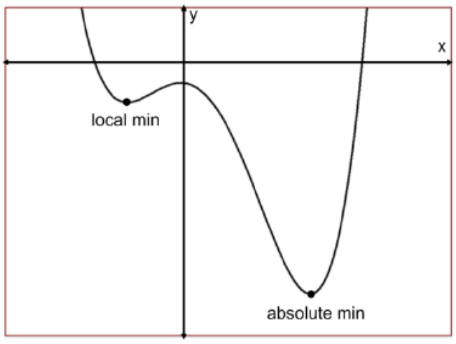


Absolute Maximum – a function has an absolute maximum at c if $f(c) \geq f(x)$ for all values of x in the domain. The maximum value of the function is $f(c)$.



Absolute Minimum – a function has an absolute minimum at c if $f(c) \leq f(x)$ for all values of x in the domain. The minimum value of the function is $f(c)$.

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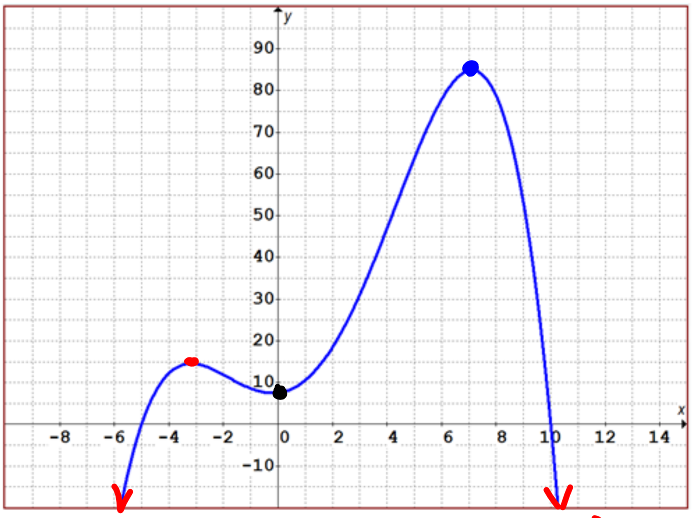
Example 1: For the following graph, answer the questions below.



a) Identify the local maximum points. $(-3, 15)$

b) Identify the local minimum points. $(0, 8)$

c) Identify the absolute maximum and absolute minimum values.

- Abs max $(7, 85)$
- no abs min

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Critical Number – is a number, c, in the domain of $f(x)$ such that $f'(c) = 0$ or $f'(c)$ is undefined. If c is a critical number, $[c, f(c)]$ is a critical point and usually corresponds to a local or absolute extrema.

Example 2: Find the critical numbers for each function.

a) $f(x) = 2x^2 + 6x - 5$

$f'(x) = 4x + 6$

$0 = 4x + 6$

$\frac{-6}{4} = 4x$

$-\frac{3}{2} = x$

b) $y = x^3 - 5x^2 - 8x + 2$

-24
-10

$y'(x) = 3x^2 - 10x - 8$

$0 = 3x^2 - 12x + 2x - 8$

$0 = 3x(x-4) + 2(x-4)$

$0 = (3x+2)(x-4)$

$x = -\frac{2}{3}$ $x = 4$

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Example 3: Determine the absolute and local extreme values of the function $y = x^3 - 6x^2 + 8x$ on the interval $0 \le x \le 6$.

$f'(x) = 3x^2 - 12x + 8$

Q.F. $x_1 = 0.85$

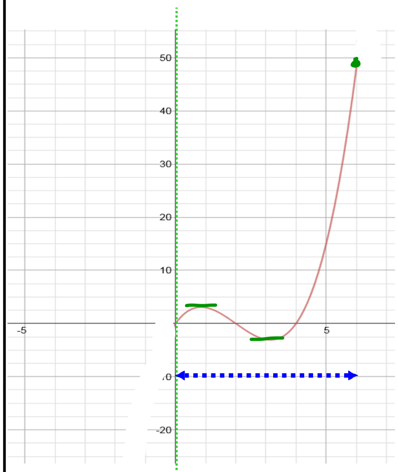
$x_2 = 3.15$

$f(0) = 0$

$f(6) = 48$

	$0<x<0.85$	0.85	$0.85<x<3.15$	3.15	$3.15<x<6$
$f'(x)$	+	0	−	0	+
$f(x)$	↗	↔	↘	↔	↗

we have extreme values at $x=0.85$ and $x=3.15$



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Example 4: Use the critical points to sketch the function $h(x) = x^4 - 8x^3 + 16x^2 - 5$.

