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2.1 Demonstrate the validity of the following identities by means of truth tables:

a) DeMorgan's theorem for three variables: (x + y + z)' = x'y'z' and (xyz)' = x' + y' + z'

$(x+y+z)' = x'y'z'$

x	y	z	$(x+y+z)$	$(x+y+z)'$	x'	y'	z'	$x'y'z'$
0	0	0	0	1	1	1	1	1
0	0	1	1	0	1	1	0	0
0	1	0	1	0	1	0	1	0
0	1	1	1	0	1	0	0	0
1	0	0	1	0	0	1	1	0
1	0	1	1	0	0	1	0	0
1	1	0	1	0	0	0	1	0
1	1	1	1	0	0	0	0	0

$\therefore$ Since both columns are logically equivalent $(x+y+z)' = x'y'z'$

$(xyz)' = x' + y' + z'$

x	y	z	xyz	$(xyz)'$	x'	y'	z'	$(x'+y'+z')$
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	0	1
0	1	0	0	1	1	0	1	1
0	1	1	0	1	1	0	0	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	0	1	0	1
1	1	0	0	1	0	0	1	1
1	1	1	1	0	0	0	0	0

$\therefore$ Since both columns are logically equivalent

$(xyz)' = (x'+y'+z')$

b) The distributive law: $x + yz = (x + y)(x + z)$

x	y	z	yz	$x+yz$	$x+y$	$x+z$	$(x+y)(x+z)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

$\therefore$ Since both columns are logically equivalent

$$x + yz = (x + y)(x + z)$$

c) The distributive law: $x(y + z) = xy + xz$

x	y	z	$(y+z)$	$x(y+z)$	xy	xz	$(xy+xz)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	1	1	1
1	1	0	1	1	0	0	1
1	1	1	1	1	1	1	1

2.4 Reduce the following Boolean expressions to the indicated number of literals

a) * x'z' + xyz + xz' to three literals

$$
\begin{aligned}
x'z' + xyz + xz' &= z'(x'+x) + xyz \\
&= z' + xyz \\
&= (z'+z)(z'+xy) \\
&= z' + xy
\end{aligned}
$$

b) * (x'y' + z)' + z + xy + wz to three literals

$$
\begin{aligned}
(x'y'+z)' + z + xy + wz &= (x'y')'z' + z + xy + wz \\
&= \left((x+y)\cdot z' + z\right) + xy + wz \\
&= (x+y+z)(z+z') + xy + wz \\
&= x + xy + y + z + wz \\
&= x(1+y) + y + z(1+w) \\
&= x + y + z
\end{aligned}
$$

c) * w'x(z' + y'z) + x(w + w'yz) to one literal

$$
\begin{aligned}
w'x(z'+y'z) + x(w+w'yz) &= x(w'z' + w'y'z + w + w'yz) \\
&= x(w'z' + w + w'z(y'+y)) \\
&= x(w'z' + w + w'z) \\
&= x(w'(z'+z) + w) \\
&= x(w'+w) \\
&= x
\end{aligned}
$$

d) * (w' + y)(w' + y')(w + x + y'z) to four literals

$$
\begin{aligned}
(w'+y)(w'+y')(w+x+y'z) &= (w' + w'y' + w'y)(w + x + y'z) \\
&= w'w + w'x + w'y'z + ww'y' + w'xy' + w'y'z \\
&\quad + w'wy + w'yz + w'yy'z \\
&= w'y'z + w'x + x'yz' + w'xy \\
&= w'y'z + w'x(1+y) + w'xy \\
&= w'y'z + w'x(1+y) \\
&= w'y'z + w'x \\
&= w'(x+y'z)
\end{aligned}
$$

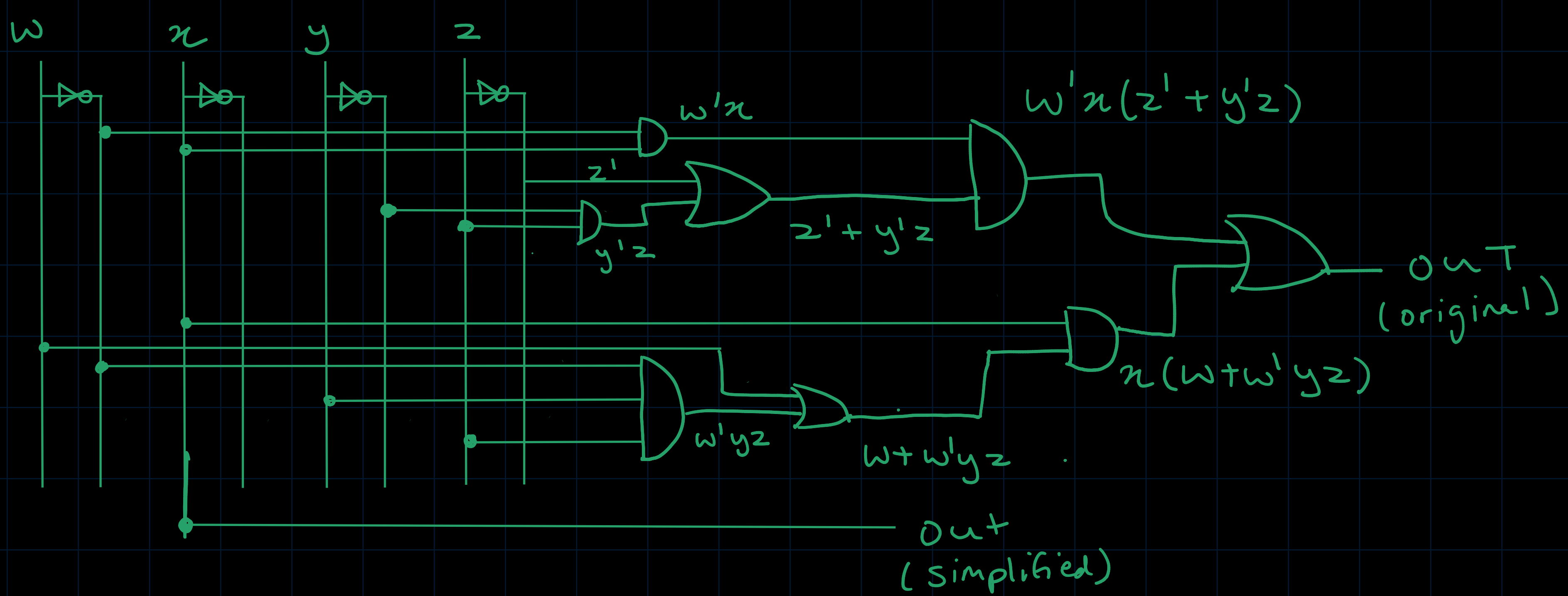
e) wxy'z + w'xz + wxyz   to two literals

