Université d'Ottawa · University of Ottawa

Faculté de Génie - Faculty of Engineering
ITI1100C Digital Systems I -Assignment 1

1) Convert the following numbers with the indicated bases to decimal:

$$(4310)_5 = 4 * 5^3 + 3 * 5^2 + 1 * 5^1 = 580_{10}$$

$$(198)_{12} = 1 * 12^2 + 9 * 12^1 + 8 * 12^0 = 260_{10}$$

$$(435)_8 = 4 * 8^2 + 3 * 8^1 + 5 * 8^0 = 285_{10}$$

$$(345)_6 = 3 * 6^2 + 4 * 6^1 + 5 * 6^0 = 137_{10}$$

2) What is the largest binary number that can be expressed with 16 bits? What are the equivalent decimal and hexadecimal numbers?

16-bit binary: 1111_1111_1111_1111
Decimal equivalent: $2^{16} - 1 = 65{,}535_{10}$
Hexadecimal equivalent: $FFFF_{16}$

3) Convert the hexadecimal number 64CD to binary, and then convert it from binary to octal.

$$64CD_{16} = 0110\_0100\_1100\_1101_2 = 110\_010\_011\_001\ \_101 = (62315\ )_8$$

4) Convert the decimal number 431 to binary in two ways:

(a) Results of repeated division by 2 (quotients are followed by remainders):

431_{10} = 215(1); 107(1); 53(1); 26(1); 13(0); 6(1) 3(0) 1(1)
Answer: $1111\_1010_2 = FA_{16}$

(b) Results of repeated division by 16:

431_{10} = 26(15); 1(10) (Faster)
Answer: FA = 1111_1010

5) Express the following numbers in decimal:

(a) $10110.0101_2 = 16 + 4 + 2 + .25 + .0625 = 22.3125$

(b) $16.5_{16} = 16 + 6 + 5*(.0615) = 22.3125$

(c) $26.24_8 = 2 * 8 + 6 + 2/8 + 4/64 = 22.3125$

(d) $DADA.B_{16} = 14*16^3 + 10*16^2 + 14*16 + 10 + 11/16 = 60{,}138.6875$

(e) $1010.1101_2 = 8 + 2 + .5 + .25 + .0625 = 10.8125$

6) Convert the following binary numbers to hexadecimal and to decimal:

(a) $1.10010_2 = 0001.1001_2 = 1.9_{16} = 1 + 9/16 = 1.563_{10}$

(b) $110.010_2 = 0110.0100_2 = 6.4_{16} = 6 + 4/16 = 6.25_{10}$

Reason: 110.010_2 is the same as 1.10010_2 shifted to the left by two places.

7) Perform the following division in binary: 111011 ÷ 101.

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       1011.11
101 | 111011.0000
      101
      01001
        101
        1001
         101
         1000
          101
          0110
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The quotient is carried to two decimal places, giving 1011.11

Checking: $111011_2 / 101_2 = 59_{10} / 5_{10} \cong 1011.11_2 = 58.75_{10}$

8) Do the following conversion problems:

(a) Convert 27.315 to binary:

	Integer Quotient		Remainder	Coefficient
27/2 =	13	+	½	$a_0 = 1$
13/2	6	+	½	$a_1 = 1$
6/2	3	+	0	$a_2 = 0$
3/2	1	+	½	$a_3 = 1$
½	0	+	½	$a_4 = 1$

$27_{10} = 11011_2$

	Integer		Fraction	Coefficient
.315 x 2 =	0	+	.630	$a_{-1} = 0$
.630 x 2 =	1	+	.26	$a_{-2} = 1$
.26 x 2 =	0	+	.52	$a_{-3} = 0$
.52 x 2 =	1	+	.04	$a_{-4} = 1$

$.315_{10} \cong .0101_2 = .25 + .0625 = .3125$

$27.315 \cong 11011.0101_2$

(b) 2/3 ≅ .6666666667

	Integer		Fraction	Coefficient
.6666_6666_67 x 2	= 1	+	.3333_3333_34	$a_{-1} = 1$
.3333333334 x 2	= 0	+	.6666666668	$a_{-2} = 0$
.6666666668 x 2	= 1	+	.3333333336	$a_{-3} = 1$
.3333333336 x 2	= 0	+	.6666666672	$a_{-4} = 0$
.6666666672 x 2	= 1	+	.3333333344	$a_{-5} = 1$
.3333333344 x 2	= 0	+	.6666666688	$a_{-6} = 0$
.6666666688 x 2	= 1	+	.3333333376	$a_{-7} = 1$
.3333333376 x 2	= 0	+	.6666666752	$a_{-8} = 0$

$.6666666667_{10} \cong .10101010_2 = .5 + .125 + .0313 + ..0078 = .6641_{10}$

$.101010102 = .1010\_1010_2 = .AA_{16} = 10/16 + 10/256 = .6641_{10}$ (Same as (b)).

