MAT 1348 – Winter 2023

Exercises 3 – Solutions

Professor: Antoine Poirier

Questions are taken from Discrete Mathematics 8th edition, by Kenneth H. Rosen.

QUESTION 1 **(1.3 # 21)**. Use the laws of logic to show that $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$.

Solution: $\neg(p \leftrightarrow q) \equiv \neg((p \wedge q) \vee (\neg p \wedge \neg q)) \equiv \neg(p \wedge q) \wedge \neg(\neg p \wedge \neg q) \equiv (\neg p \vee \neg q) \wedge (\neg\neg p \vee \neg\neg q) \equiv (p \to \neg q) \wedge (p \vee q) \equiv (p \to \neg q) \wedge (\neg q \to p) \equiv p \leftrightarrow \neg q$

QUESTION 2 **(1.3 # 23)**. Use the laws of logic to show that $\neg p \leftrightarrow q \equiv p \leftrightarrow \neg q$.

Solution: $\neg p \leftrightarrow q \equiv (\neg p \to q) \wedge (q \to \neg p) \equiv (\neg\neg p \vee q) \wedge (\neg q \vee \neg p) \equiv (p \vee q) \wedge (\neg q \vee \neg p) \equiv (\neg q \to p) \wedge (p \to \neg q) \equiv p \leftrightarrow \neg q$

QUESTION 3 **(1.3 # 27)**. Use the laws of logic to show that $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$.

Solution: $(p \to r) \wedge (q \to r) \equiv (\neg p \vee r) \wedge (\neg q \vee r) \equiv (\neg p \wedge \neg q) \vee r \equiv \neg(p \vee q) \vee r \equiv (p \vee q) \to r$

QUESTION 4 **(1.3 # 29)**. Use the laws of logic to show that $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$.

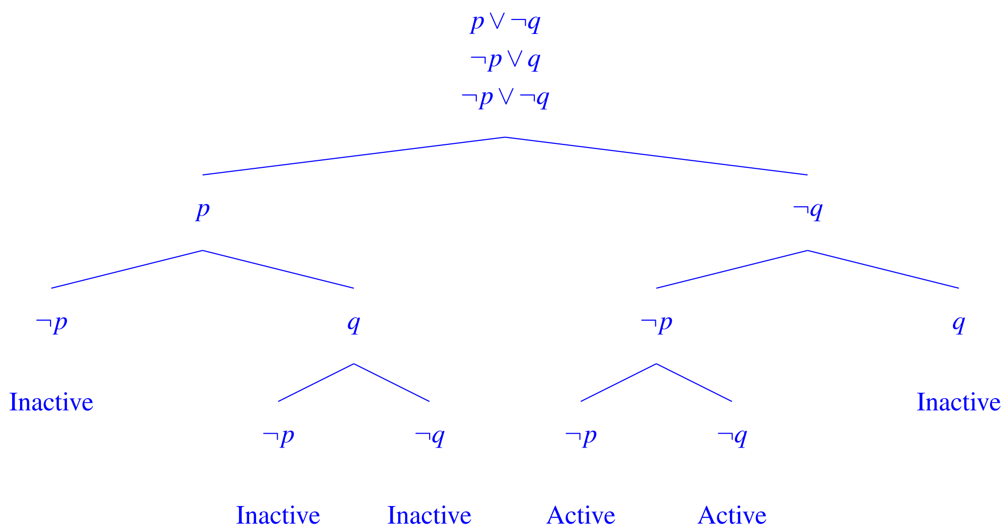
Solution: $(p \to r) \vee (q \to r) \equiv (\neg p \vee r) \vee (\neg q \vee r) \equiv (\neg p \vee \neg q) \vee r \equiv \neg(p \wedge q) \vee r \equiv (p \wedge q) \to r$

QUESTION 5 **(1.3 # 65)**. Determine if the following sets are consistent.