$$wxy'z + w'xz + wxyz = wxz(y'+y) + w'xz$$
$$= wxz + w'xz$$
$$= xz(w+w')$$
$$= xz$$

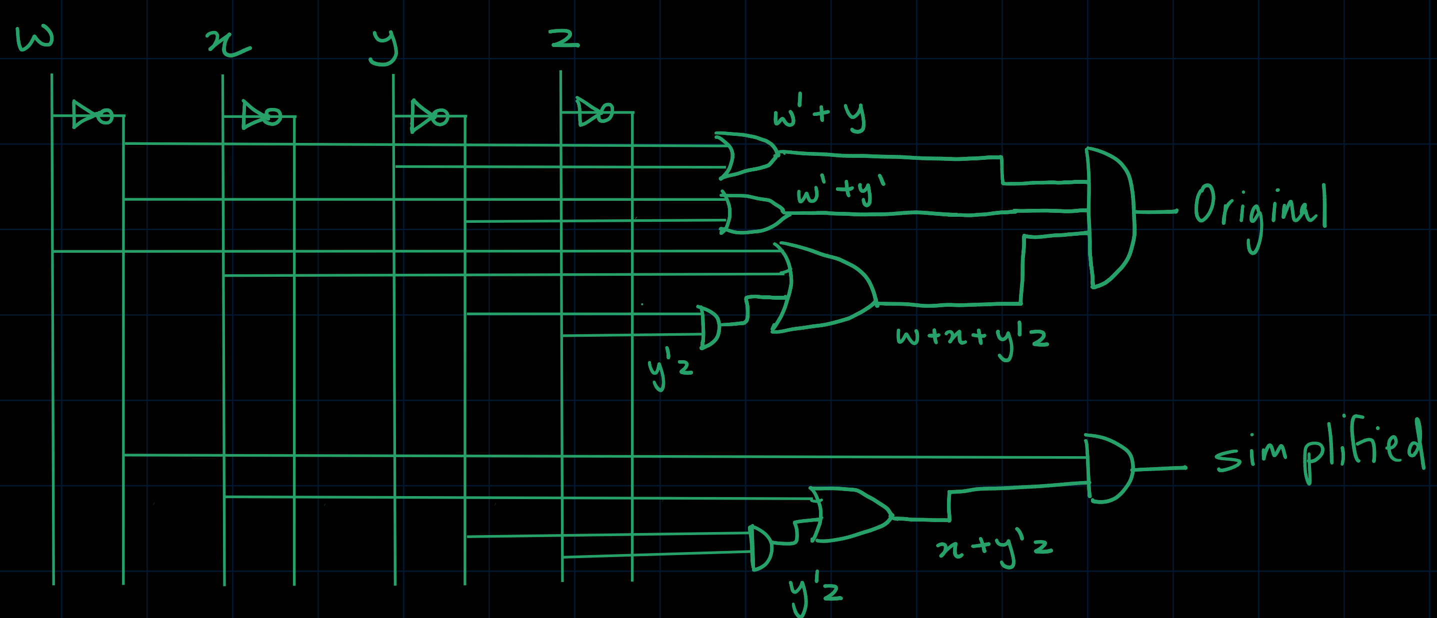
2.7 Draw logic diagrams of the circuits that implement the original and simplified expressions from 2.4 (c), (d), (e)

