GNG 1105E – Engineering Mechanics

CHAPTER S2 – FORCE SYSTEMS

Assigned readings (S2B)

2/7 Rectangular components (3-D)

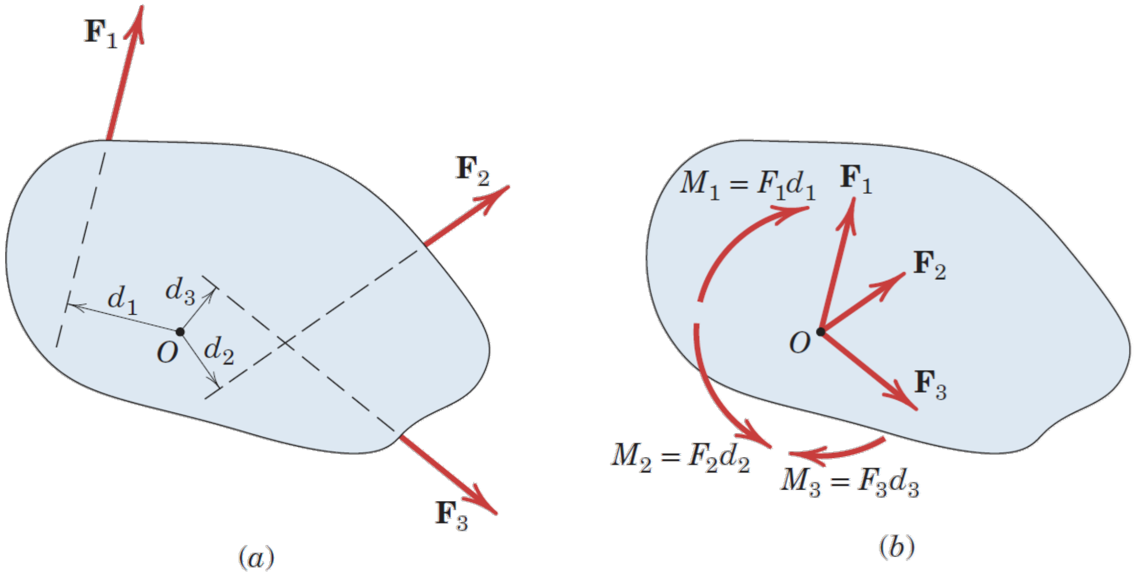
2/8 Moment and couple

2/9 Resultants (3-D)

2/6 Resultants

Finding the resultant and its line of action:

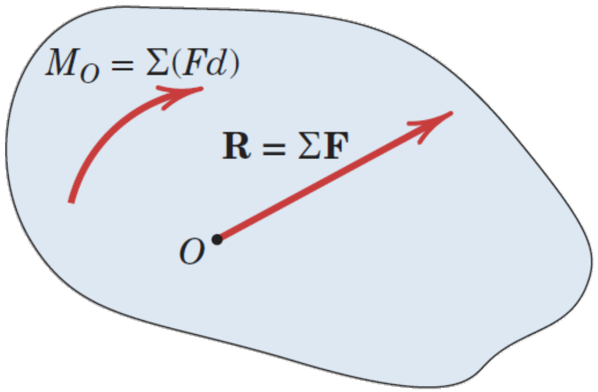
1. Move all the forces to a convenient reference point. Remember to include a couple for each force to ensure that the net tendency to **translate** and **rotate** is equivalent



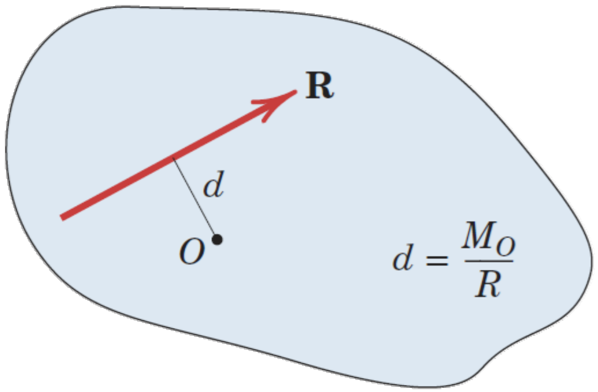
2/6 Resultants

2. Add all the forces to find the **resultant force** and add all couples to find the **resultant couple**. This will reduce the system of forces to an **equivalent force-couple system**

