Lesson 4 – The Quotient Rule

PART A: Recall the previous lesson where we proved that the derivative of the product of two functions was not the product of the derivatives. As you can imagine, similar logic applies to the derivative of a quotient.

If $m(x) = \frac{f(x)}{g(x)}$, then $m'(x) \neq \frac{f'(x)}{g'(x)}$

PART B: How TO take the derivative of a quotient.

The Quotient Rule

If $m(x) = \frac{f(x)}{g(x)}$, then $m'(x) = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$, where $g(x) \neq 0$

or

In Leibniz notation, $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$

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"To take the derivative of a quotient, take the derivative of the top times the bottom minus the derivative of the bottom times the top, all over the bottom squared."

OR

Think of the numerator as the high function and the denominator as the low function. The derivative of a quotient is "Low d-high minus high d-low over the square of what's below"

Example 1: Determine an equation for the tangent to the curve $y = \frac{x^2-3}{5-x}$ at $x = 2$.

$$y'(x) = \frac{(2x)(5-x) - (x^2-3)(-1)}{(5-x)^2}$$

$$y'(x) = \frac{10x - 2x^2 - (-x^2+3)}{(5-x)^2}$$

$$= \frac{10x - 2x^2 + x^2 - 3}{(5-x)^2}$$

$$y'(x) = \frac{-x^2+10x-3}{(5-x)^2}$$

$$y'(2) = \frac{-2^2+10(2)-3}{(5-2)^2}$$

$$= \frac{-4+20-3}{9}$$

$$y'(2) = \frac{13}{9}$$

$y = mx + b$

$y(2) = \frac{2^2-3}{5-2} = \frac{4-3}{3} = \frac{1}{3}$

$\frac{1}{3} = \frac{13}{9}(2) + b$

$\frac{1}{3} = \frac{26}{9} + b$

$\times 3$ $\frac{1}{3_{\times 3}} - \frac{26}{9} = b$

$\frac{3-26}{9} = b$

$-\frac{23}{9} = b$

$\therefore y = \frac{13}{9}x - \frac{23}{9}$

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Example 2: Find $\frac{dy}{dx}$ if $y = \frac{\sqrt{x}}{1+2x}$

$$y = \frac{x^{1/2}}{1+2x}$$

$$y'(x) = \frac{\left(\frac{1}{2}x^{-1/2}\right)(1+2x) - (\sqrt{x})(2)}{(1+2x)^2}$$

$$= \frac{\left(\frac{1}{2\sqrt{x}}\right)(1+2x) - 2\sqrt{x}}{(1+2x)^2}$$

$$= \frac{\frac{1}{2\sqrt{x}} + \frac{2x}{2\sqrt{x}} - \frac{2\sqrt{x}}{1} \times \frac{2\sqrt{x}}{2\sqrt{x}}}{(1+2x)^2}$$

$$= \frac{1+2x-4x}{2\sqrt{x}} \times \frac{1}{(1+2x)^2}$$

$$= \frac{1-2x}{2\sqrt{x}(1+2x)^2} \quad \frac{\sqrt{x}}{\sqrt{x}}$$

$$= \frac{\sqrt{x}(1-2x)}{2x(1+2x)^2}$$

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