(1) $\{(p \vee \neg q), (\neg p \vee q), (\neg p \vee \neg q)\}$
(2) $\{(p \to q), (p \to \neg q), (\neg p \to q), (\neg p \to \neg q)\}$
(3) $\{(p \leftrightarrow q), (\neg p \leftrightarrow q)\}$

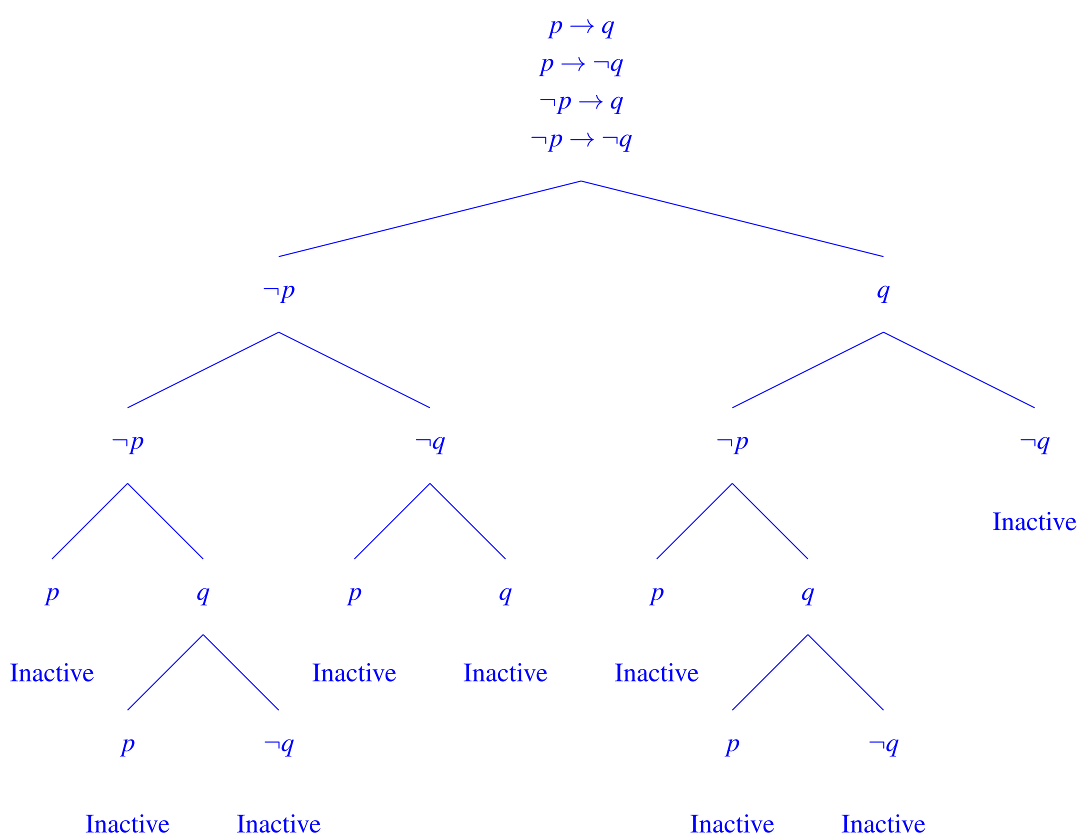
Solution:

(1) We build the following truth tree.



The two active branches indicate that all propositions are true when p and q are false. The set is consistent.

(2) We build the following truth tree.



All branches are inactive, so the set is not consistent.

(3) We build the following truth tree.

$p \leftrightarrow q$
$\neg p \leftrightarrow q$

p, q		$\neg p$, $\neg q$	
$\neg p$, q	p, $\neg q$	$\neg p$, q	p, $\neg q$
Inactive	Inactive	Inactive	Inactive

All branches are inactive, so the set is not consistent.

QUESTION 6 **(1.6 # 1)**. Determine if the following argument is valid

If Socrates is human, then he is mortal.
Socrates is human.

Therefore, Socrates is mortal.

Solution: Let $h =$ "Socrates is human" and $m =$ "Socrates is mortal". The argument is $((h \to m) \wedge h) \to m$. We verify that it is a tautology

$(h \to m) \wedge h) \to m \equiv \neg((h \to m) \wedge h) \vee m \equiv \neg((\neg h \vee m) \wedge h) \vee m \equiv \neg((\neg h \wedge h) \vee (m \wedge h)) \vee m \equiv \neg(F \vee (m \wedge h)) \vee m \equiv \neg(m \wedge h) \vee m \equiv \neg m \vee \neg h \vee m \equiv \neg h \vee (\neg m \vee m) \equiv \neg h \vee T \equiv T$

Since it is a tautology, we conclude the argument is valid.