3. Find the line of action of the single force that produces the same moment about point O.



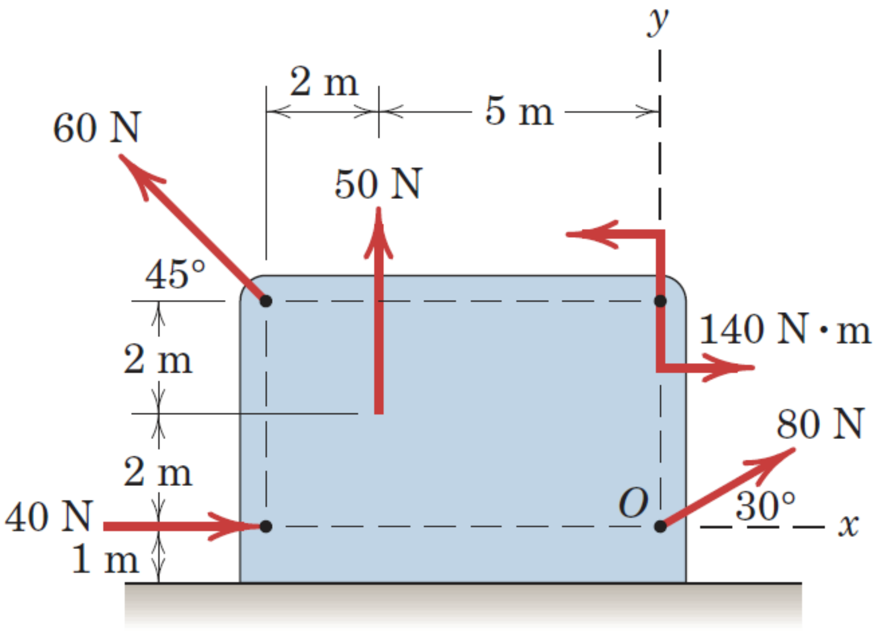
(c)



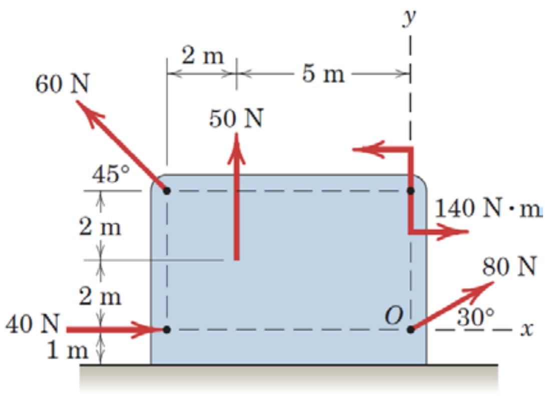
(d)

Sample problem 2/9

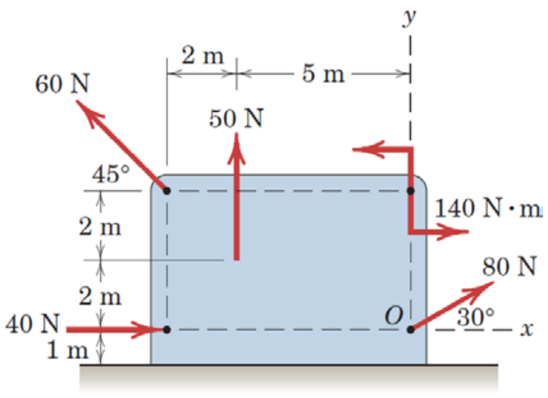
Determine the resultant of the four forces and one couple which act on the plate shown.