9) Obtain the 1's and 2's complements of the following binary numbers:

	(a)		(b)		(c)
	0001_0000		0000_0000		1101_1010
1s comp:	1110_1111	1s comp:	1111_1111	1s comp:	0010_0101
2s comp:	1111_0000	2s comp:	0000_0000	2s comp:	0010_0110

	(d)		(e)		(f)
	1010_1010		1000_0101		1111_1111
1s comp:	0101_0101	1s comp:	0111_1010	1s comp:	0000_0000
2s comp:	0101_0110	2s comp:	0111_1011	2s comp:	0000_0001

10) Find the 9's and the 10's complement of the following decimal numbers:

	(a)		(b)
	25,478,036		63,325,600
9s comp:	74,521,963	9s comp:	36,674,399
10s comp:	74,521,964	10s comp:	36,674,400

	(c)		(d)
	25,000,000		00000000
9s comp:	74,999,999	9s comp:	99999999
10s comp:	75,000,000	10s comp:	100000000

11) (a) Find the 16's complement of C3DF.

(b) Convert C3DF to binary.
(c) Find the 2's complement of the result in (b).
(d) Convert the answer in (c) to hexadecimal and compare with the answer in (a).

	C3DF	C3DF:	1100_0011_1101_1111
15s comp:	3C20	1s comp:	0011_1100_0010_0000
16s comp:	3C21	2s comp:	0011_1100_0010_0001 = 3C21

12) Perform subtraction on the given unsigned numbers using the 10's complement of the subtrahend. Where the result should be negative, find its 10's complement and affix a minus sign. Verify your answers.

(a) 2,579 → 02,579 →97,420 (9s comp) → 97,421 (10s comp)
4637 – 2,579 = 2,579 + 97,421 = 2058_{10}

(b) 1800 → 01800 → 98199 (9s comp) → 98200 (10 comp)
125 – 1800 = 00125 + 98200 = 98325 (negative)
Magnitude: 1675
Result: 125 – 1800 = 1675

(c) 4,361 → 04361 → 95638 (9s comp) → 95639 (10s comp)
2043 – 4361 = 02043 + 95639 = 97682 (Negative)
Magnitude: 2318
Result: 2043 – 6152 = -2318

(d) 745 → 00745 → 99254 (9s comp) → 99255 (10s comp)
1631 -745 = 01631 + 99255 = 0886 (Positive)
Result: 1631 – 745 = 886

13) Perform subtraction on the given unsigned binary numbers using the 2's complement of the subtrahend. Where the result should be negative, find its 2's complement and affix a minus sign.

Note: Consider sign extension with 2s complement arithmetic.

(a)
0_10010
1s comp: 1_01101
2s comp: 1_01110
0_10011
Diff: 0_00001 (Positive)
Check:19-18 = +1

(b)
0_100110
1s comp: 1_011001 with sign extension
2s comp: 1_011010
0_100010
1_111100 sign bit indicates that the result is negative
0_000011 1s complement
0_000100 2s complement
000100 magnitude
Result: -4
Check: 34 -38 = -4

(c)
0_110101
1s comp: 1_001010
2s comp: 1_001011
0_001001
Diff: 1_010100 (negative)
0_101011 (1s comp)
0_101100 (2s complement)
101100 (magnitude)
-44_{10} (result)

(d)
0_010101
1s comp: 1_101010 with sign extension
2s comp: 1_101011
0_101000
0_010011 sign bit indicates that the result is positive
Result: 19_{10}
Check: 40 – 21 = 19_{10}

14) Convert decimal +49 and +29 to binary, using the signed-2's-complement representation and enough digits to accommodate the numbers. Then perform the binary equivalent of (+29) + (-49), (-29) + (+49), and (-29) + (-49). Convert the answers back to decimal and verify that they are correct.

+49 → 0_110001 (Needs leading zero extension to indicate + value);
+29 → 0_011101 (Leading 0 indicates + value)
-49 → 1_001110 + 0_000001→ 1_001111
-29 → 1_100011 (sign extension indicates negative value)

(a) (+29) + (-49) = 0_011101 + 1_001111 = 1_101100 (1 indicates negative value.)
Magnitude = 0_010011 + 0_000001 = 0_010100 = 20; Result (+29) + (-49) = -20

(b) (-29) + (+49) = 1_100011 + 0_110001 = 0_010100 (0 indicates positive value)
(-29) + (+49) = +20

(c) Must increase word size by 1 (sign extension) to accomodate overflow of values:
(-29) + (-49) = 11_100011 + 11_001111 = 10_110010 (1 indicates negative result)
Magnitude: 01_001110 = 78_{10}
Result: (-29) + (-49) = -78_{10}

15) If the numbers $(+9,742)_{10}$ and $(+641)_{10}$ are in signed magnitude format, their sum is $(+10,383)_{10}$ and requires five digits and a sign. Convert the numbers to signed-10's-complement form and find the following sums:

+9742 → 009742 → 990257 (9's comp) → 990258 (10s) comp
+641 → 000641 → 999358 (9's comp) → 999359 (10s) comp

(a) (+9742) + (+641) → 010383

(b) (+9742) + (-641) →009742 + 999359 = 009102
Result: (+9742) + (-641) = 9102

(c) -9742) + (+641) = 990258 + 000641 = 990899 (negative)
Magnitude: 009101
Result: (-9742) + (641) = -9101

(d) (-9742) + (-641) = 990258 + 999359 = 989617 (Negative)
Magnitude: 10383
Result: (-9742) + (-641) = -10383