Université d'Ottawa · University of Ottawa

Faculté de Génie - Faculty of Engineering
ITI1100C Digital Systems I –Assignment 2 Solution

1) Obtain the truth table of the following functions, and express each function in sum of minterms and product of maxterms form:

(a) $F = (b + cd)(c + bd)\ \ bc + bd + cd + bcd = \Sigma(3, 5, 6, 7, 11, 14, 15)$

$F' = \Sigma(0, 1, 2, 4, 8, 9, 10, 12, 13)$

$F = \Pi(0, 1, 2, 4, 8, 9, 10, 12, 13)$

a b c d	F
0 0 0 0	0
0 0 0 1	0
0 0 1 0	0
0 0 1 1	1
0 1 0 0	0
0 1 0 1	1
0 1 1 0	1
0 1 1 1	1
1 0 0 0	0
1 0 0 1	0
1 0 1 0	0
1 0 1 1	1
1 1 0 0	0
1 1 0 1	1
1 1 1 0	1
1 1 1 1	1

(b) $(cd + b'c + bd')(b + d) = bcd + bd' + cd + b'cd = cd + bd'$

$= \Sigma\,(3, 4, 7, 11, 12,14, 15)$

$= \Pi\,(0, 1, 2, 5, 6, 8, 9, 10, 13)$

a b c d	F
0 0 0 0	0
0 0 0 1	0
0 0 1 0	0
0 0 1 1	1
0 1 0 0	1
0 1 0 1	0
0 1 1 0	0
0 1 1 1	1
1 0 0 0	0
1 0 0 1	0
1 0 1 0	0
1 0 1 1	1
1 1 0 0	1
1 1 0 1	0
1 1 1 0	1
1 1 1 1	1

(c) $(c' + d)(b + c')\ = bc' + c' + bd + c'd = (c' + bd)$

$= \Sigma\,(0, 1, 4, 5, 7, 8, 12, 13, 15)$

$F = \Pi\,(2, 3, 6, 9, 10, 11, 14)$

(d) $bd' + acd' + ab'c + a'c' = \Sigma\,(0, 1, 4, 5, 10, 11, 14)$

$F' = \Sigma\,(2, 3, 6, 7, 8, 9, 12, 13, 15)$

$F = \Pi\,(02, 3, 6, 7, 8, 12, 13, 15)$

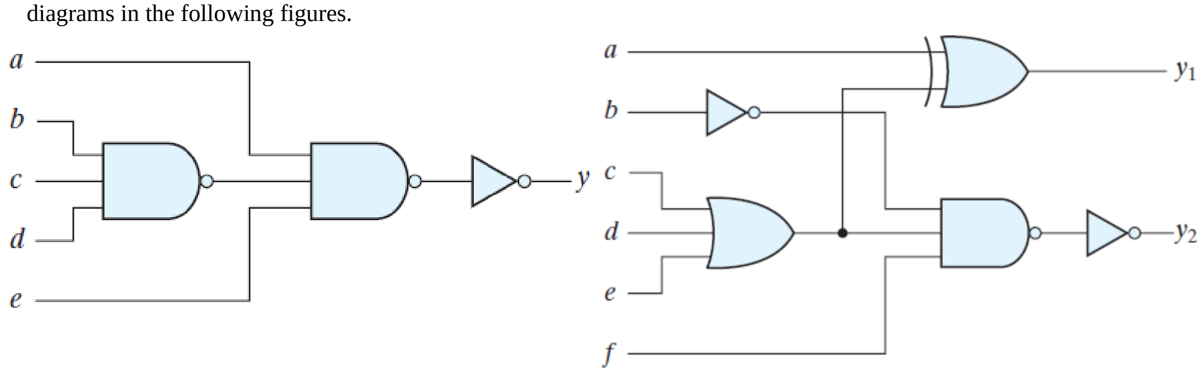
a b c d	F
0 0 0 0	1
0 0 0 1	1
0 0 1 0	0
0 0 1 1	0
0 1 0 0	1
0 1 0 1	1
0 1 1 0	0
0 1 1 1	0
1 0 0 0	0
1 0 0 1	0
1 0 1 0	1
1 0 1 1	1
1 1 0 0	1
1 1 0 1	0
1 1 1 0	1
1 1 1 1	0

2) Convert each of the following to the other canonical form:

(a) $F(x, y, z) = \Sigma(1, 3, 5) = \Pi(0, 2, 4, 6, 7)$

(b) $F(A, B, C, D) = \Pi(3, 5, 8, 11) = \Sigma(0, 1, 2, 4, 6, 7, 9, 10, 12, 13, 14, 15)$

3) Write Boolean expressions and construct the truth tables describing the outputs of the circuits described by the logic diagrams in the following figures.



