GNG 1105E – Engineering Mechanics

CHAPTER S3 – EQUILIBRIUM

Assigned readings

3/1 Introduction

3/2 System isolation and the free-body diagram

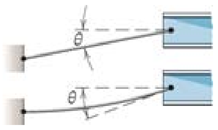
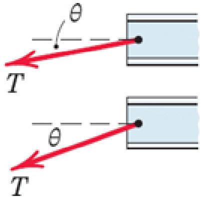
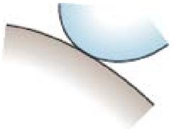
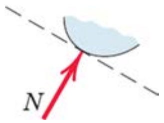
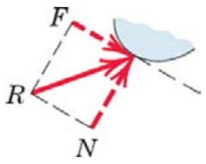
3/3 Equilibrium conditions (2-D)

3/1 Introduction

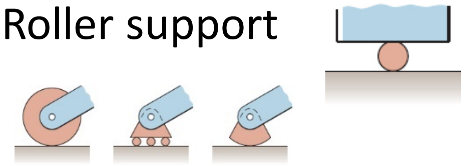
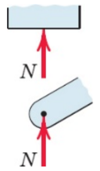
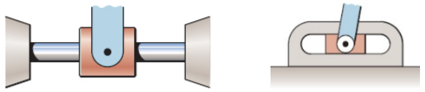
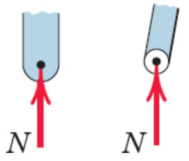
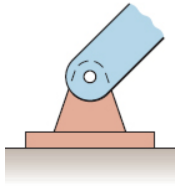
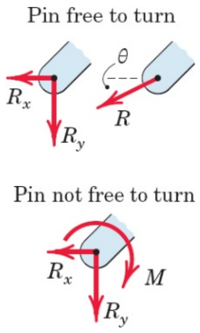
Equilibrium conditions:

These conditions are said to be **necessary** and **sufficient** to maintain equilibrium

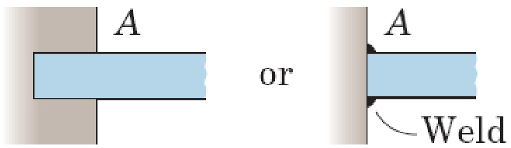
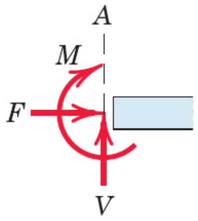
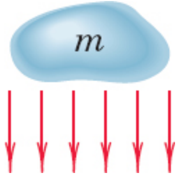
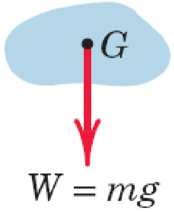
3/2 System isolation & free-body diagrams

Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential component *F* (frictional force) as well as a normal component *N* of the resultant contact force *R*.

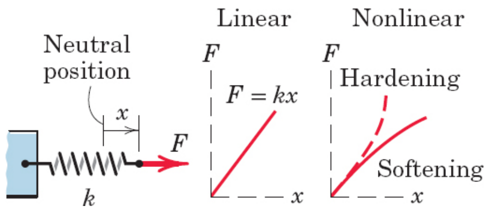
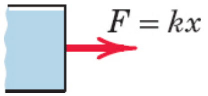
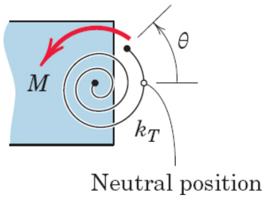
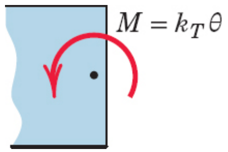
3/2 System isolation & free-body diagrams

Type of Contact and Force Origin	Action on Body to Be Isolated
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.
6. Pin connection 	 A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.

3/2 System isolation & free-body diagrams

Type of Contact and Force Origin	Action on Body to Be Isolated
7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force *F*, a transverse force *V* (shear force), and a couple *M* (bending moment) to prevent rotation.
8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass *m* is the weight *W* = *mg* and acts toward the center of the earth through the center of gravity *G*.

3/2 System isolation & free-body diagrams

Type of Contact and Force Origin	Action on Body to Be Isolated
9. Spring action 	 Spring force is tensile if the spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.
10. Torsional spring action 	 For a linear torsional spring, the applied moment M is proportional to the angular deflection θ from the neutral position. The stiffness k_T is the moment required to deform the spring one radian.

