MAT1322 midterm 1 sol

Calculus II (University of Ottawa)

University of Ottawa
Department of Mathematics and Statistics

MAT1322

Calculus II

Midterm test 1

September 25, 2019

Instructor: Vadim Kaimanovich

Duration: 75 minutes

Read the following information before starting the test:

- Verify that your copy of the test contains 5 pages, including this one.
- Write your name and student number on this page.
- Work the problems in the space provided. Use the back-pages and the blank sheet attached at the end for rough work. Do not use any other paper. Before submitting the test remove the rough work page 5.
- Show all work, clearly and in order, if you want to get full credit. Points may be taken off if it is not clear how you arrived at your answer (even if your final answer is correct).
- Please keep your written answers brief; be clear and to the point. Points may be taken off for rambling and for incorrect or irrelevant statements.
- Circle your final answers.

The Faculty of Science requires that you read and sign the following statement:

Cellular phones, calculators or other electronic devices and course notes are not allowed during this test. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the test.

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

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Last Name: ______________________________

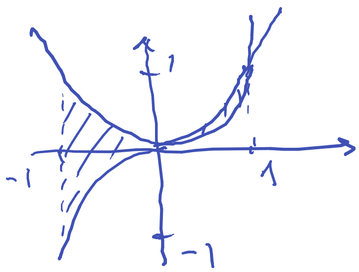
First Name: ______________________________

Student Number: ______________________________

Problem	1(2)	2(2)	3(2)	4(2)	5(2)	6(2)	Total(12)
Points							

1. Sketch the domain enclosed by the curves $y = x^2, y = x^3, x = -1, x = 1$ and find its area.

A. $\frac{7}{6}$ B. $\frac{7}{12}$ C. $-\frac{2}{3}$ D. 0 E. $\frac{3}{2}$ F. $\frac{2}{3}$ G. $\frac{2}{3}$ H. none of the above



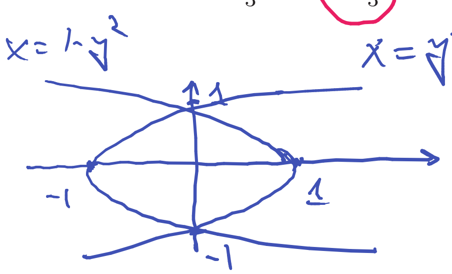
Since $x^2 \geq x^3$ on the whole interval $[-1, 1]$,

$$\text{area}(S) = \int_{-1}^{1} |x^2 - x^3| \, dx = \int_{-1}^{1} (x^2 - x^3) \, dx$$

$$= \left[\frac{x^3}{3} - \frac{x^4}{4}\right]\Bigg|_{x=-1}^{x=1} = \frac{2}{3}$$

2. Sketch the domain enclosed by the curves $x = 1 - y^2, x = y^2 - 1$ and find its area.

A. $\frac{4}{3}$ B. $\frac{8}{3}$ C. $-\frac{4}{3}$ D. 0 E. $\frac{16}{3}$ F. 1 G. $\frac{2}{3}$ H. none of the above



By slicing horizontally,

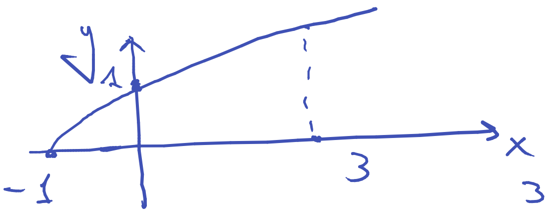
$$\text{area}(S) = \int_{-1}^{1} \left((1 - y^2) - (y^2 - 1)\right) dy$$

$$= 2\int_{-1}^{1} (1 - y^2) \, dy = 2\left(y - \frac{1}{3}y^3\right)\Bigg|_{y=-1}^{y=1}$$

$$= 2\left[\left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)\right] = \frac{8}{3}$$

3. Sketch the solid obtained by rotating the region bounded by the curves $y = \sqrt{x+1}, y = 0, x = 3$ about the x axis and find its volume.