QUESTION 7 **(1.6 # 6)**. Determine if the following argument is valid.

If it's not raining or if it's not foggy, then the race and the show will take place.
If the race takes place, then a trophy will be awarded.
A trophy has not been awarded.

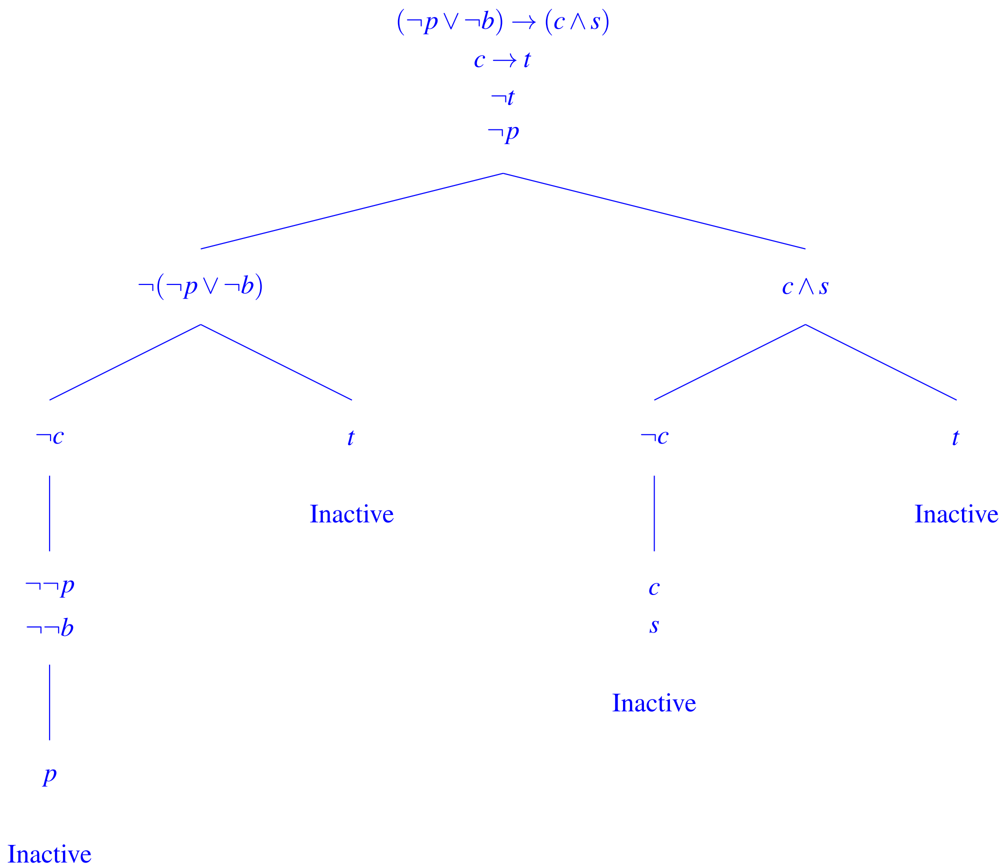
Therefore, it is raining.

Solution: Let $p =$ "it is raining", $b =$ "it is foggy", $c =$ "the race takes place", $s =$ "the show takes place" and $t =$ "a trophy is awarded". The argument becomes

