Q1. For each of the following propositions, determine its truth value, and write its negation (in English).

a. There are no black flies in Maine. (False)

Negation: It is not the case that there are no black flies in Maine.

Another possible way to negate:
There is at least one black fly in Maine.

b. All unicorns have rainbow-coloured manes. (False —assuming definition of "unicorn" includes toy unicorns)

Negation: It is not the case that all unicorns have rainbow-coloured manes.

Another possible way to negate:
There exists at least one unicorn that does not have a rainbow-coloured mane.

Q2. Let p, q, and r be the following propositions:

p : Grizzly bears have been seen in the area.
q : Hiking is safe on the trail.
r : Berries are ripe along the trail.

Write each of the following compound propositions using the variables p, q, and r, and appropriate logical connectives:

a. If $\underbrace{\text{berries are ripe along the trail}}_{r}$, then ($\underbrace{\text{hiking is safe}}_{q}$ if and only if $\underbrace{\text{grizzly bears have not been seen in the area}}_{\text{not } p}$).

Initial Simplification: if r, then (q if and only if $\neg p$)

Final Answer: $\boxed{r \rightarrow (q \leftrightarrow \neg p)}$

b. Hiking is safe on the trail $\boxed{\text{unless}}$ berries are ripe along the trail $\boxed{\text{and}}$ grizzly bears have been seen in the area.

initial simplification: q unless r and p

answer: $q \vee r \wedge p$ ← Note: by precedence, this means $q \vee (r \wedge p)$

c. Grizzly bears have been seen in the area, but hiking is safe.

answer: $p \wedge q$

d. Either hiking is safe or berries are ripe along the trail, but not both.

Initial Simplification: either q or r but not both

Final Answer: $\boxed{q \oplus r}$

d. A necessary condition for hiking to be safe is that grizzly bears have not been seen in the area.

initial simplification: A necessary condition for q is $\neg p$

translation: $q \rightarrow \neg p$ (If q is true, then all that is necessary for q must also be true).

e. A sufficient condition for grizzlies to have been seen in the area is that berries are ripe along the trail.

initial simplification: A sufficient condition for p is r

translation $r \rightarrow p$

Q3. Construct a truth table for each of the following compound propositions and determine whether it is a tautology, contradiction, or contingency:

a. $p \oplus p$

b. $(p \to q) \wedge (\neg p \to q)$

c. $\neg(p \vee q) \leftrightarrow (\neg p \wedge \neg q)$

d. $\neg(p \vee q) \leftrightarrow (\neg p \to q)$

e. $\neg p \to (q \to r)$

a)

p	$p \oplus p$
T	F
F	F

Since $p \oplus p$ is always F, it's a contradiction.

b)

p	q	$p \to q$	$\neg p \to q$	$(p\to q)\wedge(\neg p \to q)$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

Since $(p\to q)\wedge(\neg p\to q)$ is sometimes T and sometimes F, it's a contingency.

c)

p	q	$p \vee q$	$\neg(p\vee q)$	$\neg p \wedge \neg q$	$\neg(p\vee q) \leftrightarrow (\neg p \wedge \neg q)$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	T	T

$\leftarrow$ Since it is always T, it's a tautology d)

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg p \to q$	$\neg(p \vee q) \leftrightarrow (\neg p \to q)$
T	T	T	F	F	T	F
T	F	T	F	F	T	F
F	T	T	F	T	T	F
F	F	F	T	T	F	F

← Since it's always F, it's a contradiction

e)

p	q	r	$\neg p$	$q \to r$	$\neg p \to (q \to r)$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

← Since it is sometimes T and sometimes F, it's a contingency

Q4. Which of the following pairs of propositions are logically equivalent? Verify your answer with a truth table. For those which are not logically equivalent, give all counterexamples.

a. $A \oplus B$ and $\neg(A \leftrightarrow B)$

b. $p \to c$ and $\neg p \vee c$

c. $\big((x \oplus y) \wedge \neg(x \vee y)\big)$ and $(p \leftrightarrow \neg p)$.

d. $A \to B$ and $B \to A$.

e. $\neg(P \vee Q \vee R)$ and $\neg P \vee \neg Q \vee \neg R$.

f. $\neg(P \vee Q \vee R)$ and $\neg P \wedge \neg Q \wedge \neg R$.

g. $\neg(A \to B)$ and $A \wedge \neg B$.

a)

A	B	$A \oplus B$	$A \leftrightarrow B$	$\neg(A \leftrightarrow B)$	$(A \oplus B) \leftrightarrow \neg(A \leftrightarrow B)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	T	F	T

The biconditional statement $(A \oplus B) \leftrightarrow \neg(A \leftrightarrow B)$ is a tautology.

