

## 2022 MIDTERM 1 EXAM

1. A motorcycle is following a car that is traveling at constant speed on a straight highway. Initially, the car and the motorcycle are both traveling at the same speed of  $20.5 \text{ m/s}$ , and the distance between them is  $50.0 \text{ m}$ . After  $t_1 = 2.15 \text{ s}$ , the motorcycle starts to accelerate at a rate of  $4.12 \text{ m/s}^2$ . The motorcycle catches up with the car at some time  $t_2$ . How long does it take from the moment when the motorcycle starts to accelerate until it catches up with the car? In other words, find  $t_2 - t_1$ .
2. A rocket, initially at rest on the ground, accelerates straight upward from rest with constant (net) acceleration  $49.0 \text{ m/s}^2$ . The acceleration period lasts for time  $8.57 \text{ s}$  until the fuel is exhausted. After that, the rocket is in free fall. Find the maximum height reached by the rocket. Ignore air resistance and assume a constant free-fall acceleration equal to  $9.80 \text{ m/s}^2$ .
3. Two cars travel on the parallel lanes of a two-lane road. The cars' motions are represented by the position versus time graph shown in the figure. Are the two cars traveling in the same direction when they pass each other and at which of the lettered times does car 2 momentarily stop?
4. An artillery officer has the following problem. From his current position, he must shoot over a hill of height  $H$  at a target on the other side, which has the same elevation as his gun. He knows from his accurate map both the bearing and the distance  $R$  to the target and also that the hill is halfway to the target. To shoot as accurately as possible, he wants the projectile to just barely pass above the hill. Find the angle  $\theta$  above the horizontal at which the projectile should be fired.
5. A canoe has a velocity of  $0.420 \text{ m/s}$  southeast relative to the earth. The canoe is on a river that is flowing at  $0.710 \text{ m/s}$  east relative to the earth. Find the magnitude of the velocity of the canoe relative to the river.
6. A cannonball is fired horizontally from the top of a cliff. The cannon is at height  $H = 54.0 \text{ m}$  above ground level, and the ball is fired with initial horizontal speed  $v_0$ . Assume acceleration due to gravity to be  $9.80 \text{ m/s}^2$ . Assume that the cannon is fired at time  $t = 0$  and that the cannonball hits the ground at time  $t_g$ . What is the  $y$  position of the cannonball at the time  $t_g/3$ ?
7. An astronaut takes a spacewalk near the shuttle when her safety tether breaks. What should the astronaut do to get back to the shuttle?
8. Three identical blocks connected by ideal (massless) strings are being pulled along a horizontal frictionless surface by a horizontal force  $F$ . The magnitude of the tension in the

string between blocks B and C is  $T = 5.0 \text{ N}$ . What is the tension  $T_{AB}$  in the string between block A and block B?

9. Two blocks with masses  $M_1$  and  $M_2$  hang one under the other and are at rest. The tensions in the upper rope  $T_1$  and the lower rope  $T_2$  are:
10. The motion of an electron is given by  $x(t) = pt^3 + qt^2 + r$ , where  $p = -2.0 \text{ m/s}^3$ ,  $q = 1.0 \text{ m/s}^2$ , and  $r = 9.0 \text{ m}$ . Determine its velocity at  $t = 2.0 \text{ s}$ .
11. A small box of mass  $m_1 = 7.0 \text{ kg}$  is sitting on a board of mass  $m_2 = 3.0 \text{ kg}$  and length  $L = 1.0 \text{ m}$ . The board rests on a frictionless horizontal surface. The coefficient of static friction between the board and the box is  $\mu_s = 0.50$ . The coefficient of kinetic friction between the board and the box is, as usual, less than  $\mu_s$ . Find  $F_{\min}$ , the constant force with the least magnitude that must be applied to the board in order to pull the board out from under the box (which will then fall off of the opposite end of the board).
12. A small metal cylinder rests on a circular turntable that is rotating at a constant speed as illustrated in the diagram above. The small metal cylinder has a mass of  $0.20 \text{ kg}$ , the coefficient of static friction between the cylinder and the turntable is  $0.080$ , and the cylinder is located  $22 \text{ cm}$  from the center of the turntable. What is the maximum speed that the cylinder can move along its circular path without slipping off the turntable?
13. A car of mass  $M = 1200 \text{ kg}$  traveling  $50 \text{ km/h}$  enters a banked turn covered with ice. The road is banked at an angle  $\theta = 22^\circ$ , and there is no friction between the road and the car's tires. What is the radius  $r$  of the turn (assuming the car continues in uniform circular motion around the turn)?
14. Future space stations will create artificial gravity by rotating. Consider a cylindrical space station  $390 \text{ m}$  in diameter rotating about its central axis. Astronauts walk on the inside surface of the space station. What rotation period will provide "normal" gravity?

