NUMERICAL METHODS AND OPTIMIZATION

TUTO3 TOOLBOX

Methods complexity

SOLVING COST

- Diagonal system : **n** disvisions

- Triangular system: **Forward** or **Backward** resolution

n divisions and $\boldsymbol{\frac{n(n-1)}{2}}$ addition/multiplication, so in total:

$$n + 2 \times \frac{n(n-1)}{2} = n^2$$

GAUSS METHOD COST

Let A be a matrix **mxm**, then we have:

step	addition	multiplication	division
1	$(m-1)^2$	$(m-1)^2$	$m-1$
2	$(m-2)^2$	$(m-2)^2$	$m-2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$m-1$	1	1	1
total :	$6^{-1}m(m-1)(2m-1)$	$6^{-1}m(m-1)(2m-1)$	$2^{-1}n(n-1)$

Computational complexity $O\left(\frac{2n^3}{3}\right)$

LU DECOMPOSITION METHOD COST

Let A be a matrix **nxn**, then we have:

Number of divisions: $(n-1)+(n-2)+\cdots+1=n(n-1)/2$

Number of multiplications $(n-1)^2+(n-2)^2+\ldots+(1)^2=\frac{n^3}{3}-\frac{n^2}{2}+\frac{n}{6}$

Number of subtractions: $(n-1)^2+(n-2)^2+\ldots+(1)^2=\frac{n^3}{3}-\frac{n^2}{2}+\frac{n}{6}$

Computational complexity : $O\left(\frac{2n^3}{3}\right)$

CHOLESKY DECOMPOSITION METHOD COST

- Square roots : **n**
- Divisions: $\frac{\boldsymbol{n(n-1)}}{\boldsymbol{2}}$
- $\boldsymbol{2\sum_{j=1}^{n}(j-1)(n-j)}$ addition and multiplication cost
- $\boldsymbol{\sum_{j=1}^{n}(j-1)}$ addition and multiplication in the square root
- The total is $\boldsymbol{O\left(\frac{n^3}{3}\right)}$ which is the half of gauss cost.

CHOLESKY ALGORITHM DESCRIPTION

Input data: a symmetric positive definite matrix A whose elements are denoted by a_{ij}).

Output data: the lower triangular matrix L whose elements are denoted by l_{ij}).

The Cholesky algorithm can be represented in the form

$$
\begin{aligned}
l_{11} &= \sqrt{a_{11}}, \\
l_{j1} &= \frac{a_{j1}}{l_{11}}, \quad j \in [2, n], \\
l_{ii} &= \sqrt{a_{ii} - \sum_{p=1}^{i-1} l_{ip}^2}, \quad i \in [2, n], \\
l_{ji} &= \left(a_{ji} - \sum_{p=1}^{i-1} l_{ip} l_{jp} \right) / l_{ii}, \quad i \in [2, n-1], j \in [i+1, n].
\end{aligned}
$$