Lesson 1 – The Second Derivative, Velocity & Acceleration

PART A: The Relationship between Position, Velocity and Acceleration

One of the reasons for introducing derivatives was the need to calculate rates of change. We have seen that the derivative of a position function provides the velocity function which indicates the rate of change of position, usually in metres per second (m/s). We shall now look at a higher order derivate, the second derivative to determine the acceleration of an object from its position function. Acceleration is usually measured in metres per second squared (m/s^2).

For straight line motion, we have already shown that the velocity of an object is defined as the rate of change of the objects position with respect to (wrt) time:

$$v(t) = s'(t)$$

or in Leibniz notation as $v = \frac{ds}{dt}$

Velocity = rate of change of position wrt time

Acceleration is associated with a change in velocity, just as velocity is associated with a change in position. When the gas pedal in a vehicle is depressed, the car will speed up, similarly as the brake pedal is depressed, the vehicle will slow down. In both situations, the velocity will change with respect to time and therefore it has a non-zero acceleration (can be positive or negative). Instantaneous acceleration wrt time is defined as:

$$a(t) = v'(t) = s''(t)$$

or in Leibniz notation as: $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

Acceleration = rate of change of velocity wrt time

$s'(t)$
$s''(t)$

If the position function of a moving object is $s(t) = 6t^2 - t^3$

then its velocity is $v(t) = s'(t)$

$v(t) = s'(t) = 12t - 3t^2$

and its acceleration is $a(t) = v'(t)$

$a(t) = s''(t) = 12 - 6t$

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	DEFINITION	RELATIONSHIP
DISPLACEMENT $s(t)$	the distance and direction an object has moved from an origin over a period of time	$s(t)$
VELOCITY $v(t)$	the rate of change of displacement of an object with respect to time	$v(t) = s'(t)$
ACCELERATION $a(t)$	the rate of change of velocity with respect to time	$a(t) = v'(t) = s''(t)$

- velocity of an object can be determined by taking the **FIRST derivative** of the displacement equation
- acceleration can be determined by taking the **SECOND derivative** of the displacement equation

http://clem.mscd.edu/~talmanl/HTML/MovingSlopeTriangle.html

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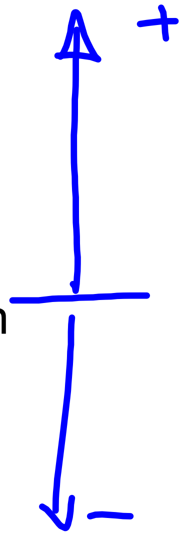
Recall:

Speed
- scalar quantity
- describes the magnitude of motion
- does **not** describe direction

Velocity
- vector quantity
- describes both magnitude and direction
- the original position is considered the origin
- positive indicates one direction from origin
- negative indicates opposite direction



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PART B: **Examples**

Example 1: Determine the first and second derivatives of each function, and then determine the value of each when the independent variable is 2.

a) $y = 3x^2 - 2x + 1$

$y'(x) = 6x - 2$

$y'(2) = 6(2) - 2$
$= 12 - 2$
$= 10$

$y''(x) = 6$

$y''(2) = 6$

b) $h(t) = 2t^3 + t^2 - 4t + 1$

$h'(t) = 6t^2 + 2t - 4$

$h'(2) = 6(2)^2 + 2(2) - 4$
$= 6(4) + 4 - 4$
$= 24$

$h''(t) = 12t + 2$

$h''(2) = 12(2) + 2$
$= 26$

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Example 2: A dragster races down a 400m strip in 8 seconds. Its distance, in metres, from the starting line after t seconds is $s = 6t^2 + 2t$.

a) Find the dragster's velocity and acceleration as it crosses the finish line. $t=8$

