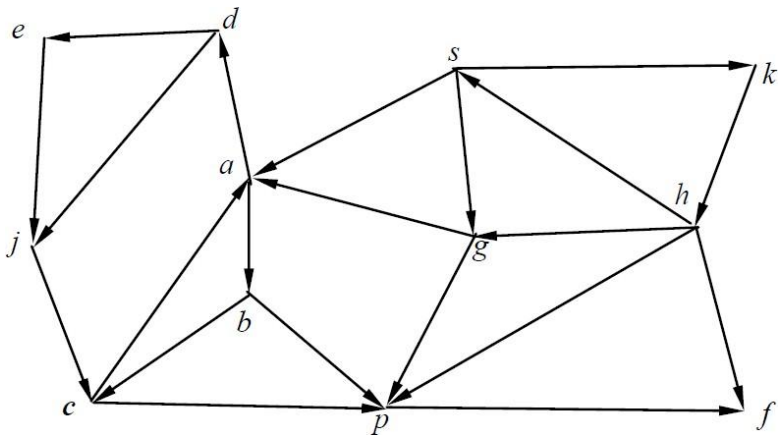
Due Date: 26th Nov 2023
20% penalty for 1 day late
40% penalty for 2 days late
Submission not allowed afterwards

CS2009: Design and Analysis of Algorithms (Fall 2023)
Assignment 4
Total Marks: 100

1. a) Conduct a DFS for the following graph. (Label each vertex u with the start time and the finish time) or (Show all steps showing stack and Visit Order). You should start the traversal from vertex a, and follow the alphabetic order whenever you need to make choices. [10 Points]



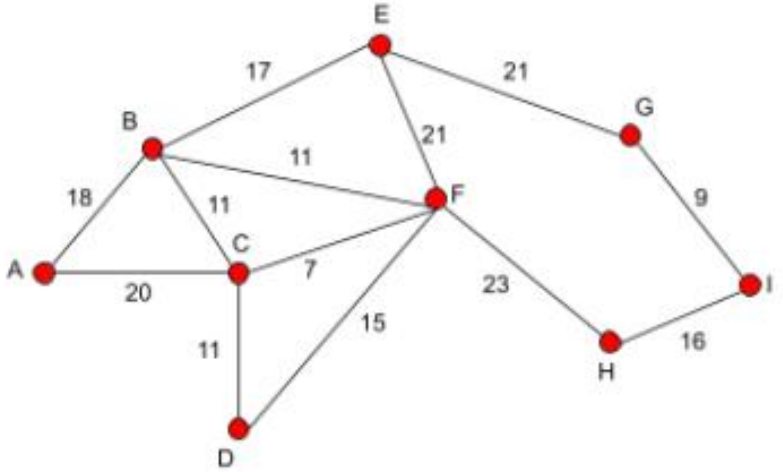
b) List all edges (back edges,forward edges, cross edges) that belong to each of the following sets: [5 Points]

c) Identify the strongly connected components and draw the component graph. [10 Points]

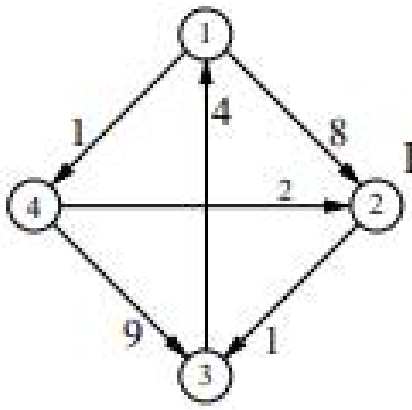
2. a) Does Kruskal's algorithm begin by selecting an edge or node? [5 Points]

(b) The diagram below represents a network of paths in a park. The number on each edge represents the length of the path in metres. Using Kruskal's algorithm, find a minimum spanning tree for the network in the diagram and state its total length. [10 Points]

(c) Find the time complexity of algorithm. [5 Points]



3. Using Floyd-Warshall, find all pairs shortest path following Figure (D^0 weight matrix is also provided). Discuss the its complexity as well [10 Points]

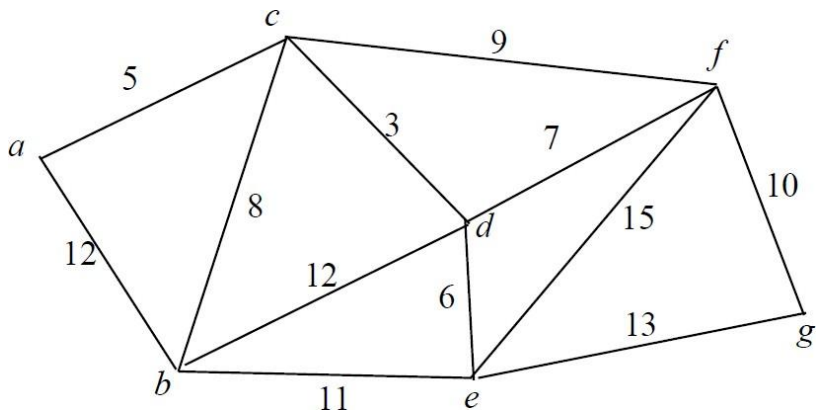


$$D^{(0)} = \begin{bmatrix} 0 & 8 & ? & 1 \\ ? & 0 & 1 & ? \\ 4 & ? & 0 & ? \\ ? & 2 & 9 & 0 \end{bmatrix}$$

? = infinity

4. Go through the lecture https://www.youtube.com/watch?v=2E7MmKv0Y24 and write summary with focus on proof of Dijkstra Algorithm [10 Points]

5. (Longest path problem) Given a weighted (undirected) graph G(V, E), the weight of an edge is called the width of the edge. The longest path is defined to be thelargest weight among all edges on the path. (An edge with the largest weight is called the heavy edge.)



A path P(u, v) is called the longest if the cost of the path is the largest among all paths from u to v.

a) Modify the Dijkstra's algorithm to compute the longest path from a given vertex $s \in V$ to every other vertex. The pseudo code is required. [10 Points]

b) For the above graph, use the algorithm in part (a) to compute the longest path from source node a to each and every other node. You need to show each step, including the initialization step. Also show the final widest path tree. [10 Points]

6. Word search puzzle problem: Given the following 2d, 4x5 grid of letters

O F O O T
V O Q U O
E O I H O
R T G H F

a) Find the word "foot" in the grid.
b) The word may be formed in any direction - up, down, left or right (not diagonals!) but all of the letters in a word must occur consecutively. Assuming the grid starts in the upper left corner with position (0,0) , and that the row is the first coordinate, “foot” would be found at 3 places in the grid: [(0,1) to (0,4)] ,[(4,4) to (0,4)] , and [(0,1) to (4,2)].
c) Assume you are provided with the above word search puzzle grid. Write a algorithm to efficiently search all occurrences of the word. Display all the coordinates (row and column) of the starting and ending positions of word occurrence. [15 Points]