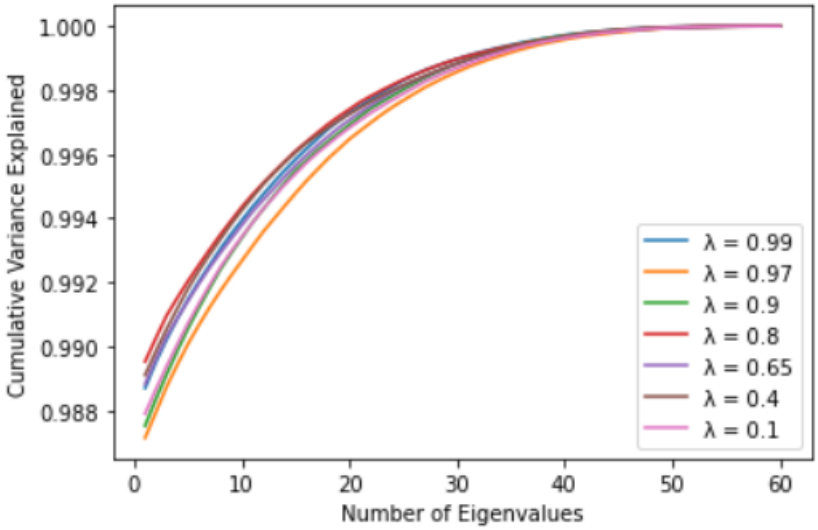
Question 1:

As the number of eigenvalues increases, the cumulative variance explained will also increase. As lambda increase, the weight will decay slower and keep more persistent, while as lambda decrease, the weight will decay faster.



Question 2:

For this question, functions chol_psd and near_psd were defined based on codes provided in class for calculating the Cholesky that assumes PSD Matrix and Near PSD Matrix, respectively. Higham's 2002 nearest psd correlation function was also defined for non-psd matrices. After that, a non-psd correlation matrix that is 500x500 was generated by using codes in class with a value of 1.0 for (n,n), 0.7357 for (1,2) and (2,1), and 0.9 for the rest, which is shown below.

```
[[1.     0.9    0.9    ... 0.9    0.9    0.9   ]
 [0.9    1.     0.7357 ... 0.9    0.9    0.9   ]
 [0.9    0.7357 1.     ... 0.9    0.9    0.9   ]
 ...
 [0.9    0.9    0.9    ... 1.     0.9    0.9   ]
 [0.9    0.9    0.9    ... 0.9    1.     0.9   ]
 [0.9    0.9    0.9    ... 0.9    0.9    1.    ]]
```

Such a matrix was put into the function near_psd and nearest_psd for fixing and a function called is_psd. Such a function was generated to determine if all eigenvalues are non-negative and confirm if the matrix is now psd since a matrix is psd if and only if its eigenvalues are all non-negative. Before using near_psd and nearest_psd, the epsilon value was set to 1e-8 to ensure that the resulting matrix has a small tolerance for numerical error during the computation. Elements in the matrix based on near_psd are all very small, while larger in the matrix based on nearest_psd. Here are two matrices calculated from near_psd and nearest_psd, respectively.

near_psd

```
[[1.00001241e+08 2.84412365e+04 2.84412365e+04 ... 2.84608913e+04
  1.75065237e+04 9.48187023e+02]
 [2.84412365e+04 1.02990192e+01 7.65601918e+00 ... 8.99390869e+00
  5.53222573e+00 2.99635995e-01]
 [2.84412365e+04 7.65601918e+00 1.02990192e+01 ... 8.99390869e+00
  5.53222573e+00 2.99635995e-01]
 ...
 [2.84608913e+04 8.99390869e+00 8.99390869e+00 ... 1.00001241e+01
  5.53604887e+00 2.99843064e-01]
 [1.75065237e+04 5.53222573e+00 5.53222573e+00 ... 5.53604887e+00
  3.78362621e+00 1.84435886e-01]
 [9.48187023e+02 2.99635995e-01 2.99635995e-01 ... 2.99843064e-01
  1.84435886e-01 1.10993209e-02]]
```

nearest_psd

```
[[1.00001241 0.89939087 0.89939087 ... 0.90001241 0.90001241 0.90001241]
 [0.89939087 1.02990192 0.76560192 ... 0.89939087 0.89939087 0.89939087]
 [0.89939087 0.76560192 1.02990192 ... 0.89939087 0.89939087 0.89939087]
 ...
 [0.90001241 0.89939087 0.89939087 ... 1.00001241 0.90001241 0.90001241]
 [0.90001241 0.89939087 0.89939087 ... 0.90001241 1.00001241 0.90001241]
 [0.90001241 0.89939087 0.89939087 ... 0.90001241 0.90001241 1.00001241]]
```

Then, these two matrices were put into the function is_psd, and they are both psd now.

```
The matrix after near psd is PSD.
The matrix after higham is PSD.
```

Both the two matrices were put into the function frobenius_norm, and the values are 100004079.86524981 and 450.1049416064457, respectively.

```
Frobenius norm of the near psd matrix:  100004079.86524981
Frobenius norm of the higham psd matrix:  450.1049416064457
```

When the size of the original matrix is 50, the run time for them is 0.001291 and 0.00099, respectively.

```
Time taken for near_psd:  0.0012919902801513672
Time taken for higham nearest_psd:  0.0009949207305908203
```

When the size of the original matrix is 100, the run time for them is 0.003978 and 0.002000, respectively.

```
Time taken for near_psd:  0.003978729248046875
Time taken for higham nearest_psd:  0.002000093460083008
```

When the size of the original matrix is 500, the run time for them is 0.050006 and 0.060023, respectively.

```
Time taken for near_psd:  0.050006866455078125
Time taken for higham nearest_psd:  0.06002378463745117
```

When the size of the original matrix is 1000, the run time for them is 0.151727 and 0.155951, respectively.

```
Time taken for near_psd:  0.15172743797302246
Time taken for higham nearest_psd:  0.15595126152038574
```

When the size of the original matrix is 2000, the run time for them is 0.570019 and 0.594579, respectively.

```
Time taken for near_psd:  0.5700197219848633
Time taken for higham nearest_psd:  0.5945794582366943
```

When the size of the original matrix is 4000, the run time for them is 3.738708 and 4.279142, respectively.

```
Time taken for near_psd:  3.73870849609375
Time taken for higham nearest_psd:  4.279142379760742
```

When the size of the original matrix is 8000, the run time for them is 72.928091 and 77.296947, respectively.

```
Time taken for near_psd:  72.92809104919434
Time taken for higham nearest_psd:  77.29694700241089
```

From the above comparisons, it is obviously that when the number of elements in a matrix is small, Higham's method will be faster, while as the number of elements increases, the near method becomes faster.