$\therefore (A \oplus B) \equiv \neg(A \leftrightarrow B)$

b)

p	c	$p \to c$	$\neg p \vee c$	$(p \to c) \leftrightarrow (\neg p \vee c)$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

The biconditional statement $(p \to c) \leftrightarrow (\neg p \vee c)$ is a tautology

$\therefore p \to c \equiv \neg p \vee c$

c)

p	x	y	$x \oplus y$	$x \vee y$	$(x \oplus y) \wedge \neg(x \vee y)$	$p \leftrightarrow \neg p$	$[(x \oplus y) \wedge \neg(x \vee y)] \leftrightarrow [p \leftrightarrow \neg p]$
T	T	T	F	T	F	F	T
T	T	F	T	T	F	F	T
T	F	T	T	T	F	F	T
T	F	F	F	F	F	F	T
F	T	T	F	T	F	F	T
F	T	F	T	T	F	F	T
F	F	T	T	T	F	F	T
F	F	F	F	F	F	F	T

The biconditional statement $[(x \oplus y) \wedge \neg(x \vee y)] \leftrightarrow [p \leftrightarrow \neg p]$ is a tautology $\therefore (x \oplus y) \wedge \neg(x \vee y) \equiv p \leftrightarrow \neg p$

d)

A	B	$A \to B$	$B \to A$	$(A \to B) \leftrightarrow (B \to A)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	F

$A \to B$ is not logically equivalent to $B \to A$

Counterexamples:

when $A = T$, $B = F$, $(A \to B) \leftrightarrow (B \to A)$ is F

when $A = F$, $B = T$, $(A \to B) \leftrightarrow (B \to A)$ is F

			①	②	③		
P	Q	R	$\neg(P \vee Q \vee R)$	$\neg P \vee \neg Q \vee \neg R$	$\neg P \wedge \neg Q \wedge \neg R$	① ↔ ②	① ↔ ③
T	T	T	F	F	F	T	T
T	T	F	F	T	F	F	T
T	F	T	F	T	F	F	T
T	F	F	F	T	F	F	T
F	T	T	F	T	F	F	T
F	T	F	F	T	F	F	T
F	F	T	F	T	F	F	T
F	F	F	T	T	F	T	T

e)+f)

$\neg(P \vee Q \vee R)$ is <u>not</u> logically equiv. to $\neg P \vee \neg Q \vee \neg R$

exercise: give the 6 counterexamples

① ↔ ③ is a tautology

$\therefore \neg(P \vee Q \vee R) \equiv \neg P \wedge \neg Q \wedge \neg R$

g)

A	B	$\neg(A \to B)$	$A \wedge \neg B$	$\neg(A \to B) \leftrightarrow A \wedge \neg B$
T	T	F	F	T
T	F	T	T	T
F	T	F	F	T
F	F	F	F	T

$\neg(A \to B) \leftrightarrow A \wedge \neg B$ is a tautology $\therefore \neg(A \to B) \equiv A \wedge \neg B$.

Q5. Considering Q4 f and g, write the negation of each of the following propositions in English.

a. $\underbrace{\text{You are at least 12 years old}}_{a}$, $\vee$ $\underbrace{\text{or you are taller than 5 feet}}_{b}$, $\vee$ $\underbrace{\text{or you have a golden ticket.}}_{c}$

Negation: $\neg(a \vee b \vee c) \equiv \neg a \wedge \neg b \wedge \neg c$

You are less than 12 years old and you are ≤ 5 feet tall, and you do not have a golden ticket.

b. In order for you to ride the roller coaster, it is necessary that you are at least 12 years old, or you are taller than 5 feet, or you have a golden ticket.

p: You can ride the roller coaster. q: $a \vee b \vee c$ from Q6a)

Q2b says $p \to q$

or $p \to a \vee b \vee c$

Negation: $\neg(p \to q) \equiv p \wedge \neg q$

$\neg(p \to a \vee b \vee c) \equiv p \wedge \neg a \wedge \neg b \wedge \neg c$

Negation: you can ride the roller coaster, and you are < 12 years old, and you are ≤ 5 feet tall, and you do not have a golden ticket.

*bonus: write the converse of Q5b as well as the contrapositive, and each of their respective negations.