c) Original $= w'x(z'+y'z) + x(w+w'yz)$

Simplified $= x$

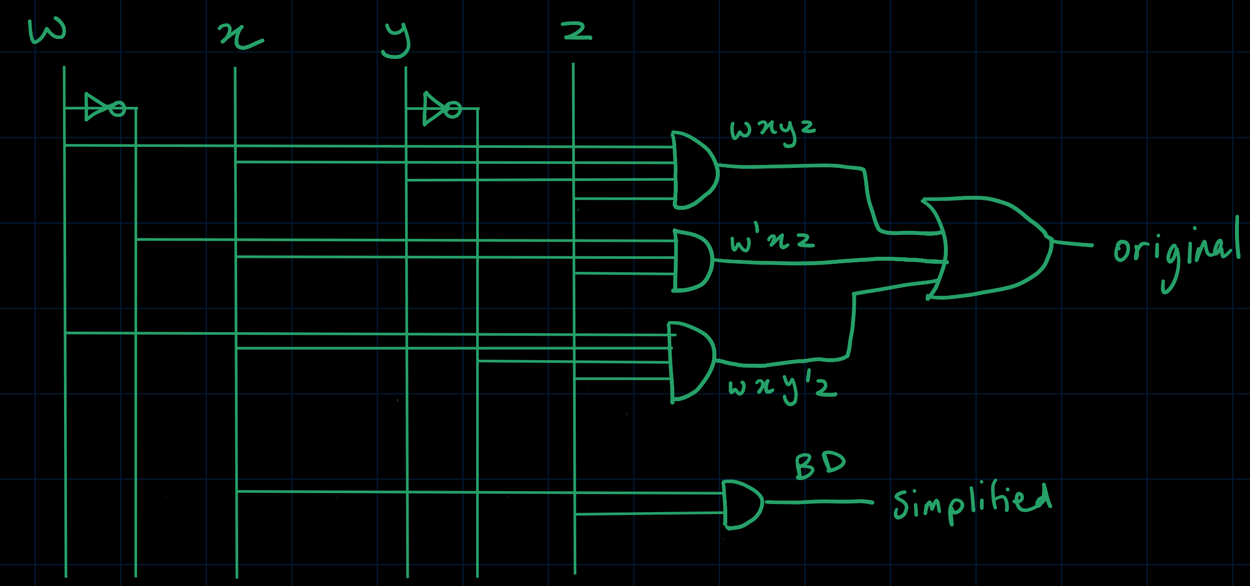


d) Original $= (w'+y)(w'+y')(w+x+y'z)$
Simplified $= w'(x+y'z)$



e) Original $wxy'z + w'xz + wxyz$

Simplified xz



Find the complement of the following expressions:

a) xy' + x'y

$F = xy' + x'y$

$F' = (xy' + x'y)' = (xy')' \cdot (x'y)'$ De Morgan's

$= (x' + y'')(x'' + y')$

$= (x' + y)(x + y')$

$= x'x + x'y' + xy + yy'$

$= xy + x'y'$

b) (a + c)(a + b')(a' + b + c')

$F = (a' + b + c')(a + b')(a + c)$

$F' = ((a' + b + c')(a + b')(a + c))$

$= (a' + b + c') + (a + b')' + (a + c)'$

$= ab'c + a'b'' + a'c'$

$F = ab'c + a'b + a'c'$

c) z + z'(v'w + xy)

$F = (v'w + xy)z' + z$

$F' = ((v'w + xy)z' + z)'$

$= ((v'w + xy)z + (z' + z))'$

$= ((v'w + xy)z)'$

$= (v'w + xy)'z'$

$= (v'w)'(xy)'z'$

$= (v + w')(x' + y')z'$

$= (vx' + vy' + w'x' + w'y')z'$

$= vx'z' + vy'z' + w'x'z' + w'y'z'$

2.11 List the truth table of the function:

a) $F = xy + xy' + y'z$

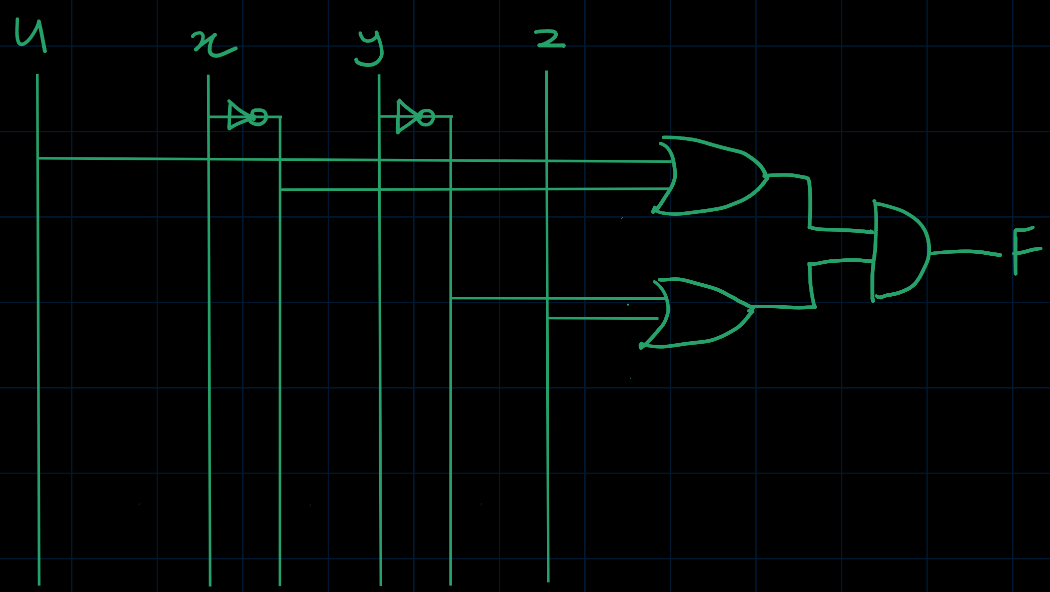
x	y	z	xy	xy'	$y'z$	F
0	0	0	0	0	0	0
0	0	1	0	0	1	1
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	0	1	0	1
1	0	1	0	1	1	1
1	1	0	1	0	0	1
1	1	1	1	0	0	1

b) $F = ac + b'c'$

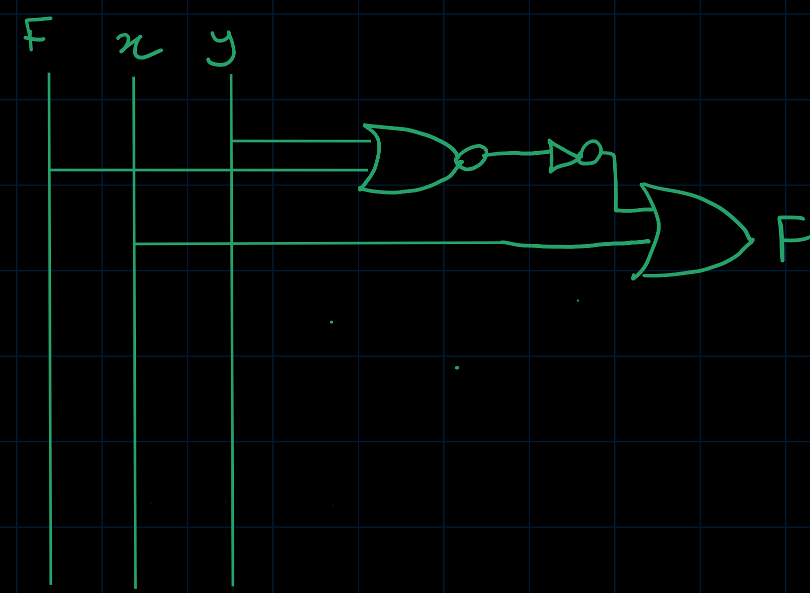
a	b	c	ac	$b'c'$	F
0	0	0	0	1	1
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	1	1
1	0	1	0	0	0
1	1	0	1	0	1
1	1	1	1	0	1

