GNG 1105E – Engineering Mechanics

CHAPTER S2 – FORCE SYSTEMS

Assigned readings

1/4 Newton's Laws

1/5 Units

1/6 Law of gravitation

1/7 Accuracy, limits, and approximations

1/8 Problem solving in statics

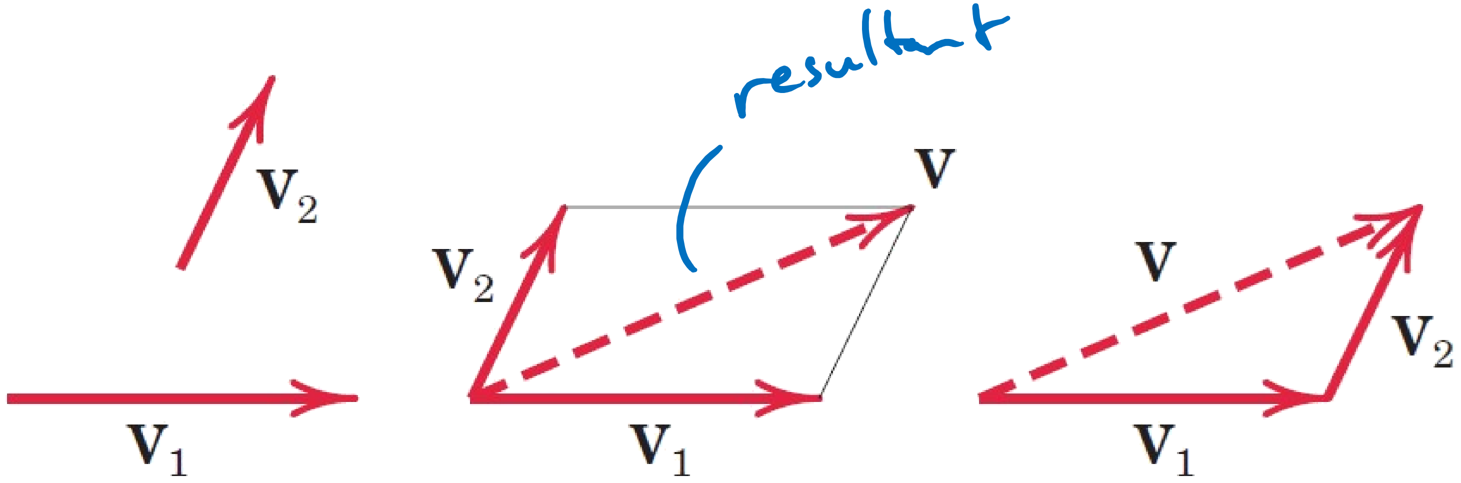
2/1 Introduction

2/2 Force

2/3 Rectangular components (2-D)

1/3 Working with vectors

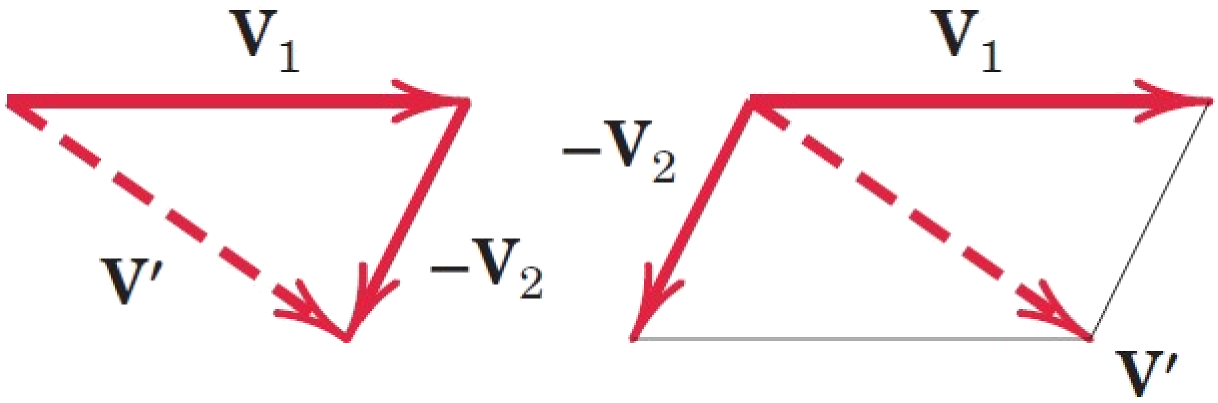
- Parallelogram Law of Addition – Vector Sum $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$
  - Two vectors, $\mathbf{V}_1$ and $\mathbf{V}_2$, treated as free vectors, may be replaced by their equivalent vector $\mathbf{V}$, which is the diagonal of the parallelogram formed by $\mathbf{V}_1$ and $\mathbf{V}_2$. This is called a *vector sum*.



