GNG 1105E – Engineering Mechanics

CHAPTER S5 – DISTRIBUTED FORCES

Assigned readings

5/1 Introduction

5/2 Center of mass

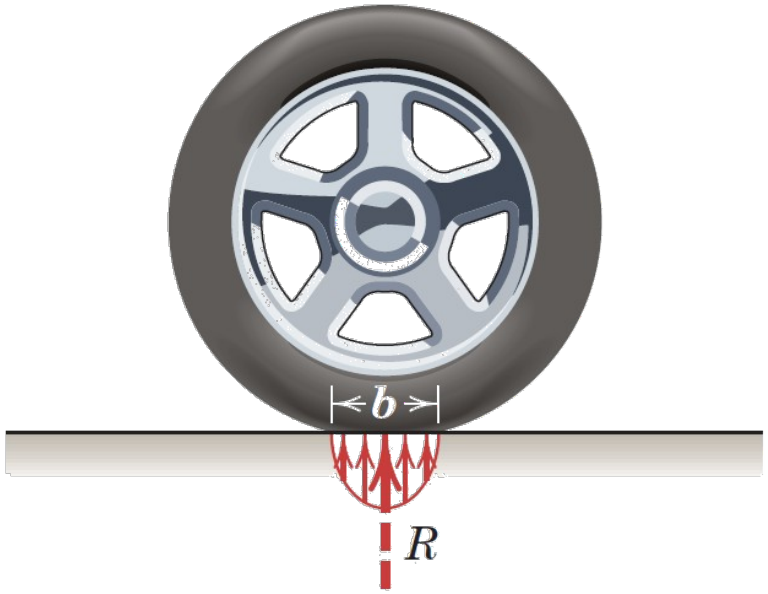
5/3 Centroids of lines and areas (NOT volumes)

5/4 Composite bodies and figures

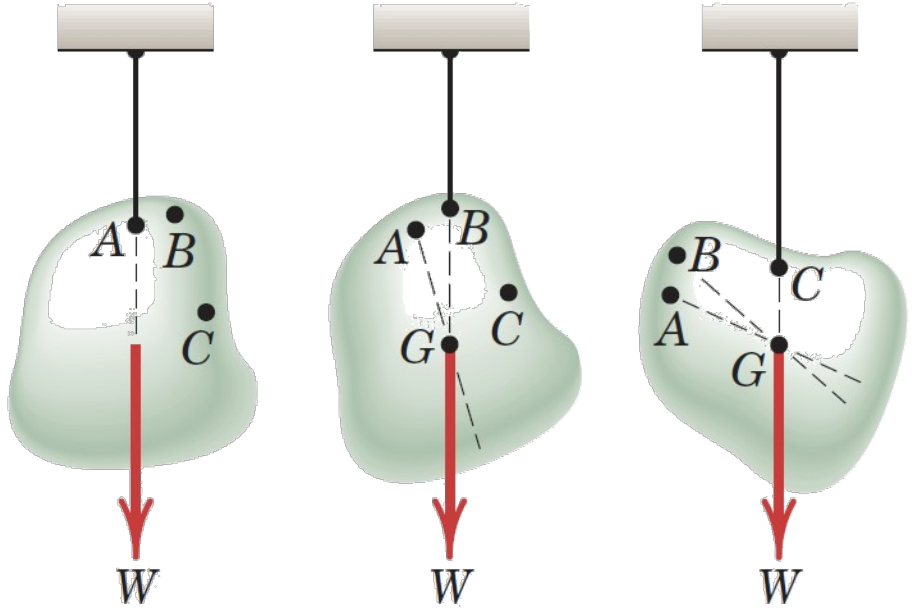
5/1 Introduction

In previous chapters, we have considered forces to be **concentrated** along a line of action

