Q1:

The time to maturity using calendar days is 14/365 which is about 0.0384.

```
Time to Maturity is: 0.038356164383561646
```

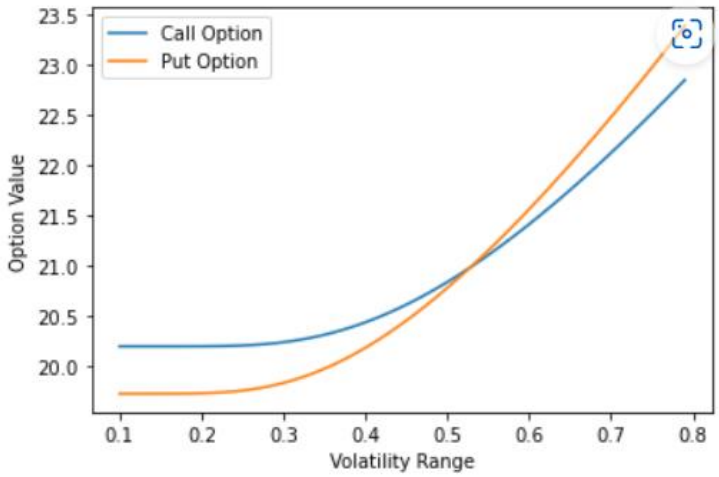
The value of the call is shown below if the strike price for the call is 145 and the implied volatility changes from 10% to 80% with the difference of 1% between each change.

```
[20.20263825343511, 20.2026382537764, 20.202638261809398, 20.202638356730773, 20.202639033177178, 20.202642333530264, 20.202654396052964, 20.20268964838479, 20.202776082453966, 20.202960395335225, 20.20331168303548, 20.203922808647434, 20.20490922506184, 20.20640560706076, 20.208560981596065, 20.21153313104685, 20.215482954832737, 20.220569298350682, 20.226944564385718, 20.234751252390794, 20.244119442817407, 20.25516515880932, 20.26798948974175, 20.28267834115229, 20.299302674597698, 20.31791911142281, 20.33857079080252, 20.361288390906367, 20.38609124025 99, 20.412988462992672, 20.44198011603939, 20.47305828832549, 20.506208141617208, 20.541408880284536, 20.578634643010304, 20.617855313781746, 20.659037252610148, 20.70214394858573, 20.747136599307822, 20.793974621607646, 20.842616098944603, 20.893018171023215, 20.945137371131068, 20.99892991650586, 21.054351956756562, 21.111359785021946, 21.16991001618237, 21.2299597360622, 21.291466625190637, 21.354389060328486, 21.41868619663481, 21.484318033029993, 21.551245463027428, 21.619430313039558, 21.688835369928626, 21.759424399357172, 21.831162156302042, 21.904014388924807, 21.977947836840116, 22.05293022468797, 22.128930251798963, 22.205917578634967, 22.283862810597626, 22.36273747971464, 22.442514024644154, 22.523165769375666, 22.604666900950477, 22.68699244647975, 22.770118249694846, 22.85402094723038]
```

The value of the call is shown below if the strike price for the put is 185 and the implied volatility changes from 10% to 80% with the difference of 1% between each change.

```
[19.732209387421904, 19.73220943786646, 19.732209990277056, 19.732213563218977, 19.732229279698288, 19.73228111522016, 19.732418468651332, 19.732725823212604, 19.73332827851695, 19.734391333686034, 19.73611523620923, 19.738725516799576, 19.742461754298233, 19.747566380005622, 19.75427478662496, 19.762807425629006, 19.773364103807694, 19.786120369911544, 19.801225703545896, 19.81880314614017, 19.838950010937594, 19.86173934509435, 19.88722187055407, 19.915428187698012, 19.94637107920667, 19.980047797513095, 20.016442256601806, 20.05552707803642, 20.097265462982648, 20.141612877867658, 20.18851855240993, 20.23792679616659, 20.289778144401026, 20.34401034672186, 20.400559213168208, 20.459359332 66321, 20.520344678352927, 20.58344911354655, 20.64860681093529, 20.715752596623474, 20.784822229333997, 20.855752624004793, 20.928482027910945, 21.00295015644113, 21.079098294745194, 21.156869370644387, 21.236208003463958, 21.317060532799104, 21.399375030655559, 21.483101299908 47, 21.568190861590438, 21.654596933144234, 21.74227439945082, 21.831179778164085, 21.92127118064417, 22.012508269574028, 22.104852214168204, 22.198265643731787, 22.29271260019962, 22.388158490175044, 22.484570036[截图(Alt + A)]31915232477726, 22.6801632907097, 22.77928460063812, 22.879250681105717, 22.980034136389037, 23.08160861303753, 23.183948757988375, 23.28703017801172, 23.39082940051766]
```