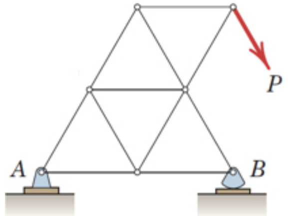
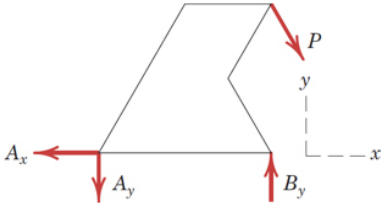
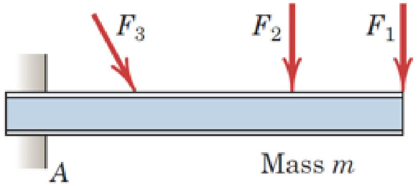
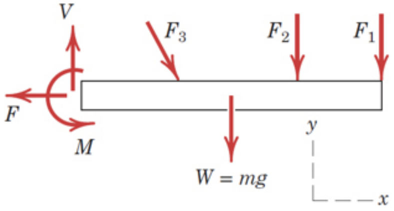
3/2 System isolation & free-body diagrams

Most important step in solving mechanics problems!

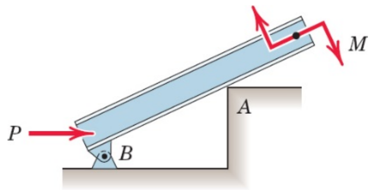
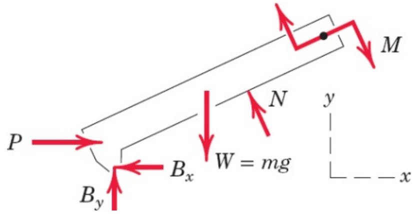
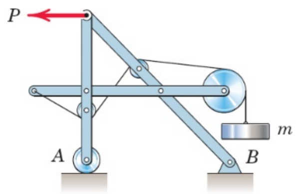
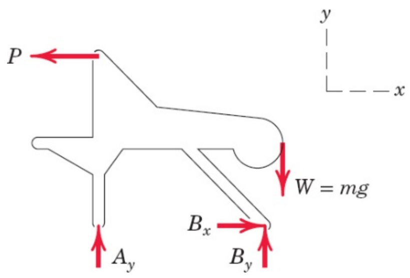
Isolate the system including external boundaries

Identify **ALL** forces which act on the isolated system and represent them in the diagram

3/2 System isolation & free-body diagrams

Mechanical System	Free-Body Diagram of Isolated Body
1. Plane truss Weight of truss assumed negligible compared with *P* 	
2. Cantilever beam 	

3/2 System isolation & free-body diagrams

Mechanical System	Free-Body Diagram of Isolated Body
4. Beam Smooth surface contact at *A*. Mass *m* 	
4. Rigid system of interconnected bodies analyzed as a single unit Weight of mechanism neglected 	

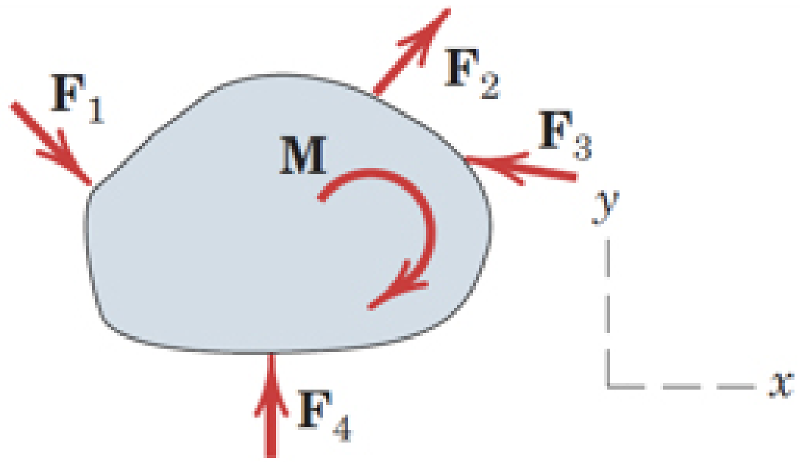
3/3 Equilibrium conditions

Scalar format:

