Lesson 2 - Vector Addition

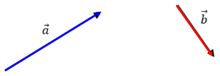
PART A: Different methods of vector addition

Addition of vectors allows us to determine the combined effect of two forces on an object, the effect of wind on the velocity of a plane relative to the ground and many other physical situations where there is a combined effect of vectors.

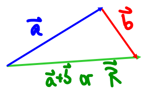
When you add two or more vectors, you are finding a single vector, called the **Resultant.** However, it is important to note that the magnitude of $\vec{a}+\vec{b}$ is less than or equal to the combined magnitudes of $\vec{a}$ and $\vec{b}$. This can also be written as:

$$|\vec{a}+\vec{b}| \leq |\vec{a}| + |\vec{b}|$$

Consider two vectors, $\vec{a}$ and $\vec{b}$



Think of $\vec{a}+\vec{b}$ as $\vec{a}$ followed by $\vec{b}$. Translate $\vec{b}$ so that the tail of $\vec{b}$ touches the head of $\vec{a}$.



$\vec{R}$ is used for the resultant vector

Find the sum by drawing a vector from the tail of $\vec{a}$ to the head of $\vec{b}$ and measuring the result. The new vector is the resultant $\vec{a}+\vec{b}$.

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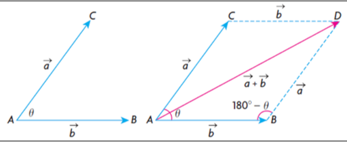
Parallelogram Law for Adding Two Vectors



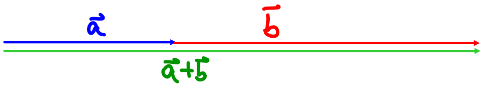
To determine the sum of the two vectors $\vec{a}$ and $\vec{b}$, complete the parallelogram formed by these two vectors when placed TAIL to TAIL (yes I know that is not what we learned initially, but stay with me). Their sum is the vector $\overrightarrow{AD}$, the diagonal of the constructed parallelogram.

Apr 27-9:06 AM

PART B: Adding *Parallel* Vectors

Vectors $\vec{a}$ and $\vec{b}$ are parallel and have the same direction

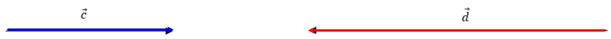
To find $\vec{a}+\vec{b}$, place the tail of $\vec{b}$ at the head of $\vec{a}$.



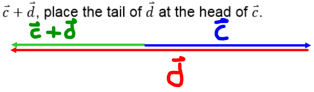
The resultant is the vector from the tail of the first vector, $\vec{a}$, to the tip of the second vector $\vec{b}$. To add the parallel vectors having the same direction, add their magnitudes. The resultant vector has the same direction as the original vectors.

Apr 25-11:02 PM

Vectors $\vec{c}$ and $\vec{d}$ are parallel but are in opposite directions.



To find $\vec{c}+\vec{d}$, place the tail of $\vec{d}$ at the head of $\vec{c}$.



The resultant is the vector from the tail of $\vec{c}$ to the head of $\vec{d}$. The magnitude of the resultant is equal to the magnitude of $\vec{c}$ minus the magnitude of $\vec{d}$. The resultant has the same direction as that of $\vec{d}$.

Apr 25-11:05 PM

PART C: *Subtracting* Vectors

The relationship between addition and subtraction with vectors is similar to the relationship between addition and subtraction with scalars. To subtract $\vec{u}-\vec{v}$, add the opposite of $\vec{v}$ to $\vec{u}$. In other words, $\vec{u}-\vec{v}$ is equivalent to $\vec{u}+(-\vec{v})$.

Adding Opposite Vectors and the Zero Vector

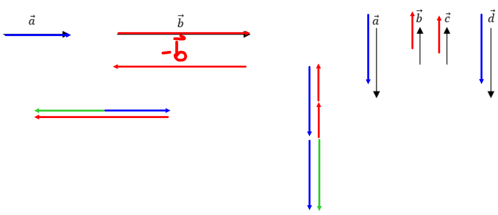
When you add two opposite integers, the result is zero. A similar result occurs when you add two opposite vectors.