In my opinion, for near psd method, it can deal matrices with a large number of elements more efficiently and accurately, while when the number of elements is relatively small, it is slower. For Higham's method, it can deal with matrices with a small number of elements more efficiently, while when the number of elements is relatively large, it is much slower.

Question 3:

Here are correlation matrix and variance generated by Standard Pearson correlation/variance and Exponentially weighted λ = 0. 97 respectively:

Standard Pearson correlation matrix:

	SPY	AAPL	MSFT	AMZN	TSLA	GOOGL	GOOG
SPY	1.000000	0.645752	0.714473	0.601967	0.477340	0.670342	0.683933
AAPL	0.645752	1.000000	0.608202	0.631967	0.441892	0.484549	0.495649
MSFT	0.714473	0.608202	1.000000	0.503177	0.417453	0.800604	0.808346
AMZN	0.601967	0.631967	0.503177	1.000000	0.227097	0.609568	0.616625
TSLA	0.477340	0.441892	0.417453	0.227097	1.000000	0.237469	0.254928
...	...	...	...	...	...	...	...
LMT	0.149402	-0.006243	0.001711	-0.074725	0.064082	-0.027067	0.019735
SYK	0.606975	0.298389	0.316639	0.264377	0.185279	0.359861	0.365131
GM	0.502788	0.215402	0.165700	0.057277	0.258044	0.185495	0.198629
TFC	0.560600	0.156530	0.090006	0.132547	0.225387	0.151731	0.153002
TJX	0.577184	0.293308	0.247382	0.406752	0.204355	0.307883	0.322024

	FB	NVDA	BRK-B	...	PNC	MDLZ	MO
SPY	0.641302	0.661717	0.372492	...	0.487906	0.324941	-0.102969
AAPL	0.376906	0.525702	0.003171	...	0.154941	-0.057519	-0.293476
MSFT	0.458262	0.578644	-0.041255	...	0.034687	0.010835	-0.211391
AMZN	0.467283	0.656592	-0.041217	...	0.068033	0.061785	-0.205990
TSLA	0.300915	0.427731	-0.048458	...	0.176572	-0.187884	-0.226491
...	...	...	...	...	...	...	...
LMT	0.408407	-0.192917	0.221126	...	0.157735	0.077381	0.135597
SYK	0.368264	0.205058	0.333976	...	0.399497	0.190449	0.103621
GM	0.218433	0.278999	0.448889	...	0.537812	0.191455	0.158622
TFC	0.281351	0.192739	0.742954	...	0.917762	0.387408	0.187204
TJX	0.410282	0.231681	0.199419	...	0.314820	0.307979	0.017582

	ADI	GILD	LMT	SYK	GM	TFC	TJX
SPY	0.703049	0.117348	0.149402	0.606975	0.502788	0.560600	0.577184
AAPL	0.565991	0.039200	-0.006243	0.298389	0.215402	0.156530	0.293308
MSFT	0.454979	-0.169047	0.001711	0.316639	0.165700	0.090006	0.247382
AMZN	0.518099	0.018856	-0.074725	0.264377	0.057277	0.132547	0.406752
TSLA	0.383221	0.093457	0.064082	0.185279	0.258044	0.225387	0.204355
...	...	...	...	...	...	...	...
LMT	0.066449	0.034229	1.000000	0.161134	0.199808	0.191557	0.153498
SYK	0.464296	0.093868	0.161134	1.000000	0.415376	0.440972	0.480617
GM	0.384554	0.074076	0.199808	0.415376	1.000000	0.503537	0.431925
TFC	0.306719	0.180406	0.191557	0.440972	0.503537	1.000000	0.324314
TJX	0.405381	0.048648	0.153498	0.480617	0.431925	0.324314	1.000000

Standard Pearson variance vector:

SPY	0.000078
AAPL	0.000257
MSFT	0.000255
AMZN	0.000261
TSLA	0.002052
...	
LMT	0.000313
SYK	0.000260
GM	0.000730
TFC	0.000301
TJX	0.000277

Exponentially weighted λ = 0.97 correlation matrix:

	SPY	AAPL	MSFT	AMZN	TSLA	GOOGL	GOOG
SPY	0.802946	0.481834	0.495695	0.501155	0.362934	0.492004	0.504527
AAPL	0.481834	0.851891	0.383828	0.575075	0.248513	0.350049	0.350851
MSFT	0.495695	0.383828	0.874554	0.408406	0.263077	0.808060	0.813082
AMZN	0.501155	0.575075	0.408406	1.082290	0.019474	0.652190	0.643200
TSLA	0.362934	0.248513	0.263077	0.019474	1.060506	0.059944	0.084138
...	...	...	...	...	...	...	...
LMT	0.132709	-0.023633	-0.056777	-0.169832	0.124690	-0.140926	-0.045883
SYK	0.489862	0.261180	0.205215	0.225756	0.083575	0.217985	0.220727
GM	0.406806	0.127895	0.048856	0.027067	0.206864	0.012972	0.029973
TFC	0.481766	0.148298	-0.015762	0.052135	0.183381	-0.040467	-0.040305
TJX	0.508132	0.253550	0.170168	0.401953	0.265935	0.243950	0.255141