$$(\neg p \vee \neg b) \to (c \wedge s)$$
$$c \to t$$
$$\neg t$$

$$p$$

We build the truth tree

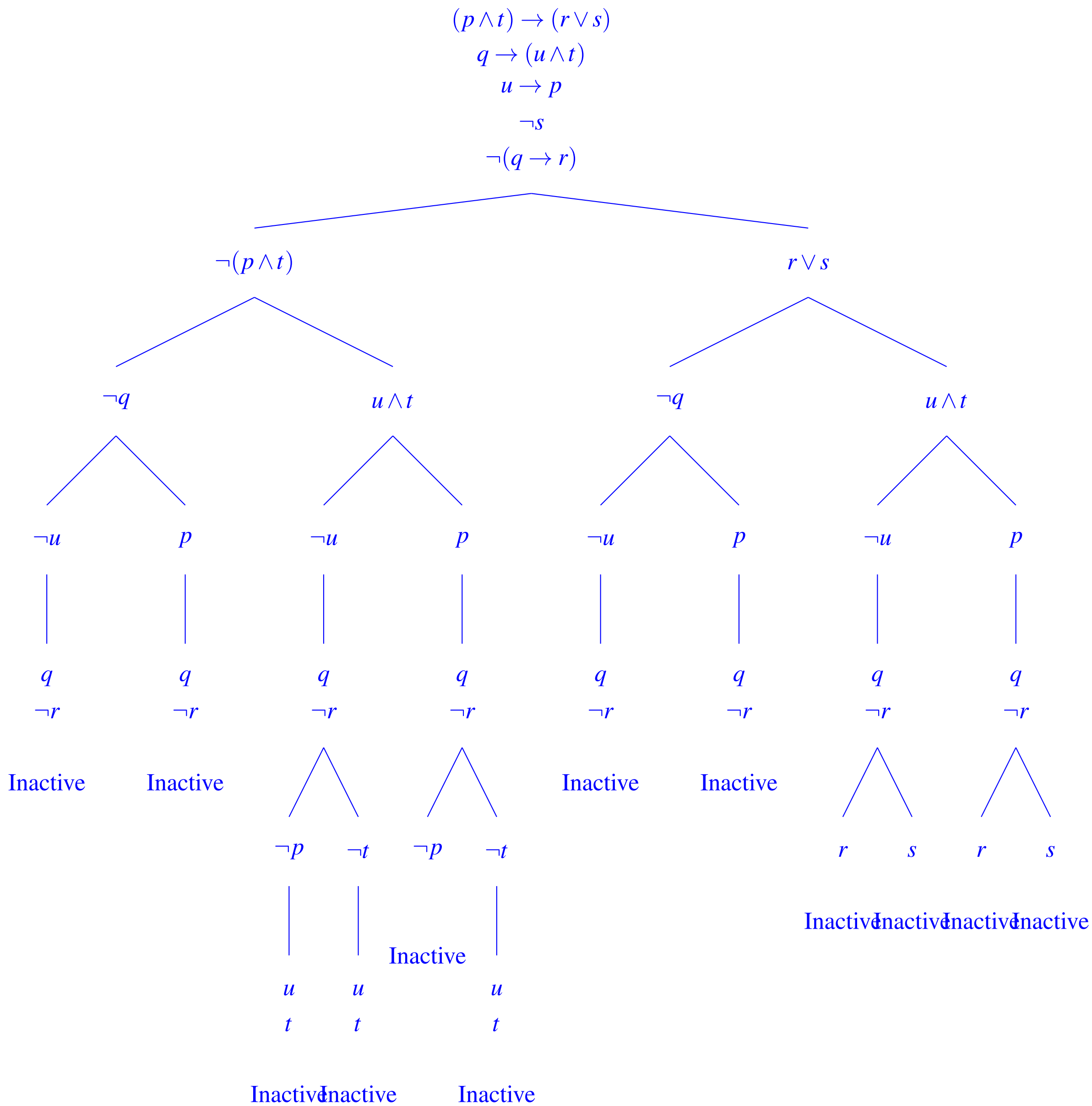


All branches are inactive, therefore the argument is valid.

QUESTION 8 **(1.6 # 12)**. Show that the following argument is valid.

$$\begin{array}{c} (p \wedge t) \to (r \vee s) \\ q \to (u \wedge t) \\ u \to p \\ \neg s \\ \hline q \to r \end{array}$$

Solution: We build the truth tree:



All branches are inactive, therefore the argument is valid.

QUESTION 9 **(1.6 # 31)**. Determine if the following argument is valid:

It does not rain or Yvette has her umbrella.
Yvette does not have her umbrella or she is not wet.
It rains or Yvette is not wet.

