

Example 3: Find the velocity and the acceleration of the displacement function $s(t) = -5t^3 + 2t^2 - 7t$ when $t = 3$. State whether the object is speeding up or slowing down at $t = 3$.

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Rate of Change			
	Negative	0	Positive
S	<p>if $s(t) < 0$ position is to the LEFT (below) the origin</p>	at origin	<p>if $s(t) > 0$ position is to the RIGHT (above) the origin</p>
V	<p>- slope of s is negative - s is decreasing - moving in neg. direction (left/down)</p>	<p>stopped $v = 0$</p>	<p>- slope of s is positive - s is increasing - moving in positive direction (moving right/up)</p>
a	<p>- slope of v is negative</p> <p><u>Case 1 $v(t) > 0$</u> $a(t) < 0$ - velocity is decreasing - slowing down</p> <p><u>Case 2 $v(t) < 0$</u> $a(t) < 0$ - velocity is increasing - speeding up</p>	<p>velocity is constant</p>	<p>- slope of v is positive</p> <p>- velocity is increasing - acceleration (if $v(t) > 0$) (speeding up)</p> <p>- velocity is decreasing - deceleration (if $v(t) < 0$) (slowing down)</p>

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In General...

accelerating \neq speeding up

a & v same sign \Rightarrow speeding up

a & v opposite sign \Rightarrow slowing down

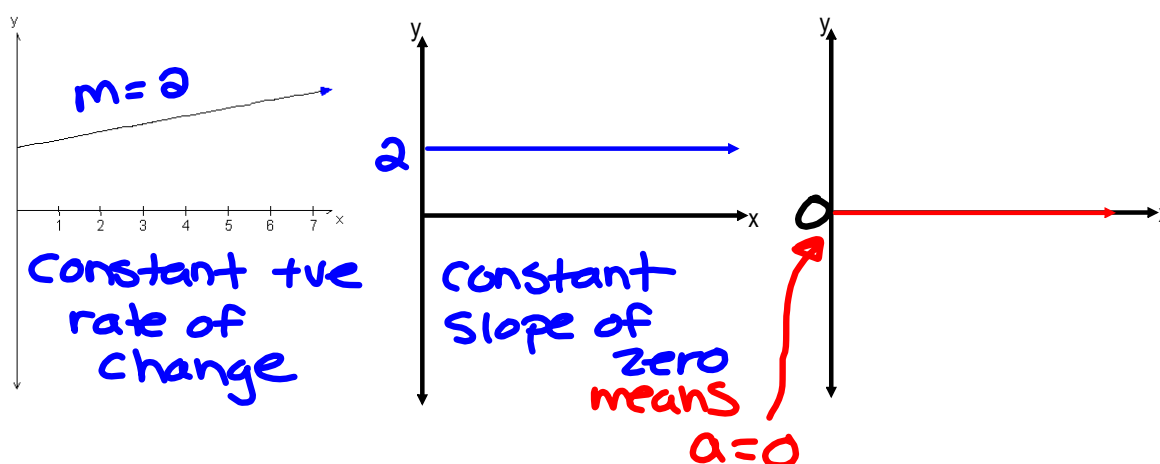
s & v same sign \Rightarrow moving away from origin

