# GNG 1105E – Engineering Mechanics

CHAPTER D2 – KINEMATICS OF PARTICLES

# Assigned readings

2/1 Introduction

2/2 Rectilinear motion

# Dynamics

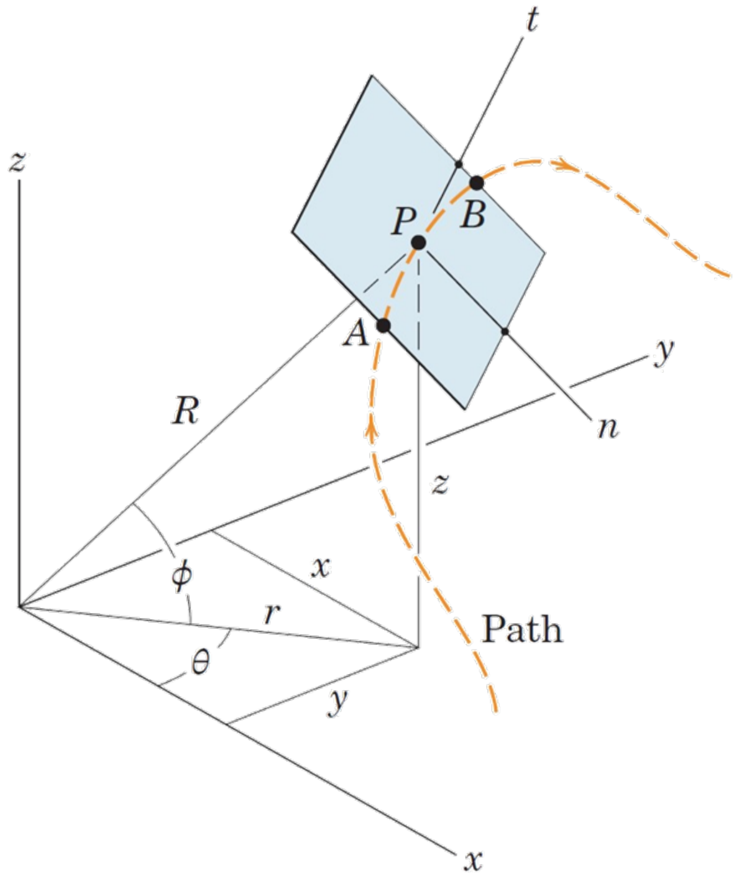
Dynamics is the branch of mechanics which deals with the motion of bodies under the action of forces

Dynamics has two distinct parts:

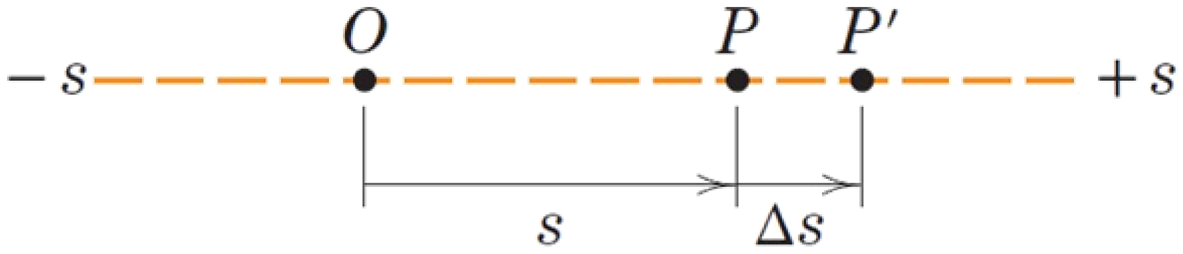
- **Kinematics** deals with the study of motion without reference to the forces which cause the motion
- **Kinetics** relates the action of forces on bodies to their resulting motions

# 2/1 Introduction

- Kinematics is the “geometry of motion”
- Particle Motion
- Choice of Coordinates
- Reference Frame