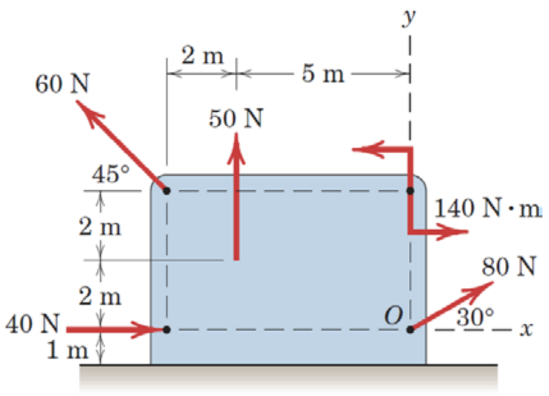
Sample problem 2/9



Sample problem 2/9



Sample problem 2/9

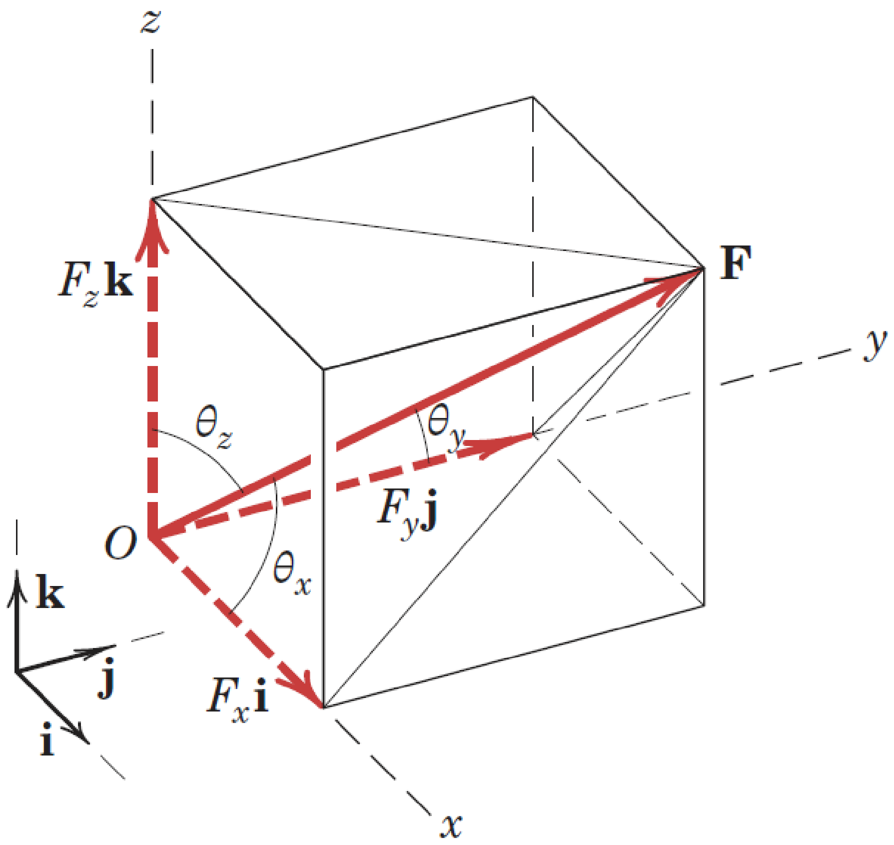


2/7 Rectangular components (3-D)

$$F_x = F\cos\theta_x \qquad F = \sqrt{F_x^{\,2} + F_y^{\,2} + F_z^{\,2}}$$

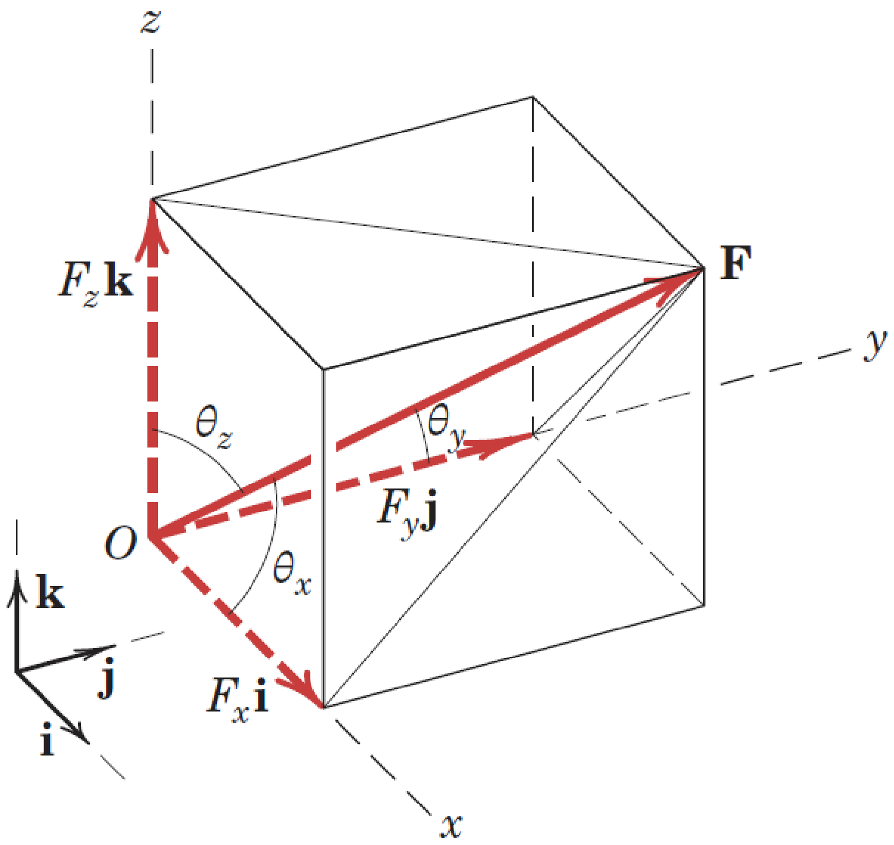
$$F_y = F\cos\theta_y \qquad \mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