(a) (b)

(a) $y = a(bcd)'e = a(b' + c' + d')e$

$$y = a(b' + c' + d')e = ab'e + ac'e + ad'e$$
$$= \Sigma(17, 19, 21, 23, 25, 27, 29)$$

a bcde	y	a bcde	y
0 0000	0	1 0000	0
0 0001	0	**1 0001**	1
0 0010	0	1 0010	0
0 0011	0	**1 0011**	1
0 0100	0	1 0100	0
0 0101	0	**1 0101**	1
0 0110	0	1 0110	0
0 0111	0	**1 0111**	1
	0		0
0 1000	0	1 1000	0
0 1001	0	**1 1001**	1
0 1010	0	1 1010	0
0 1011	0	**1 1011**	1
0 1100	0	1 1100	0
0 1101	0	**1 1101**	1
0 1110	0	1 1110	0
0 1111	0	1 1111	0

(b) $y_1 = a \oplus (c + d + e) = a'(c + d + e) + a(c'd'e') = a'c + a'd + a'e + ac'd'e'$

$y_2 = b'(c + d + e)f = b'cf + b'df + b'ef$

$y_1 = a\ \ (c + d + e) = a'(c + d + e) + a(c'd'e') = a'c + a'd + a'e + ac'd'e'$

$y_2 = b'(c + d + e)f = b'cf + b'df + b'ef$

a'-c---	*a'--d--*	*a'---e-*	*a-c'd'e'-*
001000 = 8	000100 = 8	000010 = 2	100000 = 32
001001 = 9	000101 = 9	000011 = 3	100001 = 33
001010 = 10	000110 = 10	000110 = 6	110000 = 34
001011 = 11	000111 = 11	000111 = 7	110001 = 35
001100 = 12	001100 = 12	001010 = 10	
001101 = 13	001101 = 13	001011 = 11	
001110 = 14	001110 = 14	001110 = 14	
001111 = 15	001111 = 15	001111 = 15	
011000 = 24	010100 = 20	010010 = 18	
011001 = 25	010101 = 21	010011 = 19	
011010 = 26	010110 = 22	010110 = 22	
011011 = 27	010111 = 23	010111 = 23	
011100 = 28	011100 = 28	011010 = 26	
011101 = 29	011101 = 29	011001 = 27	
011110 = 30	011110 = 30	011110 = 30	
011111 = 31	011111 = 31	011111 = 31	

-b' c--f	-b' -d-f	-b' --ef
001001 = 9	001001 = 9	000011 = 3
001011 = 11	001011 = 11	000111 = 7
001101 = 13	001101 = 13	001011 = 11
001111 = 15	001111 = 15	001111 = 15
101001 = 41	101001 = 41	100011 = 35
101011 = 43	101011 = 43	100111 = 39
101101 = 45	101101 = 45	101011 = 51
101111 = 47	101111 = 47	101111 = 55

4) Simplify the following Boolean expressions to a minimum number of literals:

(a) $ABC + A'B + ABC' = AB + A'B = B$

(b) $x'yz + xz = (x'y + x)z = z(x + x')(x + y) = z(x + y)$

(c) $(x + y)'(x' + y') = x'y'(x' + y') = x'y'$

(d) $xy + x(wz + wz') = x(y + wz + wz') = x(w + y)$

(e) $(BC' + A'D)(AB' + CD') = BC'AB' + BC'CD' + A'DAB' + A'DCD' = 0$

(f) $(a' + c')(a + b' + c') = a'a + a'b' + a'c' + c'a + c'b' + c'c' = a'b' + a'c' + ac' + b'c' = c' + b'(a' + c')$
$= c' + b'c' + a'b' = c' + a'b'$

