Lesson 5 - An Algorithm for Curve Sketching

PART A: Putting it all together

In order to make curve sketching a smoother process, we can break it down into steps, combining your already vast knowledge of functions with your newly acquired knowledge of derivatives, tangents, critical points, points of inflection, concavity and asymptotes.

Sketching the Graph of a Polynomial or Rational Function

1. Determine the domain.
2. Determine the *x*- and *y*-intercepts.
3. Determine VA and HA, and identify the behaviour around them.
4. Determine the first derivative and use it to identify any critical points and intervals of increase and decrease.
5. Determine the second derivative, and use it to identify inflection points and intervals of concavity.
6. Sketch the function.

Note: *not all steps will have to be used for all functions.*

Example 1: Sketch the function $f(x) = x^3 - 3x^2 + 4x$.

1) Domain $\{x \in \mathbb{R}\}$

2) Intercepts

x-int

$0 = x^3 - 3x^2 + 4x$

$0 = x(\underbrace{x^2 - 3x + 4}_{\text{no sol}^n})$

$x = 0$

$(0, 0)$

y-int

$y = 0$

$(0, 0)$

3) Asymptotes NA

4) 1st Derivative Test

$f'(x) = \underbrace{3x^2 - 6x + 4}_{}$

no sol^n

$(b^2 - 4ac < 0)$

$\therefore$ no extreme values

5) 2nd Derivative Test

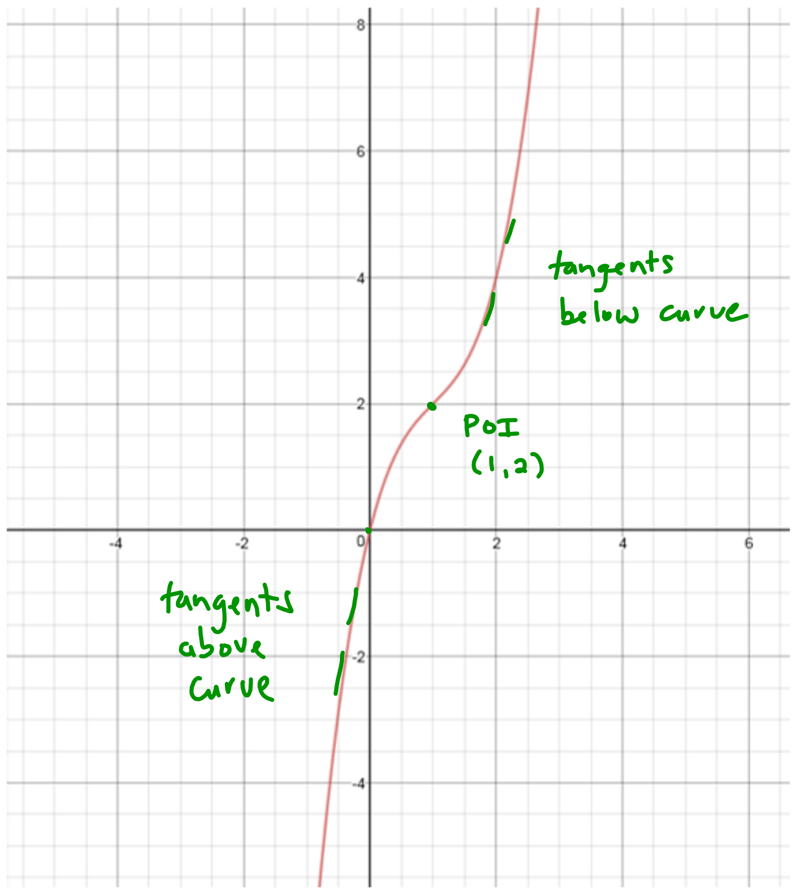
$f''(x) = 6x - 6$

$0 = 6x - 6$

$\therefore x = 1$

	$x < 1$	$x = 1$	$x > 1$
T.V.	0	1	2
$f''(x)$	$-$	0	$+$
$f(x)$	C.D.	POI	C.U.
	$\cap$	$(1, 2)$	$\cup$

6) Sketch



Example 2: Sketch the function $f(x) = \frac{x^2+9x+18}{x^2}$.

1) Domain: $D: \{x \in \mathbb{R} \mid x \neq 0\}$

2). Intercepts:

x-int (set $y=0$)

$0 = \frac{x^2+9x+18}{x^2}$

$0 = (x+6)(x+3)$

$x=-6,\ x=-3$

$(-6,0),\ (-3,0)$

y-int (set $x=0$)

DNE

3). Asymptotes:

V.A. $\rightarrow x=0$

Values of x	x+6	x+3	x^2	f(x)	f(x)→
0^-	+	+	+	+	$+\infty$
0^+	+	+	+	+	$+\infty$

∴ The graph will have the following behaviour around the V.A.

$$\left.\begin{array}{l}\lim\limits_{x\to 0^-} f(x) = +\infty \\ \lim\limits_{x\to 0^+} f(x) = +\infty\end{array}\right\} \therefore \lim_{x\to 0} f(x) = \infty$$

H.A.

$$\lim_{x\to\infty} \frac{x^2+9x+18}{x^2}$$

$$= \lim_{x\to\infty} \frac{x^2/x^2 + 9x/x^2 + 18/x^2}{x^2/x^2}$$

$$= \lim_{x\to\infty} \frac{1 + \cancel{9/x}^{\,0} + \cancel{18/x^2}^{\,0}}{1}$$

$$= 1$$

$\left(\lim\limits_{x\to -\infty} f(x) = 1\right) \therefore$ H.A. $\rightarrow y=1$

	T.V.	f(x)	f(x)→H.A.
$x\to+\infty$	1000	>1	above
$x\to-\infty$	-1000	<1	below

4) 1st Derivative Test

$$f'(x) = \frac{-9(x+4)}{x^3}$$

	x<-4	x=-4	-4<x<0	x=0	x>0
-	−	−	−	N/A	−
x+4	−	0	+		+
x^3	−	−	−		+
f'(x)	−	0	+	DNE	−
f(x)	↘	→	↗	DNE	↘

L. Min
(-4,-0.125)

5) 2nd Derivative Test

$$f''(x) = \frac{18x+108}{x^4}$$

	x<-6	x=-6	x>-6
T.V.	-7	-6	-5
f''(x)	−	0	+
f(x)	C.D.	P.O.I. (-6,0)	C.U.

6)