Here is the graph of the value of the call and the pull under different volatility:



The value of the call is shown below if the strike price for the call is 185 and the implied volatility changes from 10% to 80% with the difference of 1% between each change.

```
[2.2387951172077294e-09, 5.2683359642551386e-08, 6.050939401202346e-07, 4.178035815414734e-06, 1.9894515159178238e-05, 7.173003704136031e-05, 0.00020908346819131438, 0.0005164380294934365, 0.001118893333810031, 0.0021819485029210828, 0.0039058510261217316, 0.006516131616473242, 0.010252369115114734, 0.015356994822515402, 0.022065401441829335, 0.030598040445890762, 0.041154718624552444, 0.053910984728419376, 0.069016 31836274857, 0.08659376095705618, 0.10674062575448495, 0.12952995991124006, 0.15501248537097734, 0.18321880251490 62, 0.21416169402355134, 0.24783841232996018, 0.2842328714186646, 0.3233176928532906, 0.36505607779954374, 0.40940349268453247, 0.4563091672267774, 0.505717410983463, 0.5575687592179328, 0.6118009615387372, 0.6683498279851037, 0.7271499474800684, 0.7881352931697947, 0.8512397283634243, 0.916397425752173, 0.9835432114403595, 1.0526128441508824, 1.1235432388216857, 1.1962726427278128, 1.2707407712579872, 1.3468889095620753, 1.4246599854612576, 1.5039986182808285, 1.5848511476160034, 1.6671656454724548, 1.7508919147253188, 1.8359814764073121, 1.9223875479611436, 2.010065014267731, 2.098970392980977, 2.18906179546104, 2.2802988843908985, 2.3726428289850503, 2.4660562585486616, 2.5605032150164817, 2.6559491049918975, 2.752360651713019, 2.8497058472945938, 2.9479539055265604, 3.0470752154550027, 3.147041295922584, 3.2478247512059397, 3.3493992278543914, 3.451739372805257, 3.554820792828572, 3.658620015334556]
```

The value of the call is shown below if the strike price for the put is 145 and the implied volatility changes from 10% to 80% with the difference of 1% between each change.

```
[5.5510616778175994e-12, 3.468356454859508e-10, 8.379827173221607e-09, 1.0330124318509267e-07, 7.797476269147093e-07, 4.080100718132891e-06,
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56184962, 0.03211299896126607, 0.04148118938784773, 0.052526905379788325, 0.06535123631219886, 0.08004008772277249, 0.09666442116817597, 0.1
1528085799326337, 0.13593253737296873, 0.1586501374768048, 0.18345298683033384, 0.21035020956311978, 0.23934186260985424, 0.270420034895950
7, 0.30356988818765274, 0.33877062685500725, 0.3759963895807612, 0.4152170603522176, 0.45639899918062987, 0.4995056951561825, 0.544498345878
301, 0.5913363681781103, 0.6399778455150482, 0.6903799175936545, 0.7424991177015166, 0.7962916630763193, 0.8517137033270288, 0.9087215315924
198, 0.9672717627528336, 1.0273214826326686, 1.0888283717610925, 1.1517508068989741, 1.2160479432052789, 1.2816797796004629, 1.3486072095978
763, 1.416792059609996, 1.4861971164990777, 1.5567861459276138, 1.6285239028724803, 1.7013761354952912, 1.775309583410582, 1.850291971258446
5, 1.9262919983694324, 2.0032793252054333, 2.081224557168099, 2.1600992262850873, 2.239875771214617, 2.3205275159461145, 2.4020286475209396,
2.484354193050219, 2.5674799962653054, 2.6513826938008513]
```
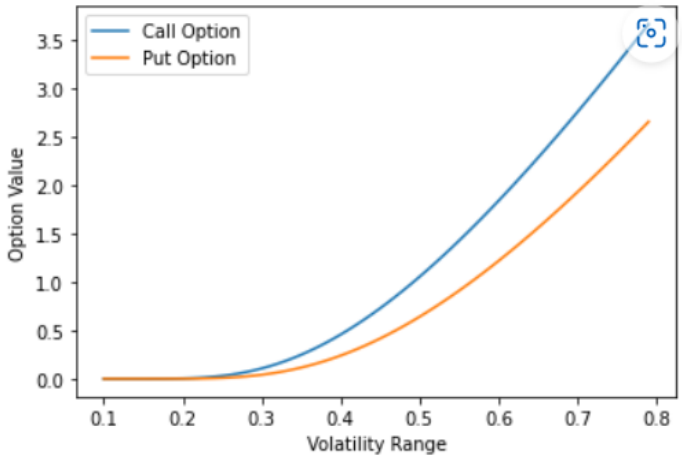