2.13 Draw Logic Diagrams to implement the following Boolean expressions:

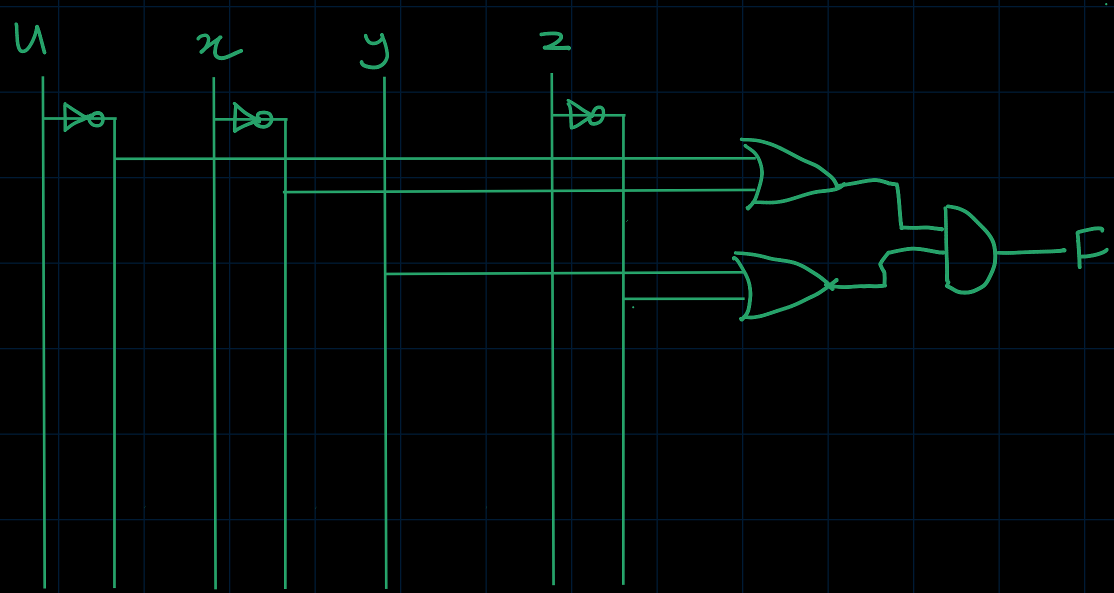
a) $F = (u + x')(y' + z)$



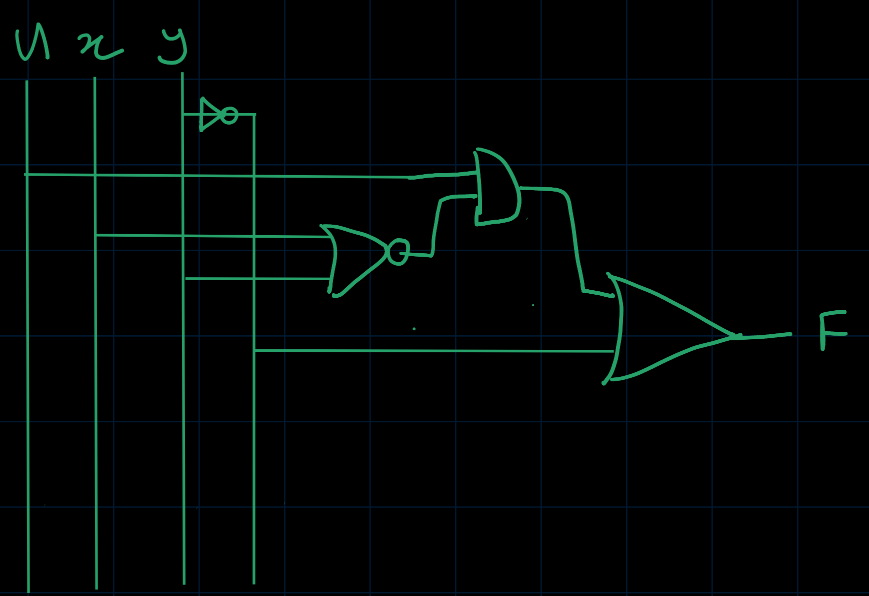
b) $F = (u \oplus y)' + x$