1/3 Working with Vectors

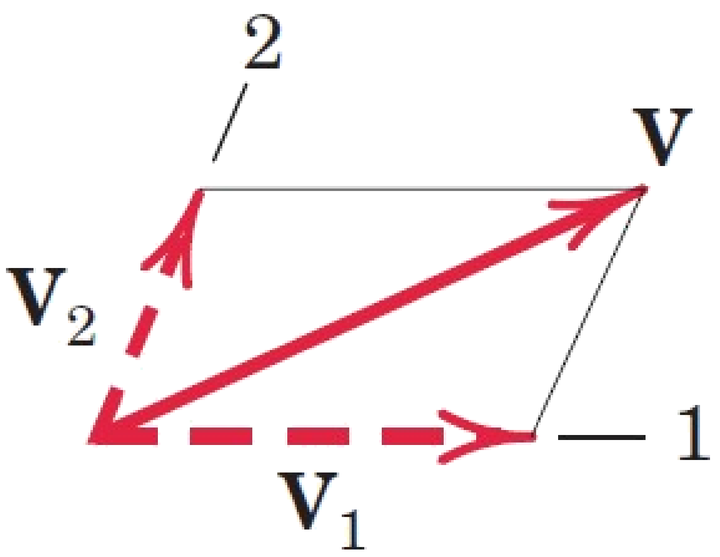
Vector difference (i.e. adding a negative):

$$\boldsymbol{V}' = \boldsymbol{V}_1 - \boldsymbol{V}_2$$

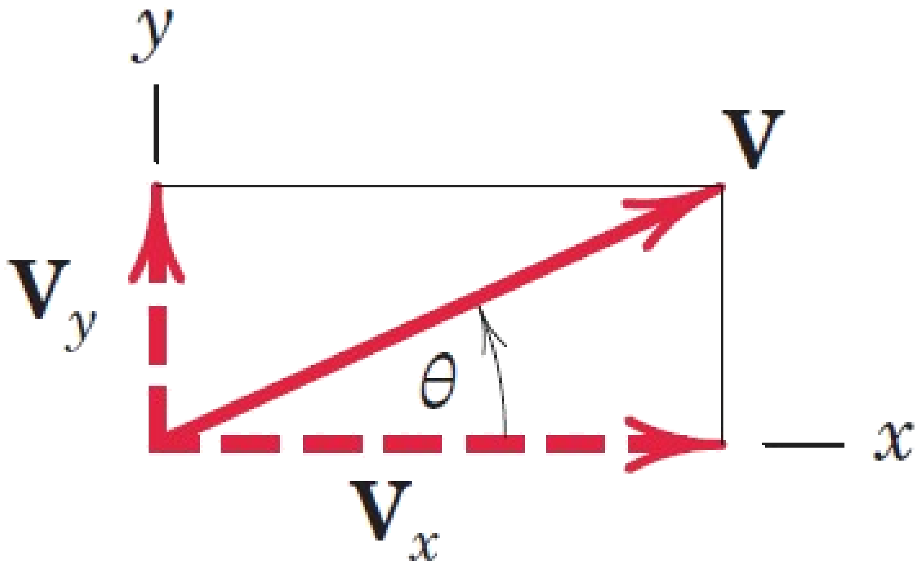


1/3 Working with Vectors

Vector components:

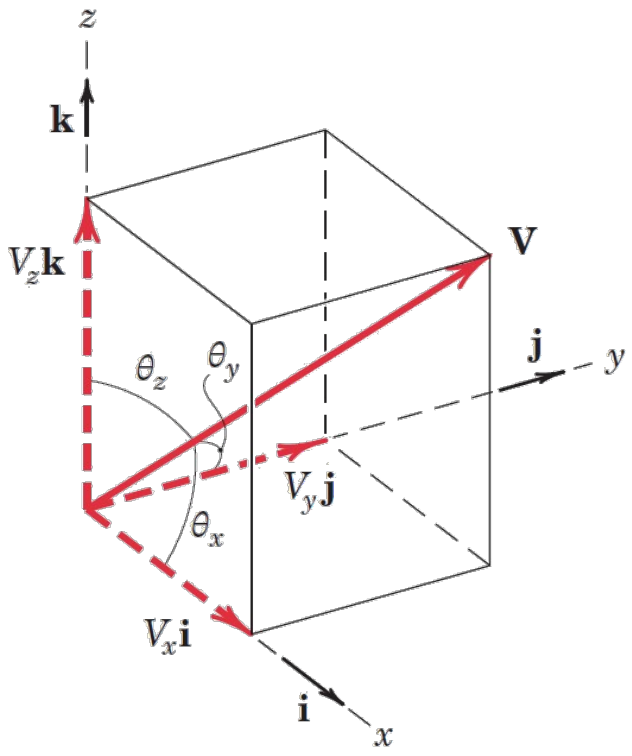


Rectangular components:



1/3 Working with Vectors

- Unit Vector Representation, $\mathbf{V} = V\boldsymbol{n}$
  - A unit vector $\boldsymbol{n}$ has a magnitude of one (unity) and points in the direction of a vector
- Three-Dimensional Vectors and Direction Cosines
  - $\mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}$
  - $V_x = V \cos \theta_x$
  - $V_y = V \cos \theta_y$
  - $V_z = V \cos \theta_z$
- Pythagorean Theorem (Vector Magnitude)



1/4 Newton’s Laws

First Law:

- A particle remains at rest or continues to move with uniform velocity (in a straight line with constant speed) if there is no unbalanced force acting on it

1/4 Newton’s Laws

Second Law:

- The acceleration of a particle is proportional to the vector sum of forces acting on it and is in the direction of this vector sum
- Later you will see that this law can be stated as follows:

1/4 Newton’s Laws

Third Law:

- The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear (they lie on the same line)

1/5 Units

Kinetic units are used to quantify each of the four fundamental concepts introduced earlier

Units for length, time, and mass can be defined arbitrarily and are referred to as base units

A unit for force must be chosen in accordance with the equation ***F=ma*** and is referred to as a derived unit

Kinetic units selected in this way are said to form a consistent system of units

1/5 Units

International System of Units (SI Units)

- In this system, the base units are the units of length, mass, and time
- Length → metres (m)
- Mass → kilograms (kg)
- Time → seconds (s)

1/5 Units

The unit of force is a derived unit and is called the Newton (N)

A Newton is defined as a force which gives an acceleration of 1 m/s^2 to a mass of 1 kg

