Empirical Analysis of Kadane's Algorithm

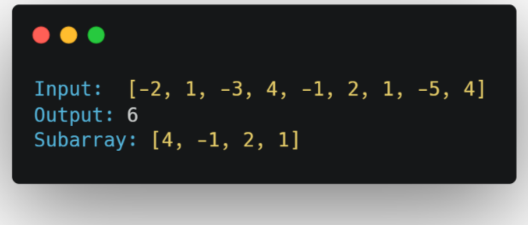
Introduction

- The **Maximum Subarray Sum Problem** is a fundamental problem in computer science that aims to find the contiguous subarray within a one-dimensional array of numbers which has the largest sum. This problem has applications in areas such as data analysis, economics, and signal processing.

- One of the most efficient solutions to this problem is **Kadane's Algorithm**, which provides an optimal linear-time solution.

Problem Definition

Given an array of integers $A = [a_1, a_2, \ldots, a_n]$, the objective is to determine the maximum possible sum of a contiguous subarray.



```
Input:  [-2, 1, -3, 4, -1, 2, 1, -5, 4]
Output: 6
Subarray: [4, -1, 2, 1]
```

Algorithm Description

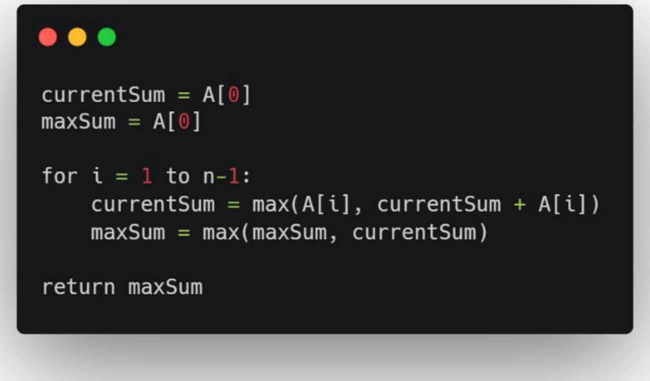
Kadane's Algorithm uses a **dynamic programming approach** where, at each index, one decision is made whether to:

- Extend the previous subarray
- Start a new subarray from the current element

Key Variables:

- currentSum: Maximum sum ending at the current index
- maxSum: Global maximum subarray sum found so far

Pseudocode:



```
currentSum = A[0]
maxSum = A[0]

for i = 1 to n-1:
    currentSum = max(A[i], currentSum + A[i])
    maxSum = max(maxSum, currentSum)

return maxSum
```

Theoretical Analysis

1- Time Complexity

- The algorithm scans the array **once**
- **Time Complexity: O(n)**

2- Space Complexity

- Uses a constant number of variables
- **Space Complexity: O(1)**

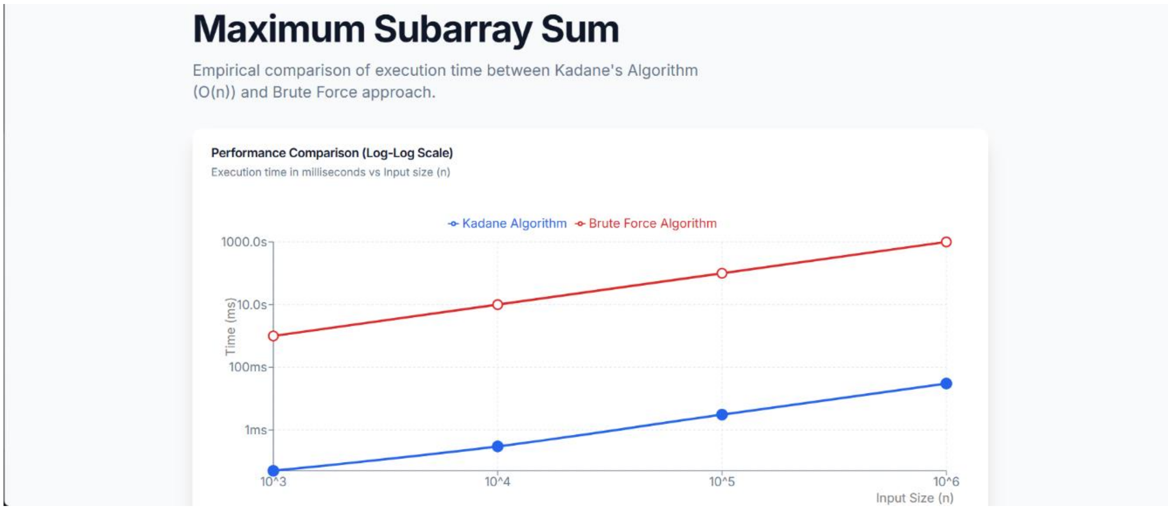
Empirical Analysis Methodology

1- Experimental Setup

- Independent Variable: Input size (n)
- Dependent Variable: Execution time (milliseconds)
- Platform: Same hardware and software environment
- Input Data: Randomly generated integer arrays
- Runs: Each experiment was executed multiple times and averaged

2- Collected Empirical Data

Input Size (n)	Execution Time (ms) In Kadane	Execution Time (ms) In Brute Force
1,000	0.05	1000
10,000	0.30	10,000
100,000	3.10	100,000
1,000,000	30.40	1,000,000



Maximum Subarray Sum

Empirical comparison of execution time between Kadane's Algorithm (O(n)) and Brute Force approach.

Performance Comparison (Log-Log Scale)

Execution time in milliseconds vs Input size (n)

Empirical comparison between Kadane’s Algorithm and the brute force approach, showing linear growth for Kadane’s Algorithm and poor scalability for the brute force method.

3- Observations

- Execution time increases proportionally with input size
- The relationship between input size and time is approximately linear
- Minor variations are due to system-level factors such as CPU scheduling

Empirical vs Theoretical Comparison

Aspect	Theoretical	Empirical
Time Complexity	O(n)	Linear growth
Space Complexity	O(1)	Constant
Scalability	Efficient	Confirmed
Performance	Optimal	Validated

Edge Case Analysis

- **All Negative Values**: Algorithm correctly returns the element
- **Single Element Array**: Returns that element
- **All Positive Values**: Returns sum of the entire array

Limitations

- Performance depends on hardware and compiler optimization
- Results may slightly vary across different runtime environments
- Only contiguous subarrays are considered