2023 MIDTERM 2 EXAM

1. A 6.0-kg block slides from point A down a frictionless curve to point B. After the block passes point B, a friction force opposes the motion of the block so that it comes to a stop 2.5 m from B. Calculate the coefficient of kinetic friction between the block and the surface after position B.

2. A 50% efficient hydroelectric plant generates electricity from water falling a height of 110 m and at a rate of R kg/s. If the maximum possible power output of the plant is 27.0 MW, what is R?

3. An L-shaped piece, represented by a shaded area on the figure, is cut from a metal plate of uniform thickness. The point that corresponds to the center of mass of the L-shaped piece is

4. A helium atom (mass = 4m) moving with speed V collides elastically with a deuterium (hydrogen-2) atom (mass = 2m) at rest. Calculate the percentage change in the kinetic energy of the helium atom after the collision.

5. A 1.0-kg mass is acted on by a net force of 4.0 N and a 3.0-kg mass is acted on by a net force of 3.0 N, in the directions shown. The acceleration of the center of mass of this system is:

6. The force exerted on a body of mass 10 kg varies with time according to F = 20t + 10, where the units are SI. If the velocity of the body was zero at t = 0, its velocity at t = 5.0 s is :

7. A 2.0-kg sphere attached to an axle by a spring is displaced from its rest position to a radius of 20 cm from the axle centerline by a standard mass of 20 kg, as in Figure 1. The same 2.0-kg sphere is also displaced 20 cm from the axle centerline, as in Figure 2, when the sphere is rotated at a speed of:

8. The moment of inertia of the wheel in the figure is 0.50 kg · m2 , and the bearing is frictionless. The radius of the wheel is 0.50 m. The acceleration of the 15-kg mass is:

9. You are given two hoops, which are (1) brass and (2) wood, and two cylinders, which are (3) brass and (4) wood; each has radius R. If all are released from the same starting line at the same time, the one(s) that reach the bottom first are:

10. A hoop of radius 3.05 m has a mass of 145 kg. The hoop rolls without slipping along a horizontal plane. If the center of mass of the hoop has a speed of 0.305 m/s, the work required to bring the hoop to rest is:

11. A man is walking north carrying a suitcase that contains a spinning wheel mounted on an axle attached to the front and back of the case. The angular velocity of the wheel points north. The man now begins to turn to walk east. As a result, the front of the suitcase:

12. The graph shows the square of the period versus the length of a simple pendulum on a certain planet. The acceleration due to gravity on that planet is:

13. You accidentally knock a full bucket of water off the side of the well shown in the figure above. The bucket plunges 15 m to the bottom of the well. Attached to the bucket is a light rope that is wrapped around the crank cylinder. How fast is the handle turning when the bucket hits bottom. The mass of the bucket plus water is 12 kg. The cylinder has a radius of 0.080 m and mass of 4.0 kg.

14. A damped oscillator is released from rest with an initial displacement of 10.00 cm. At the end of the first complete oscillation the displacement reaches 9.05 cm. When four more oscillations are completed, what is the displacement reached?

1. Chapter 6

$m = 6kg$

$h = 1m$

$d_{fromB} = 2.5m$

$\mu_k = ?$

From A to B:

Conservation of energy:

$\Delta E = E_f - E_i$

$E_f = E_i$

$E_{k_i} + U_{g_i} = E_{kf} + U_{gf}$

$\frac{1}{2}mv^2 = mgh$

$W = \Delta E_K$

$W = E_{kf} - E_{ki}$

$W = 0 - \frac{1}{2}mv^2$

From B:

$W = F \cdot d \quad W = \Delta E$

$F \cdot d = E_{kf} - E_{ki}$

$-\mu_k F_N \cdot d_{fromB} = 0 - \frac{1}{2}mv^2 \qquad * \xleftarrow{f_k}$

$-\mu_k \cdot mg \cdot d_{fromB} = -mgh$

$\mu_k = \frac{h}{d_{fromB}}$

$\mu_k = \frac{1m}{2.5m}$

$\mu_k = 0.4m \quad \boxed{B)}$

2. Chapter 6:

$h = 110m \quad e = 50\% = 0.50$

$R(\frac{kg}{s}) \quad P_{max} = 27mW \quad (2.7 \times 10^6 W)$

$R = ?$

$P = \frac{W}{\Delta t} \qquad * R = \frac{m}{\Delta t}$

$P = \frac{0.50\, mgh}{\Delta t}$

$P = 0.5Rgh$

$R = \frac{P}{0.5gh}$

$R = \frac{2.7 \times 10^6}{0.5(9.8)(110)}$

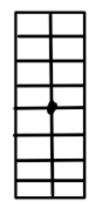
$R = 5009.276438 \frac{kg}{s}$

$R = 50 \times 10^3 \frac{kg}{s} \quad \boxed{D)}$

3. Chapter 8:

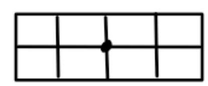
Piece 1: $m_1 = 16$

$x_{cm} = 1 \quad y_{cm} = 4$



Piece 2: $m_2 = 8$

$x_{cm} = 4 \quad y_{cm} = 1$



$r_{cm} = \frac{m_1 x_1 + \dots x_n r_n}{m_1 + \dots + m_n}$

$x_{cm} = \frac{16 \cdot 1 + 8 \cdot 4}{16 + 8} = \frac{48}{24} = 2$

$y_{cm} = \frac{16 \cdot 4 + 8 \cdot 1}{16 + 8} = \frac{72}{24} = 3$

$r = (2, 3) = 4 \quad \boxed{D)}$

4. Chapter 8

$m_1 = 4m \quad V_{1i} = V$ elastic collision

$m_2 = 2m \quad V_{2i} = 0$

Percentage change:

$$R = \frac{K_{1f} - K_{1i}}{K_{1i}}$$

$$R = \frac{\frac{1}{2}4mV_{1f}^2 - \frac{1}{2}4mV_{1i}^2}{\frac{1}{2}4mV_{1i}^2}$$

$$R = \frac{\frac{1}{2}4mV_{1f}^2}{\frac{1}{2}4mV_{1i}^2} - 1$$

$$R = \frac{V_{1f}^2}{V_{1i}^2} - 1$$

$$R = \frac{\frac{1}{9}V^2}{V^2} - 1$$

$$R = \frac{1}{9} - 1$$

$$R = -\frac{8}{9}$$

$$R = -0.8\overline{8}$$

$$R = -89\% \quad \boxed{A)}$$

$$*V_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)V_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)V_{2i}$$

$$V_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)V_{1i}$$

$$V_{1f} = \left(\frac{4m - 2m}{4m + 2m}\right)V_{1i}$$

$$V_{1f} = \frac{2}{6}V_{1i}$$

$$V_{1f} = \frac{1}{3}V_{1i} \Rightarrow V_{1f} = \frac{1}{3}V$$

5. Chapter 10

$m_1 = 1\text{kg} \quad m_2 = 3\text{kg}$

$\vec{F_1} = 4N\hat{j} \quad \vec{F_2} = 3N\hat{i}$

$\vec{a}_{cm} = ?$

$$\vec{a}_{cm} = \frac{m_1\vec{a_1} + m_2\vec{a_2}}{m_1 + m_2} = \frac{\vec{F_1} + \vec{F_2}}{m_1 + m_2} = \frac{4N\hat{j} + 3N\hat{i}}{1\text{kg} + 3\text{kg}} = \frac{3}{4}\hat{i} + 1\hat{j}$$

$$a_{cm} = \sqrt{\frac{3}{4}^2\hat{i} + 1\hat{j}^2} = \frac{5}{4} = 1.25\,m/s^2$$

$$\theta = \arctan\left(\frac{a_{ycm}}{a_{xcm}}\right) = \arctan\left(\frac{4}{3}\right) = 53.13010235^\circ$$