From F=ma:

$$1N = (1kg)\left(\frac{1m}{s^2}\right) = 1kg \cdot m/s^2$$

2/2 Force

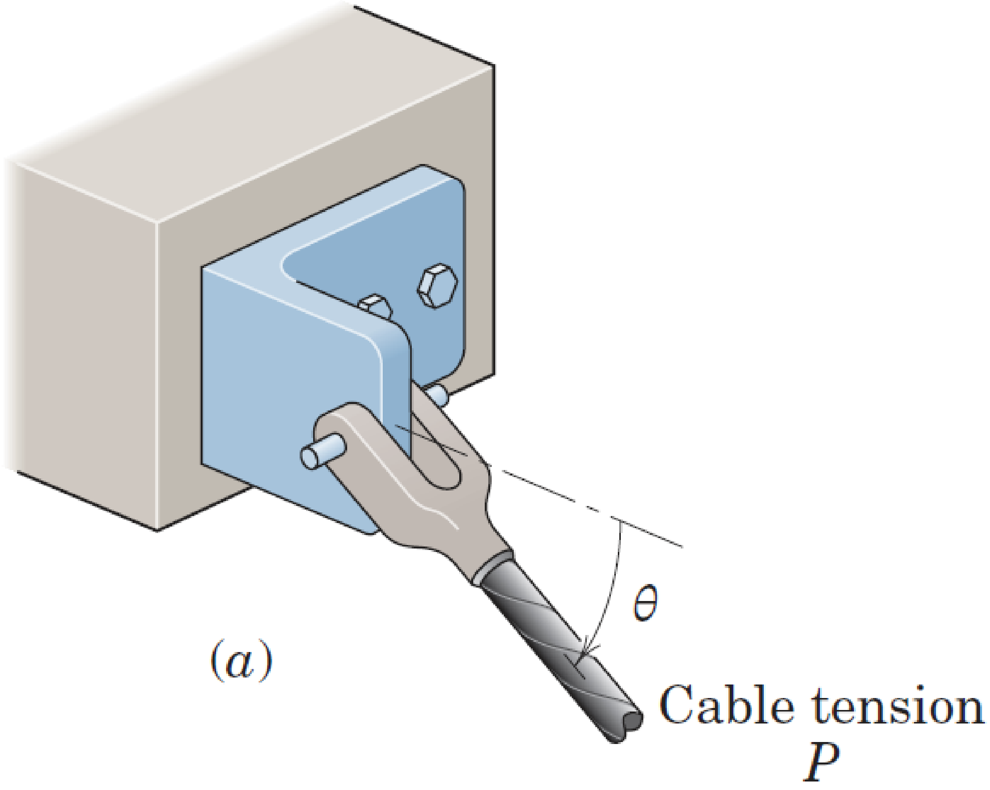
A force is defined as an **action** of one body on another

A force is a **vector** – its effect depends on the **direction** and **magnitude** of the action

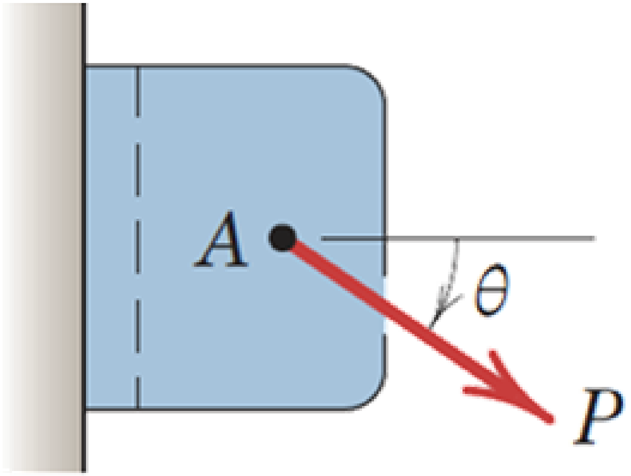
- Therefore, forces may be combined according to the parallelogram law

In general, we also need to define the **point of application** of the force

2/2 Force



(a)