	FB	NVDA	BRK-B	...	PNC	MDLZ	MO
SPY	0.544687	0.535832	0.345555	...	0.390786	0.330744	-0.116829
AAPL	0.287727	0.372305	0.069967	...	0.140435	0.001227	-0.286567
MSFT	0.327952	0.451707	-0.103954	...	-0.027926	-0.031921	-0.233916
AMZN	0.423849	0.627273	-0.046343	...	-0.015707	0.121103	-0.195052
TSLA	0.284327	0.313148	-0.022512	...	0.120515	-0.161930	-0.276196
...	...	...	...	...	...	...	...
LMT	0.601749	-0.341225	0.158757	...	0.143183	-0.008121	0.079992
SYK	0.317438	0.151172	0.310875	...	0.312952	0.215181	0.134949
GM	0.253824	0.274208	0.355979	...	0.353543	0.178065	0.159501
TFC	0.190672	0.096463	0.631047	...	0.767373	0.440096	0.136260
TJX	0.429509	0.202568	0.158213	...	0.204031	0.316260	0.040975

	ADI	GILD	LMT	SYK	GM	TFC	TJX
SPY	0.610433	0.109184	0.132709	0.489862	0.406806	0.481766	0.508132
AAPL	0.443396	0.122074	-0.023633	0.261180	0.127895	0.148298	0.253550
MSFT	0.193600	-0.348418	-0.056777	0.205215	0.048856	-0.015762	0.170168
AMZN	0.421473	0.043264	-0.169832	0.225756	0.027067	0.052135	0.401953
TSLA	0.280344	0.117509	0.124690	0.083575	0.206864	0.183381	0.265935
...	...	...	...	...	...	...	...
LMT	0.098841	0.050471	1.695970	0.099980	0.209940	0.161964	0.172002
SYK	0.460669	0.133832	0.099980	0.882323	0.301739	0.388082	0.415016
GM	0.450508	0.138933	0.209940	0.301739	0.905420	0.371654	0.393392
TFC	0.324307	0.315057	0.161964	0.388082	0.371654	0.878198	0.285376
TJX	0.443168	0.091242	0.172002	0.415016	0.393392	0.285376	0.985789

Exponentially weighted λ = 0.97 variance vector:

SPY	0.000068
AAPL	0.000219
MSFT	0.000234
AMZN	0.000189
TSLA	0.001621
...	
LMT	0.000141
SYK	0.000215
GM	0.000608
TFC	0.000248
TJX	0.000210

Each of them is combined to form 4 different covariance matrices:

Pearson correlation + Pearson variance:

	SPY	AAPL	MSFT	AMZN	TSLA	GOOGL	\
SPY	0.000078	0.000166	1.820476e-04	0.000157	0.000980	0.000166	
AAPL	0.000051	0.000257	1.549699e-04	0.000165	0.000907	0.000120	
MSFT	0.000056	0.000157	2.547999e-04	0.000131	0.000857	0.000198	
AMZN	0.000047	0.000163	1.282095e-04	0.000261	0.000466	0.000151	
TSLA	0.000037	0.000114	1.063670e-04	0.000059	0.002052	0.000059	
...	...	...	...	...	...	...	
LMT	0.000012	-0.000002	4.360001e-07	-0.000019	0.000132	-0.000007	
SYK	0.000048	0.000077	8.067947e-05	0.000069	0.000380	0.000089	
GM	0.000039	0.000055	4.222031e-05	0.000015	0.000530	0.000046	
TFC	0.000044	0.000040	2.293356e-05	0.000035	0.000463	0.000038	
TJX	0.000045	0.000076	6.303293e-05	0.000106	0.000419	0.000076	

	GOOG	FB	NVDA	BRK-B	...	PNC	MDLZ
SPY	0.000163	0.000261	0.000944	3.371054e-05	...	0.000127	0.000032
AAPL	0.000118	0.000153	0.000750	2.869805e-07	...	0.000040	-0.000006
MSFT	0.000193	0.000186	0.000826	-3.733593e-06	...	0.000009	0.000001
AMZN	0.000147	0.000190	0.000937	-3.730153e-06	...	0.000018	0.000006
TSLA	0.000061	0.000122	0.000610	-4.385441e-06	...	0.000046	-0.000019
...	...	...	...	...	...	...	...
LMT	0.000005	0.000166	-0.000275	2.001194e-05	...	0.000041	0.000008
SYK	0.000087	0.000150	0.000293	3.022489e-05	...	0.000104	0.000019
GM	0.000047	0.000089	0.000398	4.062450e-05	...	0.000140	0.000019
TFC	0.000036	0.000114	0.000275	6.723745e-05	...	0.000238	0.000038
TJX	0.000077	0.000167	0.000331	1.804741e-05	...	0.000082	0.000030

	MO	ADI	GILD	LMT	SYK	GM	\
SPY	-0.000018	0.000175	0.000011	4.679721e-05	0.000158	0.000367	
AAPL	-0.000051	0.000141	0.000004	-1.955567e-06	0.000078	0.000157	
MSFT	-0.000037	0.000113	-0.000016	5.359812e-07	0.000082	0.000121	
AMZN	-0.000036	0.000129	0.000002	-2.340601e-05	0.000069	0.000042	
TSLA	-0.000040	0.000095	0.000009	2.007243e-05	0.000048	0.000188	
...	...	...	...	...	...	...	
LMT	0.000024	0.000017	0.000003	3.132292e-04	0.000042	0.000146	
SYK	0.000018	0.000115	0.000009	5.047179e-05	0.000260	0.000303	
GM	0.000028	0.000096	0.000007	6.258566e-05	0.000108	0.000730	
TFC	0.000033	0.000076	0.000017	6.000118e-05	0.000115	0.000367	
TJX	0.000003	0.000101	0.000005	4.808010e-05	0.000125	0.000315	

	TFC	TJX
SPY	0.000169	0.000160
AAPL	0.000047	0.000081
MSFT	0.000027	0.000069
AMZN	0.000040	0.000113
TSLA	0.000068	0.000057
...	...	...
LMT	0.000058	0.000043
SYK	0.000133	0.000133
GM	0.000152	0.000120
TFC	0.000301	0.000090
TJX	0.000098	0.000277

Pearson correlation + EW variance:

	SPY	AAPL	MSFT	AMZN	TSLA	GOOGL	\
SPY	0.000068	0.000141	1.671793e-04	0.000114	0.000774	0.000121	
AAPL	0.000044	0.000219	1.423131e-04	0.000120	0.000717	0.000088	
MSFT	0.000049	0.000133	2.339897e-04	0.000095	0.000677	0.000145	
AMZN	0.000041	0.000138	1.177383e-04	0.000189	0.000368	0.000110	
TSLA	0.000033	0.000097	9.767973e-05	0.000043	0.001621	0.000043	
...	...	...	...	...	...	...	
LMT	0.000010	-0.000001	4.003908e-07	-0.000014	0.000104	-0.000005	
SYK	0.000041	0.000065	7.409016e-05	0.000050	0.000300	0.000065	
GM	0.000034	0.000047	3.877207e-05	0.000011	0.000418	0.000034	
TFC	0.000038	0.000034	2.106051e-05	0.000025	0.000365	0.000027	
TJX	0.000039	0.000064	5.788486e-05	0.000077	0.000331	0.000056	

	GOOG	FB	NVDA	BRK-B	...	PNC	MDLZ
SPY	0.000121	0.000193	0.000690	2.894474e-05	...	0.000115	2.346277e-05
AAPL	0.000087	0.000113	0.000548	2.464088e-07	...	0.000036	-4.153262e-06
MSFT	0.000143	0.000138	0.000603	-3.205758e-06	...	0.000008	7.823323e-07
AMZN	0.000109	0.000141	0.000684	-3.202805e-06	...	0.000016	4.461232e-06
TSLA	0.000045	0.000090	0.000446	-3.765452e-06	...	0.000042	-1.356644e-05
...	...	...	...	...	...	...	...
LMT	0.000003	0.000123	-0.000201	1.718277e-05	...	0.000037	5.587367e-06
SYK	0.000064	0.000111	0.000214	2.595187e-05	...	0.000094	1.375165e-05
GM	0.000035	0.000066	0.000291	3.488123e-05	...	0.000126	1.382428e-05
TFC	0.000027	0.000085	0.000201	5.773180e-05	...	0.000216	2.797332e-05
TJX	0.000057	0.000123	0.000242	1.549596e-05	...	0.000074	2.223805e-05

	MO	ADI	GILD	LMT	SYK	GM
SPY	-0.000012	0.000133	0.000007	2.108791e-05	0.000130	0.000306
AAPL	-0.000035	0.000107	0.000002	-8.812240e-07	0.000064	0.000131
MSFT	-0.000025	0.000086	-0.000009	2.415255e-07	0.000068	0.000101
AMZN	-0.000024	0.000098	0.000001	-1.054729e-05	0.000057	0.000035
TSLA	-0.000027	0.000073	0.000005	9.045104e-06	0.000040	0.000157
...	...	...	...	...	...	...
LMT	0.000016	0.000013	0.000002	1.411483e-04	0.000035	0.000122
SYK	0.000012	0.000088	0.000005	2.274376e-05	0.000215	0.000253
GM	0.000019	0.000073	0.000004	2.820255e-05	0.000089	0.000608
TFC	0.000022	0.000058	0.000010	2.703792e-05	0.000095	0.000306
TJX	0.000002	0.000077	0.000003	2.166601e-05	0.000103	0.000263

	TFC	TJX
SPY	0.000139	0.000121
AAPL	0.000039	0.000062
MSFT	0.000022	0.000052
AMZN	0.000033	0.000086
TSLA	0.000056	0.000043
...	...	...
LMT	0.000048	0.000032
SYK	0.000109	0.000101
GM	0.000125	0.000091
TFC	0.000248	0.000068
TJX	0.000080	0.000210

EW correlation + EW variance:

	SPY	AAPL	MSFT	AMZN	TSLA	GOOGL	GOOG
SPY	0.000055	0.000105	0.000116	0.000095	0.000588	0.000089	0.000089
AAPL	0.000033	0.000186	0.000090	0.000109	0.000403	0.000063	0.000062
MSFT	0.000034	0.000084	0.000205	0.000077	0.000427	0.000146	0.000143
AMZN	0.000034	0.000126	0.000096	0.000205	0.000032	0.000118	0.000113
TSLA	0.000025	0.000054	0.000062	0.000004	0.001720	0.000011	0.000015
...	...	...	...	...	...	...	...
LMT	0.000009	-0.000005	-0.000013	-0.000032	0.000202	-0.000026	-0.000008
SYK	0.000033	0.000057	0.000048	0.000043	0.000136	0.000040	0.000039
GM	0.000028	0.000028	0.000011	0.000005	0.000335	0.000002	0.000005
TFC	0.000033	0.000032	-0.000004	0.000010	0.000297	-0.000007	-0.000007
TJX	0.000035	0.000055	0.000040	0.000076	0.000431	0.000044	0.000045

	FB	NVDA	BRK-B	...	PNC	MDLZ	MO
SPY	0.000164	0.000559	0.000027	...	0.000092	2.388182e-05	-0.000014
AAPL	0.000087	0.000388	0.000005	...	0.000033	8.856941e-08	-0.000034
MSFT	0.000099	0.000471	-0.000008	...	-0.000007	-2.304878e-06	-0.000028
AMZN	0.000127	0.000654	-0.000004	...	-0.000004	8.744370e-06	-0.000023
TSLA	0.000086	0.000326	-0.000002	...	0.000028	-1.169235e-05	-0.000033
...	...	...	...	...	...	...	...
LMT	0.000181	-0.000356	0.000012	...	0.000034	-5.863858e-07	0.000010
SYK	0.000095	0.000158	0.000024	...	0.000074	1.553743e-05	0.000016
GM	0.000076	0.000286	0.000028	...	0.000083	1.285743e-05	0.000019
TFC	0.000057	0.000101	0.000049	...	0.000180	3.177773e-05	0.000016
TJX	0.000129	0.000211	0.000012	...	0.000048	2.283598e-05	0.000005

	ADI	GILD	LMT	SYK	GM	TFC	TJX
SPY	0.000116	0.000006	0.000019	0.000105	0.000247	0.000120	0.000107
AAPL	0.000084	0.000007	-0.000003	0.000056	0.000078	0.000037	0.000053
MSFT	0.000037	-0.000019	-0.000008	0.000044	0.000030	-0.000004	0.000036
AMZN	0.000080	0.000002	-0.000024	0.000048	0.000016	0.000013	0.000085
TSLA	0.000053	0.000007	0.000018	0.000018	0.000126	0.000046	0.000056
...	...	...	...	...	...	...	...
LMT	0.000019	0.000003	0.000239	0.000021	0.000128	0.000040	0.000036
SYK	0.000087	0.000007	0.000014	0.000189	0.000184	0.000096	0.000087
GM	0.000085	0.000008	0.000030	0.000065	0.000551	0.000092	0.000083
TFC	0.000061	0.000018	0.000023	0.000083	0.000226	0.000218	0.000060
TJX	0.000084	0.000005	0.000024	0.000089	0.000239	0.000071	0.000207

