MAT 1348 – Winter 2024

Exercices 1 – Solutions

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Questions are taken from Discrete Mathematics 8th edition, by Kenneth H. Rosen.

QUESTION 1 **(1.1 # 1)**. Which of the following are propositions? What are their truth values?

(1) Boston is the capital of Massachusetts.
(2) Miami is the capital of Florida.
(3) $2+3=5$
(4) $5+7=10$
(5) $x+2=11$
(6) Answer this question.

Solution:

(1) Yes, T
(2) Yes, F
(3) Yes, T
(4) Yes, F
(5) No
(6) No

QUESTION 2 **(1.1 # 3)**. What is the negation of the following propositions?

(1) Linda is younger than Sanjay.
(2) Mei wins more money than Isabelle.
(3) Moshe is taller than Monica.
(4) Abby is richer than Ricardo.

Solution:

(1) Linda is not younger than Sanjay.
(2) Mei does not win more money than Isabelle.
(3) Moshe is not taller than Monica.
(4) Abby is not richer than Ricardo.

QUESTION 3 **(1.1 # 5)**. What is the negation of the following propositions?

(1) Mei has an MP3 player.
(2) There is no polution in New Jersey.
(3) $2+1=3$
(4) Summer in Maine is hot and sunny.

Solution:

(1) Mei does not have an MP3 player.
(2) There is some polution in New Jersey.
(3) $2+1\neq 3$
(4) Summer in Maine is not hot or it is not sunny.

QUESTION 4 **(1.1 # 11)**. Let p and q be the propositions "Swimming is allowed at the beach" and "sharks have been spotted near the beach", respectively. Write each of the following proposition in English.

(1) $\neg q$
(2) $p \wedge q$
(3) $\neg p \vee q$
(4) $p \to \neg q$
(5) $\neg q \to p$
(6) $\neg p \to \neg q$
(7) $p \leftrightarrow \neg q$
(8) $\neg p \wedge (p \vee \neg q)$

Solution:

(1) Sharks have not been spotted near the beach.
(2) Swimming is allowed at the beach and sharks have been spotted at the beach.
(3) Swimming is not allowed at the beach or sharks have been spotted.
(4) If swimming is allowed at the beach, then sharks have not been spotted.
(5) If sharks have not been spotted near the beach, then swimming is allowed.
(6) If swimming is not allowed at the beach, then sharks have not been spotted.
(7) Swimming is allowed at the beach if and only if sharks have not been spotted.
(8) Swimming is not allowed at the beach, and either swimming is allowed or sharks have not been spotted.

QUESTION 5 **(1.1 # 31)**. How many rows contain the truth table of each of the following propositions?

(1) $p \to \neg p$
(2) $(p \vee \neg r) \wedge (q \vee \neg s)$
(3) $q \vee p \vee \neg s \vee \neg r \vee \neg t \vee u$
(4) $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$

Solution:

(1) 2
(2) 16
(3) 64
(4) 16

QUESTION 6 **(1.1 # 33)**. Write the truth table of each of the following propositions.

(1) $p \wedge \neg p$
(2) $p \vee \neg p$
(3) $(p \vee \neg q) \to q$
(4) $(p \vee q) \to (p \wedge q)$
(5) $(p \to q) \leftrightarrow (\neg q \to \neg p)$
(6) $(p \to q) \to (q \to p)$

Solution:

(1)

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

(2)

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

(3)

p	q	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \to q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

(4)

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \to (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

(5)

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$	$(p \to q) \leftrightarrow (\neg q \to \neg p)$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

(6)

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

QUESTION 7 **(1.3 # 11)**. Use truth tables to show that each of the following propositions are tautologies.

(1) $(p \wedge q) \to p$
(2) $p \to (p \vee q)$
(3) $\neg p \to (p \to q)$
(4) $(p \wedge q) \to (p \to q)$
(5) $\neg(p \to q) \to p$
(6) $\neg(p \to q) \to \neg q$

Solution:

(1)

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

(2)

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

(3)

p	q	$\neg p$	$p \to q$	$\neg p \to (p \to q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

(4)

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

(5)

p	q	$p \to q$	$\neg(p \to q)$	$\neg(p \to q) \to p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

(6)

p	q	$p \to q$	$\neg(p \to q)$	$\neg q$	$\neg(p \to q) \to \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

QUESTION 8 **(1.3 # 19)**. Is $(\neg q \wedge (p \to q)) \to \neg p$ a tautology?