The zero vector is written as $\mathbf{0}$. It has no specific direction.

Example 1: Add or subtract the following

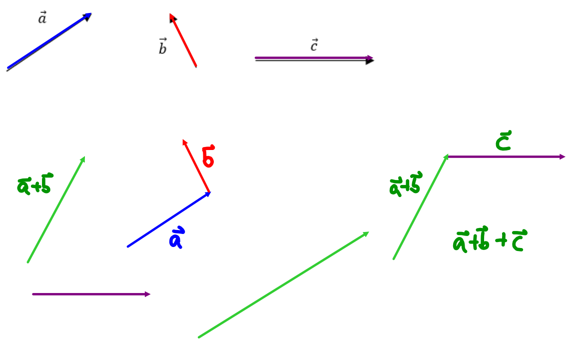
a) $\vec{a}-\vec{b}$ $= \vec{a}+(-\vec{b})$

b) $\vec{a}+\vec{b}+\vec{c}+\vec{d}$



Apr 25-11:06 PM

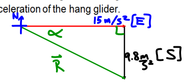
Example 2: Given the vectors $\vec{a}, \vec{b}$ and $\vec{c}$, construct $\vec{a}+\vec{b}$ and $(\vec{a}+\vec{b})+\vec{c}$.



Apr 25-11:21 PM

Example 3: A hang glider is travelling horizontally (right) with an acceleration of $15\,m/s^2$. The force of gravity is acting on the hang glider at $9.8\,m/s^2$ vertically downward.

a) Draw a scale diagram showing the acceleration vectors and the resultant acceleration of the hang glider.



b) Determine the magnitude and direction of the resultant acceleration of the hang glider.

① Magnitude of resultant found by using pyth. thᵐ.

$|\vec{R}| = \sqrt{15^2 + 9.8^2}$

$|\vec{R}| \doteq 17.9\ m/s^2$

② Direction; if using true bearing, need θ,
; if using quadrant bearing, need γ

True bearing

$\theta = 90° + \alpha$

$\tan\alpha = \frac{9.8}{15}$

$\boxed{\angle\alpha \doteq 33°}$

$\therefore \theta = 90 + 33$
$= 123°$

Quadrant Bearing

$\angle\gamma = 90 - \alpha$
$= 90 - 33$
$= 57°$

$\therefore$ The resultant velocity of the hang glider is $17.9\ m/s^2$ on a bearing of $123°$ (true bearing)

or

S57°E (Quadrant bearing)

Apr 25-11:23 PM

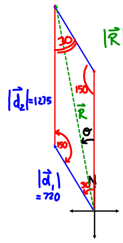
Example 4: An airplane leaves the airport travelling $N30^{\circ}W$ at $720\,km/h$. After 1 hour, the airplane then turns north and travels another 1.5 hours at $850\,km/h$. What is the displacement of the airplane after 2.5 hours?

$\vec{d_1} = 720\text{km [N30°W]}$

$d_2 = 850 \frac{km}{\cancel{hr}} \times 1.5\cancel{hr}$

$\boxed{d_2 = 1275\text{ km}}$

$\vec{d_2} = 1275\text{ km [N]}$



$|\vec{R}| = 720^2 + 1275^2 - 2(720)(1275)\cos 150°$

$|\vec{R}| = \sqrt{720^2 + 1275^2 - 2(720)(1275)\cos 150°}$

$|\vec{R}| \doteq 1932\text{ km}$

$\frac{\sin\theta}{720} = \frac{\sin 150}{1932}$

$\theta = \sin^{-1}\left(\frac{720}{1932}\sin 150\right)$

$\theta \doteq 10.7°$

$\therefore$ The displacement of the plane is 1932 km [N 10.7° W].

Apr 25-11:22 PM