# 2/2 Rectilinear motion



# 2/2 Rectilinear motion

Displacement

Velocity

Acceleration

# 2/2 Rectilinear motion

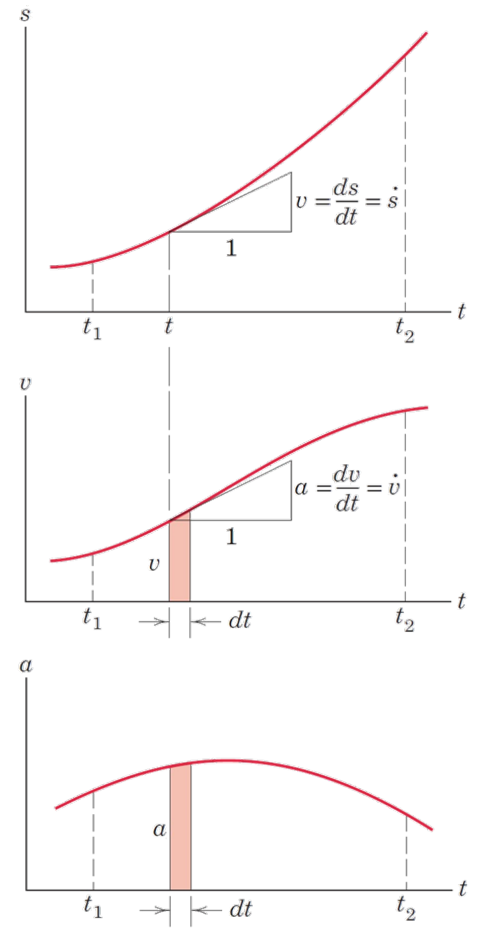
Other relationships:

# 2/2 Rectilinear motion

Functions of time

- Velocity at time *t* is the slope of the position curve at time *t*.
- Acceleration at time *t* is the slope of the velocity curve at time *t*.
- The area under the *v*-*t* curve during the interval $t_1$ to $t_2$ is the net displacement of the particle during that time interval.

$$\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v\, dt \quad \text{or} \quad s_2 - s_1 = (\text{area under } v - t \text{ curve})$$



# 2/2 Rectilinear motion

- The area under the *a*-*t* curve during the interval $t_1$ to $t_2$ is the net change in velocity of the particle during that time interval.

$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\, dt \quad \text{or} \quad v_2 - v_1 = \left(\text{area under } a - t \text{ curve}\right)$$

# 2/2 Rectilinear motion

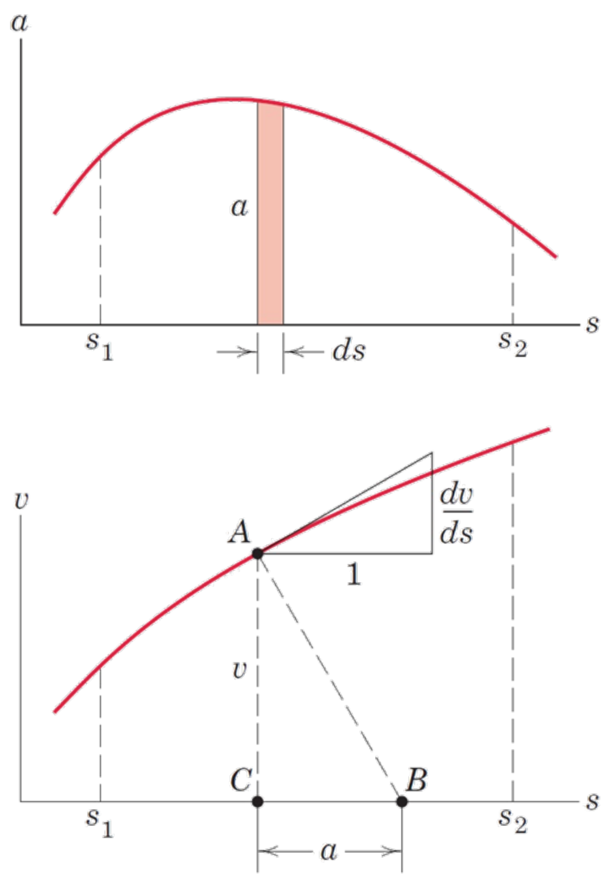
Functions of position

- The area under the *a*-*s* curve between the positions $s_1$ and $s_2$ is one-half the difference of the squared velocities of the particle at the two positions.