EW correlation + Pearson variance:

	SPY	AAPL	MSFT	AMZN	TSLA	GOOGL	GOOG
SPY	0.000063	0.000124	0.000126	0.000131	0.000745	0.000122	0.000120
AAPL	0.000038	0.000219	0.000098	0.000150	0.000510	0.000087	0.000084
MSFT	0.000039	0.000099	0.000223	0.000107	0.000540	0.000200	0.000194
AMZN	0.000039	0.000148	0.000104	0.000282	0.000040	0.000161	0.000153
TSLA	0.000028	0.000064	0.000067	0.000005	0.002177	0.000015	0.000020
...	...	...	...	...	...	...	...
LMT	0.000010	-0.000006	-0.000014	-0.000044	0.000256	-0.000035	-0.000011
SYK	0.000038	0.000067	0.000052	0.000059	0.000172	0.000054	0.000053
GM	0.000032	0.000033	0.000012	0.000007	0.000425	0.000003	0.000007
TFC	0.000038	0.000038	-0.000004	0.000014	0.000376	-0.000010	-0.000010
TJX	0.000040	0.000065	0.000043	0.000105	0.000546	0.000060	0.000061

	FB	NVDA	BRK-B	...	PNC	MDLZ	MO
SPY	0.000222	0.000765	0.000031	...	0.000101	3.274544e-05	-0.000020
AAPL	0.000117	0.000531	0.000006	...	0.000036	1.214415e-07	-0.000050
MSFT	0.000133	0.000645	-0.000009	...	-0.000007	-3.160323e-06	-0.000041
AMZN	0.000172	0.000895	-0.000004	...	-0.000004	1.198980e-05	-0.000034
TSLA	0.000116	0.000447	-0.000002	...	0.000031	-1.603191e-05	-0.000048
...	...	...	...	...	...	...	...
LMT	0.000245	-0.000487	0.000014	...	0.000037	-8.040200e-07	0.000014
SYK	0.000129	0.000216	0.000028	...	0.000081	2.130407e-05	0.000024
GM	0.000103	0.000391	0.000032	...	0.000092	1.762939e-05	0.000028
TFC	0.000078	0.000138	0.000057	...	0.000199	4.357187e-05	0.000024
TJX	0.000175	0.000289	0.000014	...	0.000053	3.131144e-05	0.000007

	ADI	GILD	LMT	SYK	GM	TFC	TJX
SPY	0.000152	0.000010	0.000042	0.000127	0.000297	0.000145	0.000141
AAPL	0.000110	0.000011	-0.000007	0.000068	0.000093	0.000045	0.000070
MSFT	0.000048	-0.000033	-0.000018	0.000053	0.000036	-0.000005	0.000047
AMZN	0.000105	0.000004	-0.000053	0.000059	0.000020	0.000016	0.000112
TSLA	0.000070	0.000011	0.000039	0.000022	0.000151	0.000055	0.000074
...	...	...	...	...	...	...	...
LMT	0.000025	0.000005	0.000531	0.000026	0.000153	0.000049	0.000048
SYK	0.000114	0.000013	0.000031	0.000230	0.000220	0.000117	0.000115
GM	0.000112	0.000013	0.000066	0.000079	0.000661	0.000112	0.000109
TFC	0.000081	0.000030	0.000051	0.000101	0.000271	0.000265	0.000079
TJX	0.000110	0.000009	0.000054	0.000108	0.000287	0.000086	0.000273

Each of them is simulated 25000 draws using Direct Simulation, PCA with 100% explained, PCA with 75% explained, and PCA with 50% explained:

Simulated Pearson correlation + Pearson variance covariance using Direct Simulation:

```
[[-0.00348129 -0.00110685  0.00729153 ...  0.00687786 -0.02922015
  -0.0102809 ]
 [-0.0035189  -0.00691351 -0.00565045 ...  0.0276371   0.00148613
  -0.01313002]
 [ 0.00046533  0.0153679   0.01562065 ...  0.00492832  0.00146873
  -0.01429966]
 ...
 [-0.00484012  0.01888502 -0.01527155 ... -0.00209276 -0.0145537
   0.01568292]
 [-0.00366558 -0.01110665 -0.00775899 ... -0.0250617  -0.006153
  -0.01460141]
 [ 0.0004498  -0.00496417  0.03016988 ...  0.03129259 -0.02855127
   0.0054494 ]]
```

Simulated Pearson correlation + EW variance covariance using Direct Simulation:

```
[[ 0.00192388 -0.00356679 -0.00797458 ...  0.03137455  0.01400501
   0.0112288 ]
 [ 0.00230111  0.00494453 -0.00616073 ... -0.00642735 -0.00921861
  -0.00245282]
 [ 0.00214959  0.01148501  0.00712619 ... -0.02333252 -0.01089991
   0.0055757 ]
 ...
 [-0.00587553 -0.02673083 -0.02117623 ... -0.02604934 -0.0029087
  -0.0188734 ]
 [ 0.00879962  0.02038075  0.02736675 ...  0.02889262  0.02328844
   0.01467118]
 [ 0.00304853  0.02198765  0.01465971 ... -0.01993597 -0.01102804
  -0.00687819]]
```

Simulated EW correlation + EW variance covariance using Direct Simulation:

```
[[-0.00052305  0.00370761 -0.00418165 ...  0.04713677  0.00761398
   0.00696358]
 [-0.00567678  0.00585246 -0.00918808 ...  0.00815274 -0.02224288
  -0.01901263]
 [-0.00216543 -0.01691328  0.0012357  ... -0.00748441 -0.01390294
   0.00911094]
 ...
 [-0.01047005 -0.01317507 -0.00343537 ... -0.05458934 -0.0314656
  -0.01842067]
 [-0.00087653  0.01220912 -0.00746592 ...  0.00600961  0.00815823
  -0.00611253]
 [-0.00105465 -0.00116799  0.00747417 ...  0.02737483 -0.00418903
   0.00348496]]
```

