MAT1322 FE version 61

Calculus II (University of Ottawa)

INSTRUCTIONS:

- Detailed instructions for the format and technical requirements of this exam have been announced in Brightspace and the course syllabus.

- **What is authorized?** You may consult your hand-written 1-page (both sides) cheat sheet (which must be included in the scanned copy of your work). You are permitted to use a basic scientific calculator. You may use non-electronic writing implements and up to 16 pieces of your own blank paper (lined or graph paper is fine). You require a device to access the exam in Brightspace and a device to join Zoom and provide the proctors with a wide view of yourself, your hands and papers and workspace. During the exam, you should not be using these devices for any other purpose.

- **Academic Fraud warning:** Do not communicate with anyone during the exam (except possibly the proctors). Close all your browsers and course documents – aside from your 1-page cheat sheet, this is a **closed-book exam**. You are **not** allowed to access course content in Brightspace. You are **not** allowed to upload, share or otherwise distribute your exam questions <u>ever</u>. You are **not** allowed to search for answers online or ask others for solutions. You are **not** allowed to use any mathematical software other than your basic scientific calculator.

  Allegations of fraud may be filed against students who appear not to be respecting the terms of this exam.

  Keep in mind: proctors expect to see students writing on paper and looking at their screen throughout the test-writing phase. The need to scroll through the questions or type a message to a proctor in Zoom **must be minimal (i.e. very infrequent)**.

- **Keep your eye on the Zoom Chat during the test-writing phase:** If issues arise, Zoom chat (or a Zoom invitation to join a breakout room) is the only way a proctor will communicate with you individually. For example, a proctor may ask you to change your camera angle via Zoom chat. If you ignore this request, then you may be flagged and you may need to write a deferred exam.

- **To ask a question:** If you have a question, use a Direct Message in the Zoom chat to let your professor know "I have a question." Be patient when awaiting a reply and continue working on your exam. You may be invited into a breakout room to ask your question without disrupting your classmates. In the main Zoom room, please keep your microphone on MUTE to avoid disrupting your classmates.

- **Connection issues:** If you briefly disconnect from Zoom, rejoin the Zoom meeting as soon as possible and notify your prof (via Zoom chat Direct Message) when you have reconnected. If your connection to Zoom is inadequate to allow you to be reasonably proctored, then you may need to contact the Faculty of Science to request that your exam be deferred.

- **Duration of Test-writing Phase:** 170 minutes

- This exam consists of 11 multiple-choice questions and 5 long-answer questions.

- The exam is worth a total of **50 points**.

- All your answers must be written on your own blank paper (lined paper or graph paper is fine).

- When (and only when) authorized by a proctor, you will need to scan and upload a copy of your work before the exam deadline.

- **MULTIPLE-CHOICE QUESTIONS**

  Questions 1–11 are **multiple-choice questions** worth 22 points total (2 points each). You may write your scrap work on your paper but it will not be graded. When you reach your answer, clearly indicate the question number and write the letter of your response beside the question number:

  For example: (*write out your scrap work, but it will not be graded*)

  (*clearly indicate your final choice*) **Q1.** [letter of your choice]

- **LONG-ANSWER QUESTIONS**

  Questions 12–16 are **long-answer questions** worth a total of 28 points. For long-answer questions, **all of your work must be justified and your steps must be written in a clear and logical order**. Clearly indicate Question numbers.

  For example: **Q12(a).** [write a fully justified solution].

- **STUDENT DECLARATION**

  Within your scanned pdf document, you must include a <u>hand-written</u> declaration, to indicate that you will honour the terms of the exam, as follows:

  ''I,   [your name printed clearly] , promise to uphold my academic integrity.
  I wrote my solutions without help from any unauthorized resource or person.

- Below your statement, you must include **your signature** and **the date**.

- **Scan-and-Upload Phase**: When the test-writing phase ends, you will be given 10 minutes to scan and create a pdf copy of your work which you will upload and submit in the Brightspace Assignment for the Final Exam.

  If possible, stay on Zoom during the scan-and-upload phase. If you have any issues, let your prof know via Direct Message Zoom chat. Continue trying to scan and upload your work while waiting for a reply.