$$\int_{v_1}^{v_2} v\,dv = \int_{s_1}^{s_2} a\,ds \quad \text{or} \quad \frac{1}{2}\left(v_2^{\,2} - v_1^{\,2}\right) = \left(\text{area under } a - s \text{ curve}\right)$$

- A line drawn perpendicular to the slope of the *v*-*s* curve at a position *s*, can be extended to the position axis to give the acceleration of the particle at that position.

$$\frac{\overline{CB}}{v} = \frac{dv}{ds} \quad \text{or} \quad \overline{CB} = v\frac{dv}{ds} = a$$



# 2/2 Rectilinear motion

If position is given as a function of time, $s(t)$, then…

- Differentiate once to obtain velocity as a function of time, $v(t)$
- Differentiate a second time to obtain acceleration as a function of time, $a(t)$
- The functions for position, velocity, and acceleration are easily plotted and evaluated at times of interest to obtain desired information.
- If position is not given as a function of time, it must be determined by successive integrations of the acceleration, which is determined by the forces which act on the particle.

# 2/2 Rectilinear motion

Case 1: Constant acceleration:

$$\int_{v_0}^{v} dv = a\int_0^t dt \quad \text{or} \quad v = v_0 + at$$

$$\int_{s_0}^{s} ds = \int_0^t \left(v_0 + at\right) dt \quad \text{or} \quad s = s_0 + v_0 t + \frac{1}{2}at^2$$

$$\int_{v_0}^{v} v\, dv = a\int_{s_0}^{s} ds \quad \text{or} \quad v^2 = {v_0}^2 + 2a\left(s - s_0\right)$$

# 2/2 Rectilinear motion

Case 2: Acceleration as a function of time

$$\int_{v_0}^{v} dv = \int_0^t f(t)\,dt \quad \text{or} \quad v = v_0 + \int_0^t f(t)\,dt$$

$$\int_{s_0}^{s} ds = \int_0^t v\,dt \quad \text{or} \quad s = s_0 + \int_0^t v\,dt$$

# 2/2 Rectilinear motion

Case 3: Acceleration as a function of velocity

$$t = \int_0^t dt = \int_{v_0}^{v} \frac{dv}{f(v)}$$

$$\int_{v_0}^{v} \frac{v\,dv}{f(v)} = \int_{s_0}^{s} ds \quad \text{or} \quad s = s_0 + \int_{v_0}^{v} \frac{v\,dv}{f(v)}$$