$$a_{cm} = 1.25\,m/s^2,\ 53^\circ \quad \boxed{E)}$$

6. Chapter 8

First, deriving Force as a function of time:

$$F(t) = \int_0^t 20t + 10$$

$$F(t) = 10t^2 + 10t$$

Next, using the impulse equation:

$$F \cdot t = m \cdot v$$

$$10t^2 + 10t = 10 \cdot v$$

And letting $t = 5$:

$$v = \frac{10(5^2) + 10(5)}{10} \Rightarrow v = 30\,m/s \quad \boxed{B)}$$

7. Chapter 10

$m = 2kg \quad r = 2cm = 0.2m$

$M = 20kg$

$kr = Mg$

$k = \frac{Mg}{r}$ (1)

$\frac{m_1 V_1^2}{r} = kr$

$\frac{m_1 V_1^2}{r} = \frac{Mg}{r} r$ *from (1)

$m_1 V_1^2 = Mgr$

$V_1 = \sqrt{\frac{Mgr}{m_1}}$

$V_1 = \sqrt{\frac{20 \cdot 9.8 \cdot 0.2}{2}}$

$V_1 = 4.427188724 \text{ m/s}$

$V_1 = 4.4 \text{ m/s}$ $\boxed{A)}$

8. Chapter 10.

$I = 0.5 kg \cdot m^2$

$r = 0.5m \quad m = 15kg$

$a = ?$

$\Sigma F = ma$

$F_N - T = ma$

$mg - T = ma$

$\tau = TR = I\alpha = I\left(\frac{\alpha}{R}\right)$

$TR = I\left(\frac{\alpha}{R}\right)$

$T = I\frac{a}{r^2}$

$mg - I\frac{a}{r^2} = ma$

$mg = a\left(\frac{I}{r^2} + m\right)$

$a = \frac{mg}{\frac{I}{r^2} + m}$

$a = \frac{15 \cdot 9.8}{\frac{0.5}{0.5^2} + 15}$

$a = 8.647 m/s^2$

$a = 8.6 m/s^2$ $\boxed{C)}$

9. Chapter 9/10

$V_{cm} = \sqrt{\frac{2gh}{1 + \frac{I}{mg^2}}} \quad * c = \frac{I}{mg^2}$

$V_{cm} = \sqrt{\frac{2gh}{1+c}}$

$C_{hollow} = 1$ (hollow hoop, coefficient of $I = MR^2$)

$C_{solid} = \frac{1}{2}$ (solid cylinder, coefficient of $I = MR^2$)

$V_{cm_{solid}} = \sqrt{\frac{2gh}{1.5}} > V_{cm_{hollow}} = \sqrt{\frac{2gh}{2}}$

So, objects 3 and 4. $\boxed{E)}$

10. Chapter 10.

$r = 3.05m \quad V = 0.305 m/s$

$m = 145kg$

$W = \Delta E_K = \frac{1}{2}MV_{cm}^2 + \frac{1}{2}\underbrace{(MR^2)}_{I}\underbrace{\left(\frac{V_{cm}}{R}\right)^2}_{\omega}$

$= \frac{1}{2}MV_{cm}^2 + \frac{1}{2}MR^2 \cdot \frac{V_{cm}^2}{R^2}$

$= \frac{1}{2}MV_{cm}^2 + \frac{1}{2}MV_{cm}^2$

$W = MV_{cm}^2$

$W = 145(0.305)^2$

$W = 13.488625J$

$W = 13.5J$ $\boxed{B)}$

11. Chapter 10

Determine the Concept In turning toward the east, you redirect the angular momentum vector from north to east by exerting a torque on the spinning wheel. The force that you must exert to produce this torque (use a right-hand rule with your thumb pointing either east of north and note that your fingers point upward) is upward. That is, the force you exert on the front end of the suitcase is upward and the force the suitcase exerts on you is downward. Consequently, the front end of the suitcase will dip downward. $\boxed{(d)}$ is correct.