*Facing up

$h'(x) = 4x^3 - 24x^2 + 32x$

$= 4x(x^2 - 6x + 8)$

$= 4x(x-4)(x-2)$

$x = 0$ $f(0) = -5$

$x = 2$ $f(2) = 11$

$x = 4$ $f(4) = -5$

	$x<0$	0	$0<x<2$	2	$2<x<4$	4	$x>4$
$4x$	−		+		+		+
$x-4$	−		−		−		+
$x-2$	−		−		+		+
$h'(x)$	−	0	+	0	−	0	+
$h(x)$	↘	↔	↗	↔	↘	↔	↗

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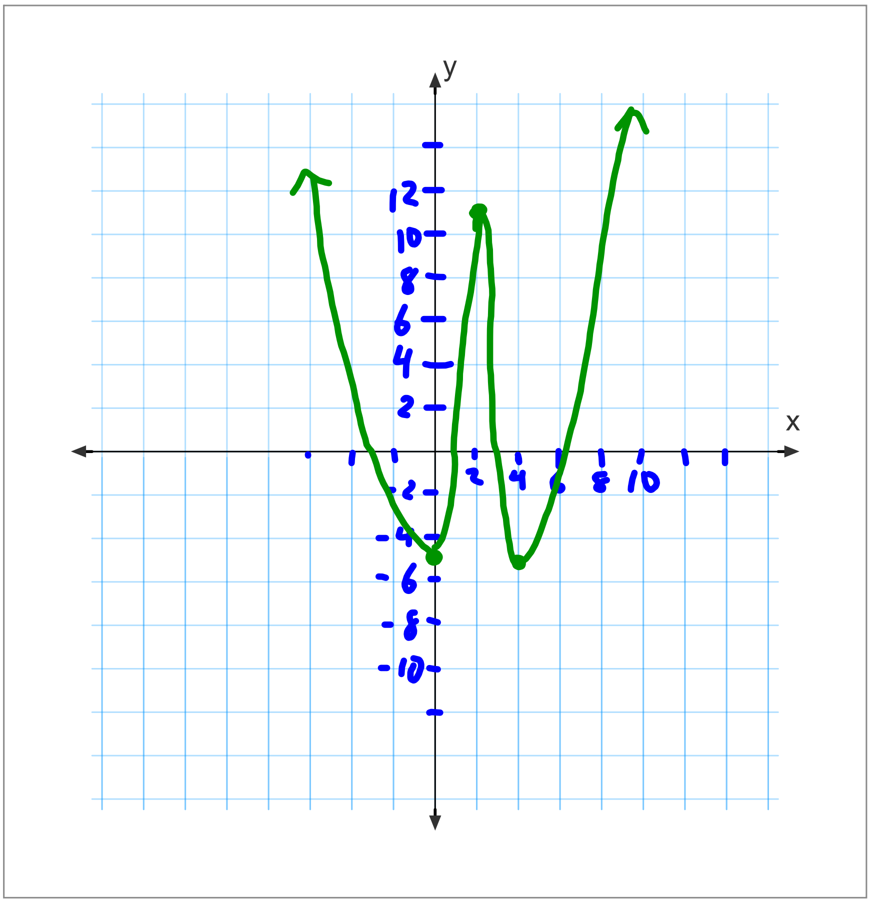

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Example 5: The monthly revenue in dollars from selling a total of x pairs of headphones is given by the function $R(x) = 2400x - 0.2x^2$.

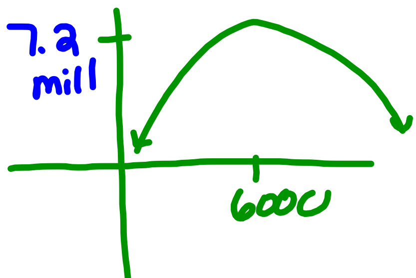
a) Determine the number of pairs of headphones that would need to be sold to maximize revenue.

$R'(x) = 2400 - 0.4x$

$0 = 2400 - 0.4x$

$\frac{0.4x}{0.4} = \frac{2400}{0.4}$

$x = 6000$



∴ 6000 headphones will maximize revenue.

	$0<x<6000$	6000	$x>6000$
$f'(x)$	+	0	−
$f(x)$	↗	↔	↘

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b) Determine the maximum revenue.

$R(6000) = 2400(6000) - 0.2(6000)^2$

$= \$7.2$ million

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