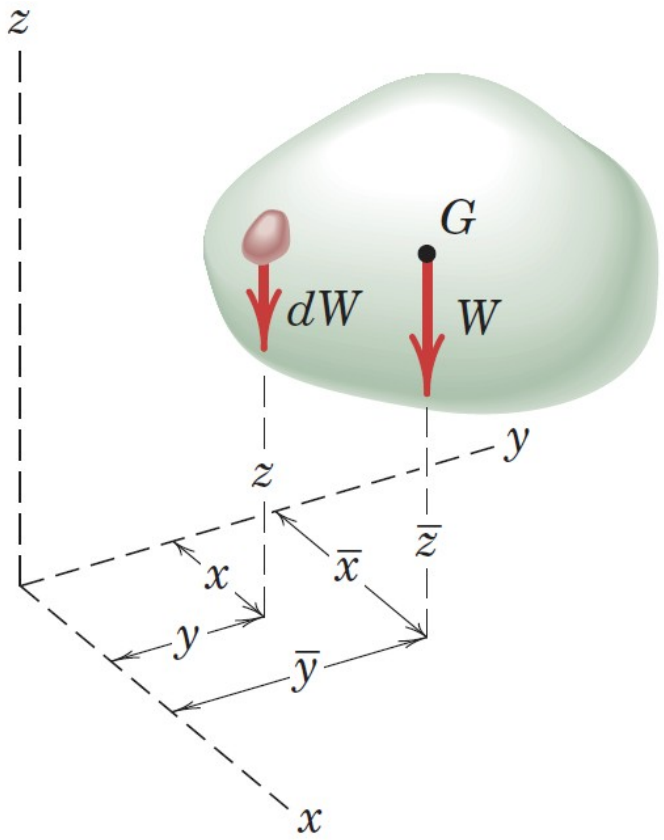
In reality, forces are **distributed** over a larger contact area



5/2 Center of mass



5/2 Center of mass



$$\bar{x} = \frac{\int x\,dW}{W} \qquad \bar{y} = \frac{\int y\,dW}{W} \qquad \bar{z} = \frac{\int z\,dW}{W}$$

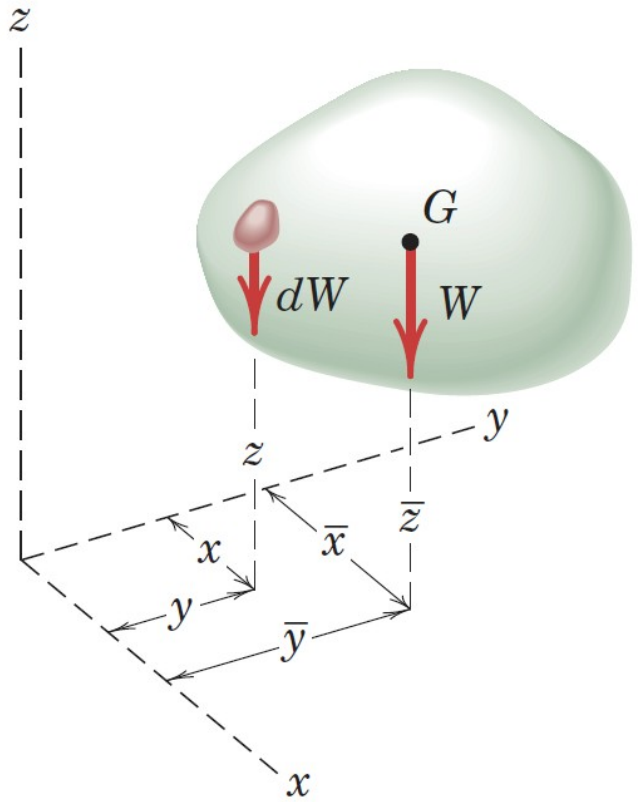
5/2 Center of mass

Substitute $W = mg$ and $dW = g\,dm$:

$$\bar{x} = \frac{\int x\,dm}{m} \quad \bar{y} = \frac{\int y\,dm}{m} \quad \bar{z} = \frac{\int z\,dm}{m}$$

Substitute $m = \rho V$ and $dm = \rho\,dV$:

$$\bar{x} = \frac{\int x\rho\,dV}{\int \rho\,dV} \quad \bar{y} = \frac{\int y\rho\,dV}{\int \rho\,dV} \quad \bar{z} = \frac{\int z\rho\,dV}{\int \rho\,dV}$$



