TuTo 2 :

Exercise 1 : Seperate the roots

(1) $(x^3-6x^2+9x)(e^x \sin(x)-1)=0$. $]-\pi,\pi[$.

then $x^3-6x^2+9x=0$ or $e^x\sin(x)-1=0$.

$\Longrightarrow x(x-3)^3=0 \Longrightarrow x_1=0$, $x_2=3$ (double) .

$e^x\sin x-1=0$, Graphically we plot $e^x\sin x=1$.

we find 2 intersections : $x_3\in]\frac{1}{4},1[$ and $x_4\in]3.05,\pi[$

(2) $\tan z=z$ $\mathbb{R}_+^*$.

- the graph of f intersects with x-axis ~~infinitely many times~~ at an infinite number of points $z_k\in]k\pi,k\pi+\frac{\pi}{2}[$ $k=1,2,\ldots$

(the graphs are in Toolbox)

Exercise 2 : $e_n=|x_n-\alpha|\leq\frac{b-a}{2^n}$.

(1) Calculate $\sqrt{2}$ with bisection method . , we solve $x^2=2$ $\Rightarrow x^2-2=0$

we have $a<\sqrt{2}<b$

(a) we choose m between a and b

(b) we calculate m^2 .

(c) if $m^2>2$, replace b by m, else replace a by m .

we take $a=1$, $b=2$., $m=1,5$. and $f(x)=x^2-2$.

$f(1)<0$ and $f(2)>0$, $f(1,5)>0$. (here $2,25>2$)

then : $b=1,5$, $a=1$, $m=1,25$.

$f(1,25)=-0,43$, then $a=1.25$ and $b=1.5$, $m=1.375$

$f(1.375)=-0.109<0$, then :

$a=1.375$, $b=1.5$, $m=1.4375$,

$f(m)=0.0664>0$, then : $a=1.375$, $b=1.4375$

$m=1.40625$.

$\boxed{\sqrt{2}\simeq 1.40625}$

(2) The equation after substitution :

$$\underbrace{2.4700202 4414 \times 10^{10} - N - \sqrt{N^2 + 1.542564 \times 10^{20}}}_{f(N)} = 0$$

we plot the graph of f we find 1 root $N^* \in [0, 2\times10^{10}]$.
we take $a=0$, $b=2\times10^{10}$, we perform 3 iterations.
(Graph is in Toolbox).

k	a	m	b	f(a)	f(m)	f(b)
0	0	10^{10}	2×10^{10}	+	−	−
1	0	0.5×10^{10}	10^{10}	+	+	−
2	0.5×10^{10}	0.75×10^{10}	10^{10}	+	+	−
3	0.75×10^{10}	0.875×10^{10}	10^{10}			

$\to N^*$

(3) Approximation accurate to 1% $\Rightarrow e_R \leq 0.01$

$$e_R \leq \frac{b-a}{2^{n+1}}, \quad N^* \simeq \frac{a+b}{2} \Rightarrow \frac{e_A}{N^*} \leq 0.01$$

do it again avec e_A only!

$$e_R \leq \frac{b-a}{2^{n+1}} \leq 0.01$$

$$2^{n-1} \geq \frac{100(b-a)}{\cancel{(b+a)}} \to (n-1)\ln 2 \geq \ln\left(\frac{100(b-a)}{b+a}\right)$$

$$n \geq \frac{\ln\left[\frac{100(b-a)}{\cancel{(b+a)}}\right]}{\ln 2} \cancel{+1} = 40.64$$

$\boxed{n = 41 \text{ iterations}}$

Exercise 3 :

(1) the equation $f(x) = \frac{x^2}{4} - \sin(x) = 0$.

(a) we verify that $f(x) = 0$ is equivalent to $\varphi(x) = x$ for each $\varphi_i(x)$.

(a) $x = x + \frac{x^2}{4} - \sin x \Leftrightarrow \frac{x^2}{4} = \sin(x)$ ✓

(b) $x = \sin^{-1}\left(\frac{x^2}{4}\right) \Leftrightarrow \sin(x) = \frac{x^2}{4}$ ✓

(c) $x = 2\sqrt{\sin(x)} \Leftrightarrow \frac{x}{2} = \sqrt{\sin(x)} \Leftrightarrow \frac{x^2}{4} = \sin(x)$ ✓

(d) $x = 2\sqrt{\frac{3x^2}{4}\sin x} \Leftrightarrow \frac{x^2}{4} = \frac{3x^2}{4}\sin x$ X.

(b) from the graph we observe that only $|\varphi_3'(x^*)| < 1$.
then we should take the iterations $x_{n+1} = 2\sqrt{\sin(x_n)}$
for the iterations, we know that:

$e_n \leq$ [illegible] $10^{-4} \Longrightarrow$

with $e_n = \frac{|x_n - x_{n-1}|}{|x_n|}$, $n = 7$, $\boxed{x_n = 1{,}933}$

$e_n \leq \frac{K^n}{1-K} |x_1 - x_0| \Rightarrow k$ constante de contraction.

Iterations (in the Tool box).

How to find k: we have ~~f'(x)~~

$\varphi_3'(x) = \frac{1}{\sqrt{\sin(x)}} \cdot \cos(x)$

φ_3' is decreasing $\forall x \in [1.5, 2.5]$.

then:

$\varphi_3'(2{,}5) \leq \varphi_3'(x) \leq \varphi_3'(1{,}5)$.

$\sup |\varphi_3'(x)| \leq \varphi_3'(1{,}5) = k = 0.070649$.

Exercise 4: $e_A = 0.001 = 10^{-3}$, 3 significant digits.

. In order to have neither profit nor a loss, we solve the equation: the cost = the profit.

the profit is $4p$, the cost is $C(p)$.

So we solve:

$$4p = 2p^{2/3} + 3p + 200 \iff \underbrace{2p^{2/3} - p + 200}_{f(p)} = 0.$$

$$f'(p) = \frac{2}{3}p^{-\frac{1}{3}} - 1.$$

Iterations in Toolbox.

$x_0 = 250$.

$$x_1 = 250 - \frac{f(250)}{f'(250)} = 250 + \frac{\cancel{32}.60}{0,9832} = 1.99\ldots$$

~~$x_2 = 1229 - \frac{f(1229)}{f'(1229)}$~~

~~error = 1~~

~~while error > tol.~~

~~$x = x_0$~~

~~$x_0 = x - \frac{f(x)}{f'(x)}$~~

~~error = abs$(x_0 - x)$~~

$p^* = 211.9240$

We take $p = 212$

We choose x_0 such that $f(x_0)\, f''(x_0) > 0$.

error in Newton method : An upper bound.

$$e_n = |x_n - \alpha| \leqslant C\,|x_1 - x_2|^2$$

$$C = \frac{\max_{[a,b]} |f''(x)|}{2\min_{[a,b]} |f'(x)|}.$$