# 6. Differentiation: Basic Rules and Product & Quotient Rules

**Lec 5 mini review.**

**The Derivative at a Point**:

$$\boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

**slope of a tangent at $x = a$**

**instantaneous rate of change at $a$**

**differentiability**
**how functions fail to be differentiable**

**differentiability $\Longrightarrow$ continuity**

**The Derivative as a Function**:

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

**relationship between $f(x)$ and $f'(x)$**

**domain of $f'(x)$**

**notation for derivative of $y = f(x)$:**
$f'(x)$ $y'$ $\frac{dy}{dx}$ $\frac{df}{dx}$ $\frac{d}{dx}f(x)$ $D_x(f(x))$

**higher-order derivatives:** $f'$ $f''$ $f'''$ $f^{(4)}$

## BASIC RULES

**Constants**

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**Powers**

$n = 0$

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$n = 1$

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$n = 2$

[*] These notes are solely for the personal use of students registered in MAT1320. 

$n \in \mathbb{N}$

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- Later, we'll use *Logarithmic Differentiation* to prove the power rule actually works just as well for any power $n \in \mathbb{R}$.
- For now, we will state the rule and use it!

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**The Power Rule** ($n \in \mathbb{R}$)

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**Constant Multiples**

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**Sums and Differences**

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**Example 6.1.** Find the first and second derivatives of each of the following functions.

**a.** $g(x) = 2x^3 + -\sqrt{x} - 5$

**b.** $f(x) = \dfrac{\pi + \sqrt[3]{x} - x^8}{x^{4/3}}$

# Derivative of Exponential Functions

Let $f(x) = b^x$ for some base $b > 0$, $b \neq 1$. Then

$$\frac{d}{dx} f(x) = \lim_{h \to 0} \frac{b^{x+h} - b^x}{h} = \lim_{h \to 0} b^x \left( \frac{b^h - 1}{h} \right) = \lim_{h \to 0} b^x \left( \frac{b^{0+h} - b^0}{h} \right) = b^x \left( \lim_{h \to 0} \frac{b^{0+h} - b^0}{h} \right) = b^x \big( f'(0) \big)$$

**Derivative of the Natural Exponential Function**

**Example 6.2.** Let $f(x)$ be a piecewise function defined as follows, where $a$, $b$, and $c$ are constants:

$$f(x) = \begin{cases} ae^x + x + 1 & \text{if } x \leq 0 \\ 3 & \text{if } x = 0 \\ bx + c & \text{if } x > 0 \end{cases}$$

Find all values of the constants $a$ and $b$ for which

- $f(x)$ is continuous at $x = 0$
- $f(x)$ is differentiable at $x = 0$

**Example 6.3.** Find the derivative of $h(t) = e^{t+1}$.

## THE PRODUCT RULE

**The Product Rule**

**Example 6.4.** Find the derivative of $y = (2e^x + x^2)\sqrt{x}$.  What is $y'(4)$ ?

## The Quotient Rule

**The Quotient Rule**

**Example 6.5.** Differentiate each of the following:

**a.** $q(x) = \dfrac{4x^{5/2} + 8e^x}{x^3 + 2x - \sqrt{2}}$

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**b.** $g(x) = h(x)\left(\dfrac{x^2}{f(x)}\right)$ where $h(x)$ and $f(x)$ are differentiable functions.

If $h(2) = 10$, $f(2) = 1$, $h'(2) = 2$, and $f'(2) = -1$, then what is $g'(2)$ ?

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## STUDY GUIDE

**Constant Multiple Rule**:
for any $k \in \mathbb{R}$, $\boxed{\dfrac{d}{dx}[kf(x)] = kf'(x)}$

**Sum/Difference Rule**:
$\boxed{\dfrac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)}$

**Constant Rule**:
for any $c \in \mathbb{R}$, $\boxed{\dfrac{d}{dx}[c] = 0}$

**Power Rule**:
for any $n \in \mathbb{R}$, $\boxed{\dfrac{d}{dx}[x^n] = nx^{n-1}}$

**Derivative of $e^x$**
$\boxed{\dfrac{d}{dx}[e^x] = e^x}$

**Product Rule**:
$\boxed{\dfrac{d}{dx}[fg] = f'g + fg'}$

**Quotient Rule**:
$\boxed{\dfrac{d}{dx}\left[\dfrac{f}{g}\right] = \dfrac{f'g - fg'}{g^2}}$