s & v opposite sign \Rightarrow moving towards origin

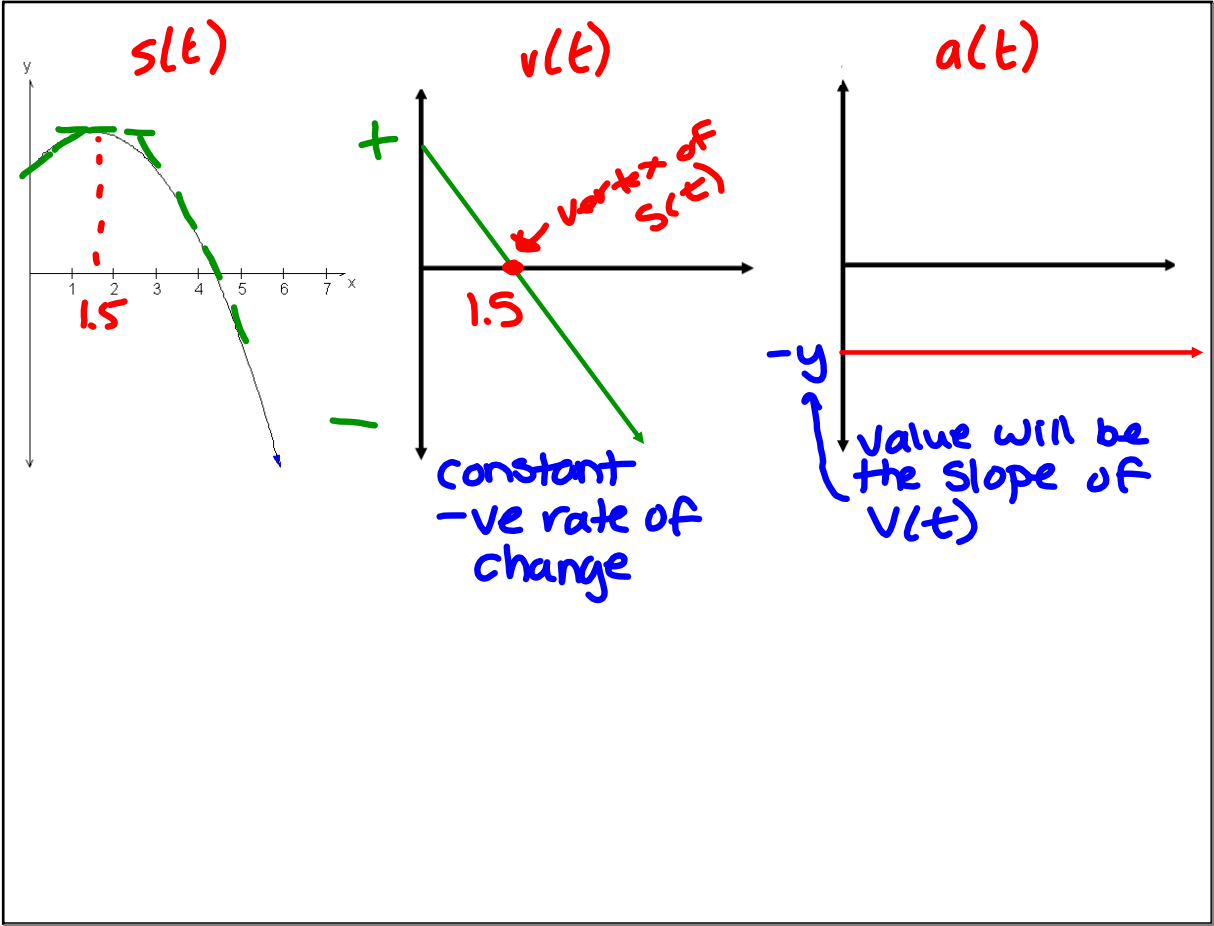
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Example 4

