Lesson 3 – Optimization Problems

PART A: What is Optimization and how do I do it?

Optimization is the process of finding the maximum or minimum value for a particular application over a given interval of values.

When solving optimization problems, we can follow the steps listed below:

- Identify what the question is asking. (deliverables)
- Define the variables, draw a diagram if it helps.
- Identify the quantity to be optimized and write an equation.
- Define the independent variable. Express all the other variables in terms of the independent variable.
- Define the function in terms of the independent variable.
- Identify and state any restrictions on the independent variable.
- Differentiate the function.
- Use the algorithm for extreme values to find the absolute maximum or minimum value in the domain.
- Use your result to answer the original problem.

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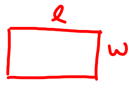
Example 1: A cattle rancher has purchased five 150m rolls of wire fencing to build a rectangular corral. She will use all of the fencing. What dimensions will produce the greatest possible area?

Maximize Area - Provide dimensions (length and width)

Let l rep length Let A rep Area
Let w rep width Let P rep perimeter



Write an equation for area, since that is the quantity to be maximized.

$A = lw$ (1)

Now we need another equation that allows us to write the area equation in terms of only 1 variable.

$P = 2l + 2w$

$150(5) = 2l + 2w$

$750 = 2l + 2w$

Now we can isolate for either length or width.

$\frac{750 - 2w}{2} = \frac{2l}{2}$ $l = 375 - w$ (2)

Now, sub equation 2 into equation 1.

$A(w) = (375 - w)(w)$

$A(w) = 375w - w^2$

Set restrictions on the domain.

$D = \{w \in \mathbb{R} \mid 0 < w < 375\}$

Differentiate area with respect to w.

$A'(w) = 375 - 2w$

Set A'(w) equal to zero and solve for w.

$0 = 375 - 2w$

$\frac{2w}{2} = \frac{375}{2}$

$w = 187.5$ m

Check end points of interval and value for w to verify if it is a max/min.

$A(0) = (375-0)(0) = 0$

$A(375) = (375-375)(0) = 0$

$A(187.5) = (375-187.5)(187.5) = 35156.25$ m^2 max area

Sub equation 3 into equaion 2 to for for length.

$l = 375 - 187.5$

$l = 187.5$ m

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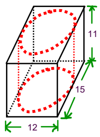
Example 2: A cylindrical storage tank with a capacity of 1000 m^3 is to be constructed in a warehouse that is 12 m by 15 m and has a height of 11 m. The specifications call for the base to be made of sheet metal, which costs $\$100/m^2$, the top of sheet metal which costs $\$50/m^2$, and the wall of sheet metal which costs $\$80/m^2$. Find the proportions of the tank to minimize cost of construction.

Note: we will assume the tank is an upright, right cylindrical tank.

Minimize Cost → provide dimensions of cylinder (r, h)



let C rep cost of tank
" r " radius of cyl.
" h " height " "

Given info: $V = 1000\ m^3$

Cost
top = $\$50/m^2$
bottom = $\$100/m^2$
side = $\$80/m^2$

Since we are minimizing cost, we need an equation for overall cost:

$$C = 50(\pi r^2) + 100(\pi r^2) + 80(2\pi r h)$$
$$C = 150\pi r^2 + 160\pi r h \quad ①$$

Now we need to write cost in terms of one variable so we need an equation that relates h to r.

$$V = \pi r^2 h$$
$$\frac{1000}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$
$$h = \frac{1000}{\pi r^2} \quad ②$$

Sub equation 2 into equation 1, find the derivative and set restrictions on the domain.

$$C(r) = 150\pi r^2 + 160\cancel{\pi}\cancel{r}\left(\frac{1000}{\cancel{\pi} r^{\cancel{2}}}\right)$$
$$C(r) = 150\pi r^2 + \frac{160000}{r}$$
$$C(r) = \frac{150\pi r^3 + 160000}{r}$$
$$C'(r) = \frac{(450\pi r^2 + 0)(r) - (150\pi r^3 + 160000)(1)}{r^2}$$
$$0 = \frac{450\pi r^3 - 150\pi r^3 - 160000}{r^2}$$
$$0 = 300\pi r^3 - 160000$$
$$160000 = 300\pi r^3$$
$$\sqrt[3]{\frac{160000}{300\pi}} = r$$
$$5.54 = r$$

min radius is determined by the maximum height of the cylinder:

~~max h=11m, find radius, r~~

$$r = \sqrt{\frac{1000}{\pi h}} \leftarrow 11m$$
$$r = 5.38$$

max radius determined by size restriction of warehouse:

r=6m

$$D = \{r \in R \mid 5.38 < r < 6\}$$

Since our goal is to mimimize Cost, we plug in the result we obtained from setting the derivative to zero, into the cost function along with the end points of our domain. The smallest value (lowest cost) is the radius we will recommend.

$$C(5.38) = \$43\,379.50$$
$$C(5.54) = \$43\,343.94 \leftarrow \text{minimum cost when } r = 5.54$$
$$C(6) = \$43631.27$$

Determine the corresponding height by plugging in r=5.54 into equation 2.

$$h = \frac{1000}{\pi r^2} = \frac{1000}{\pi(5.54)^2} = 10.4m$$

Concluding statement:

I would recommend the cylindrical tank be constructed with a radius of 5.54m and a height of 10.4m in order to minimize cost.

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