Calculus II Practice Exam

Calculus II (University of Ottawa)

Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

MAT 1722 C – Examen Final

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7 décembre, 2014

Nom :_______________________________ Prénom :_______________________

d'étudiant :____________________ # de siège :__________

Prenez le temps de lire tout le document avant de commencer et lisez chaque question attentivement. N'oubliez pas que certaines questions vallent plus de point que d'autre. Notez les questions que vous vous sentez confiant de répondre et répondez à ceux-ci en premier : vous ne devez pas répondre les questions dans l'ordre qu'ils sont écrite.

- La durée de cet examen est **180 minutes**.
- Cet examen est divisé en deux parties :
  - Partie A comprends 10 questions à choix multiples qui valent chacune 4 points. Encercler la réponse correcte. Les réponses numériques sont arrondies à la dernière décimale indiquée. Il n'y aura aucun point partiel pour les questions à choix multiples. Il y a des pages supplémentaires à la fin de l'examen pour travailler les questions à choix mutliples. Ces pages ne seront pas corrigées mais il doivent être remisent á la fin de l'examen.
  - Partie B comprends 6 questions à développement qui valent chacune 10 points. La bonne réponse nécessite une justification écrite lisiblement et logiquement; vous devez me convaincre que vous savez pourquoi votre solution est la bonne. Dessinez des boites autours de vos réponses finales.
- Utilisez l'espace spécifié pour répondre à chacune des questions. Si jamais l'espace ne vous suffit pas ou que vous utilisez l'endos de la page, veuillez l'indiquer clairement où se trouve votre réponse ainsi que la suite du développement, s'il y a lieu.
- Cet examen est á livre fermé et vos notes de cours ne seront pas allouées. L'utilisation de téléphone cellulaire, pagette ou tout autre appareil qui peut transmettre ou stocker de l'information **ne sont pas permis.**
- Seules les calculatrices approvées par la Faculté des Sciences (TI-30X, TI-34X, Casio FX-260X et Casio FX-300X) seront permises .

Bonne Chance!

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1. [**4 points**] What is the value of the improper integral $\displaystyle\int_1^\infty 2xe^{-x^2}\,dx$?

A) 0 B) $4e^{-2}$ C) $3e^{-2}$ D) e^{-1} E) $3e$ F) ∞

2. [**4 points**] Determine the volume of the solid whose base is the region in the xy-plane bounded by $y = x$ and $y = x^2$ and with cross-sections perpendicular to the x-axis being squares.

A) 1/30 B) 1/15 C) 2/3 D) 3/4 E) 5/6 F) 5/4

3. [**4 points**] Use Euler's Method with step size $h = 0.1$ to estimate $y(0.2)$, where y is the solution of the initial value problem $y' = y^2 + x,\ y(0) = 1$.

A) 1.573 B) 2.589 C) 1.231 D) 2.623 E) 3.634 F) 2.653

4. [**4 points**] A population of bacteria starts with 1000 individuals and grows at a rate proportional to its size. After one hour, there are 1500 individuals in the population. After how many hours will the population be 2000?

A) 3.64 B) 1.68 C) 1.72 D) 3.76 E) 1.71 F) 3.84

5. [4 points] Determine $y(1)$, where $y(t)$ is the solution of the separable differential equation

$$\frac{dy}{dt} = 2ty, \quad y(0) = 1.$$

A) $\sqrt{3}$ B) e C) $\sqrt{8}$ D) $\ln(3)$ E) $\ln(5)$ F) $e + 1$

6. [4 points] Determine the first three nonzero terms of the Maclaurin series of $(1 - x)e^x$.

A) $1 - \frac{1}{2}x^2 - \frac{1}{3}x^3$ B) $1 - \frac{1}{2}x^2 - \frac{4}{3}x^3$ C) $1 - \frac{1}{2}x^2 + \frac{2}{3}x^3$

D) $1 - \frac{1}{2}x - \frac{1}{3}x^2$ E) $1 - \frac{1}{2}x - \frac{4}{3}x^2$ F) $1 - \frac{1}{2}x + \frac{2}{3}x^2$

7. [**4 points**] Find the power series representation for $f(x) = \dfrac{x}{x^2+16}$ and determine the radius of convergence R.

A) $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{16^{n+1}}, \quad R = 4$ B) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n+1}}{16^{n+1}}, \quad R = 2$ C) $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{4^{n+1}}, \quad R = 4$

D) $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{n}}{16^{n}}, \quad R = 1$ E) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n+1}}{16^{n}}, \quad R = 4$ F) $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+2}}{4^{n+1}}, \quad R = 1$

8. [**4 points**] The radius r and height h of a cylinder vary as functions of time t. Set $S = 2\pi rh + 2\pi r^2$, the function giving its surface area. Given that at time $t = 2$ we have

$$r = 5 \text{ cm}, \quad h = 20 \text{ cm}, \quad \frac{dr}{dt} = 1 \text{ cm/s} \quad \text{and} \quad \frac{dh}{dt} = 2 \text{ cm/s},$$

determine the rate of change $\dfrac{dS}{dt}$ of the surface area at that instant.

