

## 2. Function Review

### Lec 1 mini review.

<b>functions:</b>	independent variable, dependent variable, domain, range
<b>graphs:</b>	vertical line test, symmetry (even/odd), periodicity, transformations intervals of increase/decrease
<b>linear functions:</b>	slope, intercepts
<b>polynomials:</b>	any degree $n \geq 0$ (constant, linear, quadratic, cubic,...), coefficients
<b>power functions:</b>	$f(x) = x^n$
<b>rational functions</b>	<b>algebraic functions</b>
	<b>root functions:</b> $f(x) = x^{1/n} = \sqrt[n]{x}$
	<b>absolute value:</b> $f(x) =  x $

## COMPOSITION

Let  $f$  and  $g$  be functions. If all numbers in the range of  $g$  are in the domain of  $f$ , then the **COMPOSITION**  $f \circ g$  is a function defined by

**Example 2.1.** Find the composition  $f \circ g$ , where  $f(x) = \frac{x-1}{x+1}$  and  $g(x) = \frac{1}{\sqrt{x}}$

## INVERSE

**Horizontal Line Test:** Let  $y = f(x)$  be a function. If every horizontal line crosses the graph of  $f$  at most once, then  $f(x)$  is a **ONE-TO-ONE (INJECTIVE)** function.

**Inverse:** Let  $y = f(x)$  be a function. If  $f$  passes the Horizontal Line Test, then the map  $f^{-1}$  defined by the rule

is a function called the **INVERSE** of  $f$ .

### Composition of a function with its inverse:

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**Example 2.2.** Find the inverse of  $g(x) = \frac{2x-1}{3x+2}$  and verify that  $(g \circ g^{-1})(x) = x = (g^{-1} \circ g)(x)$ .

**1. Write  $y = g(x)$ :**

**2. Interchange  $x$ 's and  $y$ 's:**

**3. Isolate 'new'  $y$ :**

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**Exercise 2.3.** Use the table to evaluate each of the following expressions.

$x$	1	2	3	4	5	6
$f(x)$	3	1	4	2	6	5
$g(x)$	5	3	2	6	2	3

a.  $f^{-1}(1)$

b.  $(g \circ f)(3)$

c.  $(f \circ g)(6)$

d.  $(f \circ f^{-1})(4)$

e.  $g(g(1))$

f.  $(g \circ f)(1)$

g.  $(f^{-1} \circ f^{-1})(4)$

h.  $(f^{-1} \circ f)(6)$

i.  $(g \circ f^{-1})(1)$

**Example 2.4.** Find the inverse of  $f(x) = \sqrt{x-2}$  and sketch the graphs of  $f$  and  $f^{-1}$ .

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## CATALOGUE OF IMPORTANT FUNCTIONS: EXPONENTIAL & LOGARITHMIC

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**Exponential  
Functions:**

**Natural Base:**

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**Laws of  
Exponents:**

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**Example 2.5.** Solve for  $x$  in the equation  $2^{x+3} = 16^{2x-1}$ .

**Logarithmic  
Functions:**

**Natural  
Logarithm:**

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**Laws of Logs:**

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**Inverse Relationship  
between  $a^x$  and  $\log_a(x)$ :**

**Example 2.6.** Solve for  $x$  in the equation  $\log(x + 1) + \log(x + 4) = 1$ .

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**Example 2.7.** A bacterial population grows according to the model  $b(t) = 1.8^t b_0$  where  $t$  represents time in hours,  $b(t)$  represents the number of bacteria in the population at time  $t$ , and  $b_0$  represents the initial population at time  $t = 0$  (assume  $b_0 > 0$ ).

How long will it take for the initial population to triple in size?

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## CATALOGUE OF IMPORTANT FUNCTIONS: TRIGONOMETRIC & INVERSE TRIG

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### Trigonometric Ratios:

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#### Basic Trigonometric Functions

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sine:

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cosine:

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tangent:

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## Reciprocal Trig Functions

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cosecant:

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secant:

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cotangent:

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## Useful Trig Identities

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## Inverse Trig Functions

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arcsine:

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arccosine:

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arctangent:



**Example 2.8.** Find the domain of  $g(x) = \frac{\cos(x)}{\frac{1}{2} - \sin(x)}$ .