A. 4π B. 8 C. 8π D. -4π E. -8π F. 0 G. H. none of the above



$$\mathrm{Vol}(S) = \pi \int_{-1}^{3} \left(\sqrt{x+1}\right)^2 dx = \pi \int_{-1}^{3} (x+1)\, dx$$

$$= \pi \left(\frac{x^2}{2} + x\right)\Big|_{x=-1}^{x=3} =$$

$$= \pi \left[\left(\frac{9}{2} + 3\right) - \left(\frac{1}{2} - 1\right)\right] = 8\pi$$

4. Find the average value of the function $f(x) = \frac{x^2}{\sqrt{x^3+1}}$ on the interval $[0, 2]$.

A. $\frac{2}{3}$ B. $-\frac{2}{3}$ C. $\frac{4}{3}$ D. $-\frac{4}{3}$ E. 2 F. 0 G. none of the above

$$\bar{f} = \frac{\int_0^2 f(x)\, dx}{2-0} = \frac{1}{2} \int_0^2 f(x)\, dx, \quad \text{where}$$

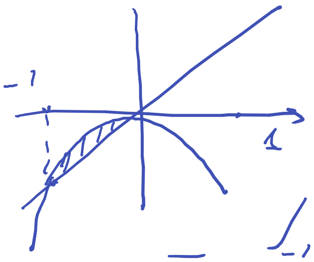
$$\int_0^2 f(x)\, dx = \int_0^2 \frac{x^2}{\sqrt{x^3+1}}\, dx = \frac{1}{3} \int_0^2 \frac{d(x^3+1)}{\sqrt{x^3+1}}$$

$$= \frac{1}{3} \int_1^9 \frac{dt}{\sqrt{t}} = \frac{2}{3}\sqrt{t}\Big|_{t=1}^{t=9} = \frac{2}{3}(3-1) = \frac{4}{3},$$

$$\text{whence } \bar{f} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}$$

5. Sketch the domain bounded by the curves $y = x$ and $y = -x^2$ and find its centroid.

A. $(\frac{1}{2}, \frac{2}{5})$ B. $\frac{1}{3}$ C. -1 D. $(\frac{1}{2}, -\frac{2}{5})$ E. $(-\frac{1}{2}, -\frac{2}{5})$ F. $(\frac{1}{2}, -\frac{1}{2})$ G. none of the above



$$\text{area}(S) = \int_{-1}^{0} (-x^2 - x)\,dx = \left(-\frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_{x=-1}^{x=0} = 0 - \left(\frac{1}{3} - \frac{1}{2}\right) = \frac{1}{6}$$

$$\bar{x} = \frac{\int_{-1}^{0} x(-x^2 - x)\,dx}{\text{area } S} = \frac{\int_{-1}^{0} (-x^3 - x^2)\,dx}{\text{area } S} = \frac{\frac{1}{4} - \frac{1}{3}}{\frac{1}{6}} = -\frac{1}{2}$$

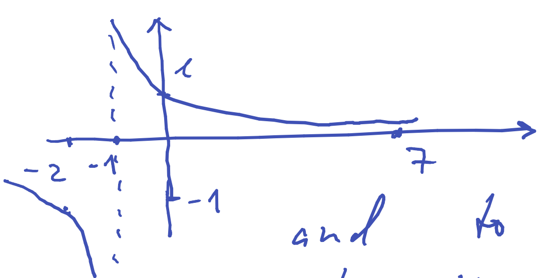
$$\bar{y} = \frac{\frac{1}{2}\int_{-1}^{0} (x^4 - x^2)\,dx}{\text{area } S} = \frac{\frac{1}{2}\left(\frac{1}{5} - \frac{1}{3}\right)}{\frac{1}{6}} = 3\left(\frac{1}{5} - \frac{1}{3}\right) = -\frac{2}{5}$$

6. Sketch the graph of the function $f(x) = 1/\sqrt[3]{x+1}$. Is the integral $I = \int_{-2}^{7} f(x)\,dx$ proper or improper? Is it convergent or divergent? If convergent, what is its value?

A. proper, divergent, $I = \frac{9}{2}$ B. proper, convergent, $I = 0$ C. improper, convergent, $I = \frac{9}{2}$

D. improper, divergent E. improper, convergent, $I = -\frac{9}{2}$ F. improper, convergent, $I = \frac{15}{2}$

G. none of the above



The discontinuity point $x = -1$ is in the middle of the integration interval $[-2, 7]$. Therefore, one has to split the interval and to investigate separately

$$I_1 = \int_{-2}^{-1} (x+1)^{-1/3}\,dx \quad \text{and} \quad I_2 = \int_{-1}^{7} (x+1)^{-1/3}\,dx$$

Since $\int (x+1)^{-1/3}\,dx = \frac{3}{2}(x+1)^{\frac{2}{3}} + C$, one has

$I_1 = \frac{3}{2}(x+1)^{2/3}\Big|_{x=-2}^{x=-1} = -\frac{3}{2}$ and

$I_2 = \frac{3}{2}(x+1)^{2/3}\Big|_{x=-1}^{x=7} = 6$ are both convergent, whence

I is also convergent with $I = I_1 + I_2 = \frac{9}{2}$

extra page for calculations (please remove it when submitting the test!)