Here is the graph of the value of the call and the pull under different volatility:



From these two graphs, compared with the call option with a strike price smaller than the current stock price, the option value for the call option with a strike price larger than the current stock price is smaller (about 20.6 and 1 when the volatility is 0.5). In contrast, as the volatility increases from 0.1 to 0.8, the increase rate of the option value is larger for the call option with a strike price larger than the current stock price (0 to 3.6 and 20.2 to 22.7). Similarly, compared with the put option with a strike price larger than the current stock price, the option value for the put option with a strike price smaller than the current stock price is smaller (about 20.5 and 0.5 when the volatility is 0.5). In contrast, as the volatility increases from 0.1 to 0.8, the increase rate of the option value is larger for the put option with a strike price smaller than the current stock price (0 to 2.5 and 19.7 to 23.4). The reason behind it is that when the strike price is smaller than the current price for a call option and the strike price is larger than the current price for a put option, these two options are already in the money, which means that if the buyer exercise now, he will gain immediately while for a call option with a strike price larger than the current price and a put option with a strike price smaller than the current price, these two options are out of the money which means that the buyer will not exercise and the option value is zero now. However, when the option is in the money, as the implied volatility increases, it is more possible that the stock price will go below the strike price for a call option, and the stock price will go above the strike price for a put option. The buyer can gain for sure if they exercise now, while they may lose as time passes. Therefore, the option value will not increase a lot. However, when the option is out of the money, as the

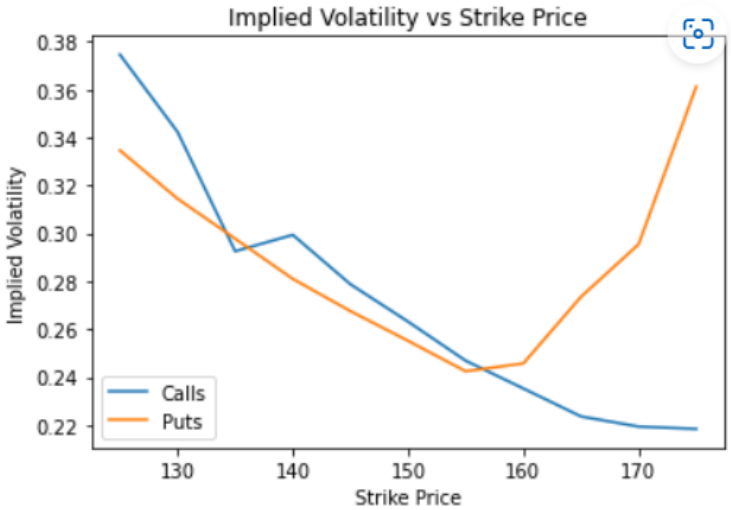
implied volatility increases, it is more possible that the stock price will go above the strike price for a call option, and the stock price will go below the strike price for a put option. The buyer cannot gain if they exercise now (they will not exercise), while they may gain as time passes. Therefore, the option value will increase a lot.

As the supply decreases and demand increases, the implied volatility will tend to increase since such an option becomes popular so that people are willing to pay more to buy it. Therefore, people holding the option are eager to sell the option at a high price which causes the price to go up quickly and increases the volatility of the price as well. Theoretically, such a price can increase forever to infinity, and the volatility can be huge. However, if the supply increases and demand decreases, the implied volatility will tend to decrease since such an option becomes unattractive, so people are unwilling to buy it. Therefore, people holding the option are willing to sell the option at a lower price while the lowest price is 0, which means that the volatility under this situation is finite and can be small.

