Q1:

For r under normal distribution with mean of 0 and assumes standard deviation with 0.1, and Pt-1 = 100, the expected value of price and the standard deviation 1 period ahead under classical Brownian Motion (Pt = Pt-1 + r) are 100 and 0.1 respectively.

$E[P_t] = P_{t-1} + E[r]$

$SD[P_t] = \mathrm{sqrt}(Var[P_t]) = \mathrm{sqrt}(Var[P_{t-1} + r]) = \mathrm{sqrt}(Var[P_{t-1}] + Var[r])$

For r under normal distribution with mean of 0 and assumes standard deviation with 0.1, and Pt-1 = 100, the expected value of price and the standard deviation 1 period ahead under Arithmetic Return System (Pt = Pt-1*(1+r)) are 100 and 10 respectively.

$E[P_t] = P_{t-1} * (1 + E[r])$.

$SD[P_t] = \mathrm{sqrt}(Var[P_t]) = \mathrm{sqrt}((P_{t-1})^2 * Var[r] + (Var[P_{t-1}] * (1 + E[r])^2))$

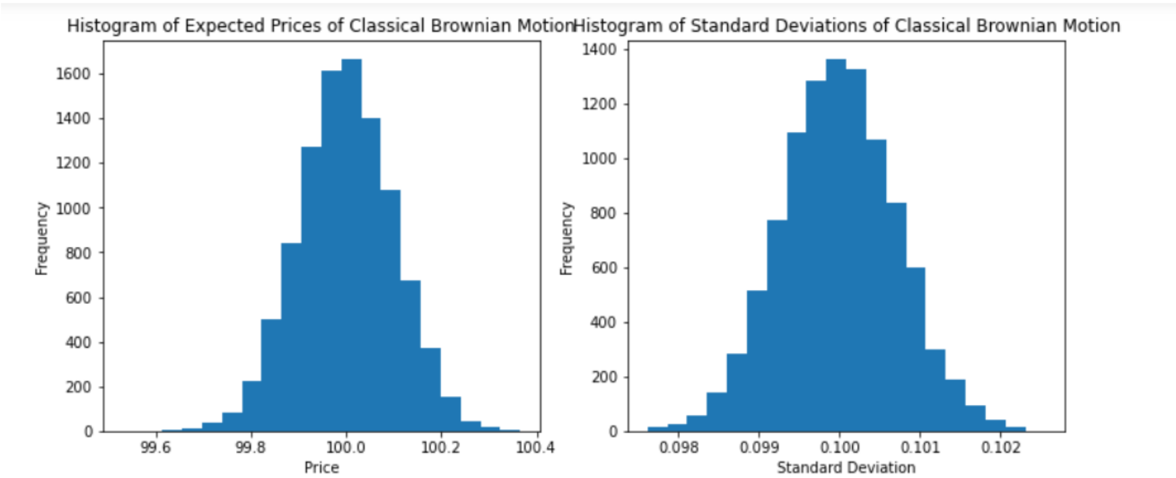
For r under normal distribution with mean of 0 and assumes standard deviation with 0.1, and Pt-1 = 100, the expected value of price and the standard deviation 1 period ahead under Geometric Brownian Motion (Pt = Pt-1*(e**rt)) are 100 and 10.08 respectively.