$\boxed{D)}$

12. Chapter 14.

$g(P) = ?$

$T = 2\pi\sqrt{\frac{L}{g(P)}}$

$T^2 = 4\pi^2 \cdot \frac{L}{g}$

$\frac{T^2}{L} = \frac{4\pi^2}{g}$

$\text{Slope} = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{60-0}{6-0} = 10$

$10 = \frac{4\pi^2}{g}$

$g = \frac{4\pi^2}{10}$

$g = 3.94784176 \text{ m/s}^2$

$g = 3.95 m/s^2$ $\boxed{C)}$

13. Chapter 10.

$h = 15\,m/s$

$m_1 = 12\,kg \rightarrow$ bucket

$r = 0.080\,m$

$m_2 = 4\,kg \rightarrow$ cylinder

$\omega = ?$

Using Conservation of energy:

$$\Delta E = E_K + U$$

$$m_1 g h_i = m_1 g h_f + \frac{1}{2} m_1 V_f^2 + \frac{1}{2} I_2 \omega_{2f}^2$$

$$* V_f = \omega_{2f} R \quad I_2 = \frac{1}{2} m_2 R^2$$

$$m_1 g (h_i - h_f) = \frac{1}{2}\left(m_1 \omega_{2f}^2 R^2 + \frac{1}{2} m_2 R^2 \omega_{2f}^2\right)$$

$$2 m_1 g (h_i - h_f) = m_1 \omega_{2f}^2 R^2 + m_2 R^2 \omega_{2f}^2$$

$$2 m_1 g (h_i - h_f) = \omega_{2f}^2 R^2 (m_1 + m_2)$$

$$\omega_{2f} = \omega = \sqrt{\frac{2 m_1 g (h_i - h_f)}{R^2 (m_1 + m_2)}}$$

$$\omega = \frac{1}{R}\sqrt{\frac{2 m_1 g h}{m_1 + \frac{m_2}{2}}} \qquad \boxed{E)}$$

$\omega = 198.4313483$

$\omega = 2.00 \times 10^2\ rad/s$

14. Chapter 14

$$x(T=0) = 10\,cm$$

$$x(T=t) = 9.05\,cm$$

$$x(T=t+4t) \Rightarrow x(T=5t) = ?$$

$$x = A e^{\left(\frac{-b}{2m}\right)t} \cos(\omega t + \phi)$$

$$x = A e^{\left(\frac{-b}{2m}\right)t} \cos\left(\frac{2\pi}{T} t + \phi\right)$$

$$x(t=0) = A\cos\phi$$

$$x(t=T) = A e^{\left(\frac{-b}{2m}\right)T} \cos\left(\frac{2\pi}{T} T + \phi\right)$$

$$x(t=T) = A e^{\left(\frac{-b}{2m}\right)T} \cos\phi$$

$$\frac{x(t=T)}{x(t=0)} = \frac{A e^{\left(\frac{-b}{2m}\right)T} \cos\phi}{A\cos\phi}$$

$$\frac{x(t=T)}{x(t=0)} = e^{\left(\frac{-b}{2m}\right)T} \quad (1)$$

$$x(t=5T) = A e^{\left(\frac{-b}{2m}\right)5T} \cos\left(\frac{2\pi}{T} 5T + \phi\right)$$

$$x(t=5T) = A\cos\phi \cdot \left[e^{\left(\frac{-b}{2m}\right)T}\right]^5$$

$$x(t=5T) = x(t=0) \cdot \left[\frac{x^5(t=T)}{x^5(t=0)}\right]$$

$$x(t=5T) = \left[\frac{x^5(t=T)}{x^4(t=0)}\right]$$

$$x(t=5T) = \left[\frac{9.05^2}{10^2}\right]$$

$$x(t=5T) = 6.070757653\ cm$$

$$= 6.07\,cm \quad \boxed{D)}$$