13. Integration by Parts and Trig Integrals

Lec 12 mini review.

Substitution: $\int F'(g(x))g'(x)dx = \int F'(u)du = F(u) + C = F(g(x)) + C$

Integrals with Even Symmetry: If $f(-x) = f(x)$, then $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

Integrals with Odd Symmetry: If $f(-x) = -f(x)$, then $\int_{-a}^{a} f(x)dx = 0$

"UNDOING" THE PRODUCT RULE

Example 13.1. $\displaystyle\int xe^x dx$

Recall the Chain Rule: $\dfrac{d}{dx}[f\big(g(x)\big)] = f'\big(g(x)\big)g'(x)$.

Some clues that might indicate we should try integration by substitution:

- we might see a composition in the integrand
- we might notice a function that could be the "inner" function $g(x)$, and its derivative will also be a factor of the integrand.
- me might see what looks like the aftermath of a Chain Rule derivative (like power chain rule, exponential chain rule, log chain rule, etc.)

In contrast, the Product Rule is $\dfrac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$.

"Undoing" the product rule is less obvious than "undoing" the chain rule:

- If we want to integrate $\int \big(f'(x)g(x) + f(x)g'(x)\big)dx$, we might be lucky enough to recognize f and g
- if the integrand is the result of a product rule, then the factors need not be related to each other whatsoever

INTEGRATION BY PARTS

Guidelines for I.B.P.

- you choose the "parts" u and v'
- when you choose u, you need to be able to calculate its derivative u'
- when you choose v', you need to be able to calculate its antiderivative v
- **Goal**: the "new" integral $\int u'v$ should be no worse than the original integral $\int uv'$

Example 13.2. $\displaystyle\int xe^x\,dx$

Example 13.3. $\int x^3 e^{2x}\,dx$

Example 13.4. $\int \ln(x)\,dx$

Example 13.5. $\int (x^4 + 2x - 9)\ln(x)\,dx$

Common "Parts"

$$\int x^n e^{kx}\,dx$$

$$\int x^n (\ln x)^m\,dx$$

Example 13.6. $\int x^3 (\ln x)^2 dx$

Sometimes, when choosing "parts" there seem to be ties...

Example 13.7. $\int e^x \sin(x)dx$

TRIG INTEGRALS

- For certain integrals involving trig functions, the trick is to make use of trig identities before integrating.

Example 13.8. $\displaystyle\int \sin^{17}(x)\,dx$

Example 13.9. $\displaystyle\int \sin^{4}(x)\,dx$

Study Guide

- $\diamond$ **strategy for integration by substitution**
- $\diamond$ **integration by parts**: $\int uv' = uv - \int u'v$
- $\diamond$ **making use of trig identities before integrating**