$$F_z = F\cos\theta_z \qquad \mathbf{F} = F(\mathbf{i}\cos\theta_x + \mathbf{j}\cos\theta_y + \mathbf{k}\cos\theta_z)$$



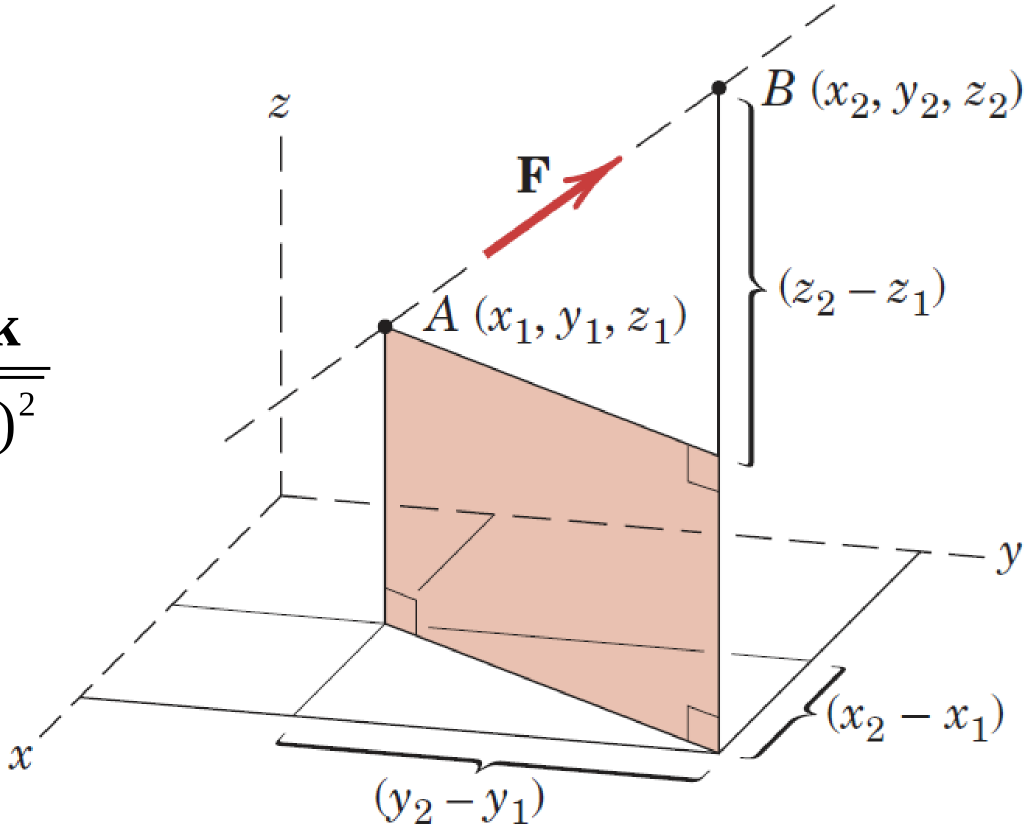
2/7 Rectangular components (3-D)

$$\mathbf{F} = F\mathbf{n}_f = F(\cos\theta_x\mathbf{i} + \cos\theta_y\mathbf{j} + \cos\theta_z\mathbf{k})$$