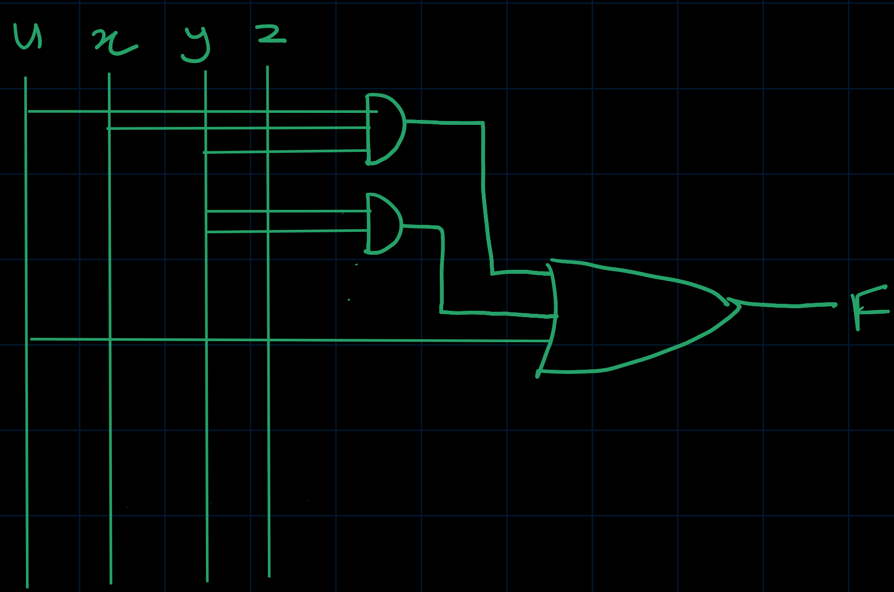
c) $F = (u' + x')(y + z')$



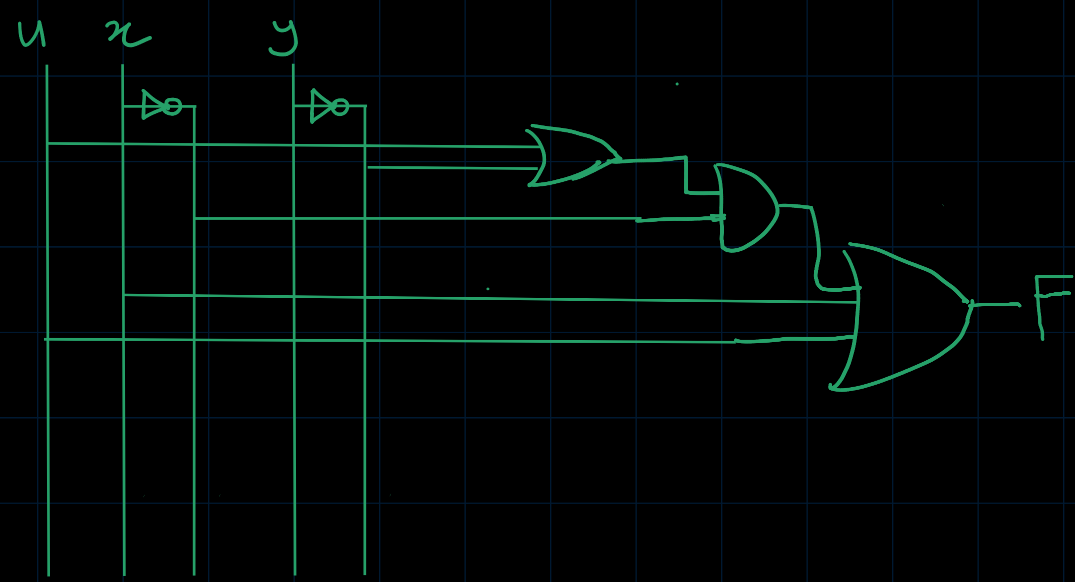
d) $F = u(x \oplus y) + y'$



e) F = u + yz + uxy



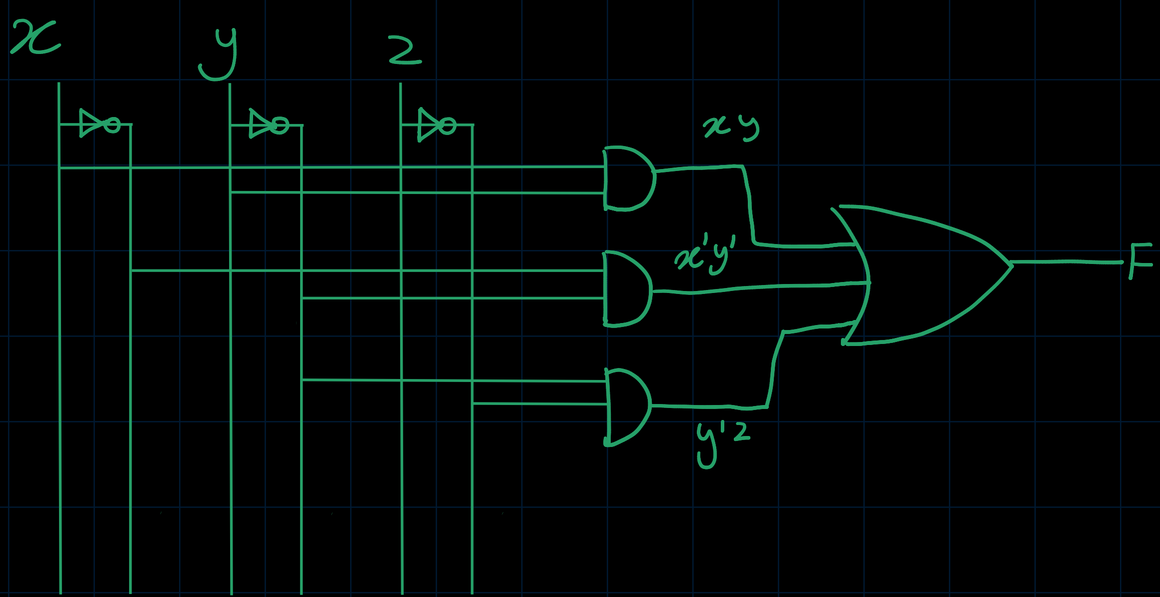
f) F = u + x + x'(u + y')