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_O = 0$$



3/3 Equilibrium conditions

Alternative equilibrium equations:

$$\sum \boldsymbol{F}_x = 0$$

$$\sum \boldsymbol{M}_A = 0$$

$$\sum \boldsymbol{M}_B = 0$$

$$\sum \boldsymbol{M}_A = 0$$

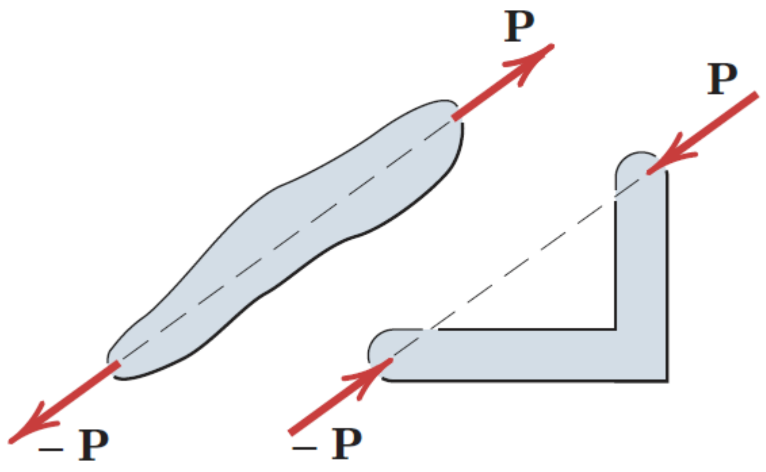
$$\sum \boldsymbol{M}_B = 0$$

$$\sum \boldsymbol{M}_C = 0$$

3/3 Equilibrium conditions

Two-force members occur when a body is in equilibrium under the action of only two forces

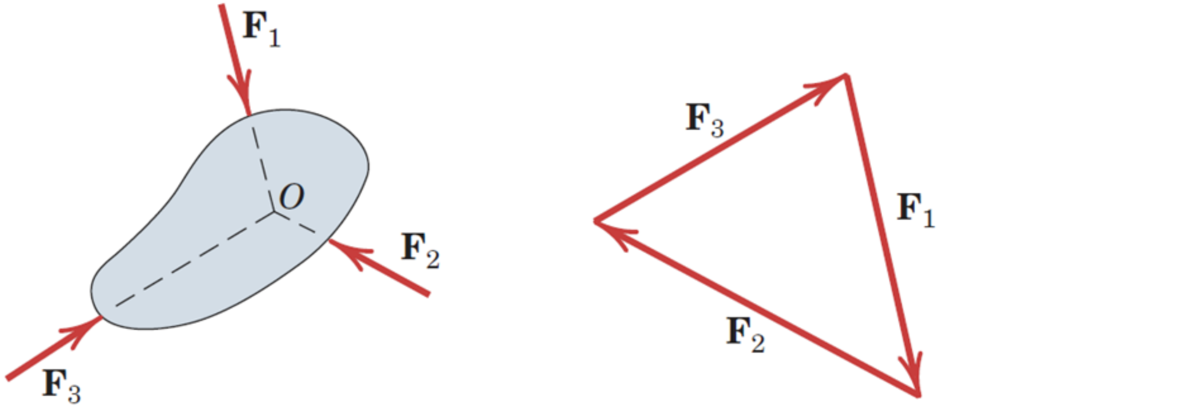
These forces must be **equal**, **opposite**, **collinear**, and independent of the object shape



3/3 Equilibrium conditions

Three-force members occur when a body is in equilibrium under the action of only three forces