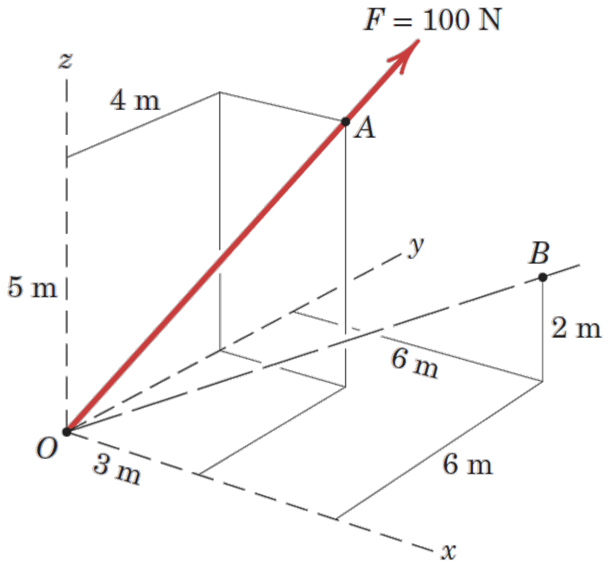
2/7 Rectangular components (3-D)

$$\mathbf{F} = F\mathbf{n}_F = F\frac{\overrightarrow{AB}}{\overline{AB}} = F\frac{(x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$



Sample problem 2/10

A force **F** with a magnitude of 100 N is applied at the origin *O* of the axes *x*-*y*-*z* as shown. The line of action of **F** passes through a point *A* whose coordinates are 3 m, 4 m, and 5 m. Determine the *x*, *y*, and *z* scalar components of **F**.



2/8 Moment and couple (3-D)

Moments in three dimensions

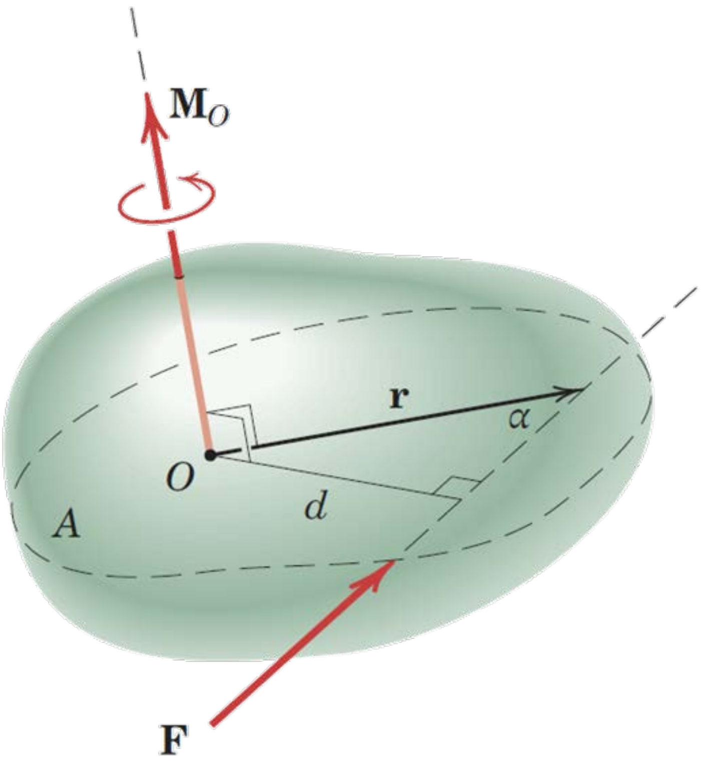
- Operate identically to moments in two dimensions
- More complicated to visualize