Q2:

Here is the implied volatility for each option and the graph for the implied volatility vs strike price for calls and puts:

	Stock	Expiration	Type	Strike	Last Price	ImpliedVol
0	AAPL	2023-04-21	Call	125	27.300	0.374604
1	AAPL	2023-04-21	Call	130	22.575	0.342360
2	AAPL	2023-04-21	Call	135	17.750	0.292533
3	AAPL	2023-04-21	Call	140	13.850	0.299367
4	AAPL	2023-04-21	Call	145	9.975	0.278753
5	AAPL	2023-04-21	Call	150	6.700	0.263151
6	AAPL	2023-04-21	Call	155	4.050	0.246839
7	AAPL	2023-04-21	Call	160	2.210	0.235255
8	AAPL	2023-04-21	Call	165	1.035	0.223583
9	AAPL	2023-04-21	Call	170	0.460	0.219361
10	AAPL	2023-04-21	Call	175	0.195	0.218373
11	AAPL	2023-04-21	Put	125	0.405	0.334626
12	AAPL	2023-04-21	Put	130	0.665	0.314484
13	AAPL	2023-04-21	Put	135	1.120	0.297783
14	AAPL	2023-04-21	Put	140	1.840	0.281005
15	AAPL	2023-04-21	Put	145	3.010	0.267542
16	AAPL	2023-04-21	Put	150	4.750	0.255144
17	AAPL	2023-04-21	Put	155	7.150	0.242428
18	AAPL	2023-04-21	Put	160	10.575	0.245712
19	AAPL	2023-04-21	Put	165	14.925	0.273505
20	AAPL	2023-04-21	Put	170	19.425	0.295425
21	AAPL	2023-04-21	Put	175	24.625	0.361250



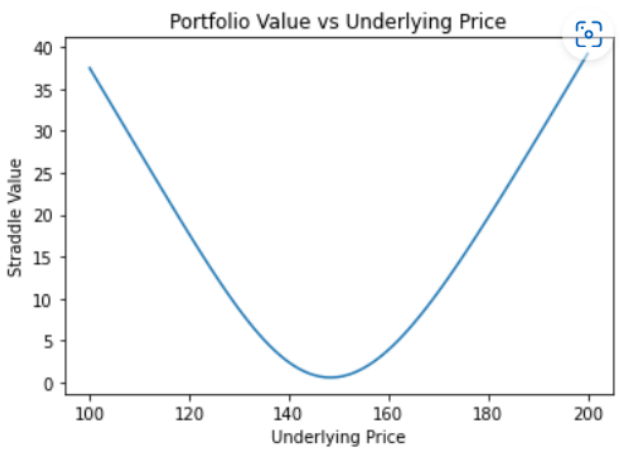
From the graph and the data above, I found that for a call option, when the strike price is below the current stock price, which is 151.03, the implied volatility will fluctuate as the strike price increases, and when the strike price is above the current stock price, the implied volatility will decrease as the strike price increases. For a put option, when the strike price is below the current stock price, the implied volatility will decrease as the strike price increases, while when the strike price is above the current stock price, the implied volatility will turn to increase as the strike price increases.

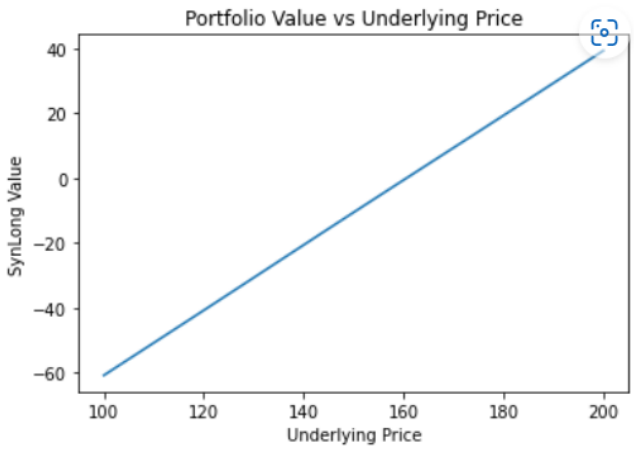
The graph can also tell us some market dynamics. First, when the option is out of the money, the implied volatility tends to be high and will decrease as the strike price move close to the current price. Also, the option's price is another crucial point to determine implied volatility. From the data, when the strike price is 175 for the call option and 125 for the put option, they are both out of the money and about 25 dollars away from the current price of 151.03. People who buy these two options will not exercise them now so that the option values for these two options are 0 to the buyer. However, the price for the call option is 0.195, while 0.405 for the put option. The reason why the buyer is willing to pay differently on two options that have no value to him now is that the implied volatility for the put option is higher (0.334626 for the put and 0.218373 for the call), which means that the stock price has more chances to fluctuate and finally be lower than the strike price and help the buyer to gain profit. Therefore, the buyer will be willing to pay more since he has more chances to earn money.

