Mat1320 Final Exam 220219 questions

Calculus I (University of Ottawa)

MULTIPLE-CHOICE QUESTIONS Questions 1–10 are multiple choice format worth 2 points each. Answers to multiple-choice questions do not need to be justified. You may write your scrap work on your paper but it will not be graded. When you reach your answer, clearly indicate the question number and write the letter of your response beside the question number: For example: *(write out your scrap work, but it will not be graded)*

(clearly indicate your final choice) $\boxed{\textbf{Q1.}\ [\texttt{letter of your choice}]}$

Q1. Which of the following is the domain of $f(x) = \dfrac{\sqrt{16-25x^2}}{\cos(x)-1}$?

A. $(-\infty, \infty)$ **B.** $(0, \frac{4}{5}]$ **C.** $[-\frac{4}{5}, 0) \cup (0, \frac{4}{5}]$ **D.** $(-\infty, 0) \cup (0, \infty)$

E. $(0, \frac{4}{5})$ **F.** $(-\frac{4}{5}, 0) \cup (0, \frac{4}{5})$ **G.** $[-\frac{4}{5}, \frac{4}{5}]$ **H.** $(-\frac{4}{5}, \frac{4}{5})$

Q2. Which of the following functions is the **inverse** of $y = 5\ln\left(\dfrac{x+3}{2}\right)$?

A. $y = \frac{2}{5}e^x - 3$ **B.** $y = 2e^{x/3} - 5$ **C.** $y = 3e^{x/5} - 2$ **D.** $y = 2e^{x/5} - 3$

E. $y = 3e^{x/2} - 5$ **F.** $y = 5e^{x/3} - 2$ **G.** $y = \frac{2}{5}e^x + 3$ **H.** $y = 5e^{x/2} - 3$

Q3. For which value of the parameter k is the following function **continuous** at $x = 1$?

$$f(x) = \begin{cases} kx + 6 & \text{if } x \le 1 \\ \dfrac{3-2x-x^2}{|1-x|} & \text{if } x > 1 \end{cases}$$

A. $k = -13$ **B.** $k = -1$ **C.** $k = -8$ **D.** $k = -4$

E. $k = -2$ **F.** $k = -3$ **G.** $k = -10$ **H.** $k = 0$

Q4. Suppose $f(t)$ and $g(t)$ are differentiable functions such that

$$f(2) = 3 \qquad g(2) = 4$$
$$f'(2) = 3 \qquad g'(2) = 2$$

Find $H'(2)$ for the function $H(t) = \dfrac{f(t)+1}{g(t)}$.

A. $-\frac{1}{2}$ **B.** $\frac{5}{9}$ **C.** $-\frac{4}{9}$ **D.** $\frac{1}{4}$ **E.** $\frac{3}{4}$ **F.** $-\frac{5}{6}$ **G.** $-\frac{1}{18}$ **H.** $-\frac{2}{3}$

Q5. Suppose a curve is implicitly defined by the following equation:

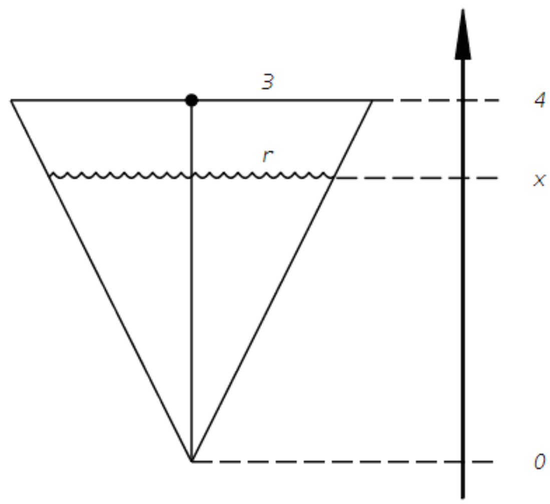
$$2xy + \cos(y-2) = 17.$$

Find the equation of the tangent line to this curve at the point $(x, y) = (4, 2)$.