Scalar approach: $M_O = Fd$

- More difficult to accomplish
- Lacks sign information

Vector approach: $\mathbf{M_O} = \mathbf{r} \times \mathbf{F}$

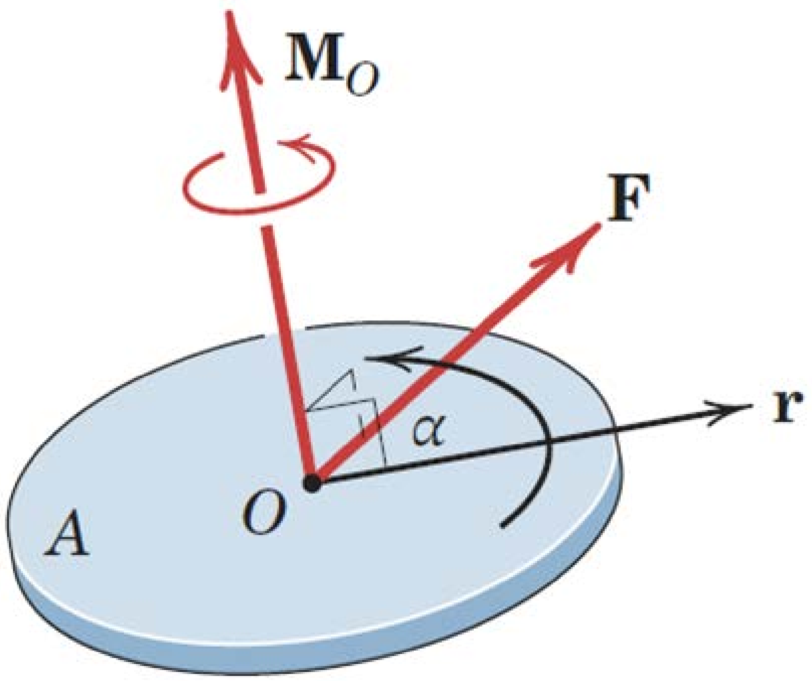
- Easy to compute
- Sign information is included automatically



2/8 Moment and couple (3-D)

Direction and sense of the moment:

- Established by right-hand rule
- Perpendicular to the plane containing **r** and **F**



$$\mathbf{i}\times\mathbf{j}=\mathbf{k} \quad \mathbf{j}\times\mathbf{k}=\mathbf{i} \quad \mathbf{k}\times\mathbf{i}=\mathbf{j}$$

$$\mathbf{j}\times\mathbf{i}=-\mathbf{k} \quad \mathbf{k}\times\mathbf{j}=-\mathbf{i} \quad \mathbf{i}\times\mathbf{k}=-\mathbf{j}$$

$$\mathbf{i}\times\mathbf{i}=\mathbf{j}\times\mathbf{j}=\mathbf{k}\times\mathbf{k}=\mathbf{0}$$

2/8 Moment and couple (3-D)

Calculating cross products via determinant:

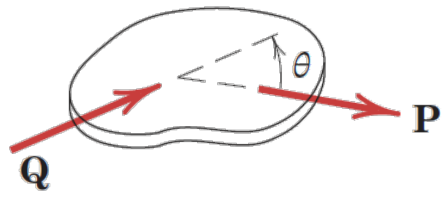
$$\mathbf{P}\times\mathbf{Q}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ P_x & P_y & P_z\\ Q_x & Q_y & Q_z\end{vmatrix}$$

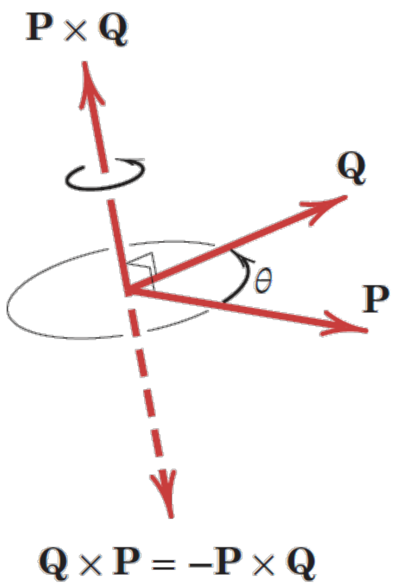
2/8 Moment and couple (3-D)

$$\mathbf{P} \times \mathbf{Q} = (P_x\mathbf{i} + P_y\mathbf{j} + P_z\mathbf{k}) \times (Q_x\mathbf{i} + Q_y\mathbf{j} + Q_z\mathbf{k})$$

$$=(P_yQ_z - P_zQ_y)\mathbf{i} + (P_zQ_x - P_xQ_z)\mathbf{j} + (P_zQ_y - P_yQ_x)\mathbf{k}$$

