Numerical Methods and Optimization

NMO – 3^{rd} Year

ENSIA — Spring 2024

LAB 3

- Solving systems of linear equations: Direct and Iterative methods -

Objectives

- Manipulate matrices and vectors on *Octave*
- Perform calculations on matrices
- Implement the resolution methods (direct and iterative) seen in the course.

Exercise 1:

Let $M = \begin{pmatrix} 4 & 2 & 0 \\ 2 & 4 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

- **Q1:** Write an Octave function to check if M is symmetric positive definite.
- **Q2:** Write an Octave function to provide LU decomposition of M.
- **Q3:** Write an Octave function to provide Cholesky decomposition of M.
- **Q4:** Write an Octave function to solve the system $Ax = b$ using the previous decomposition such that $b = \begin{pmatrix} 8 \\ 13 \\ 5 \end{pmatrix}$
- **Q5:** Write an Octave function to solve $Ax = b$ using Gauss elimination.

Exercise 2:

For the following system, we will use the methods: Jacobi and Gauss-Seidel, as an initial vector we choose $x_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$:

$$\begin{cases} 4x_1 - x_2 + x_3 = 7 \\ 4x_1 - 8x_2 + x_3 = -21 \\ -2x_1 + x_2 + 5x_3 = 15 \end{cases}$$

- Write a program that initializes the given example and, based on a choice, calls one of the two functions that you must implement according to the following declarations:
  - *function [X,niter] = jacobi(A,b,X0,nmax,tol)*
  - *function [X,niter] = gseidel(A,b,X0,nmax,tol)*

Such that:

- A the matrix of the system.
- b is the data vector.
- X_0 is the initial vector.
- n_{max} is the maximal number of iterations.
- tol is the error for the stopping criterion.