A. $y = -\frac{3}{2}x + 6$ **B.** $y = -\frac{3}{4}x + 6$ **C.** $y = -\frac{2}{3}x + 4$ **D.** $y = -\frac{1}{4}x + 2$

E. $y = -\frac{5}{2}x + 8$ **F.** $y = -\frac{1}{2}x + 4$ **G.** $y = -\frac{5}{3}x + 8$ **H.** $y = -\frac{1}{3}x + 2$

Q6. A tank is shaped like an inverted cone. Its height is 4 m and its top radius is 3 m. As shown in the diagram below, let x denote the water level in the tank (measured in m) from the bottom of the tank and let V denote the volume of water in the tank (measured in m^3).



Water is leaking from the tank.

At a certain moment, the water level in this tank is $x = 1$ m and $\dfrac{dx}{dt} = -0.03$ m/s.

Find $\frac{dV}{dt}$ at this moment in time, rounded to 3 decimal places.

Note: The volume of a cone is given by $V = \frac{1}{3}\pi r^2 x$ where r is the radius of the cone and x is its height.

A. -0.053 m^3/s **B.** -0.088 m^3/s **C.** -0.283 m^3/s **D.** -0.426 m^3/s

E. -0.094 m^3/s **F.** -0.518 m^3/s **G.** -0.349 m^3/s **H.** -0.628 m^3/s

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Q7. Suppose you know $\displaystyle\int_{-3}^{3} f(x)\,dx = 2.$ Find $\displaystyle\int_{-3}^{3} (f(x) - 4)\,dx.$

A. -4 **B.** -19 **C.** -2 **D.** -13 **E.** -1 **F.** -22 **G.** 0 **H.** -14 **I.** 1

Q8. Choose an appropriate u-**substitution** and transform the definite integral $\displaystyle\int_e^{e^3} \frac{dx}{x(\ln x)^2}$.

In terms of u, which of the following integrals is equal to the above?

A. $-\dfrac{1}{3}\displaystyle\int_1^3 u^{-2}\,du$ **B.** $-\dfrac{1}{3}\displaystyle\int_e^{e^3} u^{-2}\,du$ **C.** $\displaystyle\int_e^{e^3} u^{-2}\,du$ **D.** $\dfrac{1}{x}\displaystyle\int_1^3 u^{-2}\,du$

E. $\dfrac{1}{x}\displaystyle\int_e^{e^3} u^{-2}\,du$ **F.** $-\dfrac{1}{2}\displaystyle\int_e^{e^3} u^{-2}\,du$ **G.** $\displaystyle\int_1^3 u^{-2}\,du$ **H.** $-\dfrac{1}{2}\displaystyle\int_1^3 u^{-2}\,du$

Q9. Consider the indefinite integral: $\displaystyle\int \frac{x^3}{\sqrt{16x^2+1}}\,dx$.

To eliminate the square root before integrating, which of the following options is the appropriate **trigonometric substitution**?

A. $x = \frac{1}{4}\tan(\theta)$ **B.** $x = 4\sec(\theta)$ **C.** $x = \frac{1}{4}\sec(\theta)$ **D.** $x = 4\tan(\theta)$

E. $x = \tan(4\theta)$ **F.** $x = \frac{1}{4}\sin(\theta)$ **G.** $x = 4\sin(\theta)$ **H.** $x = \sec(4\theta)$

Q10. Find all **critical numbers** (if any) of the function $h(x) = \dfrac{e^{4x}}{x+3}$.

Only one of the following answers is correct.

A. $x = -\frac{11}{4}$ **B.** $x = -\frac{3}{4}$ **C.** $x = -\frac{1}{2}$ **D.** $x = -\frac{7}{4}$ **E.** $x = -2$

F. $x = -\frac{3}{2}$ **G.** $x = -\frac{5}{3}$ **H.** $x = -1$ **I.** $x = -3$ **J.** $x = 0$

K. h has no critical numbers.

LONG-ANSWER QUESTIONS

For long-answer questions, all of your work must be justified and your steps must be written in a clear and logical order, using correct mathematical notation. Clearly indicate Question numbers.