1.  $\vec{v}_{1i} = 20.5 \text{ m/s}$   $\vec{v}_{2i} = 20.5 \text{ m/s}$   $d_i = 50 \text{ m}$   
 $\vec{v}_{\text{motorcycle}} = 20.5 \text{ m/s}$   $\alpha = 4.12 \text{ m/s}^2$   $t_2 = ?$   
 $t_1 = 2.15 \text{ s}$   $t_2 - t_1 = ?$

Step 1: Position

Step 2: Position as a f'n of time:

$$x_{\text{car}} = x_i + v_{\text{car}} t$$

$$x_{\text{car}} = 50 + 20.5t$$

$$x_{\text{motorcycle}} = v_{\text{motorcycle}} \cdot t + \left[ \frac{1}{2} a (t_2 - t_1)^2 \right]$$

$$x_{\text{motorcycle}} = 20.5t + \left[ \frac{1}{2} (4.12) (t_2 - t_1)^2 \right]$$

NOTE: Car and motorcycle have the same position, so:

$$x_{\text{car}} = x_{\text{motorcycle}}$$

$$t_2 - t_1 = \sqrt{\frac{2(x_{\text{car}} - 20.5t)}{4.12}}$$

$$t_2 - t_1 = \sqrt{\frac{2(50 + 20.5t - 20.5t)}{4.12}}$$

$$t_2 - t_1 = 4.926646392 \text{ s}$$

$$t_2 - t_1 = 4.93 \text{ s}$$

(B)

2.

$$a = 49 \text{ m/s}^2$$

$$t = 8.57 \text{ s}$$

height reached during acceleration:

$$h = \frac{1}{2} a t^2$$

$$= \frac{1}{2} (49) (8.57)^2$$

$$h = 1799.40005 \text{ m}$$

$$h_{\text{max}} = h + h_2$$

$$= 1799.40005 + 8997.00025$$

$$h_{\text{max}} = 10796.4003 \text{ m}$$

$$h_{\text{max}} = 1.08 \times 10^4 \text{ m}$$

(D)

Velocity reached:

$$v = at$$

$$= 49(8.57)$$

$$v = 419.93 \text{ m/s}$$

At max height,  $v = 0$

$$v^2 = v_{\text{max}}^2 + 2ah_{\text{max}}$$

$$h_2 = \frac{v^2}{2a}$$

$$= \frac{(419.93)^2}{2 \cdot 9.8}$$

$$h_2 = 8997.00025 \text{ m}$$

3.

We can see on the graph the slope of car 2 is 0 at point C, which shows that the car momentarily stops. When the two cars intersect, they are not going the same direction, so: (D)

4.