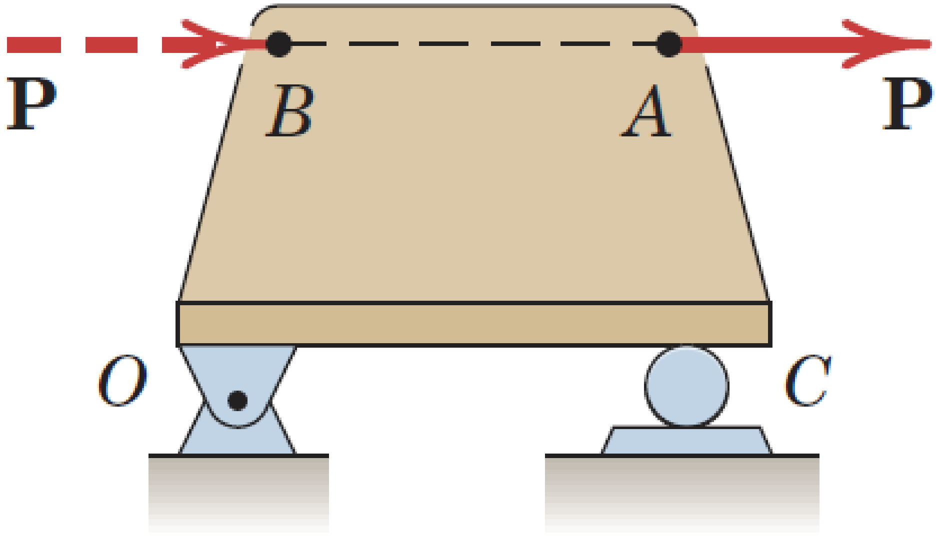


(b)

2/2 Force

Principle of Transmissibility

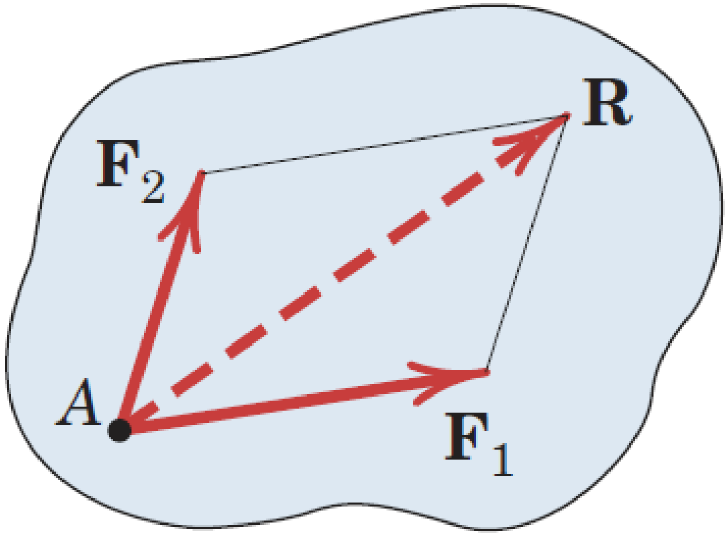
- For rigid bodies, a force can slide anywhere along its line of action without changing the net effects on the body