For example: **Q11 a).** [write a fully justified solution].

Q11. [6 points] Evaluate each of the following limits. For each you must show ALL your steps and use appropriate calculus and algebraic methods seen in class. If a limit does not exist, you must justify how you reached this conclusion. Identify the types of any indeterminate forms you encounter along the way.

a) $\displaystyle\lim_{x\to\infty} 4x(1-e^{3/x})$ b) $\displaystyle\lim_{t\to 3} \frac{\ln(t/3)}{2t-6}$ c) $\displaystyle\lim_{\theta\to 0} \frac{\sin(4\theta)-\cos(3\theta)}{6\cos(\theta)}$

Q12. [3 points] Use the limit <u>definition of the derivative</u> (first principles) to find $f'(x)$ for the function

$$f(x) = \sqrt{7 - 9x}.$$

Use appropriate notation, limit laws, and algebraic methods seen in class.

Show ALL your steps!

Q13. [6 points] Use the substitution $u = \cos(4x)$ to evaluate the following indefinite integral:

$$\int \frac{7\sin(4x)\cos(4x)}{\cos^2(4x) + 4\cos(4x) + 3}\, dx$$

– Use appropriate methods of integration to complete this computation.

– Make sure you show ALL your steps in a clearly organized fashion, and use appropriate mathematical notation throughout.

Q14. [6 points] Evaluate each of the following definite integrals. You must show ALL your steps and write your solutions in a logical order, using appropriate mathematical notation throughout.

Give an exact final answer.

a) $\displaystyle\int_0^1 (x^2 + 2)e^{4x}\, dx$

b) $\displaystyle\int_0^2 \frac{e^{2y} - 3}{e^{2y} - 6y + 12}\, dy$

Q15. [6 points] Suppose a population of buffalo is susceptible to some disease. Let $N(t)$ denote the number of buffalo infected after t weeks.

Suppose the following function models the spread of infection in this population, for $t \geq 0$ weeks:

$$N(t) = 80t^2 e^{-0.9t}$$

a) Find the derivative $N'(t)$. You do not need to simplify your answer.

b) Within the first 5 weeks (i.e. the time interval $0 \leq t \leq 5$), find the number of weeks t at which $N(t)$ attains a global maximum. Show ALL your work and justify why this is when a <u>GLOBAL</u> maximum is attained.

c) What is the global maximum value of $N(t)$ in the time interval $[0, 5]$?

Q16. [8 points] After long computations, we find the following information about a function $f(x)$.

- The domain of f, f', and f'' is $\{x \in \mathbb{R} : x \neq -2\}$
- $f'(x) = 0$ for $x = -3,\ x = 1,\ x = 3$
- $f''(x) = 0$ for $x = -4,\ x = 2$
- $\lim_{x\to -2} f(x) = -\infty$
- $\lim_{x\to -\infty} f(x) = 2$ and $\lim_{x\to\infty} f(x) = \infty$

Here is a summary of information about the signs of its first and second derivatives:

interval	$x < -4$	$-4 < x < -3$	$-3 < x < -2$	$-2 < x < 1$	$1 < x < 2$	$2 < x < 3$	$x > 3$
sign of f'	$+$	$+$	$-$	$+$	$-$	$-$	$+$
sign of f''	$+$	$-$	$-$	$-$	$-$	$+$	$+$

a) Identify the intervals of increase/decrease of f.

b) Does f have any local minima? If so, give each of the x-values at which f attains a local minimum.

c) Does f have any local maxima? If so, give each of the x-values at which f attains a local maximum.

d) Identify the intervals of on which f is concave up/down.

e) Does f have any inflection points? If so, give each of the x-values at which f has an inflection point.

f) Does f have any vertical/horizontal asymptotes? If so, identify each horizontal asymptote and each vertical asymptote.

g) From all of the above, sketch the graph of a function whose behaviour matches that of f, as established in parts a)–f).

Label each of the special x-values (local extrema or inflection points) and asymptotes you've identified in the preceding parts.

End of the Exam!