  If absolutely necessary, you may leave Zoom to scan and upload your exam. If you have any issues uploading your exam in Brightspace, you must email your exam to your professor immediately. If possible, rejoin the Zoom meeting and let your prof know right away. If a student cannot upload their work in the 10-minute window and does not notify their professor or send a backup copy of the exam by email, then late submissions may not be accepted and a request to write a deferred exam must be submitted to the Faculty of Science infosci@uOttawa.ca

Good luck!

Final Checklist:

☐ Before you start scanning, wait for explicit permission from a proctor.

☐ When authorized to scan, create a scanned pdf copy of:

☐ your STUDENT DECLARATION signed and dated

☐ your ANSWERS (CLEARLY LABELLED) to all MULTIPLE-CHOICE questions 1-11

Q1 ☐

Q2 ☐

Q3 ☐

Q4 ☐

Q5 ☐

Q6 ☐

Q7 ☐

Q8 ☐

Q9 ☐

Q10 ☐

Q11 ☐

☐ your SOLUTIONS (CLEARLY LABELLED) to all LONG-ANSWER questions 12-16

Q12 ☐

Q13 ☐

Q14 ☐

Q15 ☐

Q16 ☐

☐ both sides of your prepared CHEAT SHEET

☐ Submit your work as a single PDF file in the Brightspace Assignment for the final exam.

☐ Only the latest submission will be kept in Brightspace. Make sure you submit all your pages as a single PDF file.

☐ Use the Assignments tab to verify that you have submitted the correct document(s) in the exam's Brightspace Assignment.

Q1. Let R be the region enclosed by the curve $\quad y = 4x + e^{5x}\quad$ and the lines $x = 1$, $x = 2$, and $y = 0$.

Let S be the solid whose flat base is the region R and whose cross-sections perpendicular to the x-axis are **squares.**

Which of the following definite integrals computes the total volume of S ?

A. $\displaystyle\int_1^2 \left(16x^2 + e^{25x^2}\right)dx$ **B.** $\displaystyle\int_1^2 2\pi\left(4x^2 + xe^{5x}\right)dx$ **C.** $\displaystyle\int_1^2 \left(16x^2 + e^{10x}\right)dx$

D. $\displaystyle\int_1^2 \left(16x^2 + 8xe^{5x} + e^{10x}\right)dx$ **E.** $\displaystyle\int_1^2 \left(16x^2 + 8e^{5x} + e^{25x^2}\right)dx$ **F.** $\displaystyle\int_1^2 \pi\left(4x + e^{5x}\right)^2 dx$

Q2. A reservoir has a rectangular door located at the bottom of one of its vertical sides, as shown in the diagram below. The reservoir is $6\,\text{m}$ high and filled to the top with water. The door is $4\,\text{m}$ wide by $2\,\text{m}$ high.
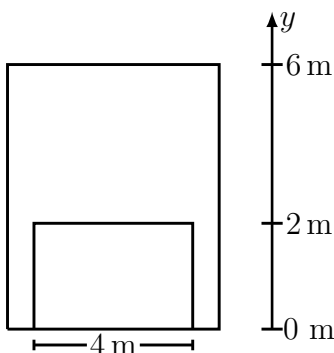
Let y represent the height from the bottom of the reservoir.

Find the **hydrostatic force** exerted by the water on the door.

Note that the density of water is 1000 kg/m^3 and the acceleration due to gravity is 9.8 m/s^2.

A. 392 000 N   **B.** 764 400 N   **C.** 1 372 000 N   **D.** 1 254 400 N

E. 2 940 000 N   **F.** 1 881 600 N   **G.** 808 500 N   **H.** 1 200 500 N

Q3. A potato with a temperature of $16°$ is put in a $230°$ oven and heats up according to the differential equation

$$\frac{dH}{dt} = k(H - 230)$$

where $H(t)$ represents the potato's temperature t minutes after being placed in the oven. After 10 minutes the temperature of the potato is $130°$C. What is the value of k to 3 decimal places?