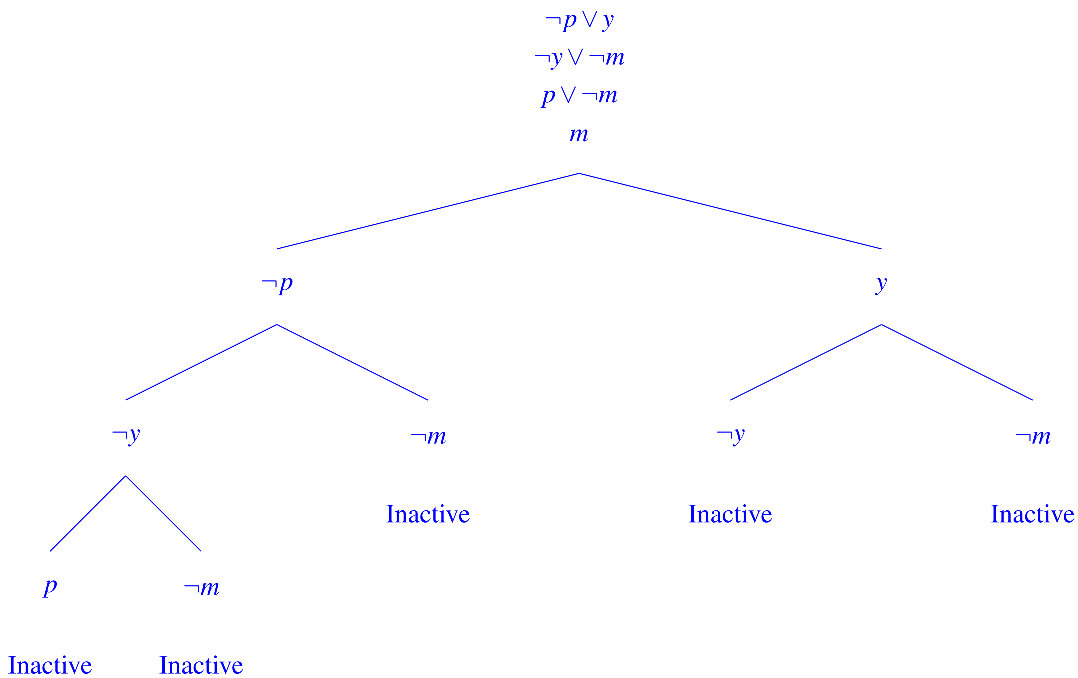
Therefore, Yvette is not wet.

Solution: Let $p =$ "It rains", $y =$ "Yvette has her umbrella", $m =$ "Yvette is wet". The argument becomes

$$\neg p \vee y$$
$$\neg y \vee \neg m$$
$$p \vee \neg m$$

$\neg m$

We build the truth tree:



All branches are inactive, therefore the argument is valid.

QUESTION 10 (). Determine if the following argument is valid:

Jasmine is skiing or it is not snowing.
It is snowing or Bart plays hockey.

Therefore, Jasmine is skiing or Bart plays hockey.

Solution: Let $j =$ "Jasmine is skiing", $n =$ "it is snowing" and $b =$ "Bart plays hockey". The argument becomes $((j \vee \neg n) \wedge (n \vee b)) \to (j \vee b)$. The argument is valid because it is a tautology: the truth table confirms that:

j	n	b	$j \vee \neg n$	$n \vee b$	$j \vee b$	$((j \vee \neg n) \wedge (n \vee b)) \to (j \vee b)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	T	T
F	T	F	F	T	F	T
F	F	T	T	T	T	T
F	F	F	T	F	F	T

QUESTION 11 (). Determine if the following argument is valid.

If you do all the exercises, then you will understand discrete math.
You understand discrete math.

Therefore, you did all the exercises.

Solution: Let $e =$ "You do all the exercises", $m =$ "You understand discrete math". The argument is

$$\begin{array}{c} e \rightarrow m \\ m \\ \hline e \end{array}$$

We build the truth tree:

$$\begin{array}{ccc} & e \rightarrow m & \\ & m & \\ & \neg e & \\ \swarrow & & \searrow \\ \neg e & & m \\ \text{Active} & & \text{Active} \end{array}$$

This tree contains active branches: the argument is therefore invalid. When e is false and m is true, the hypotheses are true, but the conclusion is false.