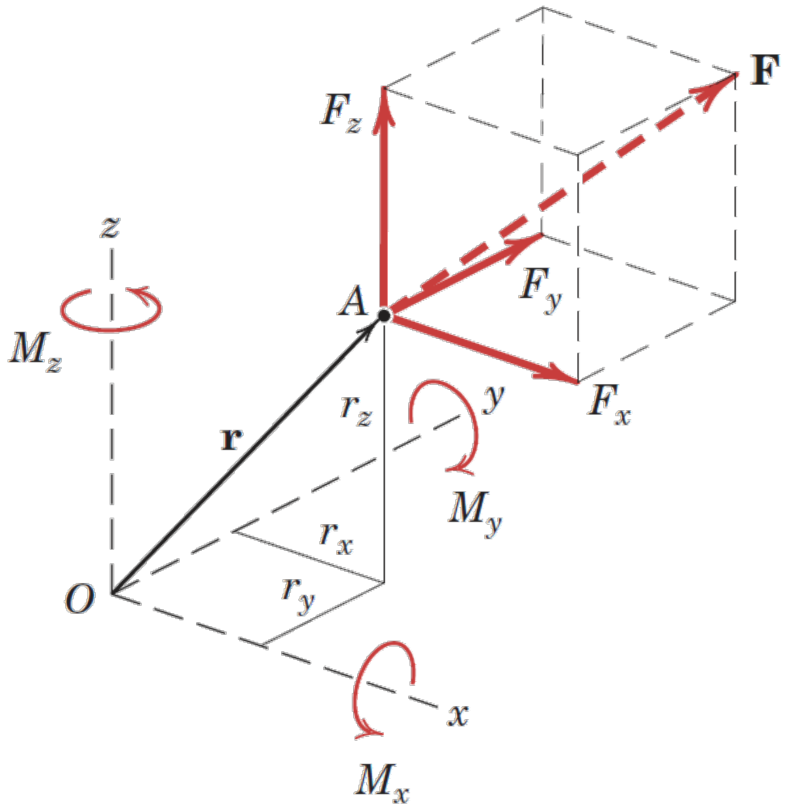
$$|\mathbf{P} \times \mathbf{Q}| = PQ \sin\theta$$





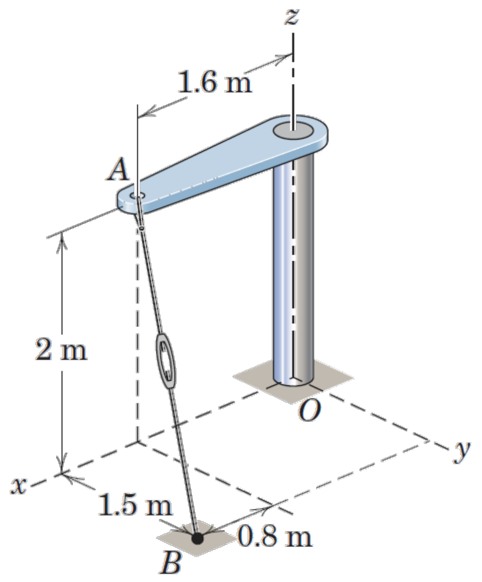
2/8 Moment and couple (3-D)

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

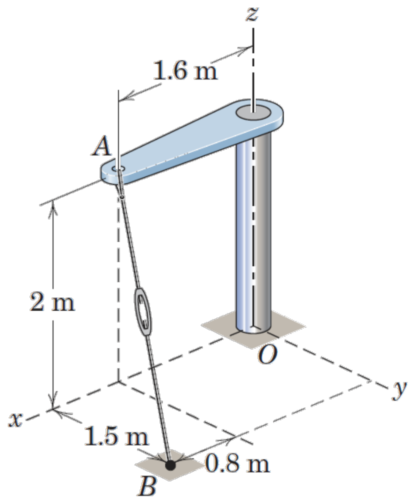


Sample problem 2/12

The turnbuckle is tightened until the tension in cable *AB* is 2.4 kN. Determine the moment about point *O* of the cable force acting on point *A* and the magnitude of this moment.