Solution: Yes. The truth table is

p	q	$\neg q$	$p \to q$	$\neg q \wedge (p \to q)$	$\neg p$	$(\neg q \wedge (p \to q)) \to \neg p$
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T

QUESTION 9 **(1.1 # 23)**. For each of the following sentences, determine if it's an "or" or an "exclusive or".

(1) In order to take the discrete math class, one must have taken a calculus class or a computer science class.
(2) When you buy an ACME car, you receive 2000$ or a loan worth 2% of the car's value.
(3) A meal for two is comprised of two items from column A or three items from column B.
(4) School is closed if there is at least 50cm of snow or if the temperature goes below -75 C°.

Solution:

(1) or
(2) exclusive or
(3) exclusive or
(4) or

QUESTION 10 **(1.3 # 17)**. Use a truth table to verify the following equivalences.

(1) $p \vee (p \wedge q) \equiv p$
(2) $p \wedge (p \vee q) \equiv p$

Solution: We have the following truth table:

p	q	$p \wedge q$	$p \vee (p \wedge q)$	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	F	F	T	F
F	F	F	F	F	F

Since column 4 is identical to column 1, we conclude that (1) is true. Since column 6 is identical to column 1, we conclude that (2) is true.

QUESTION 11 **(1.3 # 21)**. Show $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are logically equivalent.

Solution: The two compound propositions are true precisely when the truth values of p and q are different.

QUESTION 12 **(1.3 # 23)**. Show $\neg p \leftrightarrow q$ and $p \leftrightarrow \neg q$ are logically equivalent.

Solution: The two compound propositions are true precisely when the truth values of p and q are different.

QUESTION 13 **(1.3 # 25)**. Show that $\neg(p \leftrightarrow q)$ and $\neg p \leftrightarrow q$ are logically equivalent.

Solution: The two compound propositions are true precisely when the truth values of p and q are different.

QUESTION 14 **(1.3 # 27)**. Show that $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent.

Solution: For $(p \to r) \wedge (q \to r)$ to be false, one of the hypotheses must be false, and the conclusion r must be false. So, $(p \to r) \wedge (q \to r)$ is false when $p \vee q$ is true and r is false, which is precisely when $(p \vee q) \to r$ is false. Therefore, the two propositions are equivalent.

QUESTION 15 **(1.3 # 29)**. Show that $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent.

Solution: For $(p \to r) \vee (q \to r)$ to be false, the two implications must be false, so p and q must be true while r must be false. Therefore, $(p \to r) \vee (q \to r)$ is false exactly when $(p \wedge q) \to r$ is false. The two propositions are therefore equivalent.

QUESTION 16 **(1.3 # 31)**. Show that $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$ are logically equivalent.

Solution: If p and q have different truth values, then either $p \to q$ or $q \to p$ will be false. If p and q have the same truth values, then $p \to q$ and $q \to p$ will be true. We conclude the two propositions are equivalent.

QUESTION 17 **(1.3 # 33)**. Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.

Solution: The last column of the truth table only contains "T".

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	$(p \to q) \wedge (q \to r) \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

QUESTION 18 **(1.2 # 1)**. Consider the following variables:

e = "You can edit a Wikipedia article"
a = "You are an admin"

Use the previous variables to translate the following argument into logic.

"You cannot edit a Wikipedia article unless you are an admin."

Solution: $e \to a$

QUESTION 19 **(1.2 # 5)**. Consider the following variables

e = "You are eligible to become president"
a = "You are at least 35 years old"
b = "You were born in the USA"
p = "When you were born, both your parents were american citizens"
r = "You lived at least 14 years in the USA"

Use the previous variables to translate the following text into logic

"You are eligible to become president only if you are at least 35 years old, you were born in the USA or, when you were born, both parents were american citizen, and you have lived at least 14 years in the USA."

Solution: $e \to (a \wedge (b \vee p) \wedge r)$

QUESTION 20 **(1.2 # 9)**. Are the following specifications consistent?

"The system is in multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode."

Solution: This is inconsistent. The system is not in interrupt mode, so it must mean it is in multiuser state. This means the system is operating normally, and therefore the kernel is functioning. However, it is said that either the kernel is not functioning, or the system is in interrupt mode: neither are true. This makes the specifications inconsistent.