Lesson 3 - Properties of Vectors

PART A: Scalar multiplication

If $\vec{v}$ is a vector and k is a real number, then $k\vec{v}$ is also a vector such that

1. $|k\vec{v}| = |k||\vec{v}|$
2. The directions of $\vec{v}$ and $k\vec{v}$ are the same if $k > 0$ and opposite if $k < 0$

Note: dividing $\vec{v}$ by 2 is the same as multiplying $\vec{v}$ by $1/2$.

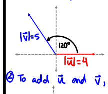
Unit Vector: a unit vector is a vector that has a magnitude of one unit. We can find a unit vector in the direction of any vector by dividing the original vector by its magnitude

If $\vec{a}$ is a non-zero vector, then $\vec{b} = \frac{1}{|\vec{a}|}\vec{a}$ is a ***unit vector*** in the direction of $\vec{a}$. length equals 1

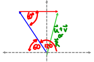
Example 1: Using a scalar to create a unit vector

Given that $|\vec{u}| = 4$ and $|\vec{v}| = 5$ and the angle between $\vec{u}$ and $\vec{v}$ is 120°, determine the unit vector in the same direction as $\vec{u} + \vec{v}$.

① Angle between two vectors → angle between them when placed tail to tail.



② To add $\vec{u}$ and $\vec{v}$, place them head to tail:



③ Determine magnitude of $\vec{R}$

$|\vec{R}| = \sqrt{4^2+5^2-2(4)(5)\cos 60^\circ}$

$|\vec{R}| = \sqrt{21}$

④ Now the unit vector will be represented by the "^" (hat) symbol:

For example: $\hat{R}$ is a unit vector in direction of $\vec{R}$

$\hat{n}$ " " " " " " " $\vec{n}$

$\hat{R} = \frac{1}{|\vec{R}|}\vec{R}$ ← definition of unit vector

$\hat{R} = \frac{1}{\sqrt{21}}\vec{R}$

∴ A direction vector in the direction of $\vec{u}+\vec{v}=\vec{R}$ can be represented by: $\hat{R} = \frac{1}{\sqrt{21}}\vec{R}$

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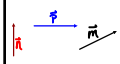
Example 2: Three unit vectors $\vec{m}$, $\vec{n}$ and $\vec{p}$, all lie on the same plane and are such that the angle between $\vec{m}$ and $\vec{n}$ is 60°, the angle between $\vec{m}$ and $\vec{p}$ is 30°, and $\vec{n}$ and $\vec{p}$ are perpendicular. Construct the vector $\vec{a} = \vec{m} + 2\vec{n} - 3\vec{p}$ graphically and determine its magnitude.



① Determine $\angle\gamma$

$\tan\gamma = 3/2$

$\angle\gamma = \tan^{-1}(3/2)$

$\boxed{\angle\gamma \doteq 56.3^\circ}$

② Determine $\angle\theta$

$\angle\theta = 180^\circ - 56.3^\circ - 60^\circ$

$\boxed{\angle\theta \doteq 63.7^\circ}$

③ Determine $|\vec{CA}|$

$|\vec{CA}| = \sqrt{3^2+2^2}$

$|\vec{CA}| = \sqrt{13}$

④ Determine $|\vec{a}|$

$|\vec{a}| = \sqrt{|\vec{m}|^2 + |\vec{CA}|^2 - 2|\vec{m}||\vec{CA}|\cdot\cos\theta}$

$= \sqrt{1^2 + (\sqrt{13})^2 - 2(1)(\sqrt{13})\cdot\cos 63.7^\circ}$

$\boxed{|\vec{a}| \doteq 3.3 \text{ units}}$

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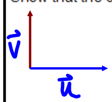
PART B: Investigation – Does the distributive property hold for vectors?

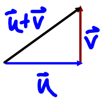
Show that the distributive property $k(\vec{u}+\vec{v}) = k\vec{u} + k\vec{v}$ is true for vectors. Let $k = 3$.

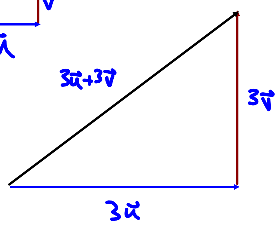


$3(\vec{u}+\vec{v}) = 3\vec{u} + 3\vec{v}$

$3|\vec{u}+\vec{v}| = |3\vec{u}+3\vec{v}|$

because of addition of vectors and the resultant it proves it is true for all vectors.





Therefore, the distributive property is true for all vectors

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p. 299-301 #2ac, 3, 4ace, 5ac, 6ce, 7a, 9, 11, 19, 22

p. 306 # 4, 6, 7, 9, 11

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