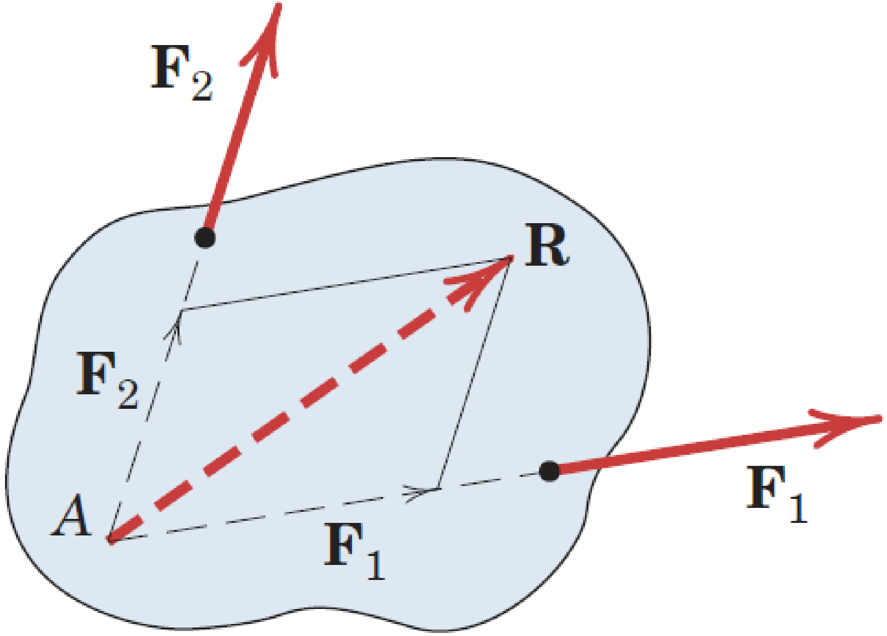


2/2 Force

Concurrent forces have lines of action that intersect at a point

When dealing with concurrent forces, we can treat the object as a **particle**





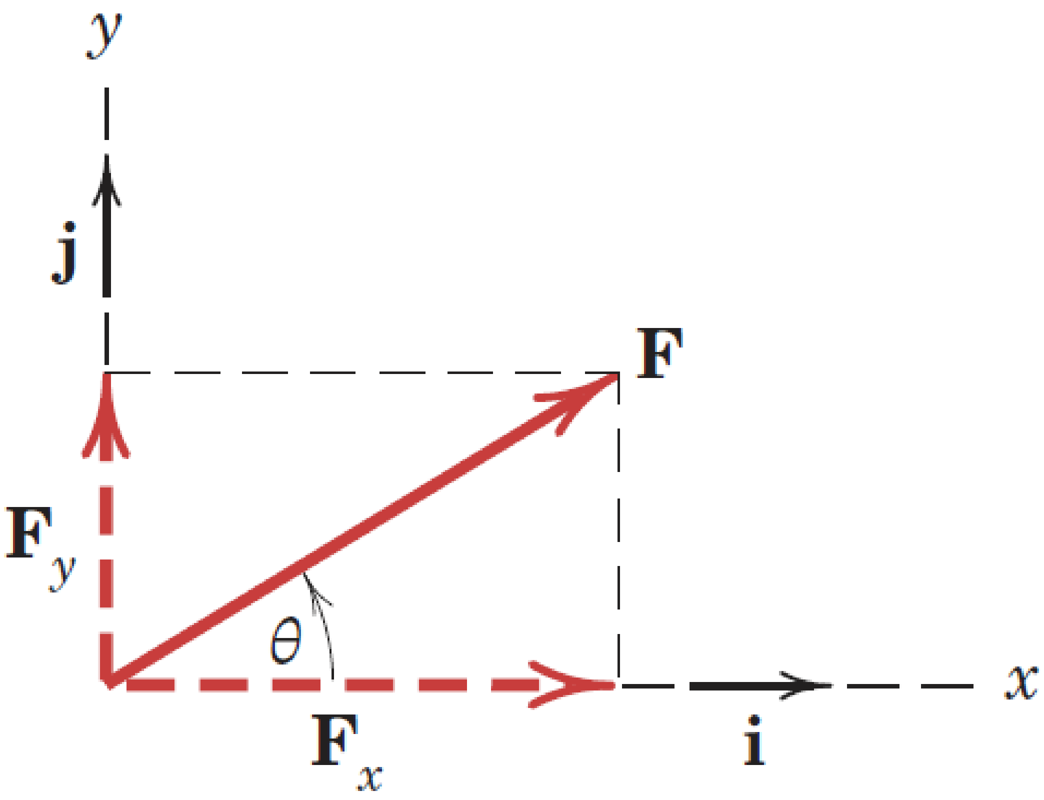
2/3 Rectangular components

Vector Components: $\mathbf{F} = \mathbf{F_x} + \mathbf{F_y}$

Scalar Components: $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$

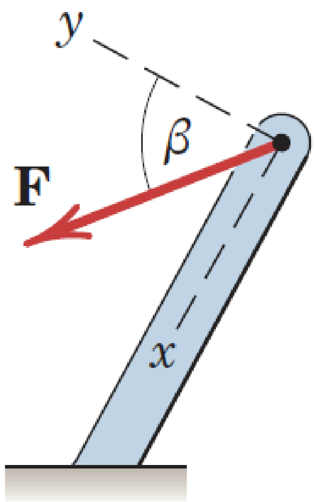
