
12. Integration by Substitution

Lec 11 mini review.

FTC Suppose f is continuous on $[a, b]$.

FTC1 If $g(x) = \int_a^x f(t)dt$, then $g'(x) = f(x)$.

FTC2 $\int_a^b f(x)dx = F(b) - F(a)$ where F is any antiderivative of f (that is, $F' = f$).

◇ **indefinite integral vs. definite integral**

◇ **the Net Change Theorem for the integral of a rate of change:** $\int_a^b F'(x)dx = F(b) - F(a)$

“UNDOING” THE CHAIN RULE

- If we think of differentiation as an operation on the set of differentiable functions, then integration (informally, “anti-differentiation”) does the opposite.
- What does “undoing” the Chain Rule look like?
- There are many different instances of the Chain Rule...

GUIDELINES FOR SUBSTITUTION

0. Inspect the integrand. Is it familiar because it's a function's derivative (give or take a constant multiple) ? If so, you should know what to do.
1. If the integrand is not an "obvious" familiar function's derivative, then consider the possibility that the integrand may be the aftermath of a Chain Rule:

2. If you see a potential "inner" function $g(x)$, then call it u (you might be wrong, but it doesn't hurt to try).

Now, compute your substitution ingredients:

3. Rewrite your integral in terms of u and du .

Note. The "old" variable (let's say it was x) should completely cancel. We should get a new (and hopefully easier) integral with respect to the "new" variable u .

4. Evaluate the new integral. Then don't forget to rewrite your answer in terms of the "old" variable.

Example 12.1. $\int 2xe^{x^2} dx.$

Example 12.2. $\int x^2 e^{x^3-9} dx.$

Example 12.3. $\int \sin(2t) dt.$

Example 12.4. $\int \tan(\theta) d\theta.$

Example 12.5. $\int \sec(x) dx.$

Example 12.6. $\int_{-\pi/2}^{2\pi} \cos(x) \sin^3(x) dx.$

Substitution Methods for Definite Integrals.

1. Solve the indefinite integral first, and completely (in terms of “old” variable), then subtract at the limits of integration.
2. Substitute the limits of integration at the same time as you perform the substitution.

Example 12.7. $\int_1^e \frac{\ln x}{x} dx.$

INTEGRALS OF FUNCTIONS WITH EVEN/ODD SYMMETRY

Let $a \in \mathbb{R}$. Suppose f is continuous on $[-a, a]$.

◇ If f has even symmetry, that is $f(-x) = f(x)$ for all $x \in [-a, a]$, then

◆ If f has odd symmetry, that is $f(-x) = -f(x)$ for all $x \in [-a, a]$, then

Example 12.8. $\int_{-1}^1 \frac{\tan x}{1 + x^2 + x^6} dx$

STUDY GUIDE

- ◇ strategy for integration by substitution
- ◇ two ways to evaluate definite integrals via substitution
- ◇ using even/odd symmetry to evaluate certain integrals