$V(t)=s'(t)= 12t+2$
$v(8)=s'(8) = 12(8)+2$
$=98$ m/s

$a(t)=v'(t)=s''(t)$
$a(t)= 12$ m/s^2

b) How fast was it moving 60 m down the strip? time @ 60m

$v(t)= 12t+2$
$v(3)= 12(3)+2$
$=36+2$
$=38$ m/s

$60=6t^2+2t$
$0=6t^2+2t-60$
Q.F. $t_1=3$
$t_2=-3.3$
inadmissable

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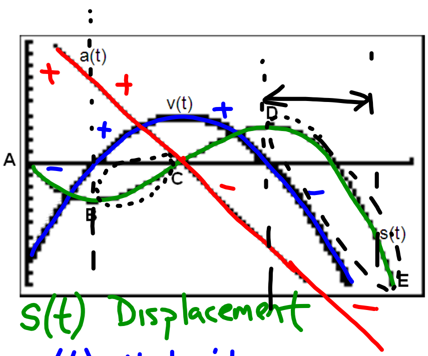
Below is a graphical representation of:

- the position function, $s(t)$
- the velocity function, $s'(t)$ or $v(t)$
- the acceleration function, $s''(t)$, $v'(t)$ or $a(t)$

Note: if an object had zero acceleration, it would *not* begin to move if *at rest*

On the graph of position, **circle** the zones where the object is **speeding up** and where the object is **slowing down**.



s(t) Displacement
v(t) Velocity
a(t) Acceleration

A - B less steep
∴ slowing down

B - C getting steeper
∴ speeding up

C - D less steep
∴ slowing down

D - E getting steeper
∴ speeding up

Now consider the graph of acceleration, does a positive acceleration always correspond to speeding up? NO Similarly, does a negative acceleration always correspond to slowing down?

To compare speeds, look at the steepness of tangents on $s(t)$ graph

*Note: $s(t)$ is being considered as *linear* motion (like a number line)

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SPEEDING UP VERSUS SLOWING DOWN

There are two possible cases for speeding up:

Case 1 B - C

An object is moving in a positive direction and the velocity becomes more positive.
$v(t) > 0$ (moving in a positive direction)
$a(t) > 0$ (the velocity is becoming more positive)
Note that $v(t) \times a(t) > 0$

Case 2 D - E

An object is moving in a negative direction and the velocity becomes more negative.
$v(t) < 0$ (moving in a negative direction)
$a(t) < 0$ (velocity is becoming more negative)
Note that again, $v(t) \times a(t) > 0$

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An object is speeding up when $v(t) \times a(t) > 0$

In others words, when velocity and acceleration are in the **SAME** direction

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There are two possible cases for slowing down:

Case 1 C - D

An object moving in a positive direction, $v(t) > 0$, and the velocity becomes less positive, $a(t)<0$. Note that $v(t) \times a(t) <0$.

Case 2 A - B

An object moving in a negative direction, $v(t) < 0$ and the velocity becomes less negative, $a(t)>0$. Note that $v(t) \times a(t) <0$.

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An object is slowing down when $v(t) \times a(t) < 0$

In others words, when velocity and acceleration are in **OPPOSITE** directions

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Example 3: Find the velocity and the acceleration of the displacement function $s(t) = -5t^3 + 2t^2 - 7t$ when $t = 3$. State whether the object is speeding up or slowing down at $t = 3$.

$V(t) = -15t^2 + 4t - 7$

$V(3) = -15(3)^2 + 4(3) - 7$

$V(3) = -15(9) + 12 - 7$

$V(3) = -130$ m/s

$a(t) = -30t + 4$

$a(3) = -30(3) + 4$

$= -90 + 4$

$= -86$ m/s^2

If $V(t) \times a(t) > 0$ speeding up

If $V(t) \times a(t) < 0$ slowing down

$(-130)(-86) = 11180$ $\because 11180 > 0$

$\therefore$ we are speeding up.

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Day 1 HW

pg. 127-129

1, 2odd, 3def, 4, 5, 6ac, 7, 8

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