Sample problem 2/12



2/9 Resultants (3-D)

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \sum \mathbf{F}$$

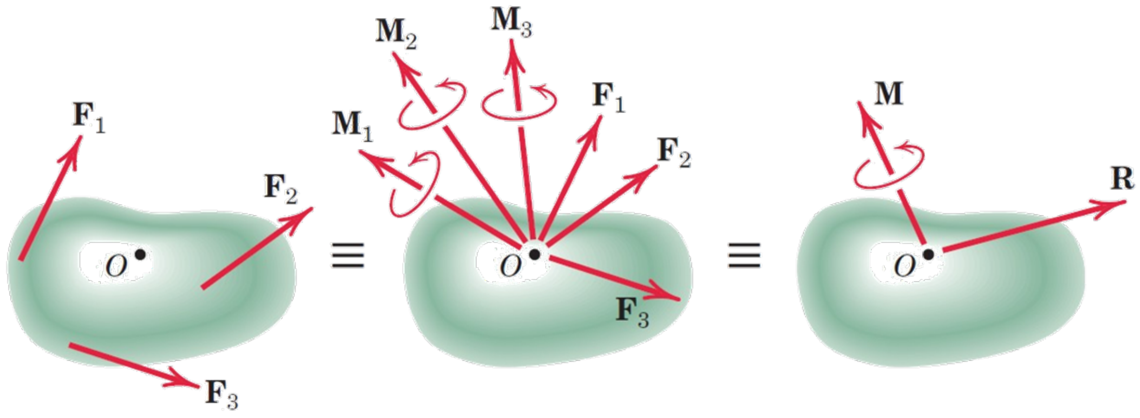
$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \cdots = \sum (\mathbf{r} \times \mathbf{F})$$

$$R_x = \sum F_x \qquad R_y = \sum F_y \qquad R_z = \sum F_z$$

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2 + (\sum F_z)^2}$$

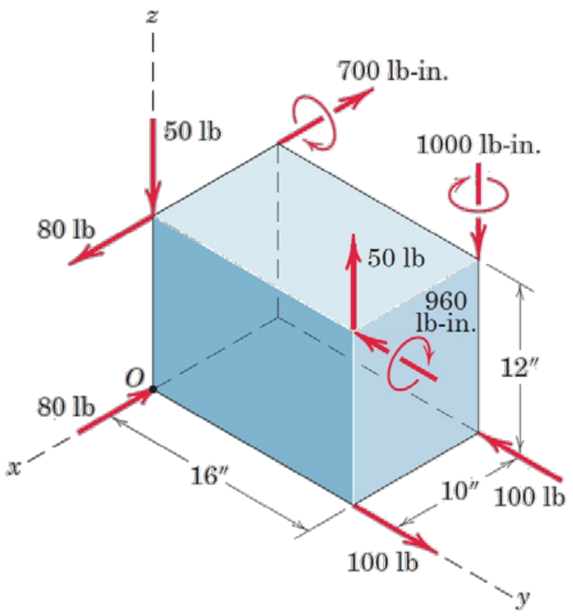
$$\mathbf{M}_x = \sum (\mathbf{r} \times \mathbf{F})_x \qquad \mathbf{M}_y = \sum (\mathbf{r} \times \mathbf{F})_y \qquad \mathbf{M}_z = \sum (\mathbf{r} \times \mathbf{F})_z$$

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2}$$



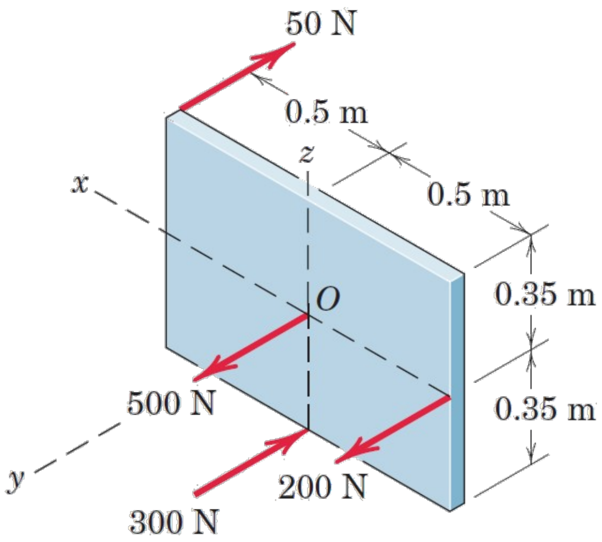
Sample problem 2/16

Determine the resultant of the force and couple system which acts on the rectangular solid.



Sample problem 2/17

Determine the resultant of the system of parallel forces which act on the plate. Solve with a vector approach.



Sample problem 2/17