Lesson 6 – Operations with Algebraic Vectors in R^3

PART A: Vectors in R^3 expressed in terms of Unit Vectors

Just as we expressed the vector $\overrightarrow{OP}$ in terms of unit vectors $\hat{\imath}$ and $\hat{\jmath}$ in R^2, we can extend the use of standard basis vectors to include a third dimension. The unit vectors $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ are chosen in R^3. The unit vectors still have a magnitude of 1, but $\hat{k}$ is a vector that lies along the *z*-axis (whereas $\hat{\imath}$ lies along the *x*-axis and $\hat{\jmath}$ lies along the *y*-axis).

In three-dimensional space, there are three *standard basis* vectors

$$\hat{\imath} = [1,0,0] \quad \hat{\jmath} = [0,1,0] \quad \hat{k} = [0,0,1]$$

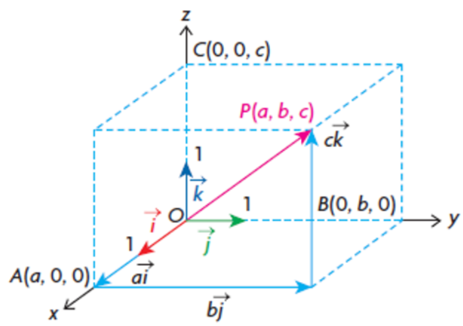
Standard basis vectors are unit vectors.

In figure 1, vector $\overrightarrow{OP} = [a,b,c]$. Like we did in R^2, we can now write this vector in terms of the standard basis vectors as follows:

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$

$\overrightarrow{OP} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

$= [a, b, c]$



The position vector $\overrightarrow{OP}$, whose tail is at the origin and whose head is located at point *P*, can be represented as either $\overrightarrow{OP} = [a,b,c]$ or $\overrightarrow{OP} = a\vec{i} + b\vec{j} + c\vec{k}$, where $O(0,0,0)$ is the origin, $P(a,b,c)$ is any point in R^3, and $\vec{i}$, $\vec{j}$ and $\vec{k}$ are the standard unit vectors along the *x*-, *y*- and *z*-axes, respectively.

Example 1: Write each vector $\overrightarrow{OA} = [4,2,-9]$, $\overrightarrow{OB} = [-7,5,3]$ and $\overrightarrow{OC} = [-2,-8,-2]$ using the *standard basis vectors*.

$\overrightarrow{OA} = 4\hat{i} + 2\hat{j} - 9\hat{k}$

$\overrightarrow{OB} = -7\hat{i} + 5\hat{j} + 3\hat{k}$

$\overrightarrow{OC} = -2\hat{i} - 8\hat{j} - 2\hat{k}$

Example 2: Write each vector $\overrightarrow{OM} = 4\vec{i} - 7\vec{j} + 3\vec{k}$ and $\overrightarrow{ON} = -5\vec{i} - 8\vec{k}$ in *component form*.

$\overrightarrow{OM} = [4,-7,3]$

$\overrightarrow{ON} = [-5,0,-8]$

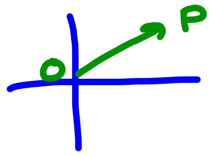
Example 3: Given $\vec{a} = -\vec{i} + 2\vec{j} + \vec{k}$, $\vec{b} = 2\vec{j} - 3\vec{k}$ and $\vec{c} = \vec{i} - 3\vec{j} + 2\vec{k}$, determine $2\vec{a} + \vec{b} - \vec{c}$.

Method 1: Standard Unit Vectors	Method 2: Components
$= 2(-\hat{i}+2\hat{j}+\hat{k}) + 2\hat{j} - 3\hat{k} - (\hat{i} - 3\hat{j} + 2\hat{k})$ $= -2\hat{i} + 4\hat{j} + 2\hat{k} + 2\hat{j} - 3\hat{k} - \hat{i} + 3\hat{j} - 2\hat{k}$ $= -3\hat{i} + 9\hat{j} - 3\hat{k}$ $= [-3, 9, -3]$	$\vec{a} = [-1,2,1]$ $\vec{b} = [0,2,-3]$ $\vec{c} = [1,-3,2]$ $= 2[-1,2,1] + [0,2,-3] - [1,-3,2]$ $= [-2,4,2] + [0,2,-3] + [-1,3,-2]$ $= [-2+0-1, 4+2+3, 2-3-2]$ $= [-3, 9, -3]$

PART B: Magnitude of the Position Vector

Given that $\overrightarrow{OP} = [a, b, c] = a\vec{i} + b\vec{j} + c\vec{k}$, we can determine

$$|\overrightarrow{OP}| = \sqrt{(a)^2 + (b)^2 + (c)^2}$$



Example 4: If $A(7, -11, 13)$ and $B(4, -7, 25)$ are two points in R^3, determine each of the following:

a) $|\overrightarrow{OA}|$

$= \sqrt{7^2 + (-11)^2 + 13^2}$

$= \sqrt{49 + 121 + 169}$

$= \sqrt{339}$

b) $|\overrightarrow{OB}|$

$= \sqrt{4^2 + (-7)^2 + 25^2}$

$= \sqrt{16 + 49 + 625}$

$= \sqrt{690}$

PART C: Magnitude of a Vector

To find the magnitude of a vector, use the formula for the distance between two points.

Magnitudes in R^3

If $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ are two points, then

$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

(**Recall**: the vector $\overrightarrow{AB} = [x_2 - x_1, y_2 - y_1, z_2 - z_1]$ is the related position vector)

Example 5: Find the position vector and the magnitude of the vector $\overrightarrow{AB}$ with $A(1,3,-6)$ and $B(7,-3,4)$.

$\overrightarrow{AB} = [7-1, -3-3, 4-(-6)]$

$\overrightarrow{AB} = [6, -6, 10]$

$|\overrightarrow{AB}| = \sqrt{6^2 + (-6)^2 + 10^2}$

$= \sqrt{172}$

$= 2\sqrt{43}$