5) Find the complement of the following expressions:

(a) $F' = (xy' + x'y)' = (xy')'(x'y)' = (x' + y)(x + y') = xy + x'y'$

(b) $F' = [(a + c)\ (a + b')(a' + b + c')]' = (a + c)' + (a + b')' + (a' + b + c')'$
$= a'c' + a'b + ab'c$

(c) $F' = [z + z'(v'w + xy)]' = z'[z'(v'w + xy)]' = z'[z'v'w + xyz']'$
$= z'[(z'v'w)'(xyz')'] = z'[(z + v + w') + (x' + y' + z)]$
$= z'z + z'v + z'w' + z'x' + z'y' + z'z = z'(v + w' + x' + y')$

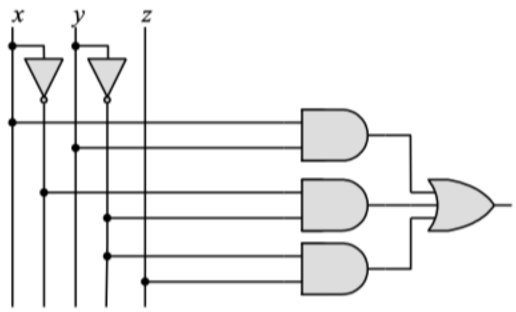
6) Given the Boolean functions F_1 and F_2, show that:

(a) $F_1 + F_2 = \Sigma\, m_{1i} + \Sigma m_{2i} = \Sigma\, (m_{1i} + m_{2i})$

(b) $F1\ F2 = \Sigma\, m_i\, \Sigma m_j$ where $m_i\, m_j = 0$ if $i \neq j$ and $m_i\, m_j = 1$ if $i = j$

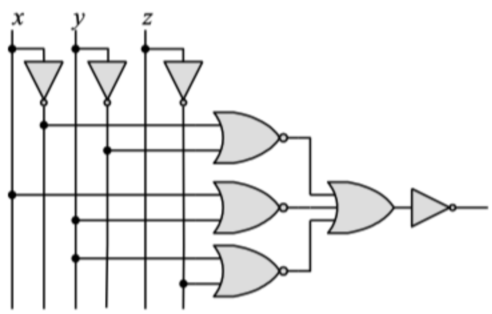
7) Implement the Boolean function $F = xy + x'y' + y'z$

(a)



$F = xy + x'y' + y'z$

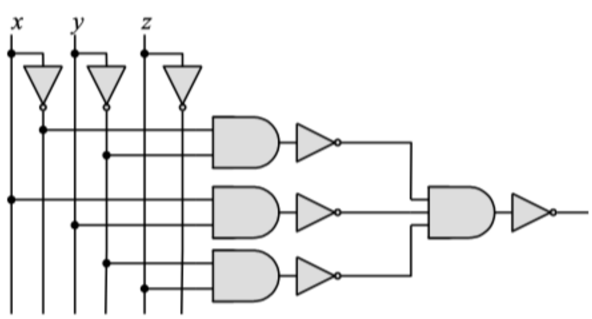
(b)



$F = xy + x'y' + y'z$

$= (x' + y')' + (x + y)' + (y + z')'$

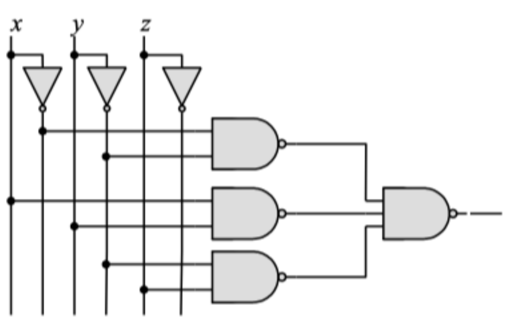
(c)