$$F_x = F\cos\theta \qquad F = \sqrt{F_x^2 + F_y^2}$$

$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

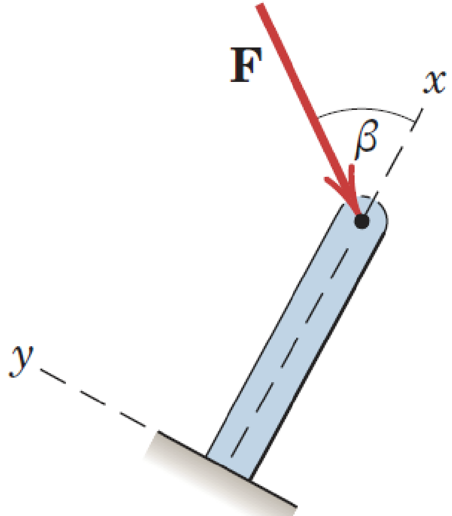


2/3 Rectangular components

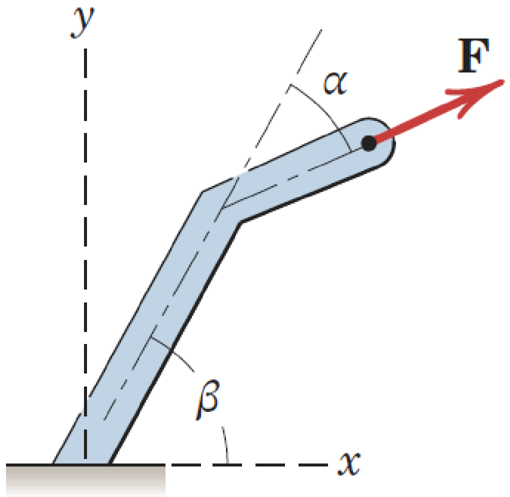
Note: x and y axes do not necessarily have to be horizontal and vertical



$$F_x = F \sin \beta$$
$$F_y = F \cos \beta$$



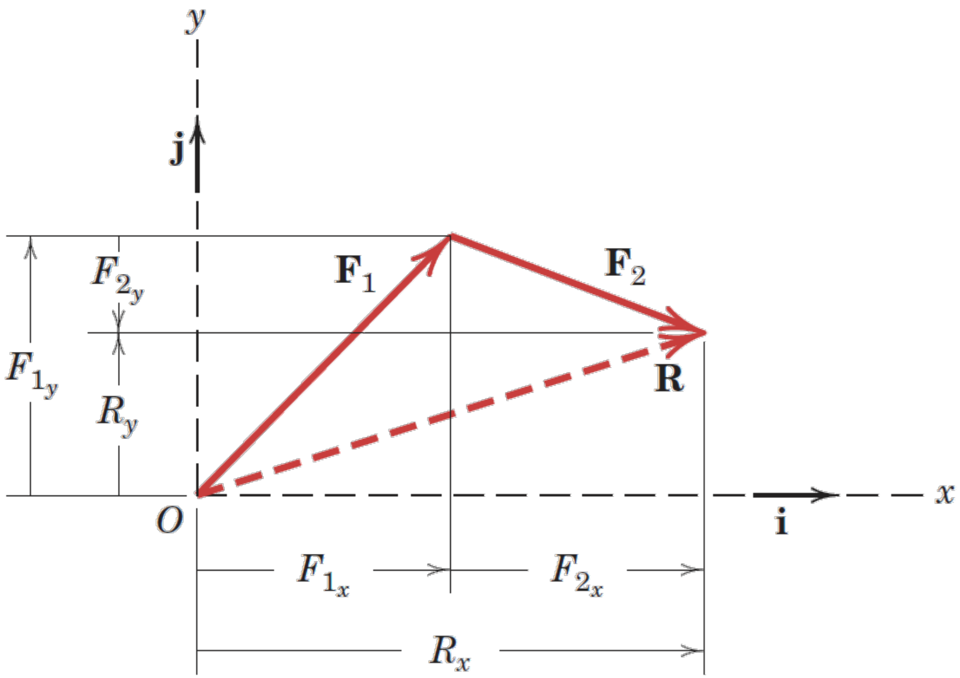
$$F_x = -F \cos \beta$$
$$F_y = -F \sin \beta$$