A) 180π B) 100π C) 80π D) 40π E) 60π F) 120π

9. [**4 points**] Match each function with the corresponding series expansion.

$\dfrac{1}{1+x^3}$ •	• $\displaystyle\sum_{n=0}^{\infty}(-1)^n x^{3n}$
$\cos(4x)$ •	• $\displaystyle\sum_{n=1}^{\infty} n^3 x^{n-1}$
$\displaystyle\int_0^x \frac{1}{1-t}dt$ •	• $\displaystyle\sum_{n=0}^{\infty}\frac{(-16)^n x^{2n}}{(2n)!}$
$\displaystyle\frac{d}{dx}\left(\sum_{n=0}^{\infty} n^2 x^n\right)$ •	• $\displaystyle\sum_{n=1}^{\infty}\frac{x^{n+1}}{n+1}$

10. [**4 points**] Find the directional derivative of the function $f(x, y) = 2xy - 3y^2$ at the point $(5, 5)$ in the direction of the vector $\vec{v} = \langle 4, 3\rangle = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$.

A) 2 B) -6 C) 10 D) 3 E) 9 F) -4

11. [10 points] Let $\mathcal{R}$ denote the region of the plane which is bounded by the curve $y = x^3$, the horizontal line $y = 8$ and the y-axis. Set $\mathcal{S}$ to be the solid of revolution obtained by rotating the region $\mathcal{R}$ about the vertical line $x = -3$.

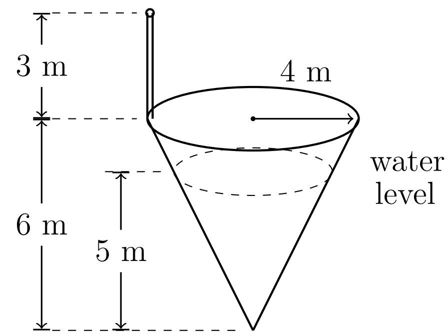
Calculate the volume of $\mathcal{S}$ by the disc/washer method or the method of cylindrical shells (your choice). **Sketch the region $\mathcal{R}$, the section of the solid $\mathcal{S}$ which intersects the xy-plane, and an element of volume (disc/washer or cylinder) with its dimensions.**

12. **[10 points]** A reservoir is built in the form of a right circular cone pointing down, as in the figure to the right.

Its height is 6 m and the radius at the base (that is, the top of the reservoir) is 4 m. It is full of water to a depth of 5 m.

We wish to pump all of this water to a height of 3 m above the reservoir.

Denote by x the height in meters measured **from the bottom of the reservoir**.



(a) What is, at first approximation, the volume ΔV of the layer of water between the heights x and $x + \Delta x$?

Response: $\Delta V \;\cong$

(b) What is, at first approximation, the work ΔW required to pump that layer of water to a height of 3 m above the reservoir? Recall that the density of water is $1000\ \mathrm{kg/m^3}$, and that $g \cong 9.8\ \mathrm{m/s^2}$.

Response: $\Delta W \;\cong$

(c) What is, in Joules, the work required to pump all the water in the reservoir to a height of 3 meters above it?

13. **[10 points]** In a certain cistern, there are 400 L of brine containing 25 kg of dissolved salt. Another brine solution which contains 0.3 kg/L of salt is introduced into the cistern at a rate of 4 L/min. The solution is kept well-mixed and, at the same time, is emptied at such a rate that the volume is unchanged. Let $Q(t)$ denote the quantity (in kg) of salt within the cistern at time t (in minutes).

(i) Find an initial value problem for $Q(t)$.

(ii) Solve it. Be sure to explain your process.

(ii) How much salt is in the cistern after one hour?

14. (a) [7 points] Determine whether the series $\sum_{n=1}^{\infty}(-1)^n \frac{\ln n}{n}$ is absolutely convergent, semi-convergent or divergent.

(b) [3 points] We wish to calculate $s = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{1+n}{n^2}$ with error no worse than 0.1. What is the number, k, of terms which we must add in order to be certain that the error in approximating s by $s_k = \sum_{n=1}^{k}(-1)^{n+1}\frac{1+n}{n^2}$ satisfies $|\text{error}| \leq 0.1$?

15. [10 points] Consider the power series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n(x-2)^n}{n}$.

Determine its radius and interval of convergence. Take care to justify the convergence or divergence of the series at the endpoints of the interval.

16. [10 points] Consider the function $f(x,y) = \dfrac{1}{x^2+4y^2-1}$.

(a) Determine the domain of the function and skecth it in the xy-plane.

(b) Find the equation of the tangent plane to the surface at the point where $(x,y) = (2,1)$.

(c) Sketch the level curves $f(x,y) = C$ for $C = 1/3, -1, 1/15$.

(d) In which direction does f change the fastest at the point where $(x,y) = (2,1)$? What is the maximal rate of change at that point?

Page supplémentaire pour vos calculs

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