5/3 Centroids

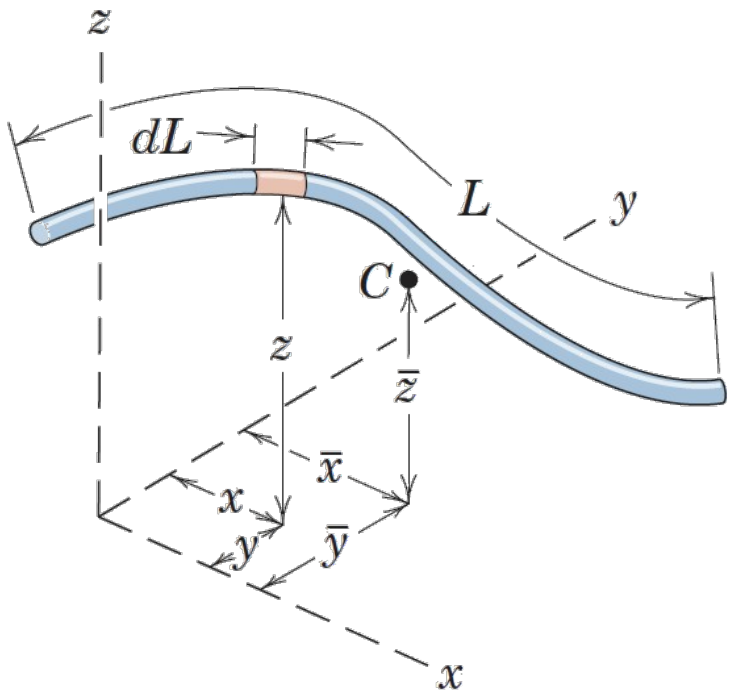
When the density of an object is uniform, its center of mass coincides with its geometric **centroid**

We will look at centroids of lines (uniform cross-sectional area) and two-dimensional areas (uniform thickness) only

- NOT volumes

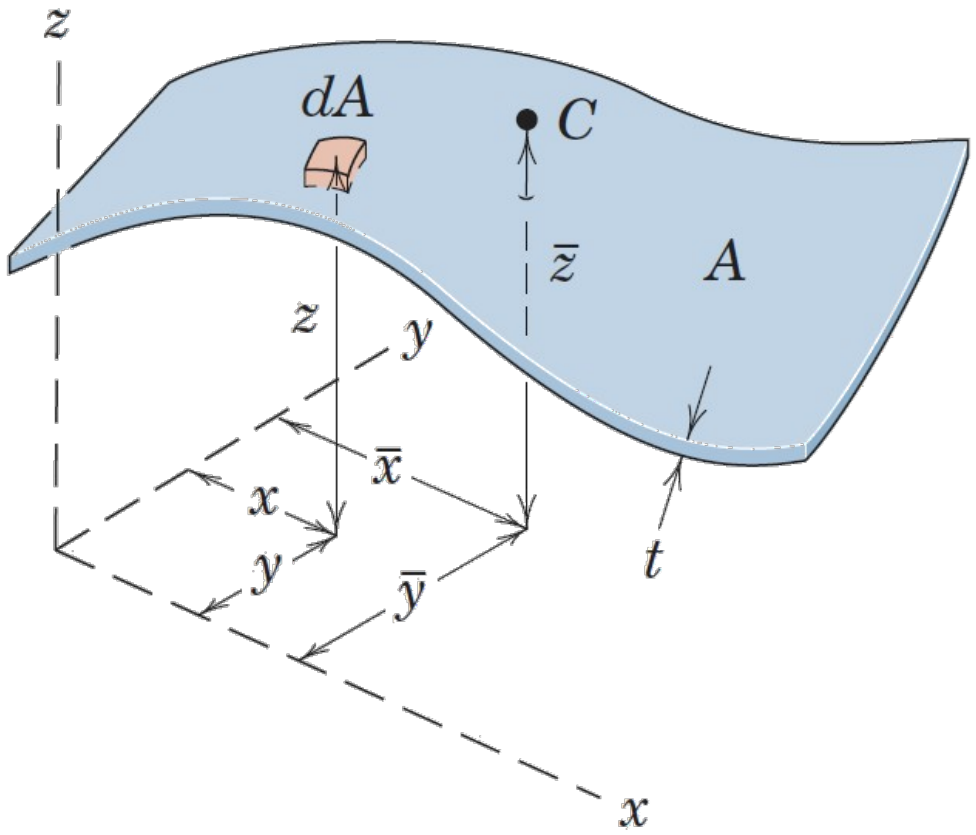
5/3 Centroids

$$\bar{x} = \frac{\int x\, dL}{L} \quad \bar{y} = \frac{\int y\, dL}{L} \quad \bar{z} = \frac{\int z\, dL}{L}$$