Motion along y-axis:

$$v_{iy} = v_i \sin \theta$$

$$a = g \text{ (due to gravity)} = -9.8$$

$$v_{fy} = \text{velocity at highest point} = 0.$$

Kinematic equation:

$$v_{fy}^2 = v_{iy}^2 + 2ah$$

\*Where h is the height.

$$0 = (v_i \sin \theta)^2 - 2gh$$

$$h = \frac{(v_i \sin \theta)^2}{2g}$$

Now, taking  $\frac{h}{x}$ :

$$\frac{\frac{v_i^2 \sin^2 \theta}{2g}}{\frac{2v_i^2 \sin \theta \cos \theta}{g}} = \frac{v_i^2 \sin^2 \theta}{2g} \cdot \frac{g}{2v_i^2 \sin \theta \cos \theta}$$

$$= \frac{\sin \theta}{4 \cos \theta}$$

$$\frac{h}{x} = \frac{1}{4} \tan \theta$$

$$\theta = \tan^{-1} \left( 4 \frac{h}{x} \right) \quad * x = R$$

$$\theta = \arctan \left( \frac{4H}{R} \right) \quad (E)$$

Motion along x-axis:

$$v_{ix} = v_o \cos \theta$$

$$x = R = \text{distance travelled.}$$

$$t = \frac{2v_i \sin \theta}{g}$$

From:

$$h = v_{iy} t + \frac{1}{2} a t^2$$

$$0 = v_{iy} t + \frac{1}{2} (-g) t^2$$

$$t = \frac{2v_i \sin \theta}{g}$$

$$\text{So, } x = v_{ix} t$$

$$= v_i \cos \theta \left( \frac{2v_i \sin \theta}{g} \right)$$

$$x = \frac{2v_i^2 \sin \theta \cos \theta}{g}$$

5. Relative Velocity

$$v_{ce} = 0.420 \text{ m/s} \quad \swarrow \text{Southeast}$$

(same w.r.t earth)

$$v_{re} = 0.710 \text{ m/s} \quad \rightarrow \text{east}$$

Step 1: Components:

$$\vec{v}_{ceX} = v_{ce} \cos(-45^\circ)$$

$$= 0.420 \cos(-45^\circ)$$

$$\vec{v}_{ceX} = 0.2969848481 \text{ m/s}$$

$$\vec{v}_{ceY} = v_{ce} \sin(-45^\circ)$$

$$= 0.420 \sin(-45^\circ)$$

$$\vec{v}_{ceY} = -0.2969848481 \text{ m/s}$$

$$V = \sqrt{0.2969848481^2 + (-0.2969848481)^2}$$

$$V = 0.508705726 \text{ m/s}$$

$$V = 0.509 \text{ m/s} \quad (A)$$

$$\vec{v}_{reX} = 0.710 \text{ m/s}$$

$$\vec{v}_{reY} = 0$$

Step 2: Relative velocity

$$\vec{v}_{xy} = \vec{v}_x - \vec{v}_y$$

$$\vec{v}_x = \vec{v}_{ceX} - \vec{v}_{reX}$$

$$= 0.2969848481 - 0.710$$

$$\vec{v}_x = -0.4130151519$$

$$\vec{v}_y = \vec{v}_{ceY} - \vec{v}_{reY}$$

$$= -0.2969848481 - 0$$

$$\vec{v}_y = -0.2969848481 \text{ m/s}$$

6.

$$\begin{aligned}
 h &= 54 \text{ m} \\
 y &= y_0 + v_{0y}t - \frac{1}{2}gt^2 \\
 0 &= H + 0 - \frac{1}{2}gt^2 \\
 0 &= H - \frac{1}{2}gt^2 \\
 t_g &= \sqrt{\frac{2H}{g}} \rightarrow t_g = \frac{2\sqrt{15}}{7} \\
 t_g &= \sqrt{\frac{2H}{g}} \\
 t_g &= \sqrt{\frac{2(54)}{9.8}} \\
 t_g &= \frac{2\sqrt{54}}{3} \\
 y &= 54 - \frac{1}{2}(9.8)\left(\frac{2\sqrt{15}}{7}\right)^2 \\
 y &= 48 \text{ m} \quad \boxed{C}
 \end{aligned}$$

7.

An astronaut whose safety tether has broken should throw an object away from the shuttle to propel herself back towards it, using the conservation of momentum.