Simulated EW correlation + Pearson variance covariance using Direct Simulation:

```
[[ 0.00263487  0.02339098  0.00144991 ...  0.02971862  0.01606164
  -0.00515601]
 [-0.00140555 -0.01204159 -0.02226277 ...  0.00760628  0.01623785
  -0.0065356 ]
 [-0.00464849 -0.01906958 -0.00805419 ... -0.01127477 -0.01055042
  -0.00335446]
 ...
 [ 0.00366429  0.01161942  0.0189554  ... -0.00303497 -0.00490557
  -0.02366064]
 [-0.00113901 -0.03655521 -0.0046612  ...  0.03759188 -0.00506086
   0.00268003]
 [-0.00207332  0.00199339 -0.00813105 ... -0.0191015   0.00411022
  -0.00149524]]
```

Simulated Pearson correlation + Pearson variance covariance using PCA(100%):

```
[[-1.30773459e-04 -9.85856598e-05  1.10161844e-05 ...  3.53660701e-29
   6.44068896e-29  3.31712378e-30]
 [ 4.58519392e-05 -1.47783997e-05 -8.71389403e-07 ... -2.63970588e-29
   3.77090723e-29  6.30370483e-30]
 [ 1.31202960e-05  2.44954959e-05  1.19234680e-05 ...  5.92018730e-29
   6.36615984e-29 -1.74716260e-30]
 ...
 [ 2.44701818e-06 -3.44212735e-05  5.23805902e-06 ...  2.08040212e-29
  -6.35715269e-29 -1.84356981e-30]
 [ 8.90405945e-06 -5.84342054e-05  8.92363010e-06 ... -2.87197335e-29
   1.46460607e-29 -7.63380171e-31]
 [ 1.14992798e-04 -1.99274723e-05  4.88661996e-06 ... -9.55928463e-29
  -3.09167474e-29  7.19893822e-30]]
```

Simulated Pearson correlation + EW variance covariance using PCA(100%):

```
[[-4.30106266e-06 -2.00196241e-05  1.06814621e-05 ... -6.14471206e-29
  -3.43601119e-29  1.08843526e-30]
 [ 1.01182346e-04  1.36308891e-06  4.34415530e-06 ...  1.52843653e-28
  -3.38065908e-29  1.74489321e-30]
 [-1.17851708e-05 -1.05199273e-05 -1.80046069e-06 ... -8.59480599e-29
   5.49178018e-29  3.19819080e-30]
 ...
 [ 2.40761937e-05  1.51846306e-05  1.34335740e-05 ...  2.58056400e-29
  -8.12626375e-29  1.91027804e-30]
 [ 4.27644598e-05 -6.35616047e-05 -3.63805305e-06 ... -3.95658942e-30
   1.78038038e-29 -2.95631400e-30]
 [-6.69756994e-05 -3.19644293e-05  4.86145028e-06 ...  3.84688911e-29
   3.01225273e-29  8.92414576e-30]]
```

Simulated EW correlation + EW variance covariance using PCA(100%):

```
[[ 2.79538000e-05 -1.25823730e-05  1.00259609e-05 ... -3.39025107e-30
  -1.38117749e-29  3.49404001e-32]
 [ 5.73615208e-05 -3.15928745e-05 -6.91392343e-06 ... -2.96241902e-29
  -2.48952861e-29  2.41675481e-30]
 [ 4.62040630e-06  1.11107907e-05  6.20123023e-06 ... -4.76860184e-29
  -6.42641405e-30 -8.12718034e-30]
 ...
 [ 3.42793189e-05 -9.08806886e-06 -1.09198655e-06 ... -3.92173783e-29
  -5.09600344e-29  6.15374893e-30]
 [-9.19323122e-05 -5.20196657e-05 -3.49246577e-06 ... -6.99319093e-29
   3.61382642e-29 -1.06582162e-30]
 [-6.54873769e-05  1.09167698e-05 -1.47289070e-05 ... -1.69344306e-29
  -4.35388675e-29  6.02794069e-30]]
```

Simulated EW correlation + Pearson variance covariance using PCA(100%):

```
[[-5.84375474e-05  2.69228501e-05 -4.15741401e-06 ...  1.69841394e-29
  -3.00723695e-29  7.68092209e-30]
 [-1.70496867e-05 -4.55721784e-06  5.91758904e-06 ...  1.43227608e-29
  -4.05252688e-29  3.49342698e-30]
 [-4.90608857e-06 -5.48780136e-05  8.36258782e-06 ... -4.01110948e-29
   3.04691799e-29  8.56697684e-31]
 ...
 [ 2.75745857e-06  5.24599444e-06  7.06167731e-07 ...  3.43518836e-29
   1.00136499e-29  2.60324312e-30]
 [-9.99221404e-05  5.10493471e-05  2.13909832e-05 ... -6.99205760e-30
   9.87834369e-29 -2.17806097e-31]
 [ 3.40435726e-05  1.45707584e-05  1.55051619e-05 ... -8.88925724e-29
  -3.51008297e-30  2.60442330e-30]]
```

Simulated Pearson correlation + Pearson variance covariance using PCA(75%):

```
[[-1.70570689e-04  2.46858813e-06 -8.39430728e-06 ...  7.04438375e-29
  -2.18267283e-29  2.04993040e-30]
 [ 1.38914728e-04 -1.98102456e-05  9.48067557e-06 ... -2.63964663e-29
  -1.33677784e-29 -4.81125730e-30]
 [ 1.48266731e-05  3.86179843e-05 -1.45459435e-05 ...  6.35759752e-29
  -3.94108541e-29  5.21307477e-30]
 ...
 [ 1.28031148e-04 -1.42481635e-06  2.36684634e-05 ...  2.44433326e-29
   3.35056049e-29  6.25858895e-30]
 [-3.56937046e-05  3.36424288e-05 -2.99731473e-07 ... -1.12827974e-30
  -7.77106317e-29 -3.43515878e-30]
 [-4.66435863e-05 -4.10775711e-05  1.59987630e-06 ... -1.55537739e-29
  -4.79768663e-30  5.16099770e-30]]
```