A. $k = -0.076$ **B.** $k = -0.062$ **C.** $k = -0.028$ **D.** $k = -0.057$

E. $k = -0.034$ **F.** $k = -0.046$ **G.** $k = -0.040$ **H.** $k = -0.037$

Q4. Determine whether the **infinite sequence** $\left\{\dfrac{3n}{\sqrt{6n^2+7}}\right\}_{n=1}^{\infty}$ converges or diverges.

If it converges, find its limit.

A. $\dfrac{3}{7}$ **B.** $\dfrac{3}{\sqrt{7}}$ **C.** $\dfrac{3}{6}$ **D.** 0 **E.** $\dfrac{3}{\sqrt{6}}$ **F.** 1 **G.** This sequence diverges.

Q5. Fact: the series $S = \displaystyle\sum_{m=3}^{\infty} \dfrac{1}{m(\ln m)^4}$ converges by virtue of the Integral Test.

According to the Remainder Estimate Theorem, if we estimate S by the partial sum S_{10}, what is the guaranteed upper bound U such that the remainder $R_{10} = S - S_{10}$ satisfies $R_{10} \leq U$?

A. $U \approx 0.0031$ **B.** $U \approx 0.0367$ **C.** $U \approx 0.0085$ **D.** $U \approx 0.0066$ **E.** $U \approx 0.0124$ **F.** $U \approx 0.0273$

Q6. Which 2 of the series below are **divergent** ? You must select 2 answers: ☐ ☐

A. $\displaystyle\sum_{k=0}^{\infty} \dfrac{2^{3k}}{3^{k+1}}$

B. $\displaystyle\sum_{m=1}^{\infty} \dfrac{1}{m^3 + \sqrt{m}}$

C. $\displaystyle\sum_{n=0}^{\infty} \dfrac{(-1)^n\, n}{n+1}$

D. $\displaystyle\sum_{N=0}^{\infty} \dfrac{(-1)^N}{N+8}$

E. $\displaystyle\sum_{n=0}^{\infty} \dfrac{2^n}{5^{2n+1}}$

F. $\displaystyle\sum_{k=0}^{\infty} \dfrac{2^k}{(k+1)!}$

Q7. Find the radius of convergence R of the power series $\displaystyle\sum_{n=0}^{\infty} \dfrac{(2n)!(x+2)^n}{(n+2)!}$.

A. $R = \dfrac{1}{2}$ **B.** $R = \dfrac{1}{4}$ **C.** $R = 2$ **D.** $R = 4$ **E.** $R = 1$ **F.** $R = \infty$ **G.** $R = 0$

Q8. Fact: the series $S = \displaystyle\sum_{n=1}^{\infty} \dfrac{(-1)^n}{n}$ converges by virtue of the Alternating Series Test.

According to the Alternating Series Estimation Theorem, what is the smallest value of N for which the error $|R_N| = |S - S_N|$ is at most 0.061 ?

A. 9 **B.** 10 **C.** 11 **D.** 12 **E.** 13 **F.** 14 **G.** 15 **H.** 16 **I.** 17

Q9. Find the tangent plane to $f(x,y) = x\ln(y) + x^3y + 1$ at the point $(x,y) = (2,1)$.

A. $z = 2x + 2y - 2$ **B.** $z = 4x + 2y - 4$ **C.** $z = 12x + 10y - 24$

D. $z = 12x + 10y - 25$ **E.** $z = 4x + 6y - 7$ **F.** $z = 3x + 2y - 1$

Q10. For the function $f(x,y) = x^3 e^{2y}$ find the **maximum value of its directional derivative** at the point $(x,y) = (1,0)$,

A. $\sqrt{27}$ **B.** $\sqrt{23}$ **C.** $\sqrt{20}$ **D.** $\sqrt{13}$ **E.** 5 **F.** $\sqrt{18}$ **G.** $\sqrt{40}$ **H.** 6

Q11. Suppose $z = f(x,y)$, $\quad x = g(t)$, $\quad$ and $\quad y = h(t)$ $\quad$ satisfy:

$$g(6) = 0 \qquad\qquad h(6) = 3 \qquad\qquad f_x(0,3) = 1$$
$$g'(6) = -1 \qquad\qquad h'(6) = -2 \qquad\qquad f_y(0,3) = 2$$