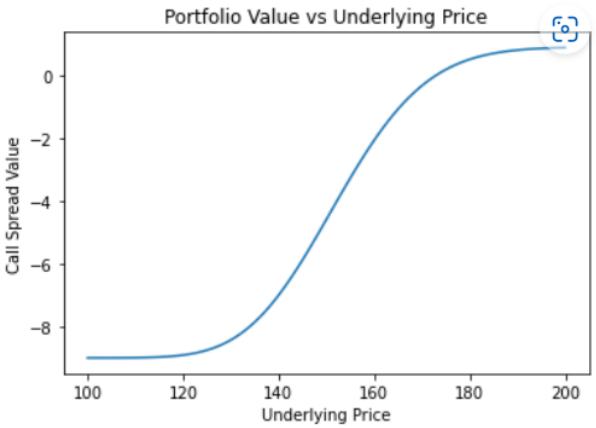
The implied volatility can also show the expectation of people about the stock market. When the option is out of the money now while the price of it is still high, it shows that people still want to buy them since they expect that the market will fluctuate a lot and they still have chances to gain from the option. However, if the option is out of the money now and the price of it is low, it shows that people do not want to own these options since they expect that the market will be steady and the option will not be likely to help them gain.

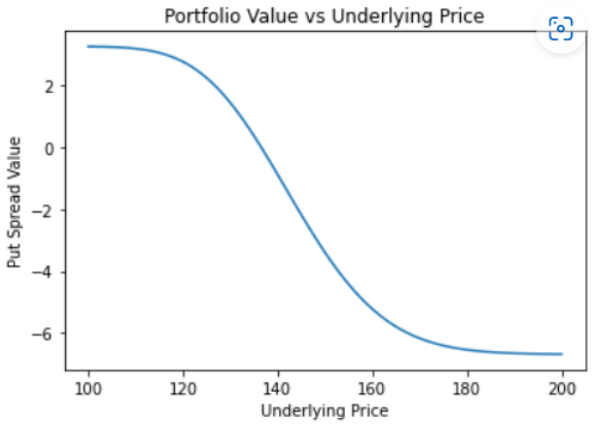
Q3:

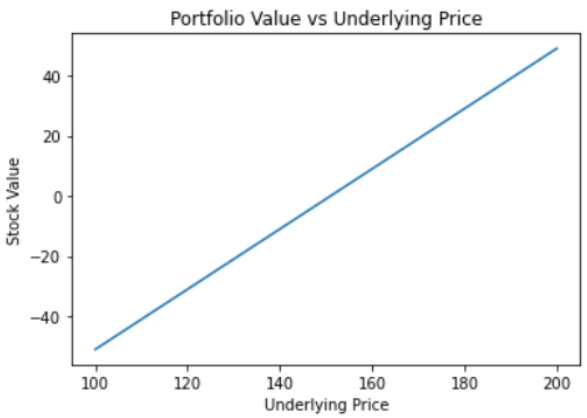
Here are the graphs for different portfolios with the portfolio value over a range of underlying values:

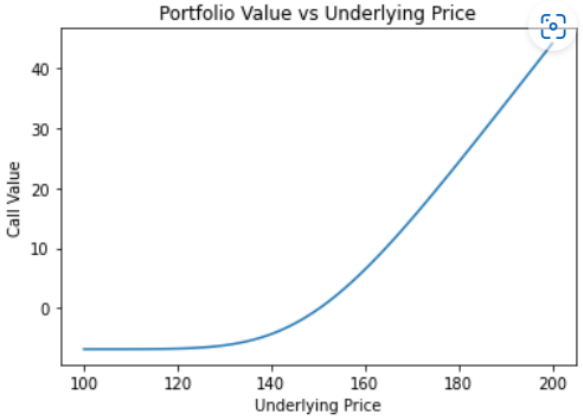


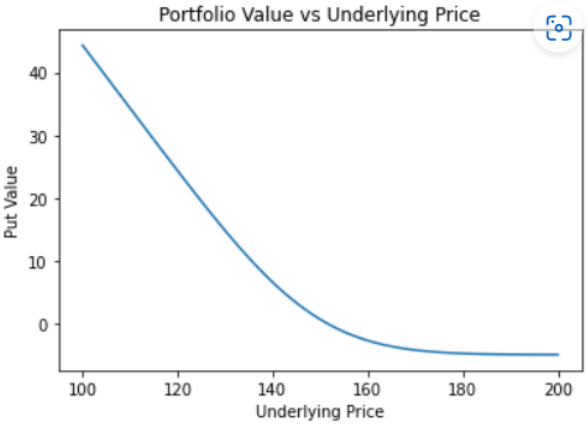


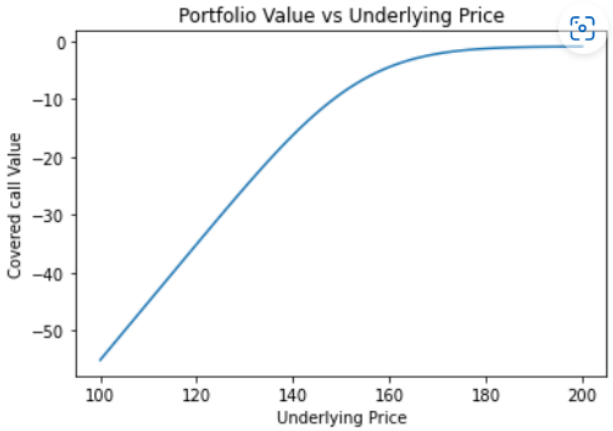

Portfolio Value vs Underlying Price
Call Spread Value
0
−2
−4
−6
−8
100
120
140
160
180
200
Underlying Price

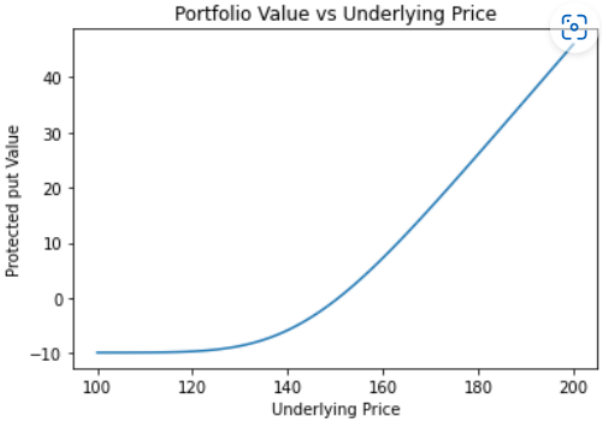

Portfolio Value vs Underlying Price
Put Spread Value
2
0
−2
−4
−6
100
120
140
160
180
200
Underlying Price


Portfolio Value vs Underlying Price
Stock Value
40
20
0
−20
−40
100
120
140
160
180
200
Underlying Price


Portfolio Value vs Underlying Price
Call Value
40
30
20
10
0
100
120
140
160
180
200
Underlying Price


Portfolio Value vs Underlying Price
Put Value
40
30
20
10
0
100
120
140
160
180
200
Underlying Price


Portfolio Value vs Underlying Price
Covered call Value
0
−10
−20
−30
−40
−50
100
120
140
160
180
200
Underlying Price


Portfolio Value vs Underlying Price
Protected put Value
40
30
20
10
0
−10
100
120
140
160
180
200
Underlying Price

Straddle:
For the shape of a straddle, when the price of the underlying asset is close to the strike price of the option, the value of the portfolio will be low since benefits from the call or the put will be low, which may not even cover the cost of buying the call and the put. However, when the underlying asset's price is far away from the option's strike price, exercising the call or the put will help the investor gain a lot, which is shown in the graph that looks like a “V”.

SynLong:
For SynLong, it involves long a call and short a put. When the underlying asset's price is low, the investor will not exercise the call while losing a lot of money from shorting the put. Therefore, the loss can be very large. However, if the underlying asset's price is high, the investor will exercise the call and not pay for shorting the put since people who long the put will not exercise. Therefore, he will earn a lot. When the price is low, the holder of SynLong loses money, and if the price is high, the holder earns money which is shown in the graph.

