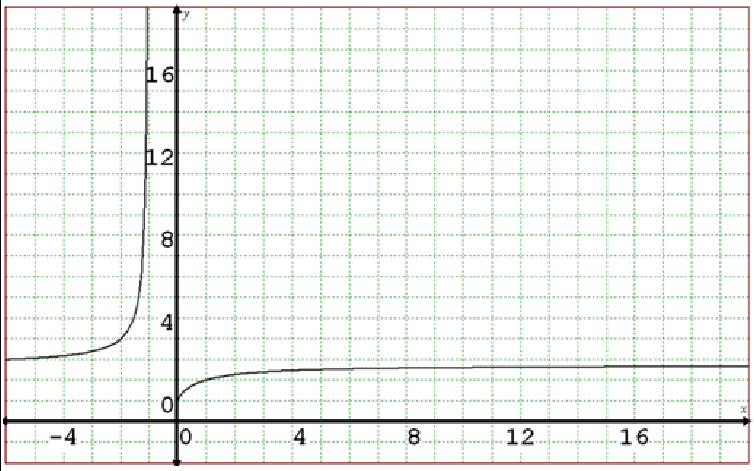
Lesson 1 - The Natural Logarithm & Derivative of $y = e^x$

PART A: The Number e

There are many letters in mathematics which represent numbers. These include π and e. To 20 decimal places, $e = 2.71828182845904523536$. The number e is named the Euler number after the Swiss mathematician Leonhard Euler.



This is the graph of $y = \left(1 + \frac{1}{x}\right)^x$.

The letter e represents the limit as x approaches infinity of this graph.

Therefore, e can be defined as follows:

$$e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$

There are many applications that use $y = e^x$ (which is called the natural exponential function).