# 2/2 Rectilinear motion

Case 4: Acceleration as a function of position

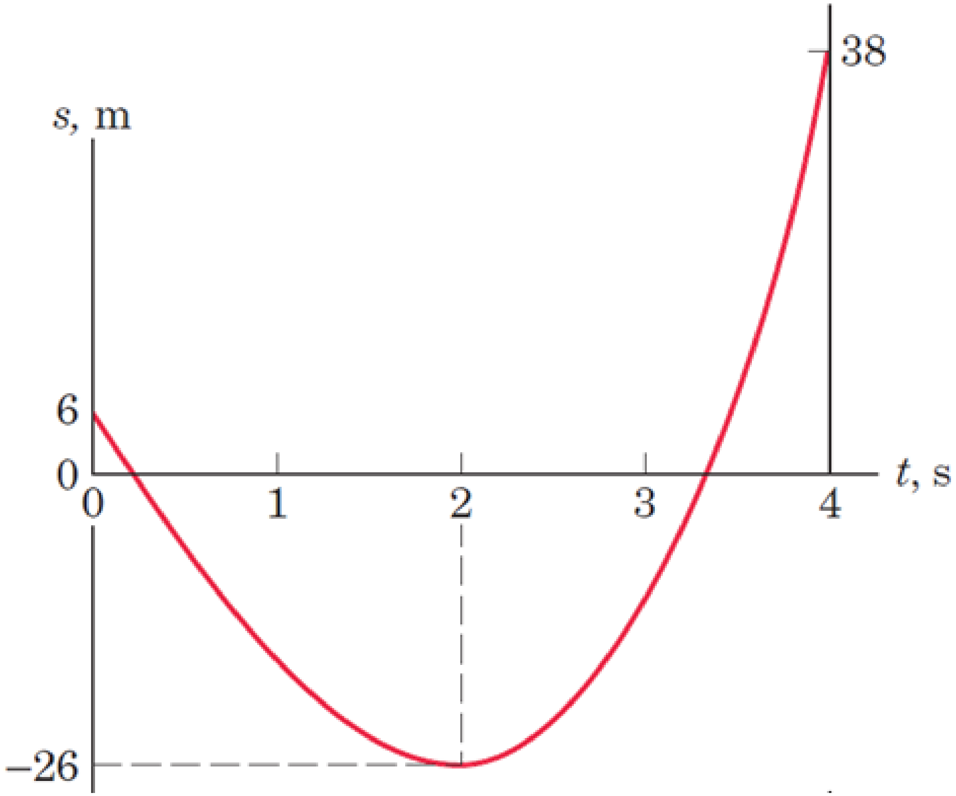
$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} f(s)\,ds \quad \text{or} \quad v^2 = {v_0}^2 + 2\int_{s_0}^{s} f(s)\,ds$$

$$\int_{s_0}^{s} \frac{ds}{g(s)} = \int_{0}^{t} dt \quad \text{or} \quad t = \int_{s_0}^{s} \frac{ds}{g(s)}$$