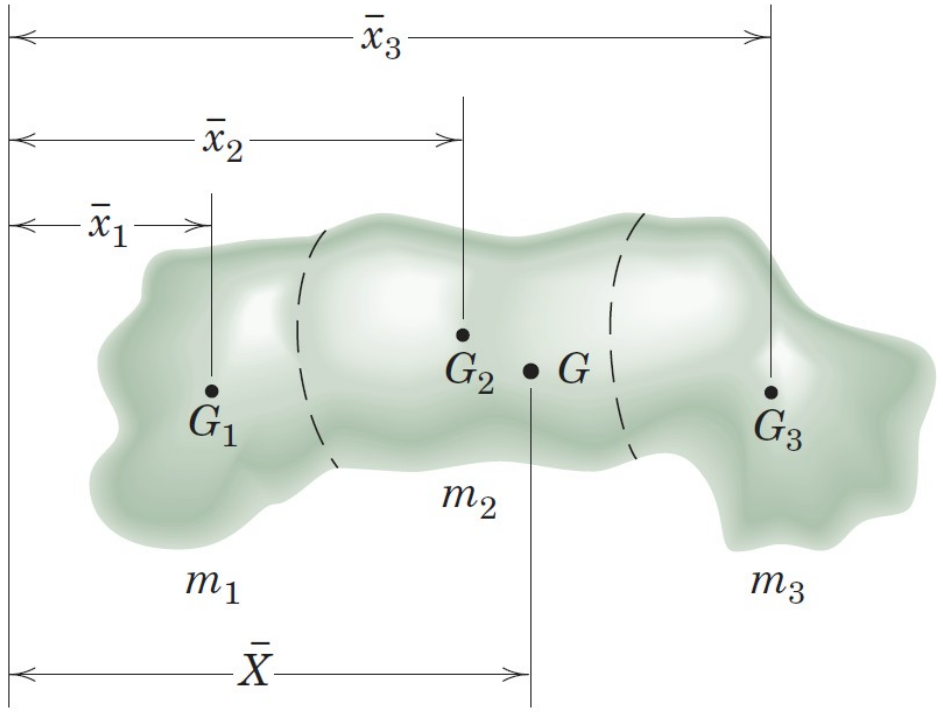
5/3 Centroids

$$\bar{x} = \frac{\int x\, dA}{A} \quad \bar{y} = \frac{\int y\, dA}{A} \quad \bar{z} = \frac{\int z\, dA}{A}$$



5/4 Composite bodies and figures

$$\bar{X} = \frac{\sum m\bar{x}}{\sum m} \qquad \bar{Y} = \frac{\sum m\bar{y}}{\sum m} \qquad \bar{Z} = \frac{\sum m\bar{z}}{\sum m}$$



5/4 Composite bodies and figures

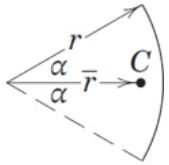
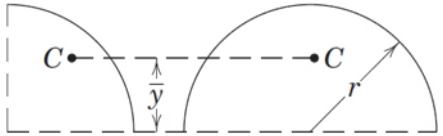
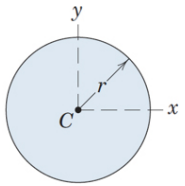
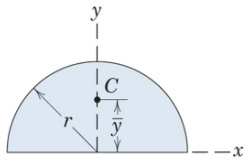
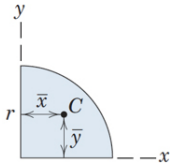
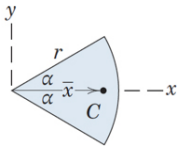
Figure	Centroid
Arc Segment 	$\bar{r} = \dfrac{r \sin \alpha}{\alpha}$
Quarter and Semicircular Arcs 	$\bar{y} = \dfrac{2r}{\pi}$
Circular Area 	$\bar{x} = \bar{y} = 0$
Semicircular Area 	$\bar{y} = \dfrac{4r}{3\pi}$