PART B: Investigation

1. A table of values is given for $f(x) = e^x$. On the grid, graph the following:

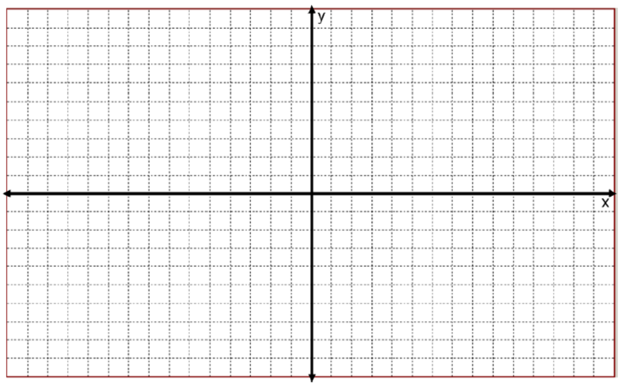
   i) $f(x) = e^x$

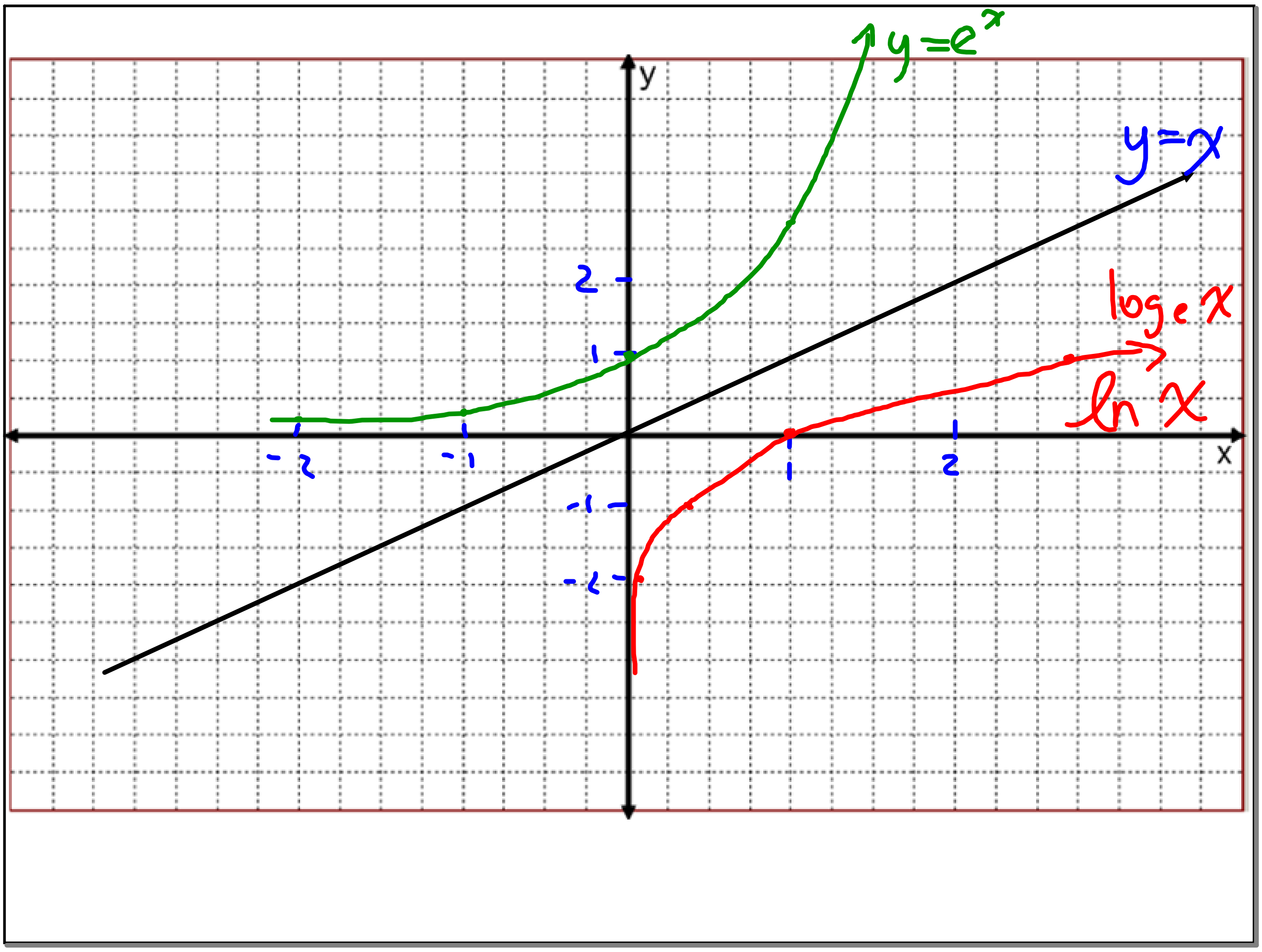
   ii) $f(x) = x$ (the line $y = x$)

   iii) The reflection of the graph of $f(x) = e^x$ in the line $y = x$

Choose an appropriate scale for the axes.

x	***e^x*** *(to the nearest hundredth)*
–2	0.14
–1	0.37
0	1
1	2.72
2	7.39





Note: Uses of Euler # *e*

1. solution to many differential equations, modelling electric circuits, spring-mass systems

2. Newtons laws of cooling/heating in solution of differential equation

$$\frac{dT}{dt} = -k(T - T_o)$$

As you might remember from Advanced Functions, the logarithm is the inverse of the exponential function. For example: $y = \log_2 x$ is the inverse of $y = 2^x$. You may not realize it, but in the investigation above, you actually graphed the inverse of $y = e^x$. The inverse of $y = e^x$ is $y = \log_e x$. As you can see from the graph, the reflection of $y = e^x$ in the line $y = x$ represents the graph of $y = \log_e x$.

The function $y = \log_e x$ can also be written as $y = \ln x$ and is called the **Natural Logarithm Function.**

2. Label the graphs above accordingly and complete the table below to compare the two graph

	$f(x) = e^x$	$g(x) = \ln(x)$
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R} \mid x > 0$
Range	$y \in \mathbb{R} \mid y > 0$	$y \in \mathbb{R}$
Asymptote	HA $\rightarrow y = 0$	VA $\rightarrow x = 0$
***x*-intercept(s)**	None	$(1, 0)$
***y*-intercept(s)**	$(0, 1)$	None
Intervals of increase	$(-\infty, \infty)$	$(0, \infty)$
Concavity	C.U.	C.D

3. Does the inverse of the natural exponential function exist and, if yes, what is it?

Yes it is a function (passes VLT) and it is called the natural logarithm ($\ln x$) pronounced "lon" x