Simulated Pearson correlation + EW variance covariance using PCA(75%):

```
[[-5.89165103e-05  1.65921574e-05  4.15021233e-06 ...  3.94127639e-29
  -1.67303110e-29 -4.11380809e-30]
 [ 3.56204477e-05 -3.39355532e-05  1.29602836e-05 ...  4.37281904e-29
   6.33684610e-29 -1.50133821e-30]
 [-2.37001621e-05  1.52658260e-05  7.17029253e-06 ...  7.13610619e-29
  -2.86169345e-29  9.59145787e-31]
 ...
 [ 5.95774271e-05  1.65167949e-05 -4.18717154e-06 ... -9.80472804e-30
  -8.87805494e-30  9.23519657e-31]
 [-1.44691743e-05 -1.56354668e-05  2.25735049e-06 ...  1.00268141e-28
  -2.26806559e-29  3.57862807e-30]
 [ 4.65038503e-05  5.05402875e-05  4.46866541e-06 ...  4.49828145e-29
  -9.47486280e-30  3.74468265e-31]]
```

Simulated EW correlation + EW variance covariance using PCA(75%):

```
[[-7.11317082e-05 -3.78386869e-06  1.39060088e-06 ...  3.24438484e-29
   1.29537785e-29  3.01395077e-30]
 [-1.40494070e-05 -5.43135411e-06 -7.69435067e-06 ...  1.81432869e-29
  -1.57506688e-29 -8.43864975e-31]
 [-6.05721968e-05 -3.67644457e-06  8.60437765e-06 ...  3.65431553e-29
  -3.33141955e-30  6.10593844e-30]
 ...
 [ 1.63017847e-05 -3.91271458e-05  1.30134335e-05 ...  1.53929648e-29
   1.80111094e-29 -3.12677296e-30]
 [-3.81466550e-06 -2.55815854e-05 -2.12572194e-05 ...  4.49213434e-30
   5.09087738e-29 -3.42509248e-30]
 [-2.83575523e-05  4.95391762e-06  1.82869660e-05 ...  1.56830796e-29
  -4.76331906e-29 -6.30834078e-30]]
```

Simulated EW correlation + Pearson variance covariance using PCA(75%):

```
[[ 4.45158928e-05 -2.57017684e-05  1.19352854e-05 ...  9.46310292e-29
   2.20641676e-29  2.37720607e-30]
 [-4.06579275e-05  3.61532432e-05 -2.72943658e-06 ...  8.50586506e-29
  -3.34102063e-29  3.95172585e-30]
 [ 1.04019819e-04  2.31405000e-05 -1.09865463e-05 ...  5.59475894e-29
  -2.47717037e-29  1.18892872e-30]
 ...
 [ 3.19729785e-05  1.82614010e-05 -5.90259363e-06 ...  7.66837852e-30
  -2.38754567e-29 -1.14688321e-30]
 [ 2.94048096e-05  2.41341701e-05  5.92956079e-06 ...  1.98603047e-29
  -1.09227729e-29 -1.45830957e-30]
 [ 4.02567382e-05  3.70092531e-05 -9.94896868e-07 ...  4.04192804e-29
  -4.77329591e-29 -2.00149804e-30]]
```

Simulated Pearson correlation + Pearson variance covariance using PCA(50%):

```
[[-6.68621080e-05 -5.69090569e-05 -1.20780845e-05 ...  7.34768256e-29
   2.95756789e-29 -1.13574151e-30]
 [ 2.70733919e-06 -1.43747789e-06 -1.45252716e-05 ... -3.81936482e-29
  -4.70358032e-29  1.32474353e-32]
 [ 6.86469937e-05  4.60355169e-05 -9.91447300e-06 ... -1.24967762e-29
   6.32117606e-30 -2.62030416e-30]
 ...
 [-6.68071741e-05  4.54371468e-06 -2.98134128e-05 ...  3.61963527e-29
  -3.41091505e-29 -9.76779739e-30]
 [-1.02465158e-05  3.37043304e-05  1.48019032e-05 ...  1.21069568e-29
   2.84167733e-29  3.47371661e-30]
 [ 6.87100845e-05 -6.89831206e-05 -4.41065861e-06 ... -4.76587439e-29
   4.71780290e-29  2.77577753e-30]]
```

Simulated Pearson correlation + EW variance covariance using PCA(50%):

```
[[ 1.64479850e-05 -3.93657739e-05  4.72236230e-06 ...  4.98645118e-29
  -2.37242672e-29  4.69349034e-31]
 [-1.04357250e-04 -8.48460377e-06 -1.21753600e-05 ... -3.27041328e-29
   2.99099072e-29  6.30481758e-30]
 [-1.05377075e-04 -1.04160398e-05 -8.23813498e-06 ... -1.09746037e-28
   4.78494404e-29  7.35336110e-31]
 ...
 [-1.50312843e-05 -5.21509181e-05 -7.19736987e-06 ... -5.49540215e-29
   5.87453863e-29 -7.60405705e-31]
 [ 5.88849470e-05 -1.37420862e-05  7.71801010e-06 ... -5.77150423e-29
   3.58928592e-29  5.26215707e-30]
 [ 9.24119289e-06 -1.61055803e-05  3.32757413e-06 ...  9.26798998e-29
  -3.91136240e-29 -3.81379245e-30]]
```

Simulated EW correlation + EW variance covariance using PCA(50%):

```
[[-6.95274160e-05 -4.24322840e-05 -1.54993714e-05 ... -4.39006079e-29
   1.41894597e-29 -1.06918486e-29]
 [-4.39336722e-06 -1.51774173e-05 -1.70967673e-05 ...  3.33550779e-29
  -3.19628709e-29 -6.87660690e-30]
 [-2.26913697e-05  2.20507648e-05  6.24339710e-06 ... -1.90943519e-29
  -3.62338539e-29 -5.38520453e-30]
 ...
 [-2.04218819e-05 -2.90649165e-05 -3.60702107e-06 ... -3.33371546e-30
  -7.31206352e-30  2.07369334e-30]
 [-2.37457573e-05 -1.26202978e-05 -1.39609387e-05 ...  5.12738680e-30
   1.05513846e-29 -5.97869643e-30]
 [-1.47182984e-05  1.14787221e-05  7.27923168e-06 ... -7.37315137e-30
   3.44435027e-31  1.63295342e-30]]
```