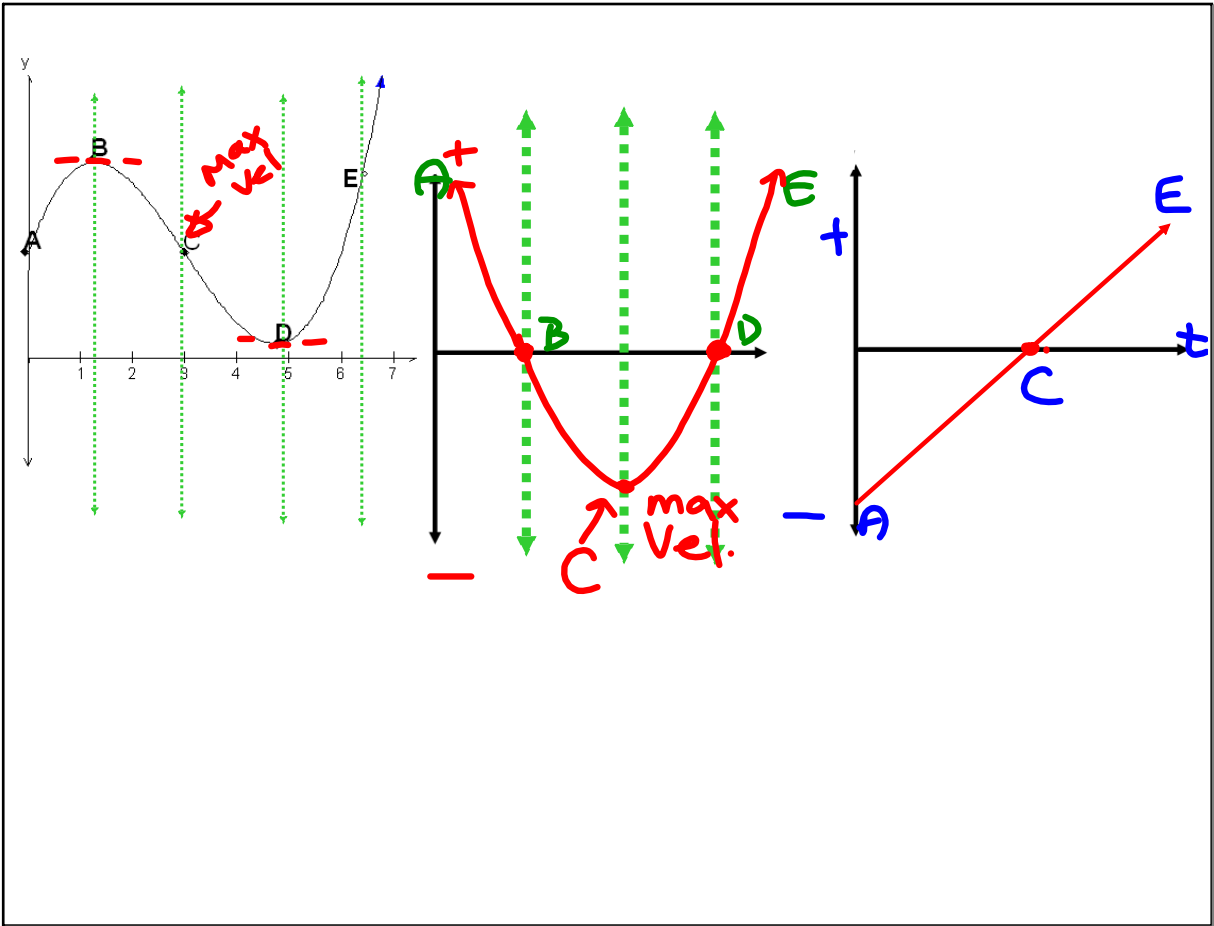
For each distance-time graph, sketch the velocity function and the acceleration function.



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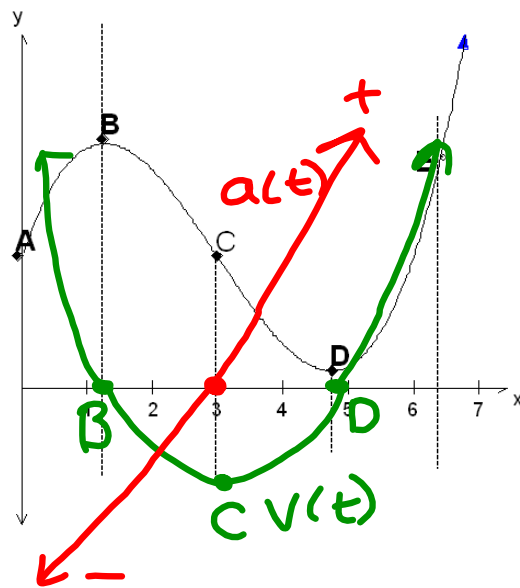


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Example 5: The graph shows the position function of an object.



a) At what point(s) on the graph is the velocity 0?

B and D

b) During which intervals is the velocity positive?

AB, DE

c) During which intervals is the velocity negative?

BC, CD or BD

d) During which intervals is the acceleration positive?

CE

e) During which intervals is the acceleration negative?