$$F_x = F \cos(\beta - \alpha)$$
$$F_y = F \sin(\beta - \alpha)$$

2/3 Rectangular components

Rectangular components can be used instead of the parallelogram law to find the resultants of 2 or more forces:



$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (F_{1_x}\mathbf{i} + F_{1_y}\mathbf{j}) + (F_{2_x}\mathbf{i} + F_{2_y}\mathbf{j})$$

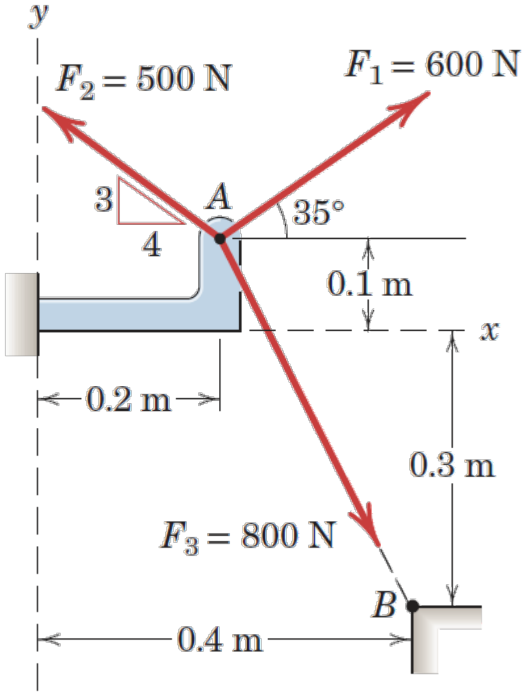
$$R_x\mathbf{i} + R_y\mathbf{j} = (F_{1_x} + F_{2_x})\mathbf{i} + (F_{1_y} + F_{2_y})\mathbf{j}$$

$$R_x = F_{1_x} + F_{2_x} = \sum F_x$$

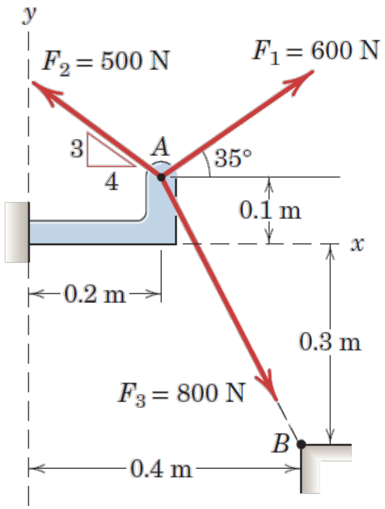
$$R_y = F_{1_y} + F_{2_y} = \sum F_y$$

Sample problem 2/1

The forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$, all act on point *A* of the bracket. Determine the *x* and *y* scalar components of each of the three forces, and their resultant force.



Sample problem 2/1



Sample problem 2/1