Figure	Centroid
Quarter-Circular Area 	$\bar{x} = \bar{y} = \dfrac{4r}{3\pi}$
Area of Circular Sector 	$\bar{x} = \dfrac{2}{3}\dfrac{r \sin \alpha}{\alpha}$

5/4 Composite bodies and figures

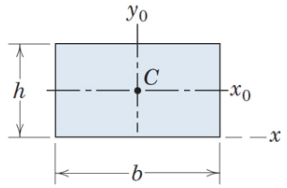
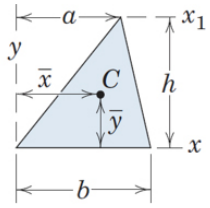
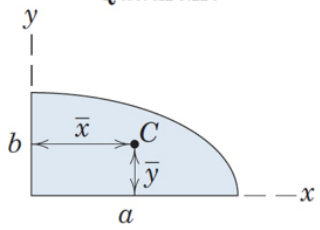
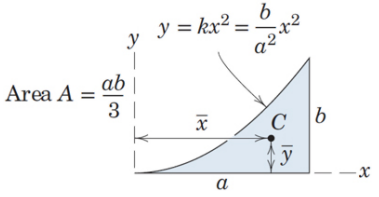
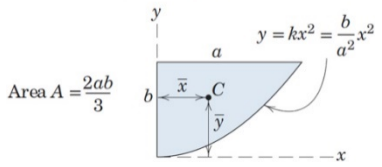
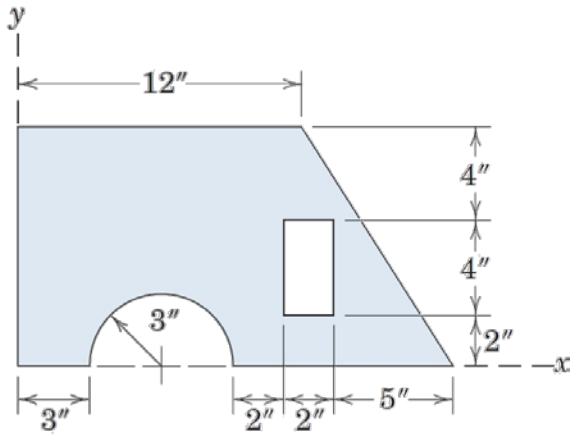
Figure	Centroid
Rectangular Area 	$\bar{x} = \bar{y} = 0$
Triangular Area 	$\bar{x} = \frac{a+b}{3}$ $\bar{y} = \frac{h}{3}$
Area of Elliptical Quadrant 	$\bar{x} = \frac{4a}{3\pi}$ $\bar{y} = \frac{4b}{3\pi}$

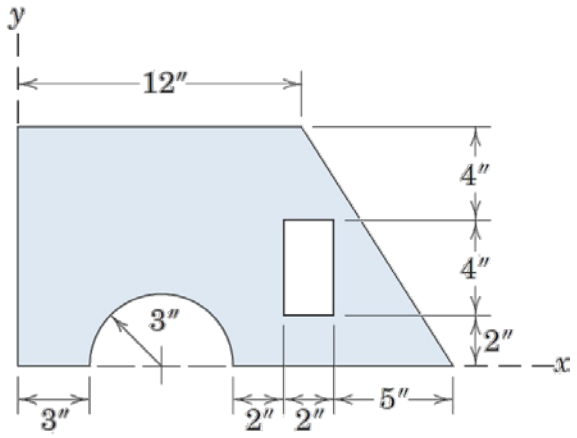
Figure	Centroid
Subparabolic Area 	$\bar{x} = \frac{3a}{4}$ $\bar{y} = \frac{3b}{10}$
Parabolic Area 	$\bar{x} = \frac{3a}{8}$ $\bar{y} = \frac{3b}{5}$

Sample problem 5/6

Locate the centroid of the shaded area.



Sample problem 5/6



Sample problem 5/6