AC

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Example 6: The position function of an object moving along a straight line is represented by the function $s(t) = 2t^3 - 2t^2 - 40t$, where t is in seconds and s is in metres.

a) What is the position of the object after 2 s and after 5 s?

$$\begin{aligned}
 s(2) &= 2(2)^3 - 2(2)^2 - 40(2) & s(5) &= 2(5)^3 - 2(5)^2 - 40(5) \\
 s(2) &= 2(8) - 2(4) - 80 & &= 250 - 50 - 200 \\
 &= 16 - 8 - 80 & &= 0 \text{ m} \\
 &= -72 \text{ m}
 \end{aligned}$$

b) What is the velocity of the object after 2 s and after 5 s?

$$\begin{aligned}
 v(t) &= 6t^2 - 4t - 40 & v(5) &= 6(5)^2 - 4(5) - 40 \\
 v(2) &= 6(2)^2 - 4(2) - 40 & &= 150 - 20 - 40 \\
 &= 24 - 8 - 40 & &= 90 \text{ m/s} \\
 &= -24 \text{ m/s}
 \end{aligned}$$

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c) When is the object stopped? What is its position at this time?

$$0 = 6t^2 - 4t - 40 \quad v=0$$

$$Q.F. \quad t_1 = -2.2 \text{ s} \leftarrow \text{reject} \quad t_2 = 2.94 \text{ s}$$

$$S(2.94) = 2(2.94)^3 - 2(2.94)^2 - 40(2.94)$$

$$= 50.8 - 17.3 - 117.6$$

$$= -84 \text{ m}$$

d) When is the object moving in a positive direction?

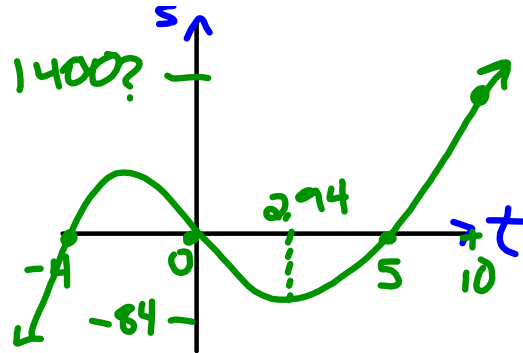
$$0 = 2t^3 - 2t^2 - 40t$$

$$0 = 2t(t^2 - t - 20)$$

$$0 = 2t(t - 5)(t + 4)$$

$$t = 0 \quad t = -4$$

$$t = 5$$



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e) Determine the total distance travelled by the object during the first 10 s.

$$S(10) = 2(10)^3 - 2(10)^2 - 40(10)$$

$$= 2(1000) - 2(100) - 400$$

$$= 2000 - 200 - 400$$

$$= 1400 \text{ m}$$

$$\text{total distance} = 84 + 84 + 1400$$

$$= 1568 \text{ m}$$

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Example 7: Starting at time $t = 0$, Johnny "The Speed Demon" accelerates down the local drag strip in his 1996 Porsche 911 Twin Turbo, and then brakes and comes to a stop. The position function for Johnny's car is given by $s(t) = 6t^2 - \frac{1}{5}t^3$, where s is in metres, and t is in seconds.

a) After how many seconds does "The Speed Demon" stop?

$$v(t) = 12t - \frac{3}{5}t^2 \quad 0 = 3t(20 - t)$$

$$0 = 12t - \frac{3}{5}t^2 \quad t = 0$$

$$0 = \frac{60t - 3t^2}{5} \quad \boxed{t = 20 \text{ seconds}}$$

b) What distance does Johnny's Porsche travel?

$$s(20) = 6(20)^2 - \frac{1}{5}(20)^3$$

$$= 2400 - 1600$$

$$= 800 \text{ m}$$

c) Is Johnny speeding up or slowing down at $t = 5$ seconds?

$$v(5) = 12(5) - \frac{3}{5}(5)^2 \quad a(t) = 12 - \frac{6}{5}t$$

$$= 60 - 15$$

$$= 45 \text{ m/s} \quad a(5) = 12 - \frac{6(5)}{5}$$

$$= 6 \text{ m/s}^2$$

$$\therefore a(t) \times v(t) > 0$$

\therefore The car is speeding up.

d) At what time does Johnny begin to apply the brakes? $a = 0$

$$0 = 12 - \frac{6}{5}t$$

$$\frac{6}{5}t = 12$$

$$6t = 60$$

$$\frac{6t}{6} = \frac{60}{6}$$

$$t = 10$$

\therefore He applies the brakes at 10 seconds

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