2.14 Implement the Boolean function F = xy + x'y' + y'z

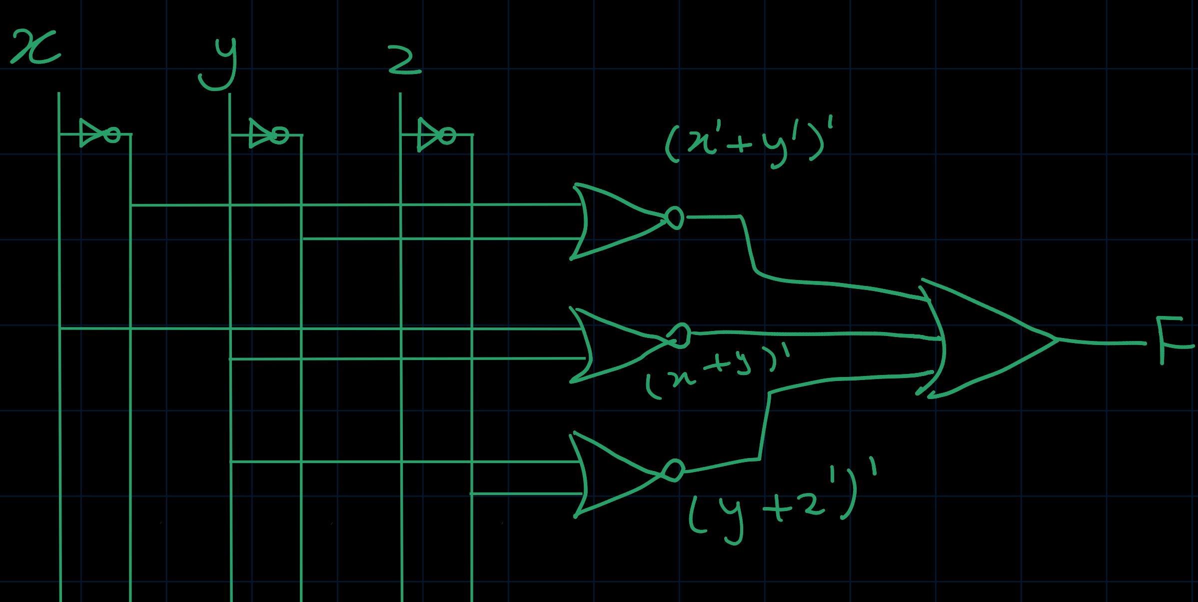
a) Win AND, OR and inverter gates

$$F = xy + x'y' + y'z$$



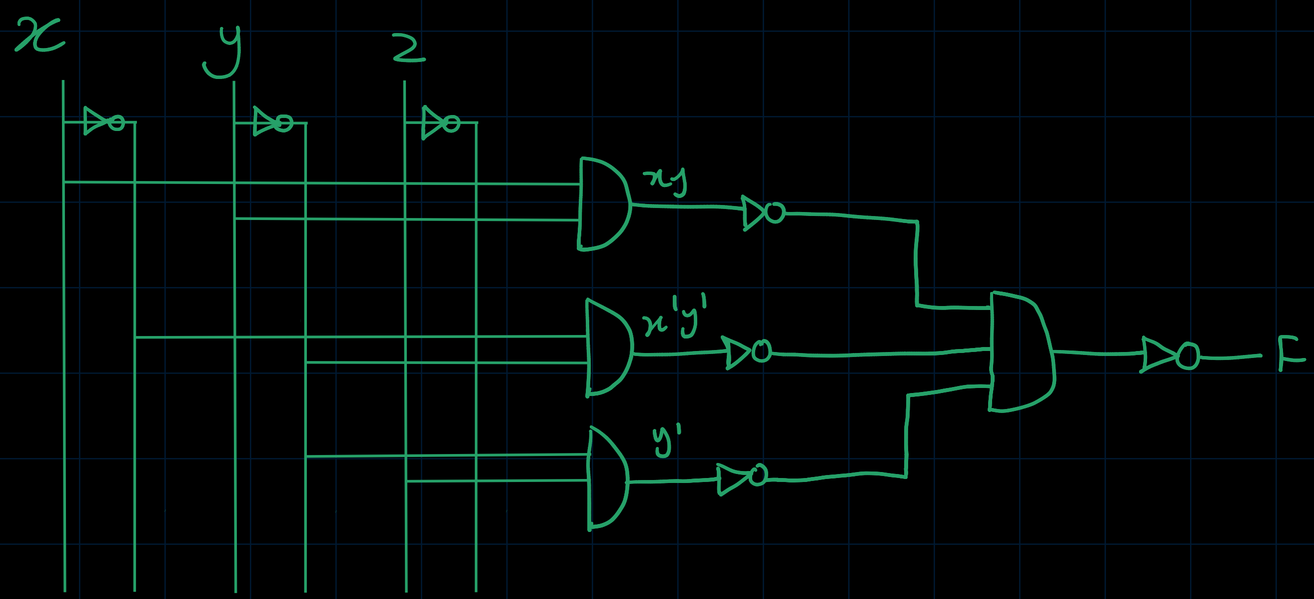
b) With OR and inverter gates

$$F = xy + x'y' + y'z = (x'+y')' + (x+y)' + (y+z')'$$



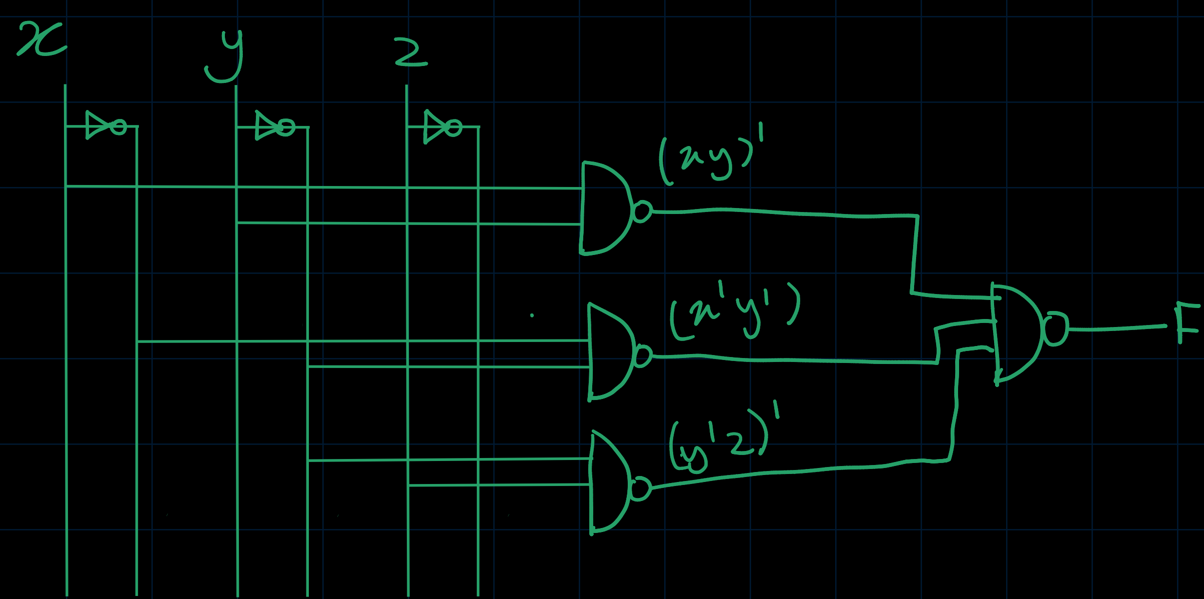
c) With AND and inverter gates

$$F = xy + x'y' + y'z = [(xy)'(x'y')'(y'z)']'$$



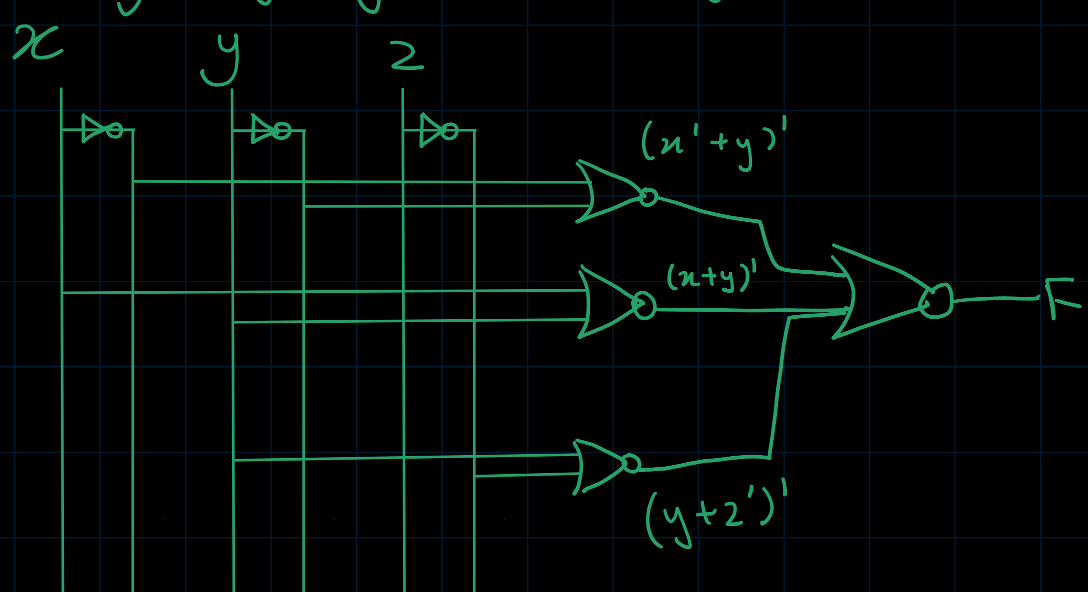
d) With NAND inverter gates

$$F = xy + x'y' + y'z = [(xy)'(x'y')'(y'z)']'$$



e) With NOR and inverter gates