$F = xy + x'y' + y'z$

$= [(xy)' (x'y')' (y'z)']'$

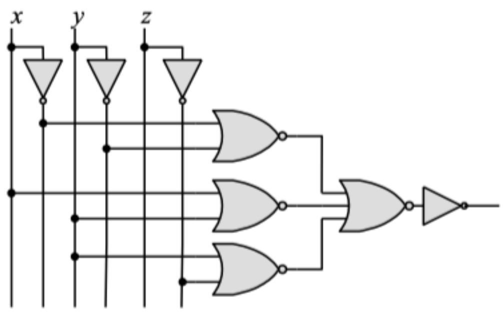
(d)



$F = xy + x'y' + y'z$

$= [(xy)' (x'y')' (y'z)']'$

(e)



$F = xy + x'y' + y'z$

$= (x' + y')' + (x + y)' + (y + z')'$

8) Simplify the following Boolean functions $\boldsymbol{T_1}$ and $\boldsymbol{T_2}$ to a minimum number of literals:

(a) $T_1 = A'B'C' + A'B'C + A'BC' = A'B'(C' + C) + A'C'(B' + B) = A'B' + A'C' = A'(B' + C')$

(b) $T_2 = T_1' = A'BC + AB'C' + AB'C + ABC' + ABC$

$= BC(A' + A) + AB'(C' + C) + AB(C' + C)$

$= BC + AB' + AB = BC + A(B' + B) = A + BC$

$\sum(3, 5, 6, 7) = \Pi(0, 1, 2, 4)$

$T_1 = A'B'C' + A'B'C + A'BC'$

$A'B' \qquad A'C'$

$T_1 = A'B' \ A'C' = A'(B' + C')$

$T_2 = A'BC + AB'C' + AB'C + ABC' + ABC$

$AC' \qquad AC$

BC

$T_2 = AC' + BC + AC = A + BC$

9) Show that a positive logic NAND gate is a negative logic NOR gate and vice versa.

Gate		NAND (Positive logic)		NOR (Negative logic)	
x y	z	x y	z	x y	z
L L	H	0 0	1	1 1	0
L H	H	0 1	1	1 0	0
H L	H	1 0	1	0 1	0
H H	L	1 1	0	0 0	1

Gate		NOR (Positive logic)		NAND (Negative logic)	
x y	z	x y	z	x y	z
L L	H	0 0	1	1 1	0
L H	L	0 1	0	1 0	1
H L	L	1 0	0	0 1	1
H H	L	1 1	0	0 0	1

$y_1 = \Sigma\,(2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35)$

$y_2 = \Sigma\,(3, 7, 9, 13, 15, 35, 39, 41, 43, 45, 47, 51, 55)$

ab cdef	y_1 y_2	*ab cdef*	y_1 y_2	*ab cdef*	y_1 y_2	*ab cdef*	y_1 y_2
00 0000	0 0	01 0000	0 0	10 0000	1 0	11 0000	0 0
00 0001	0 0	01 0001	0 0	10 0001	1 0	11 0001	0 0
00 0010	1 0	01 0010	1 0	10 0010	1 0	11 0010	0 0
00 0011	1 1	01 0011	1 0	10 0011	1 1	11 0011	0 1
00 0100	0 0	01 0100	0 0	10 0100	0 0	11 0100	0 0
00 0101	0 0	01 0101	0 0	10 0101	0 0	11 0101	0 0
00 0110	1 0	01 0110	1 0	10 0110	0 0	11 0110	0 0
00 0111	1 1	01 0111	1 0	10 0111	0 1	11 0111	0 1
00 1000	1 0	01 1000	1 0	10 1000	0 0	11 1000	0 0
00 1001	1 1	01 1001	1 0	10 1001	0 1	11 1001	0 0
00 1010	1 0	01 1010	1 0	10 1010	0 0	11 1010	0 0
00 1011	1 0	01 1011	1 0	10 1011	0 1	11 1011	0 0
00 1100	1 0	01 1100	1 0	10 1100	0 0	11 1100	0 0
00 1101	1 1	01 1101	1 0	10 1101	0 1	11 1101	0 0
00 1110	1 0	01 1110	1 0	10 1110	0 0	11 1110	0 0
00 1111	1 1	01 1111	1 0	10 1111	0 1	11 1111	0 0

10) Determine whether the following Boolean equation is true or false.

$$x'y' + x'z + x'z' = x'z' + y'z' + x'z$$

→ The solution is not provided in the manual.