$\boxed{D)}$

8.

$$\begin{aligned}
 m_A &= m_B = m_C = m \\
 F_{T(BC)} &= 5N \\
 F &= ma \\
 a &= \frac{F}{m} \\
 a &= \frac{5}{2m} \\
 \swarrow \\
 \text{two blocks}
 \end{aligned}$$

Total Horizontal Force:

$$\begin{aligned}
 T(AB) &= m \cdot a \\
 &= m \cdot \frac{5}{2} \\
 T(AB) &= \frac{5}{2}N \\
 T(AB) &= 2.5N \quad \boxed{B)}
 \end{aligned}$$

9.

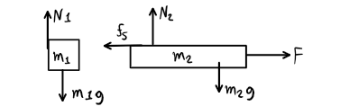
As we see in the picture, rope  $T_1$  is being held onto by two blocks: block  $m_1$  and block  $m_2$ . Combined with gravity, the tension is affected by these two blocks. For rope  $T_2$ , we see it is only held by block  $m_2$ . Therefore, we can represent the tensions as:

$$T_1 = (m_1 + m_2)g \text{ and } m_2g \quad \boxed{A)}$$

10.

$$\begin{aligned}
 X(t) &= pt^3 + qt^2 + r \\
 V(t) &= 3pt^2 + 2qt \\
 &= 3(-2)t^2 + 2(1)t \\
 V(t) &= -6t^2 + 2t \\
 \text{At } t=2: \\
 V(2) &= -6(2)^2 + 2(2) \\
 V(2) &= -20 \text{ m/s} \quad \boxed{E)}
 \end{aligned}$$

11.



$$\begin{aligned}
 F &= m_1a \\
 F &= m_1m_1g \\
 m_1a &= m_1m_1g \\
 a &= m_1g \\
 F_{min} &= (m_1 + m_2)a \\
 &= (7+3)(0.5 \cdot 9.8) \\
 F_{min} &= 49N \quad \boxed{C)}
 \end{aligned}$$

12.

$$\begin{aligned}
 m &= 0.2 \text{ kg} \\
 \mu_s &= 0.030 \\
 r &= 22 \text{ cm} = 0.22 \text{ m}
 \end{aligned}$$

Maximum speed occurs when centripetal force equals frictional.

$$\begin{aligned}
 F_c &= F_f \\
 \frac{mv^2}{r} &= \mu mg \\
 v^2 &= \frac{\mu mgr}{m} \\
 v &= \sqrt{\mu gr} \\
 v &= \sqrt{0.030 \cdot 9.8 \cdot 0.22} \\
 v &= 0.4153071153 \text{ m/s} \\
 v &= 0.42 \text{ m/s} \quad \boxed{B)}
 \end{aligned}$$

13.

$$\begin{aligned}
 m &= 1200 \text{ kg} \\
 v &= 50 \text{ km/hr} \cdot \frac{1 \text{ hr}}{3600 \text{ s}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = 13.8 \text{ m/s} \\
 \theta &= 22^\circ \\
 r &=? \\
 \tan \theta &= \frac{v^2}{rg} \\
 r &= \frac{v^2}{\tan \theta g} \\
 r &= \frac{(13.8)^2}{\tan(22^\circ)(9.8)} \\
 r &= 48.71911323 \\
 r &= 48.7 \text{ m} \quad \boxed{D)}
 \end{aligned}$$

14.

$$\begin{aligned}
 d &= 390 \text{ m}, R = 195 \\
 a &= \omega^2 R \quad \omega = \frac{2\pi}{T} \\
 \text{Let } a &= g \\
 g &= \omega^2 R \\
 \omega &= \sqrt{\frac{g}{R}} \\
 &= \sqrt{\frac{9.8}{195}} \\
 \omega &= 0.22441794153 \\
 T &= \frac{2\pi}{\omega} \\
 T &= \frac{2\pi}{0.22441794153} \\
 T &= 28.02748548 \\
 T &= 28 \text{ s} \quad \boxed{E)}
 \end{aligned}$$