Use the Chain Rule to find $\dfrac{dz}{dt}$ when $t = 6$

A. -6 **B.** -7 **C.** -14 **D.** -10 **E.** -1 **F.** -5 **G.** -12 **H.** -9
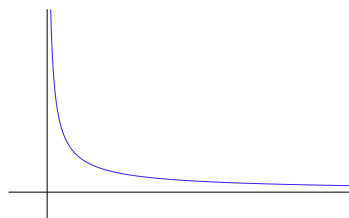

LONG-ANSWER QUESTIONS

Q12. Let R be the region that lies between the curve $y = \dfrac{5}{x^{2/3}}$ and the x-axis on the interval $[6, \infty)$.

The graph of $y = \dfrac{5}{x^{2/3}}$ is given here:

(a) [**2 points**] The area of R corresponds to the improper integral $\displaystyle\int_6^\infty \frac{5}{x^{2/3}}\,dx$.

Determine whether this improper integral converges or diverges. If it converges, find its exact value. Your answer must be well-justified.

(b) [**1 point**] Let S be the solid obtained by revolving R about the x-axis. Give an expression for the approximate volume $V(x)$ of the thin slice of S that lies between x and $x + \Delta x$, where its thickness Δx is assumed to be small.

(c) [**3 points**] Give an improper integral that computes the volume of S and determine whether it converges or diverges. If it converges, find the exact volume of S. Your answer must be well-justified and use appropriate mathematical notation.

Q13. A tank is shaped like a circular cone, with its apex pointing down (like an ice cream cone). Its height is 15 m, the radius of its top circular face is 4 m, and it is filled with water to a height of 12 m. The goal of this question is to determine the total work required to pump all of its water to a height 1 m above the top of the tank. Recall: the density of water is $\rho = 1000$ kg/m^3 and the acceleration due to gravity on Earth's surface is $g = 9.8$ m/s^2.

(a) **[3 points]** Let x be the height (in m) measured <u>from the bottom</u> of the tank (the tip of the cone). Find an expression for the approximate volume $V(x)$ of a thin layer of water between heights x and $x + \Delta x$ m. To earn full marks:

▶ You must draw and clearly label a diagram.

▶ You must show all your work *and* briefly explain your answer.

(b) **[1 point]** What is the approximate work $W(x)$ required to pump the thin layer of water (described in part (a)) to a height 1 m above the top of the tank? Briefly justify your answer.

(c) **[1 point]** Give a definite integral that computes the total work required to pump all of the water in the tank to a height of 1 m above its top.
Do **not** evaluate the integral – just write it down.

Q14. **[5 points]** For each of the following <u>convergent</u> series, find its exact sum. Show all your work !

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n + (-2)^n}{4^{2n+1}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \left(3^{1/n} - 3^{1/(n+1)}\right)$

Q15. **[5 points]** Find the Maclaurin series representation for each of the following functions.

For each series:

i. Give its Maclaurin series representation using summation notation.

ii. Explicitly write out its first four (4) nonzero terms. Write exact numbers for the coefficients (i.e. do not leave them un-simplified).

Your answers must be well-justified. Show all your steps!

(a) $f(x) = (1 - 4x)^{3/4}$ (b) $g(x) = x^3 e^{4x}$

Q16. Consider the function $z = f(x, y) = x^2 + 16y^2 + 3$.

(a) **[2 points]** What is the domain and range of f ? Briefly justify your answer.

(b) **[2 points]** Draw a contour diagram for f with the level curves $k = f(x, y)$ for $k = 3$ and $k = 19$.

Show your work and indicate the value of k corresponding to each curve in your diagram.

Do not forget the scales on the coordinate axes, and choose the scales so that the curves can be properly drawn and seen.

(c) **[1 point]** Sketch the trace of f in the xz-plane.

(d) **[2 points]** Find the directional derivative of f in the direction of the vector $\mathbf{v} = \langle 4, 3 \rangle$ at the point $(x, y) = (5, 1)$.   Show all your work!

End of the Exam!