Call spread:
For the call spread, it involves long a call with a lower strike price and short a call with a higher strike price. When the underlying asset's price is low, the investor will not exercise the first call and not pay for the second call since people who long this call will not exercise it. Therefore, the investor will gain/lose from the difference in the price between these two calls. However, when the price goes up, the investor will exercise the first call and earn money, while he will also pay for the second call since people who long this call will also exercise it. Therefore, the earnings will increase but limit to a value which is shown in the graph.

Put spread:
For the put spread, it is similar to the call price but has a reverse direction. Therefore, the graph of the put spread looks like a reverse version of the call spread in the graph.

Stock:
For the stock, it is obvious that when the stock price goes up, the investor earns money, and if it goes down, the investor loses money which is shown in the graph.

Call:
For the call, when the price of the underlying asset is low, the investor will not exercise it so that the value will be the cost of long a call, while when the price goes up, the investor will exercise it and earn money which is shown in the graph.

Put:
For the put, when the price of the underlying asset is high, the investor will not exercise it so that the value will be the cost of long a put while when the price goes down, the investor will exercise it and earn money which is shown in the graph.

Covered call:
For the covered call, it involves long a stock and short a call. When the underlying asset's price

is high, investors will gain from holding stock but pay for the call since people who long the call will exercise. Therefore, the gain from the covered call will be limited to a specific value. However, when the price of the underlying asset goes down, people will lose from holding the stock, so the lose can be really huge, which is shown in the graph.

Protected put:

For the protected put, it involves long a stock and long a put. When the price of the underlying asset is high, investors will gain from holding stock and not exercise the put. However, when the price of the underlying asset goes down, people will lose from holding a stock while gaining from holding a put. Therefore, the loss can be hedged and limited to a specific value.

Links to other topics:

These graphs can be tied to topics like VaR and the expected shortfall that we learned before. For example, for a long straddle, it involves buying a call and a put, so the cost of such a strategy will be relatively high. If the underlying asset's price remains constant or fluctuates just a little bit, the holder of the straddle will lose a lot since they will gain a little bit or even nothing from exercising the option but pay a lot for both options. Therefore, the VaR and expected shortfall will be relatively significant, which is shown in the graph.

However, a protective put involves buying a stock and a put. When the stock price goes down, the buyer will exercise the put option to hedge the loss from the stock. If the stock price increases, the buyer will not exercise the put option but gain from the stock directly. Therefore, whatever the stock price goes up or down, the holder of the protective put will have a relatively low risk of losing a lot so that the VaR and expected shortfall will be relatively smaller than the straddle strategy, which is shown in the graph that the maximum loss whatever the stock price is will be around -10.

Here are Mean, VaR and ES for

Portfolio	Mean	VaR (95%)	ES
Straddle	1.2899	0.5705	0.5686
SynLong	-9.5168	16.9741	18.8240
Call Spread	-4.2880	6.1999	6.6069
Put Spread	-3.5597	5.0398	5.3221
Stock	0.0792	7.3706	9.2100
Call	0.7365	3.0627	3.6876
Put	0.5534	2.2803	2.7522
Covered Call	-8.7257	13.3617	14.8088
Protected Pull	0.5920	4.1285	4.9821

The result matches the graph shown above.

For the straddle, the loss from holding it can be hedged from long both a call and a put. When the stock price fluctuates around the strike price, the loss will be relatively small and

smaller than the cost of long a call and a put. Therefore, the VaR and ES are quite small.
For the SynLong, if the stock price goes down, you will not exercise the call but pay for the holder of the put you short, which will cause the loss to be relatively significant. However, since the stock's minimum price is 0, the loss is still limited. Therefore, the VaR and ES are pretty large for SynLong.

For the call spread and put spread, there VaR and ES are between the straddle and SynLong. That is because both the gain and loss from holding it will be locked between two specific boundaries, which lock the risk as well.

For the stock, the VaR and ES are quite large as well but a little bit smaller than SynLong. This is because when the stock price is low, people holding the stock will only lose from the stock, but people holding the SynLong will not only lose from the stock but also pay for the call.

For the call and the put, VaR and ES are smaller for the put than call. That is because the minimum price of a stock is 0, but the maximum price of a stock is infinity. Therefore, the VaR and ES for a call and a put will be different and larger for a call.

For the covered call, VaR and ES are quite big since when the stock price goes down, the stock holder will lose a lot of money and cannot gain from shorting a call.

For the protected put, VaR and ES are much smaller since when the stock price goes down, holding the put option will hedge the downside risk from holding the stock.