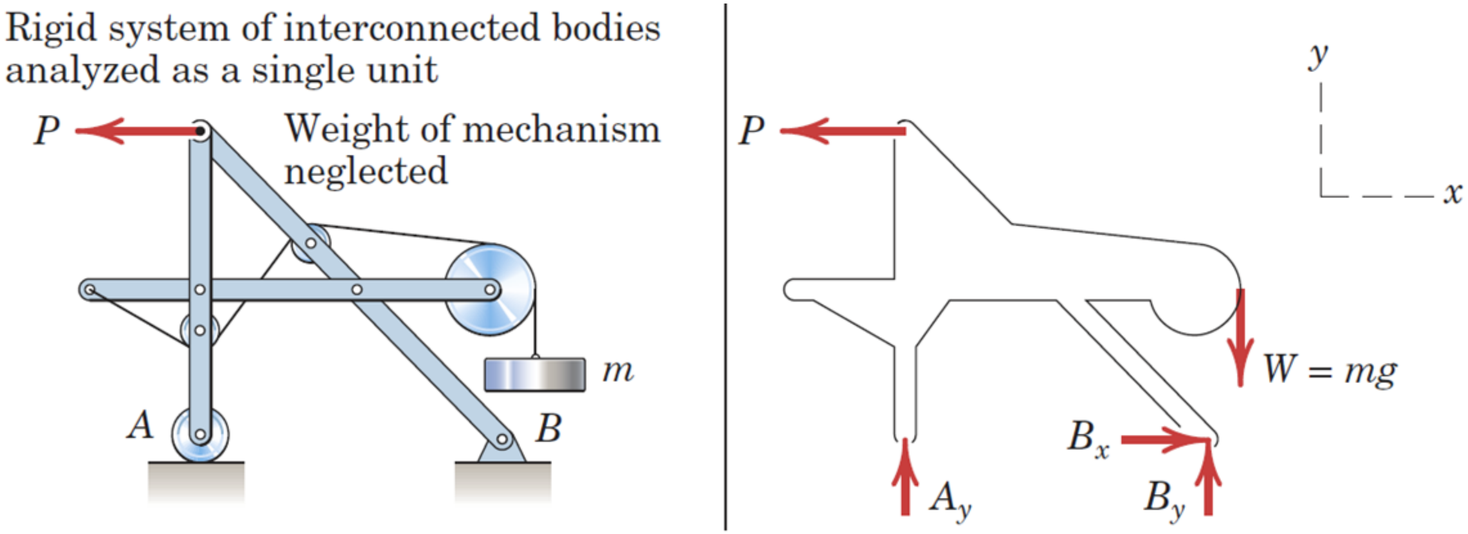
These forces must be **concurrent**, except in cases where all three forces are parallel



(*a*) Three-force member (*b*) Closed polygon satisfies $\Sigma \mathbf{F} = \mathbf{0}$

