Q1:

Here are values of greeks under the closed form:

```
            Call:
            Delta:  0.5324712004880571
            Gamma:   0.039922623383376134
            Theta:  -22.034420175464646
            Vega:   19.653415294909422
            Rho:   7.583586080244792
            Carry rho:  7.943303251116359

            Put:
            Delta:  -0.4641711613572652
            Gamma:   0.039922623383376134
            Theta:  -21.166204581630176
            Vega:   19.653415294909422
            Rho:   -7.277010958127815
            Carry rho:  -6.924416366000847
```

Here are values of greeks under the finite difference derivative assumed delta is 0.00001:

```
Call:                                   Put:
Delta:  0.5340091220773502              Delta:  -0.46551181398513103
Gamma:   0.040074610296869644           Gamma:   0.040500935938325704
Theta:  -24.898522349303672             Theta:  -18.78699699915387
Vega:   19.710179716270204              Vega:   19.710179716980747
Rho:   7.583586079817905                Rho:   -7.27701095755151
Carry rho:   7.966245676271909          Carry rho:  -6.944415967780059
```

We can find that the differences between two methods are extremely small except theta:

```
            Percentage difference between closed form and finite difference
            Delta Call: 0.29%
            Gamma Call: 0.38%
            Theta Call: 13.00%
            Vega Call: 0.29%
            Rho Call: 0.00%
            Carry Rho Call: 0.29%


            Delta Put: 0.29%
            Gamma Put: 1.45%
            Theta Put: 11.24%
            Vega Put: 0.29%
            Rho Put: 0.00%
            Carry Rho Put: 0.29%
```

This may be because that theta is the rate of change of option price with respect to time, and when using the finite derivative to calculate theta, I need to find the difference between the price with a further expiration date and the price with a closer expiration date. However, as the option approaches the maturity date, the time sensitivity will decrease. Therefore, the delta added to the time may affect the price, which is closer to the maturity date, so the difference between these two prices will be bigger and cause a bigger theta.

Here are values under binomial tree valuation with and without dividends:

```
Price of Call with no Dividend:  4.222570679674838
Price of Put with no Dividend:  3.710912626903574



Price of Call with Dividend:  4.12094731509267
Price of Put with Dividend:  4.115843684542343
```

Here are greeks for each of them with and without dividend:

```
Delta for call without dividend:  0.5339239061985523
Delta for put without dividend:  -0.4899082160170209
Gamma for call without dividend:  7925.7497542783985
Gamma for put without dividend:  5116484005.272346
Theta for call without dividend:  -24.843985953815203
Theta for put without dividend:  -19.251060318414837
Vega for call without dividend:  19.66096625012348
Vega for put without dividend:  19.635089838065944
Rho for call without dividend:  7.5832077798487765
Rho for put without dividend:  -5.960356561574009


Delta for call with dividend:  0.5188423412239018
Delta for put with dividend:  -0.5034004939119541
Gamma for call with dividend:  2.6645352591003754e-05
Gamma for put with dividend:  5.329070518200751e-05
Theta for call with dividend:  -24.890760996765234
Theta for put with dividend:  -18.51414421212283
Vega for call with dividend:  19.583442903847725
Vega for put with dividend:  19.80911943828545
Rho for call with dividend:  6.8718417230329
Rho for put with dividend:  -7.224244998971229
```

Here are the sensitivity of call and put (the change of the value of option when $1 more dividend is added):

```
Sensitivity for call:  -0.06488422533802485
Sensitivity for put:   0.5250644289516524
```

Q2:

Here are Mean, VaR and ES under General BS Model:

|  | Mean | VaR | ES |
| --- | --- | --- | --- |
| Straddle | -0.020074 | 0.030475 | 0.012751 |
| SynLong | -0.172135 | 17.395152 | 20.516734 |
| CallSpread | -0.050510 | 3.168194 | 2.278892 |
| PutSpread | 0.041372 | 2.676970 | 2.849594 |
| Stock | -0.165544 | 17.534686 | 20.716947 |
| Call | -0.096104 | 5.078616 | 5.258910 |
| Put | 0.076031 | 4.165711 | 4.396920 |
| CoveredCall | -0.103219 | 9.519598 | 10.074095 |
| ProtectedPut | -0.115470 | 7.258040 | 7.613237 |

Here are Mean, VaR and ES for the American option:

|  | Mean | VaR | ES |
| --- | --- | --- | --- |
| Straddle | 1.706808 | 1.358056 | 1.378846 |
| SynLong | -1.365078 | 19.044635 | 22.288197 |
| CallSpread | -0.369772 | 4.163668 | 4.338956 |
| PutSpread | 0.653083 | 2.531771 | 2.721417 |
| Stock | -0.165544 | 17.534686 | 20.716947 |
| Call | 0.170865 | 6.346522 | 6.537232 |
| Put | 1.535943 | 4.239485 | 4.487616 |
| CoveredCall | -0.634109 | 13.609202 | 16.729831 |
| ProtectedPut | 1.057802 | 7.453776 | 7.809109 |