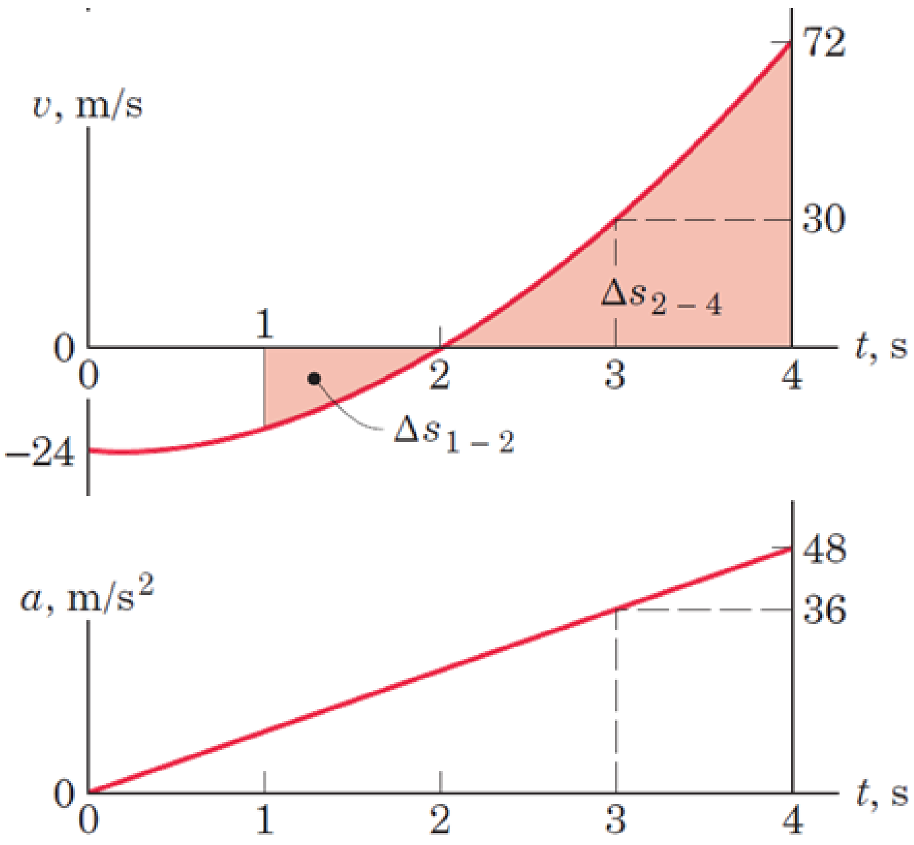
# Sample problem 2/1

The position coordinate of a particle which is confined to move along a straight line is given by $s = 2t^3 - 24t + 6$, where *s* is measured in meters from a convenient origin and *t* is in seconds. Determine (*a*) the time required for the particle to reach a velocity of 72 m/s from its initial condition at $t = 0$, (*b*) the acceleration of the particle when $v = 30$ m/s, and (*c*) the net displacement of the particle during the interval from $t = 1$ s to $t = 4$ s.

# Sample problem 2/1



# Sample problem 2/1

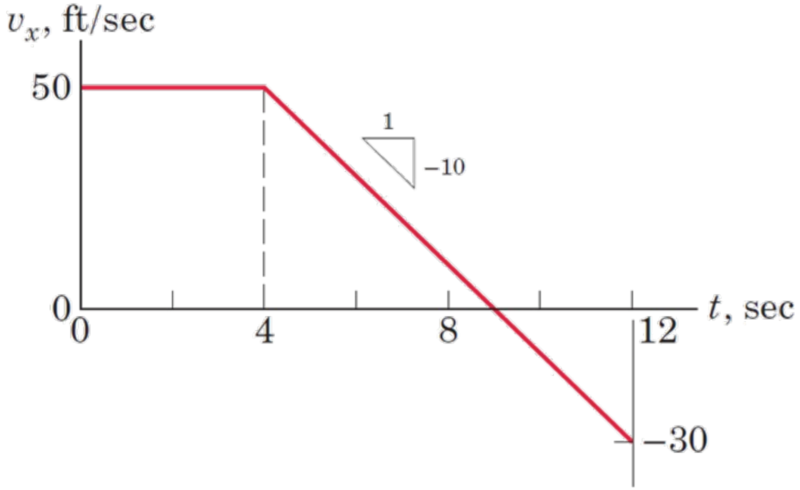


# Sample problem 2/1

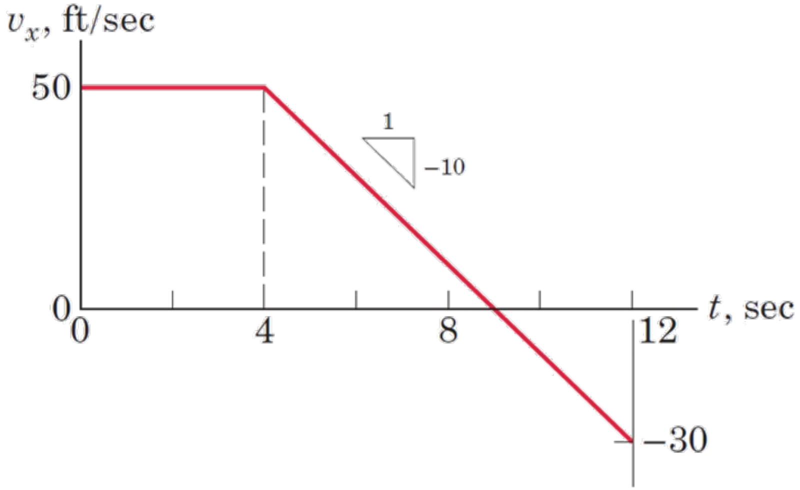
# Sample problem 2/2

A particle moves along the $x$-axis with an initial velocity $v_x = 50$ ft/sec at the origin when $t = 0$. For the first 4 seconds it has no acceleration, and thereafter it is acted on by a retarding force which gives it a constant acceleration $a_x = -10\ ft/sec^2$. Calculate the velocity and the $x$- coordinate of the particle for the conditions of $t = 8$ sec and $t = 12$ sec and find the maximum positive $x$-coordinate reached by the particle.

# Sample problem 2/2



# Sample problem 2/2



# Sample problem 2/2