Simulated EW correlation + Pearson variance covariance using PCA(50%):

```
[[ 6.70151081e-05  1.17042044e-05  1.12876991e-05 ... -5.89118624e-29
   1.48869084e-29 -1.19798509e-30]
 [ 7.30667432e-05  1.10267606e-05 -4.76566484e-05 ...  5.67499018e-29
  -1.10972397e-30 -8.55718247e-30]
 [-5.20943037e-05  9.99955357e-06  2.60707881e-05 ... -5.78051037e-29
  -1.92422065e-29 -1.84414712e-30]
 ...
 [ 8.45879082e-06  1.10555837e-05  1.55564271e-05 ...  3.43424829e-29
  -6.12084981e-29 -5.40785797e-30]
 [-6.00429739e-05  2.58940481e-06  1.21959597e-05 ... -1.00689313e-29
  -1.46246081e-29 -1.84992000e-30]
 [ 1.29301178e-05 -1.04848708e-05  2.45685996e-05 ...  7.04821807e-29
   3.85955413e-29 -7.49119041e-30]]
```

Stimulated covariance is compared with it's input matrix using Frobenius Norm:

Frobenius Norm for Pearson correlation + Pearson variance, Pearson correlation + EW variance, EW correlation + Pearson variance, and EW correlation + EW variance respectively under Direct Simulation:

```
Frobenius Norm:  1.9819513161108095
Frobenius Norm:  1.7932251863715762
Frobenius Norm:  1.9776976470016878
Frobenius Norm:  1.9776976470016878
```

Frobenius Norm for Pearson correlation + Pearson variance, Pearson correlation + EW variance, EW correlation + Pearson variance, and EW correlation + EW variance respectively under PCA(100%):

```
Frobenius Norm:  1.7236780251045427
Frobenius Norm:  0.01435382871868507
Frobenius Norm:  0.011515246869320336
Frobenius Norm:  0.012500217122160946
```

Frobenius Norm for Pearson correlation + Pearson variance, Pearson correlation + EW variance, EW correlation + Pearson variance, and EW correlation + EW variance respectively under PCA(75%):

```
Frobenius Norm:  0.009872448326144617
Frobenius Norm:  0.01435586509229508
Frobenius Norm:  0.011506146544224749
Frobenius Norm:  0.01250634931603419
```

Frobenius Norm for Pearson correlation + Pearson variance, Pearson correlation + EW variance, EW correlation + Pearson variance, and EW correlation + EW variance respectively under PCA(50%):

```
Frobenius Norm:  0.009875604396548063
Frobenius Norm:  0.014368676706386209
Frobenius Norm:  0.011508157885362743
Frobenius Norm:  0.012499768340143977
```

The run time for simulation of Pearson correlation + Pearson variance covariance using Direct Simulation is 0.053775 seconds:

..

```
Run time: 0.05377531051635742 seconds
```

The run time for simulation of Pearson correlation + EW variance covariance using Direct Simulation is 0.050666 seconds:

```
Run time: 0.05066680908203125 seconds
```

The run time for simulation of EW correlation + EW variance covariance using Direct Simulation is 0.050260 seconds:

```
Run time: 0.050260305404663086 seconds
```

The run time for simulation of EW correlation + Pearson variance covariance using Direct Simulation is 0.050260 seconds:

```
Run time: 0.052086591720581055 seconds
```

The run time for simulation of Pearson correlation + Pearson variance covariance using PCA(100%) is 0.072336 seconds:

```
Run time: 0.07233691215515137 seconds
```

The run time for simulation of Pearson correlation + EW variance covariance using PCA(100%) is 0.072336 seconds:

```
Run time: 0.0713033676147461 seconds
```

The run time for simulation of EW correlation + EW variance covariance using PCA(100%) is 0.071479 seconds:

```
Run time: 0.07147955894470215 seconds
```

The run time for simulation of EW correlation + Pearson variance covariance using PCA(100%) is 0.071479 seconds:

```
Run time: 0.07330965995788574 seconds
```

The run time for simulation of Pearson correlation + Pearson variance covariance using PCA(75%) is 0.068168 seconds:

```
Run time: 0.06816816329956055 seconds
```

The run time for simulation of Pearson correlation + EW variance covariance using PCA(75%) is 0.063912 seconds:

```
Run time: 0.06391263008117676 seconds
```

The run time for simulation of EW correlation + EW variance covariance using PCA(75%) is 0.063912 seconds:

```
Run time: 0.0670015811920166 seconds
```

The run time for simulation of EW correlation + Pearson variance covariance using PCA(75%) is 0.063912 seconds:

```
Run time: 0.06541967391967773 seconds
```

The run time for simulation of Pearson correlation + Pearson variance covariance using PCA(50%) is 0.068168 seconds:

```
Run time: 0.06903243064880371 seconds
```

The run time for simulation of Pearson correlation + EW variance covariance using PCA(50%) is 0.070382 seconds:

```
Run time: 0.07038259506225586 seconds
```

The run time for simulation of EW correlation + EW variance covariance using PCA(50%) is 0.071623 seconds:

```
Run time: 0.07162356376647949 seconds
```

The run time for simulation of EW correlation + Pearson variance covariance using PCA(50%) is 0.068779 seconds:

```
Run time: 0.06877923011779785 seconds
```

Frobenius Norm for all four covariance matrices under direct simulation(about 2) is larger than the Frobenius Norm for other covariance matrices under PCA(about 0.01). It indicates that the PCA generated a covariance matrix more accurately than direct simulation. Also, the run time for four covariance matrices under the direct simulation is about 0.05 seconds, while the run time for covariance matrices under PCA methods is about 0.07 seconds. Therefore, there may be a tradeoff between time to run and accuracy. A shorter run time will lead to less accuracy and vice versa. This may be because the increased simulation accuracy may need more calculation, so it would take a longer time to get the result.

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