Here are Mean, VaR and ES using Delta Normal:

| | Mean | VaR | ES |
|---|---|---|---|
| Straddle | 1.726881 | 1.327582 | 1.366096 |
| SynLong | -1.192944 | 1.649483 | 1.771463 |
| CallSpread | -0.319262 | 0.995474 | 2.060063 |
| PutSpread | 0.611710 | -0.145199 | -0.128177 |
| Stock | 0.000000 | 0.000000 | -0.000000 |
| Call | 0.266969 | 1.267905 | 1.278322 |
| Put | 1.459913 | 0.073774 | 0.090696 |
| CoveredCall | -0.530890 | 4.089604 | 6.655736 |
| ProtectedPut | 1.173273 | 0.195736 | 0.195872 |

It is obviously that VaR and ES using Delta Normal is much smaller. Also, for the American option with dividends, it has similar VaR and ES with using the GBSM which is what we calculated two weeks ago.

Q3:

Here is the expected annual return for each stock:

| AAPL | META | UNH | MA | MSFT | NVDA | HD | PFE | AMZN | BRK-B | PG | XOM | TSLA | JPM | V | DIS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.155897 | 0.017799 | 0.251786 | 0.221132 | 0.154706 | 0.277501 | 0.119634 | 0.076351 | -0.042604 | 0.128892 | 0.080892 | 0.51768 | -0.032989 | 0.097493 | 0.239141 | -0.154139 |

| GOOGL | JNJ | BAC | CSCO |
|---|---|---|---|
| -0.01694 | 0.12322 | -0.11141 | 0.146634 |

Here is the covariance matrix:

| | AAPL | META | UNH | MA | MSFT | NVDA | HD | PFE | AMZN | BRK-B | PG | XOM | TSLA | JPM | V | DIS | GOOGL | JNJ | BAC | CSCO |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AAPL | 0.125870 | 0.138450 | 0.037150 | 0.080627 | 0.102120 | 0.169905 | 0.065668 | 0.032485 | 0.121148 | 0.055080 | 0.036651 | 0.037400 | 0.153651 | 0.058180 | 0.070839 | 0.086706 | 0.111018 | 0.022560 | 0.065720 | 0.065598 |
| META | 0.138450 | 0.397662 | 0.016967 | 0.101652 | 0.141126 | 0.238689 | 0.098061 | 0.044733 | 0.193248 | 0.061130 | 0.033370 | 0.020594 | 0.171747 | 0.073386 | 0.084348 | 0.126038 | 0.180629 | 0.021160 | 0.088086 | 0.076111 |
| UNH | 0.037150 | 0.016967 | 0.060439 | 0.030871 | 0.036030 | 0.046162 | 0.025838 | 0.031813 | 0.034462 | 0.027949 | 0.027640 | 0.026630 | 0.038818 | 0.033057 | 0.029355 | 0.022288 | 0.029570 | 0.022799 | 0.034468 | 0.028618 |
| MA | 0.080627 | 0.101652 | 0.030871 | 0.095001 | 0.079222 | 0.136279 | 0.056341 | 0.033175 | 0.095431 | 0.047143 | 0.030916 | 0.030592 | 0.096747 | 0.057846 | 0.081746 | 0.076376 | 0.078389 | 0.017244 | 0.062896 | 0.051323 |
| MSFT | 0.102120 | 0.141126 | 0.036030 | 0.079222 | 0.126825 | 0.174559 | 0.070353 | 0.034804 | 0.132794 | 0.052257 | 0.033590 | 0.031056 | 0.130864 | 0.056264 | 0.067689 | 0.087526 | 0.119305 | 0.020162 | 0.064708 | 0.060321 |
| NVDA | 0.169905 | 0.238689 | 0.046162 | 0.136279 | 0.174559 | 0.400609 | 0.111165 | 0.045994 | 0.219608 | 0.083621 | 0.041611 | 0.054022 | 0.289079 | 0.097618 | 0.117207 | 0.156584 | 0.186520 | 0.021585 | 0.112497 | 0.097880 |
| HD | 0.065668 | 0.098061 | 0.025838 | 0.056341 | 0.070353 | 0.111165 | 0.096304 | 0.033007 | 0.096065 | 0.041975 | 0.034141 | 0.015810 | 0.077137 | 0.043210 | 0.049766 | 0.063428 | 0.069073 | 0.022186 | 0.046118 | 0.048028 |
| PFE | 0.032485 | 0.044733 | 0.031813 | 0.033175 | 0.034804 | 0.045994 | 0.033007 | 0.069957 | 0.036978 | 0.030954 | 0.027252 | 0.019820 | 0.022106 | 0.031609 | 0.030407 | 0.024327 | 0.029122 | 0.027384 | 0.030568 | 0.029297 |
| AMZN | 0.121148 | 0.193248 | 0.034462 | 0.095431 | 0.132794 | 0.219608 | 0.096065 | 0.036978 | 0.240848 | 0.065250 | 0.029913 | 0.037149 | 0.186227 | 0.070257 | 0.082711 | 0.123267 | 0.148734 | 0.022436 | 0.084565 | 0.070853 |
| BRK-B | 0.055080 | 0.061130 | 0.027949 | 0.047143 | 0.052257 | 0.083621 | 0.041975 | 0.030954 | 0.065250 | 0.049777 | 0.024642 | 0.033756 | 0.061053 | 0.046702 | 0.041746 | 0.050975 | 0.055835 | 0.019110 | 0.050416 | 0.040100 |
| PG | 0.036651 | 0.033370 | 0.027640 | 0.030916 | 0.033590 | 0.041611 | 0.034141 | 0.027252 | 0.029913 | 0.024642 | 0.047927 | 0.004758 | 0.023590 | 0.029152 | 0.029212 | 0.027476 | 0.027987 | 0.023373 | 0.028226 | 0.034248 |
| XOM | 0.037400 | 0.020594 | 0.026630 | 0.030592 | 0.031056 | 0.054022 | 0.015810 | 0.019820 | 0.037149 | 0.033756 | 0.004758 | 0.118992 | 0.042324 | 0.029124 | 0.024025 | 0.034844 | 0.031206 | 0.006583 | 0.032605 | 0.023242 |
| TSLA | 0.153651 | 0.171747 | 0.038818 | 0.096747 | 0.130864 | 0.289079 | 0.077137 | 0.022106 | 0.186227 | 0.061053 | 0.023590 | 0.042324 | 0.454471 | 0.067457 | 0.090494 | 0.127793 | 0.143091 | 0.012573 | 0.083742 | 0.066815 |
| JPM | 0.058180 | 0.073386 | 0.033057 | 0.057846 | 0.056264 | 0.097618 | 0.043210 | 0.031609 | 0.070257 | 0.046702 | 0.029152 | 0.029124 | 0.067457 | 0.082269 | 0.052656 | 0.061260 | 0.059076 | 0.018766 | 0.082293 | 0.046671 |
| V | 0.070839 | 0.084348 | 0.029355 | 0.081746 | 0.067689 | 0.117207 | 0.049766 | 0.030407 | 0.082711 | 0.041746 | 0.029212 | 0.024025 | 0.090494 | 0.052656 | 0.082537 | 0.066516 | 0.066482 | 0.016627 | 0.058255 | 0.044717 |
| DIS | 0.086706 | 0.126038 | 0.022288 | 0.076376 | 0.087526 | 0.156584 | 0.063428 | 0.024327 | 0.123267 | 0.050975 | 0.027476 | 0.034844 | 0.127793 | 0.061260 | 0.066516 | 0.136048 | 0.095142 | 0.015101 | 0.072092 | 0.052491 |
| GOOGL | 0.111018 | 0.180629 | 0.029570 | 0.078389 | 0.119305 | 0.186520 | 0.069073 | 0.029122 | 0.148734 | 0.055835 | 0.027987 | 0.031206 | 0.143091 | 0.059076 | 0.066482 | 0.095142 | 0.159651 | 0.019545 | 0.069711 | 0.064541 |
| JNJ | 0.022560 | 0.021160 | 0.022799 | 0.017244 | 0.020162 | 0.021585 | 0.022186 | 0.027384 | 0.022436 | 0.019110 | 0.023373 | 0.006583 | 0.012573 | 0.018766 | 0.016627 | 0.015101 | 0.019545 | 0.031196 | 0.017874 | 0.021732 |
| BAC | 0.065720 | 0.088086 | 0.034468 | 0.062896 | 0.064708 | 0.112497 | 0.046118 | 0.030568 | 0.084565 | 0.050416 | 0.028226 | 0.032605 | 0.083742 | 0.082293 | 0.058255 | 0.072092 | 0.069711 | 0.017874 | 0.100148 | 0.048502 |
| CSCO | 0.065598 | 0.076111 | 0.028618 | 0.051323 | 0.060321 | 0.097880 | 0.048028 | 0.029297 | 0.070853 | 0.040100 | 0.034248 | 0.023242 | 0.066815 | 0.046671 | 0.044717 | 0.052491 | 0.064541 | 0.021732 | 0.048502 | 0.084528 |

Here is the weights of each stock in the portfolio:

| | AAPL | META | UNH | MA | MSFT | NVDA | HD | PFE | AMZN | BRK-B | PG | XOM | TSLA | JPM | V | DIS | GOOGL | JNJ | BAC | CSCO |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| weight % | 0.0 | 0.0 | 22.61 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 57.53 | 0.0 | 0.0 | 12.94 | 0.0 | 0.0 | 6.92 | 0.0 | 0.0 |

The sharpe ratio of it is 1.4635 which means that the portfolio has generated 1.4635 units of excess return per unit of volatility above the risk-free rate.