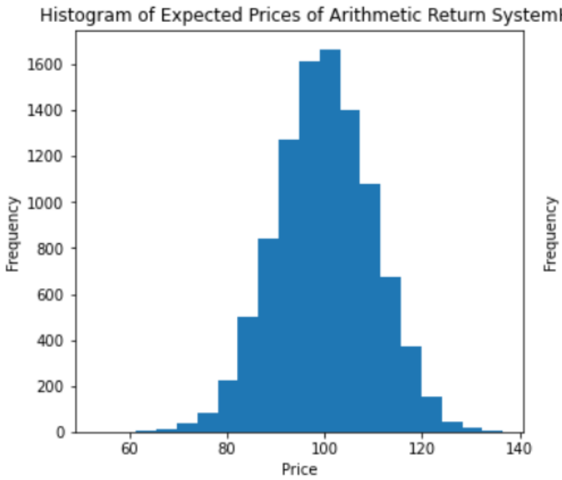
$E[P_t] = P_{t-1} * e^{(E[r]*t)}$

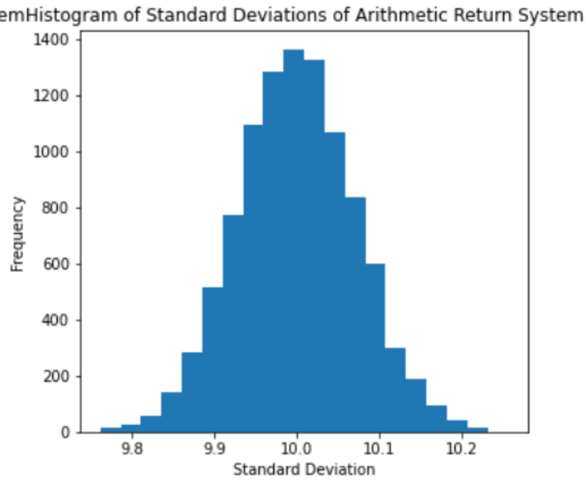
$SD[P_t] = \mathrm{sqrt}(Var[P_t]) = \mathrm{sqrt}((P_{t-1})^2 * Var[r] * (e^{(2*E[r]*t)} - 1))$

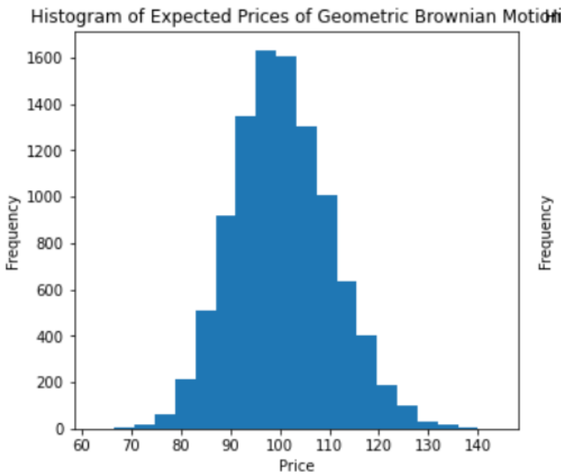
```
For Pt = Pt-1 + r, the expected value of price one period ahead is 100.00 and the standard deviation is 0.10
For Pt = Pt-1*(1+r), the expected value of price one period ahead is 100.00 and the standard deviation is 10.00
For Pt = Pt-1*(e**rt), the expected value of price one period ahead is 100.00 and the standard deviation is 10.08
```

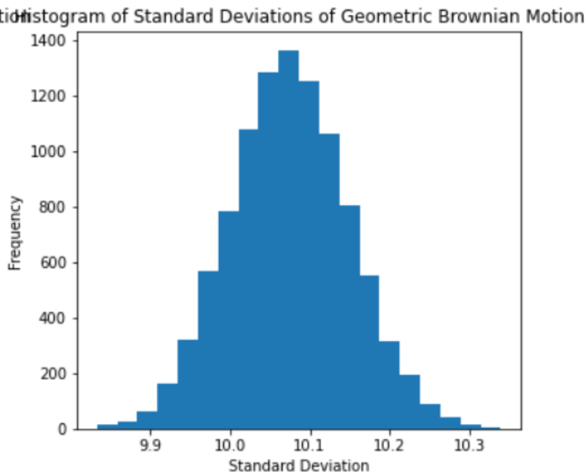
Here is are graphs to show prices and standard deviation under different types of returns with a simulation of 10000 times:











It is clear that the mean and standard deviation match my expectation above.

