Numerical Methods and Optimization

11/2/2024 – 3^{rd} Year

ENSIA — Spring 2024

Tutorial/LAB 2

- Approximate resolution of nonlinear equations of type $f(x) = 0$ - Part I - Tutorial

Exercise 1: Roots separation

Separate the roots of the following equations using a graphical method (use a calculator or *plot* in Octave), algebraic method (factorize the polynomials), or analytical method (study the variations of the function). Provide the multiplicity of the roots if possible.

(1) $\left(u^3 - 6u^2 + 9u\right)\left(e^u sin(u) - 1\right) = 0$ in $]-\pi, \pi[$ (2) $tanz = z$ in $\mathbb{R}_+^*$

Exercise 2: Bisection Method

(1) Calculate $\sqrt{2}$ with a calculator equipped only with the four basic operations.

(2) The equation $\frac{2}{\rho q \mu_0}\left(\frac{T}{T_0}\right)^{2.42} - N - \sqrt{N^2 + 4n_i^2} = 0$ can be used to determine the doping density N of doped silicon.
We have: $T_0 = 300, T = 1000K, \mu_0 = 1350cm^2V^{-1}s^{-1}, q = 1.7 \times 10^{-9}C, n_i = 6.21 \times 10^9 cm^{-3}$, and $\rho = 6.5 \times 10^6 V s\ cm\ C^{-1}$

a. Perform 3 iterations using the bisection method to obtain the doping density N.

b. How many iterations are needed to obtain an approximation accurate to 1%?

Exercise 3: Method of Successive Iterations (fixed point)

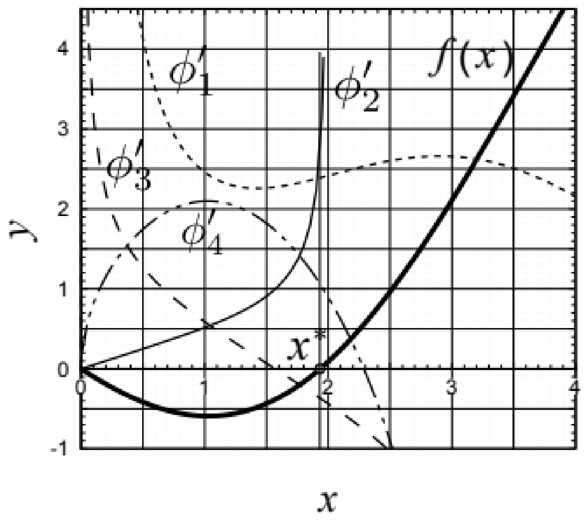
(1) The equation $f(x) = x^{2/4} - sin(x) = 0$ has a unique positive root x^*.

a. Verify that $f(x) = 0$ is equivalent to $x = \phi(x)$, then use the figure below to determine which of the four functions ϕ_i can be used to determine x^* by the method of successive iterations $x_{n+1} = \phi(x_n)$.

b. Determine x^* with 4 correct significant digits by taking $x_0 = 1.5$.

$$\phi_1(x) = x + \frac{x^2}{4} - sin\ x \qquad \phi_2(x) = sin^{-1}\left(\frac{x^2}{4}\right)$$

$$\phi_3(x) = 2\sqrt{sin\ x} \qquad \phi_4(x) = 2\sqrt{\frac{3x^2}{4} sin\ x}$$



Exercise 4: Method of Newton-Raphson

Suppose it costs $C(p)$ dollars to produce p grams per day of a certain chemical sach that:

$$C(p) = 2p^{\frac{1}{3}} + 3p + 200$$

The firm can sell any amount of the chemical at 4$ a gram. Find the break-even point of the firm, that is, how much it should produce per day in order to have neither profit nor a loss (use with an initial guess as $p_0 = 250$ and stop when error reaches 0.1%).

Exercise 5: Secant Method

The oscillating current in an electrical circuit is given by the formula $i = 9e^{-t}cos(2\pi t)$ where time t is expressed in seconds. Determine all the values of t for which $i = 3$ using the secant method.

Part II - LAB

(1) Write **Octave** programs to apply the previous methods seen in the tutorial. We will use a termination test of the form $|x_{n+1} - x_n| < \epsilon$ and ensure to include an iteration counter that will interrupt the process as soon as N_{max} iterations are performed without reaching the precision ϵ.
For example, N_{max} can be set to 100.

(2) Solve the problem of *Execise 4* using the four methods, compare the results, and conclude.