$$F = xy + x'y' + y'z = (x'+y')' + (x+y)' + (y+z')'$$



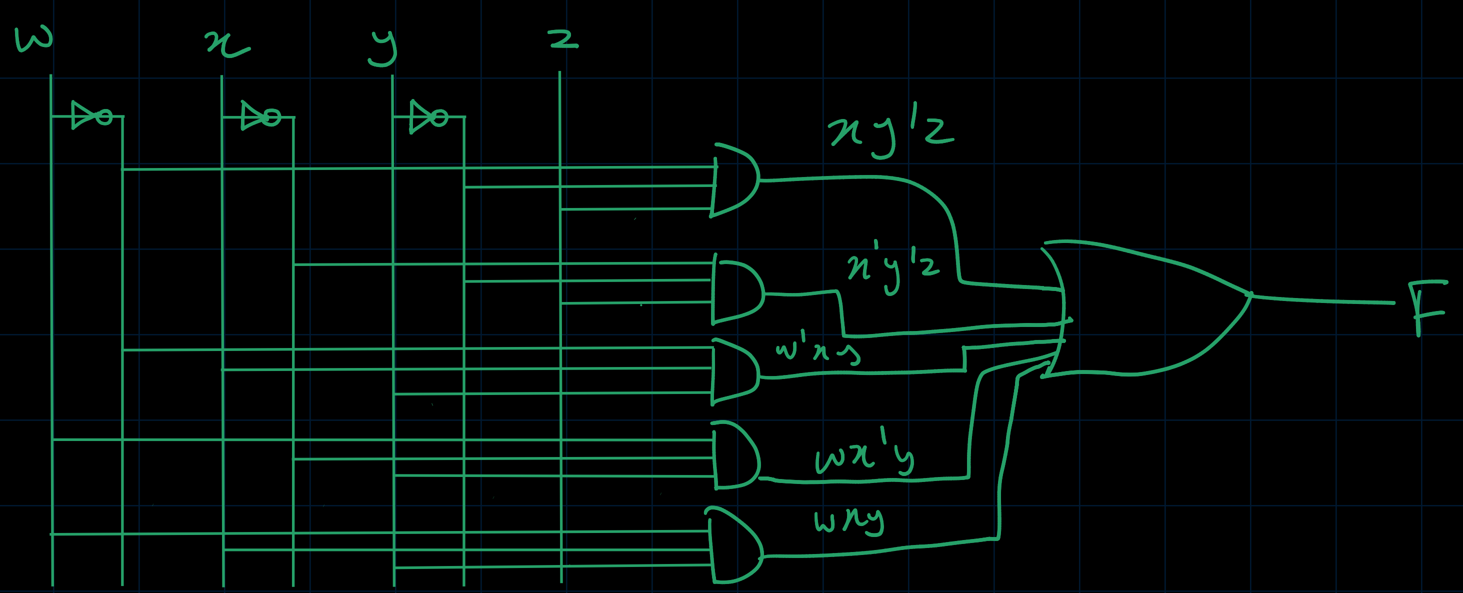
2.18 For the Boolean function

$$F = xy'z + x'y'z + w'xy + wx'y + wxy$$

a) Obtain the truth table of F

w	x	y	z	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

b) Draw a logic diagram using the original Boolean Expression



c) Use Boolean alegbra to simplify the function to a minimum number of literals

$$F = xy'z + x'y'z + w'xy + wx'y + wxy + wxy$$

$$= y'z(x+x') + xy(w'+w) + wy(x'+x)$$

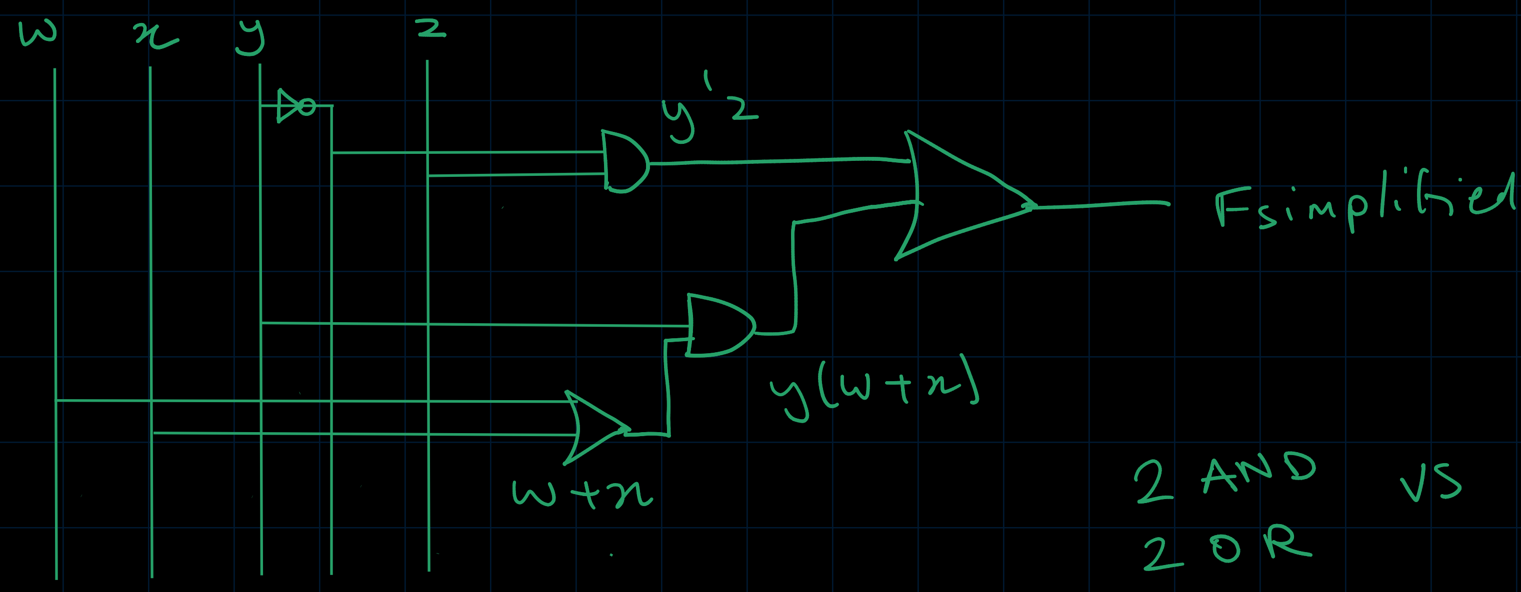
$$= y'z + xy + wy$$

$$= y'z + y(x+w)$$

d) Obtain the truth table of the function from the simplified expression and show it is the same as the one in part (a)

w	x	y	z	y'z	x+w	y(x+w)	F
0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	1
0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	0	1	0	0
0	1	0	1	1	1	0	1
0	1	1	0	0	1	1	1
0	1	1	1	0	1	1	1
1	0	0	0	0	1	0	0
1	0	0	1	1	1	0	1
1	0	1	0	0	1	1	1
1	0	1	1	0	1	1	1
1	1	0	0	0	1	0	0
1	1	0	1	1	1	0	1
1	1	1	0	0	1	1	1
1	1	1	1	0	1	1	1

e) Draw the logic diagram from the simplified expression, and compare the total number of gates with the diagram of part (b)



2 AND vs 5 AND
2 OR 1 OR