I changed the standard deviation of r to 0.2 and do all steps again:

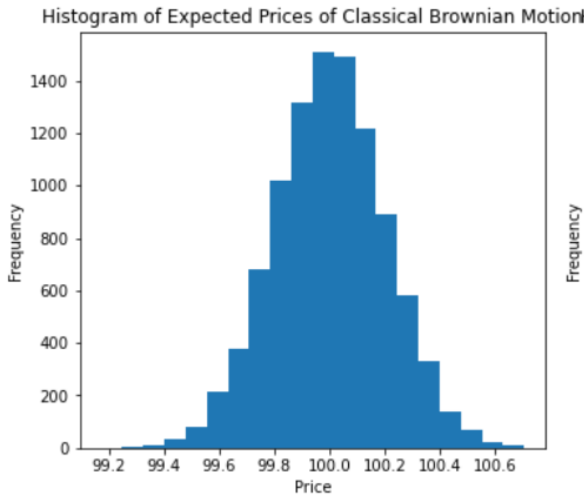
For r under normal distribution with mean of 0 and assumes standard deviation with 0.2, and Pt-1 = 100, the expected value of price and the standard deviation 1 period ahead under classical Brownian Motion (Pt = Pt-1 + r) are 100 and 0.2 respectively.

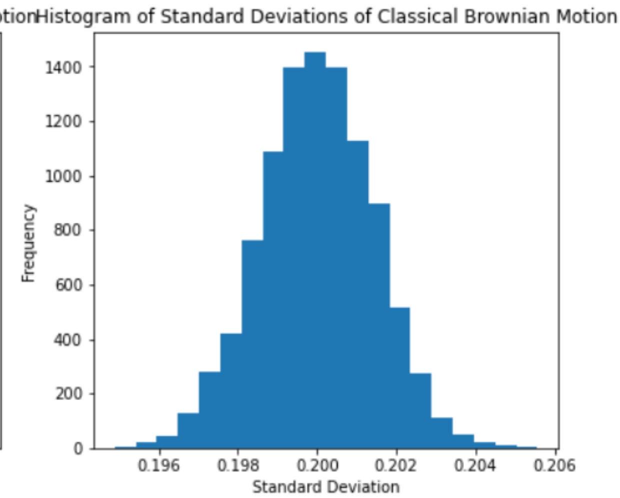
For r under normal distribution with mean of 0 and assumes standard deviation with 0.2, and Pt-1 = 100, the expected value of price and the standard deviation 1 period ahead under Arithmetic Return System (Pt = Pt-1*(1+r)) are 100 and 20 respectively.

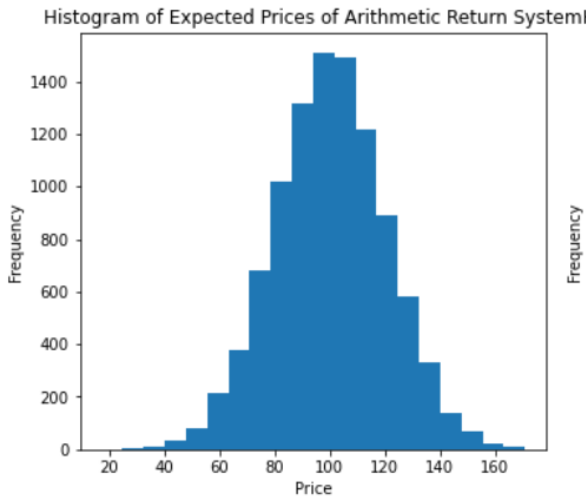
For r under normal distribution with mean of 0 and assumes standard deviation with 0.2, and Pt-1 = 100, the expected value of price and the standard deviation 1 period ahead under Geometric Brownian Motion (Pt = Pt-1*(e**rt)) are 100 and 20.61 respectively.

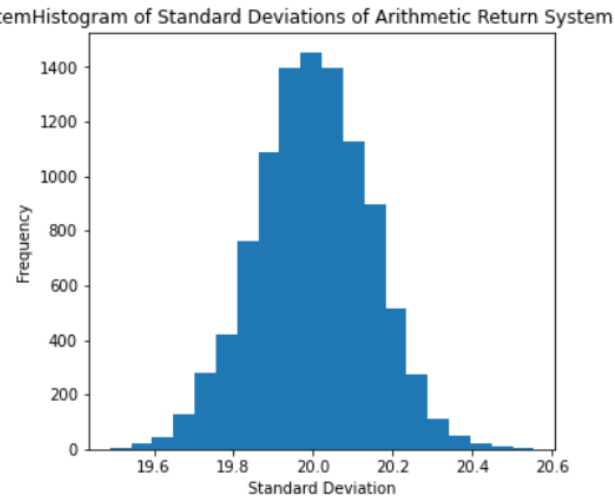
```
For Pt = Pt-1 + r, the expected value of price one period ahead is 100.00 and the standard deviation is 0.20
For Pt = Pt-1*(1+r), the expected value of price one period ahead is 100.00 and the standard deviation is 20.00
For Pt = Pt-1*(e**rt), the expected value of price one period ahead is 100.00 and the standard deviation is 20.61
```