PART C: Investigation of $y = e^x$ and its derivative

1. Follow the investigation in the book on page 227 and fill in the table below based on the investigation.

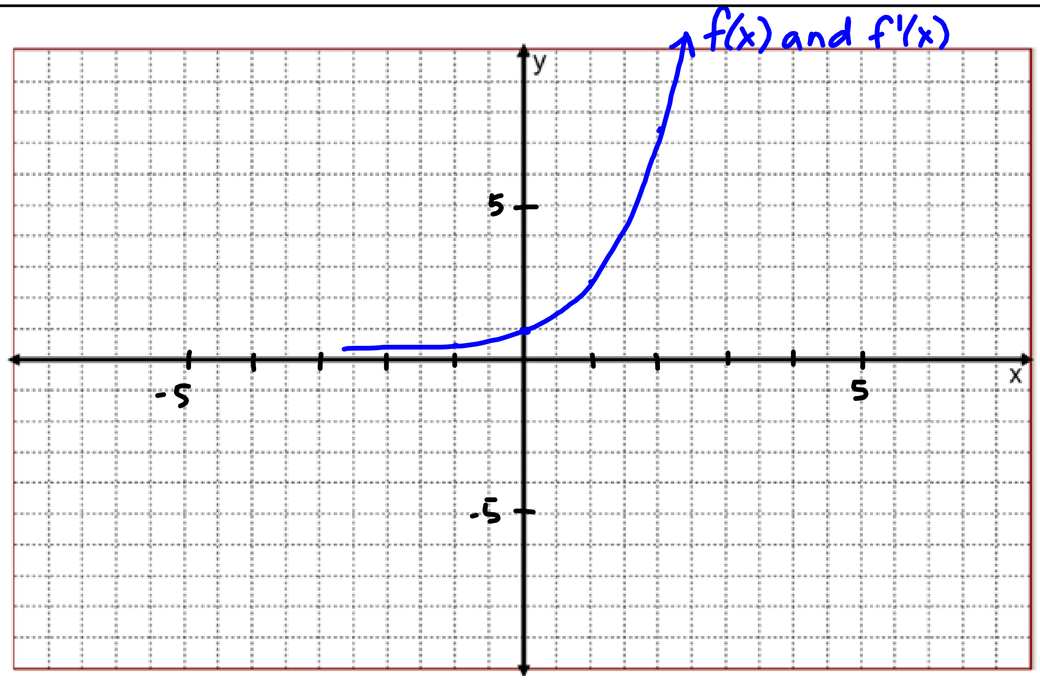
x	$f(x)$	$f'(x)$
-2	0.135	0.135
-1	0.368	0.368
0	1	1
1	2.718	2.718
2	7.389	7.389
3	20.086	20.086

2. Answer the questions from the investigation below:

D. the values of f(x) and f'(x) are the same

E.

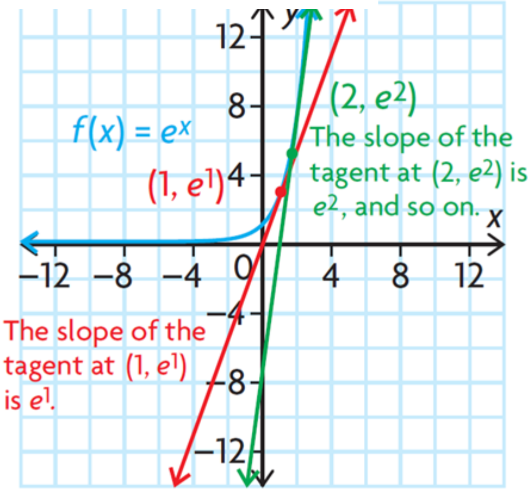
the two graphs are mapped onto each other



G. the derivative of $f(x)=e^x$ is $f'(x)=e^x$

PART D: Investigation Summary:

From the investigation, it should have been apparent that the values of $y = e^x$ and its derivative were identical. This means that the slope of the tangent at any point is the value of the function at that point.



The Derivative of $f(x) = e^x$

For the function $f(x) = e^x$, $f'(x) = e^x$

Although the derivative of $f(x) = e^x$ is equal to the original function itself, this is not the case when we have composite functions involving e^x.

The Derivative of a Composite Function involving e^x

In general, if $f(x) = e^{g(x)}$, then $f'(x) = e^{g(x)} \cdot g'(x)$ by the chain rule.

Example 1: Determine the derivative of e^{x^2-x}.

$$f(x) = e^{x^2-x}$$
$$f'(x) = e^{x^2-x} \cdot (2x-1)$$

Example 2: Determine the derivative of $x^2 \cdot e^x$.

$$f(x) = x^2 \cdot e^x$$
$$f'(x) = 2x \cdot e^x + e^x \cdot x^2$$
$$= e^x(2x + x^2)$$
$$= x \cdot e^x(2 + x)$$