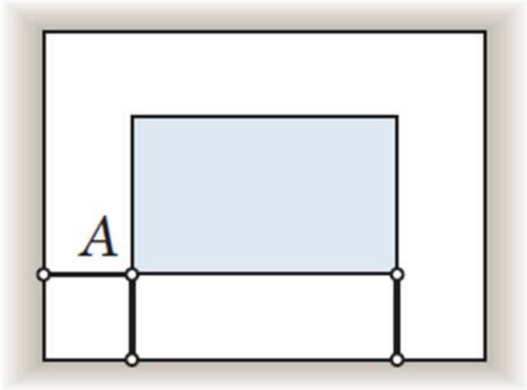
3/3 Equilibrium conditions

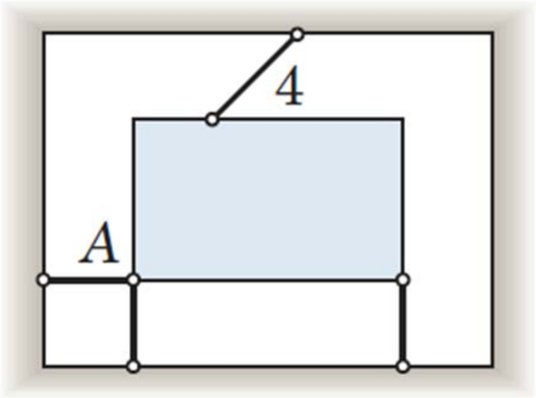
Statically determinant system



3/3 Equilibrium conditions

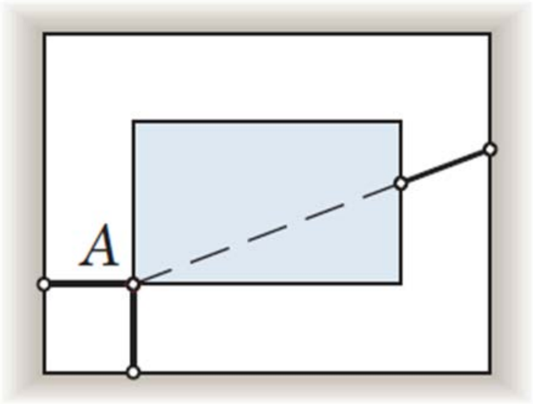
Adequacy of constraints

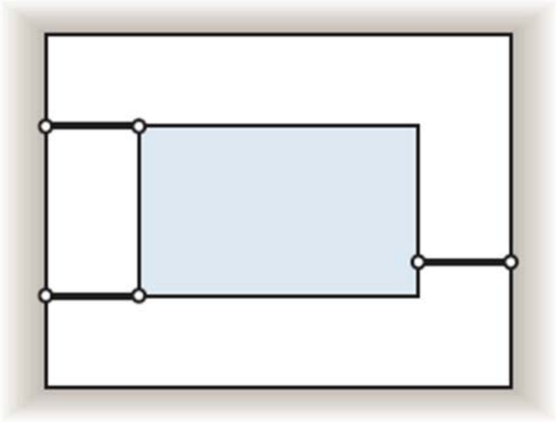




3/3 Equilibrium conditions

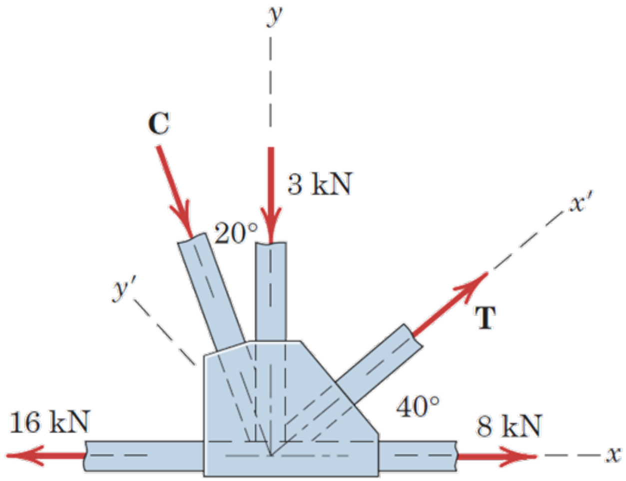
Adequacy of constraints





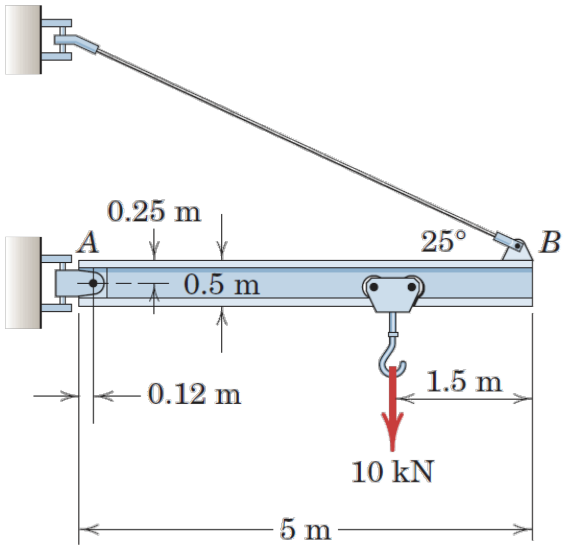
Sample problem 3/1

Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.

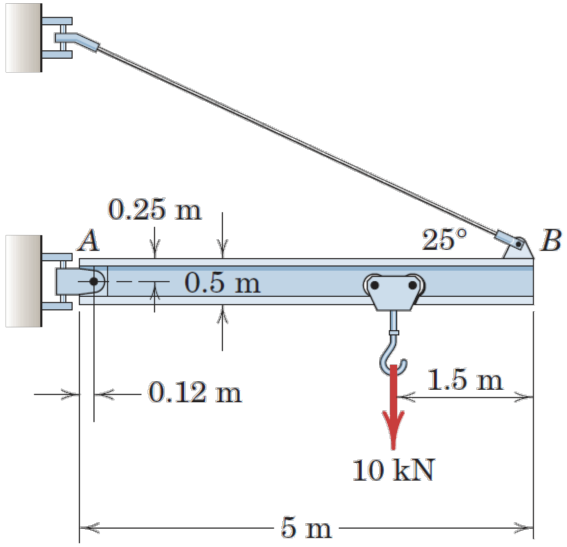


Sample problem 3/4

Determine the magnitude *T* of the tension in the supporting cable and the magnitude of the force on the pin at *A* for the jib crane shown. The beam *AB* is a standard 0.5-m I-beam with a mass of 95 kg per meter of length.



Sample problem 3/4



Recommended problems

Chapter S3 Practice Problems

- Questions 1-20