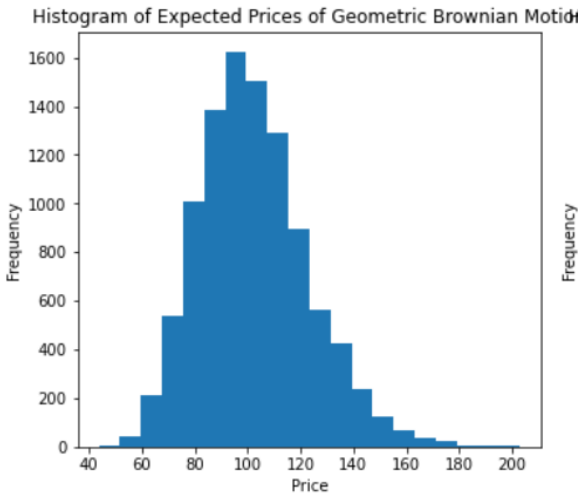
Here is are graphs to show prices and standard deviation under different types of returns with a simulation of 10000 times:

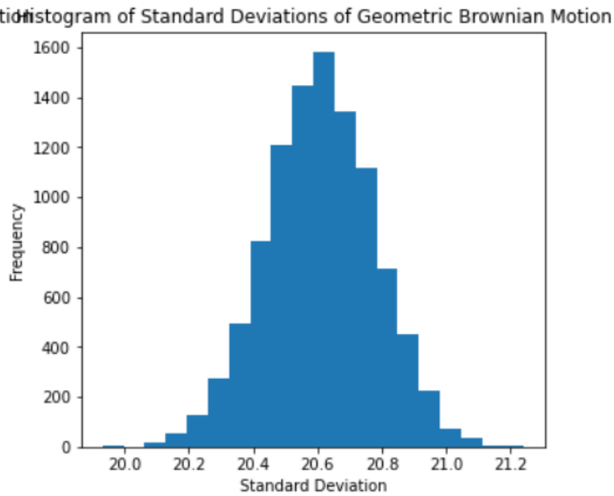












It is clear that the mean and standard deviation match my expectation above as well.

Q2:

Here are the arithmetic returns for all prices:

```
              Date       SPY      AAPL      MSFT      AMZN      TSLA  \
1    2/15/2022 0:00  0.016127  0.023152  0.018542  0.008658  0.053291
2    2/16/2022 0:00  0.001121 -0.001389 -0.001167  0.010159  0.001041
3    2/17/2022 0:00 -0.021361 -0.021269 -0.029282 -0.021809 -0.050943
4    2/18/2022 0:00 -0.006475 -0.009356 -0.009631 -0.013262 -0.022103
5    2/22/2022 0:00 -0.010732 -0.017812 -0.000729 -0.015753 -0.041366
..              ...       ...       ...       ...       ...       ...
244   2/3/2023 0:00 -0.010629  0.024400 -0.023621 -0.084315  0.009083
245   2/6/2023 0:00 -0.006111 -0.017929 -0.006116 -0.011703  0.025161
246   2/7/2023 0:00  0.013079  0.019245  0.042022 -0.000685  0.010526
247   2/8/2023 0:00 -0.010935 -0.017653 -0.003102 -0.020174  0.022763
248   2/9/2023 0:00 -0.008669 -0.006912 -0.011660 -0.018091  0.029957

        GOOGL      GOOG      META      NVDA  ...       PNC      MDLZ  \
1    0.007987  0.008319  0.015158  0.091812  ...  0.012807 -0.004082
2    0.008268  0.007784 -0.020181  0.000604  ...  0.006757 -0.002429
3   -0.037746 -0.037669 -0.040778 -0.075591  ... -0.034949  0.005326
4   -0.016116 -0.013914 -0.007462 -0.035296  ... -0.000646 -0.000908
5   -0.004521 -0.008163 -0.019790 -0.010659  ...  0.009494  0.007121
..        ...       ...       ...       ...  ...       ...       ...
244 -0.027474 -0.032904 -0.011866 -0.028053  ... -0.004694 -0.011251
245 -0.017942 -0.016632 -0.002520 -0.000521  ... -0.014451  0.003945
246  0.046064  0.044167  0.029883  0.051401  ... -0.000368 -0.016473
247 -0.076830 -0.074417 -0.042741  0.001443  ... -0.008469 -0.004456
248 -0.043876 -0.045400 -0.030039  0.005945  ... -0.016588 -0.007717

           MO       ADI      GILD       LMT       SYK        GM       TFC
1    0.004592  0.052344  0.003600 -0.012275  0.033021  0.026240  0.028572
2    0.005763  0.038879  0.009294  0.012244  0.003363  0.015301 -0.001389
3    0.015017 -0.046988 -0.009855  0.004833 -0.030857 -0.031925 -0.033380
4    0.007203 -0.000436 -0.003916 -0.005942 -0.013674 -0.004506 -0.003677
5   -0.008891  0.003243 -0.001147 -0.000673  0.008342 -0.037654 -0.002246
..        ...       ...       ...       ...       ...       ...       ...
244 -0.001277 -0.002677  0.038211  0.004134  0.002336 -0.008916 -0.005954
245  0.001066 -0.007102  0.022012  0.021826 -0.041181  0.005106 -0.009782
246 -0.008518  0.019544 -0.003590 -0.001641  0.003573  0.001451  0.008669
247 -0.001289 -0.018009 -0.004416  0.002819 -0.015526  0.004106 -0.015391
248 -0.003656  0.004275 -0.001634  0.000937 -0.014391  0.001443 -0.016619

          TJX
1    0.013237
2   -0.025984
3   -0.028763
4    0.015038
5   -0.013605
..        ...
244  0.001617
245 -0.004595
246 -0.003618
247  0.009363
248  0.005603
```

Here are the returns for META after removing the mean from the series:

```
1       0.015175
2      -0.020165
3      -0.040761
4      -0.007446
5      -0.019774
          ...
244    -0.011850
245    -0.002503
246     0.029899
247    -0.042725
248    -0.030022
Name: META, Length: 248, dtype: float64
```

Here is the VaR using a normal distribution:

0.06560156967533282

Here is the VaR using a normal distribution with an Exponentially Weighted variance (λ = 0.94):

0.05253206870346172

Here is the VaR using a MLE fitted T distribution:

0.0022964447067896376

Here is the VaR using a fitted AR(1) model:

0.06592848204123766

Here is the VaR using a Historic Simulation:

0.05223042912767907

Comparing VaR values from different simulations, I found that the VaR values using an MLE-fitted T distribution are relatively smaller than the other four simulations. I think the reason behind it is that the T distribution has fatter tails than the normal distribution, which means that the probability of enormous loss and gain is larger so that under the same confidence level (95%), the value on the x-axis which is the return will be smaller, which is a smaller VaR.

Q3:

Here are VaR values under exponentially weighted covariance and normal distribution:

I chose normal distribution to calculate VaR because it is an easy way and I found that portfolio A, B, C and all have a smaller VaR compare to VaR using Exponentially weighted covariance.

```
portfolio:A
VaR Using Exponentially weighted variance: 24302.5420929740
portfolio:B
VaR Using Exponentially weighted variance: 21551.2756024795
portfolio:C
VaR Using Exponentially weighted variance: 21453.6784730695
portfolio:ALL
VaR Using Exponentially weighted variance: 22131.3735012223
```

```
VAR Using normal distribution:
12571.119582860485
21857.25085508876
30760.215902723063
67206.27308362359
```