Problem Sheet 1

1. Suppose that Z_1, Z_2, Z_3 are zero mean random variables with

$$\text{var}(Z_1) = 1,\ \text{var}(Z_2) = 2,\ \text{var}(Z_3) = 3,$$

$$\text{cov}(Z_1, Z_2) = -0.5,\ \text{cov}(Z_2, Z_3) = 2.5,\ \text{cov}(Z_1, Z_3) = 0.$$

   Calculate each of the following:

   (a) $\mathrm{E}(Z_1^2 - Z_2 - Z_2 Z_3)$

   (b) $\text{var}(2Z_1 + 3Z_2 - Z_3)$

   (c) $\text{cov}(3Z_1 - Z_2, Z_2 - 2Z_3)$

   (d) $\text{corr}(Z_1, 2Z_2 + Z_3)$

2. Suppose $\{e_t\}$ is a normal white noise process with mean zero and variance σ_e^2. Let $\{Y_t\}$ be a process defined as:

$$Y_t = e_t + \theta e_{t-1}.$$

   (a) Find the autocovariance function and autocorrelation function of Y_t for any general θ. [Hint: Calculate $\text{cov}(Y_t, Y_{t-k})$ case-by-case for several values of k.] Also, find the autocovariance function and autocorrelation function of Y_t if $\theta = 2$.

   (b) Is the time series $\{Y_t\}$ stationary?.

3. Apply a moving average filter to Y_t, where Y_t is the natural logarithm of the Johnson and Johnson earnings data (the original data are given in the `jj` object in the `astsa` package). Specifically, let

$$V_t = \frac{Y_t + Y_{t-1} + Y_{t-2} + Y_{t-3}}{4}.$$

   The R code

```
v = filter(y, rep(1/4, 4), sides = 1)
```

   may be helpful in implementing this. Type `help(filter)` in R for more details about this R function. Plot Y_t as a line and overlay (superimpose) V_t as a dashed line, and provide this plot. Discuss whether the moving average filter captures the overall trend in the time series.

4. Construct a time series plot of the Dubuque temperature data, but include the monthly plotting symbols. Note that the temperature data and the month information are in the tempdub object in the TSA package. Type library(TSA); data(tempdub); print(tempdub) in R to see the data set.

5. (a) Simulate and plot a white noise process $e_t \overset{\sim}{\to}$ iid N(0, 1) of length $n = 100$ using the following commands in R:

```
> wn.n01 = rnorm(100,0,1)
> plot(wn.n01,ylab="White noise process",xlab="Time",type="o")
```

   (b) Repeat part (a) under the assumption that

   - $e_t \overset{\sim}{\to}$ iid $t(1)$
   - $e_t \overset{\sim}{\to}$ iid $\chi^2(4)$

   To do this, just replace the first line of the code above with `wn.t1 = rt(100,1)` and `wn.chisq4 = rchisq(100,4)`, respectively. Comment on the differences among the 3 simulated white noise processes.

(c) Repeat part (a) using $n = 200$, $n = 300$, and $n = 500$. With your plot from part (a), take your 4 standard normal white noise processes and put them in a 22 matrix of plots using the —par(mfrow=c(2,2))— command in R. Label each plot in the matrix according to the sample size used, e.g.,

```
plot(wn.n01.100,ylab="WN",xlab="Time",main="Sample.size=100",type="o"):
```

6. Suppose that Z_1 and Z_2 are uncorrelated random variables with zero mean and unit variance. Consider the process defined by

$$Y_t = Z_1 \cos(\omega t) + Z_2 \sin(\omega t) + e_t,$$

where $e_t \widetilde{\to}$ iid $N(0, \sigma_e^2)$ and $\{e_t\}$ is independent of both Z_1 and Z_2.

(a) Prove that $\{Y_t\}$ is stationary.

(b) Let Z_1 and Z_2 be independent N(0,1) random variables, and set $\sigma_e^2 = 1$ and $\omega = 0.5$. Use the following R commands to simulate n = 150 observations from the $\{Y_t\}$ process:

```
> omega = 0.5
> Z = rnorm(2,0,1)
> e.t = rnorm(150,0,1)
> Y.t = e.t*0
> for (i in 1:length(e.t)){Y.t[i] = Z[1]*cos(omega*i)
        + Z[2]*sin(omega*i) + e.t[i]}
> plot(Y.t,ylab="Trigonometric process",xlab="Time",type="o")
```

Describe the appearance of your time series. the time series appear to have a seasonality

(c) Amend the R code above to simulate a realisation of the process

$$\tilde{Y}_t = \beta_0 + \beta_1 t + Z_1 \cos(\omega t) + Z_2 \sin(\omega t) + e_t,$$

where $\beta_0 = 100$,$\beta_1 = 0.05$, $\sigma_e^2 = 1$ and $\omega = 0.5$. To do this, replace the last three lines of the R code above with

```
> Y.tilde = e.t*0
> for (i in 1:length(e.t)){Y.tilde[i] = 100 + 0.05*i + Z[1]*cos(omega*i)
         + Z[2]*sin(omega*i) + e.t[i]}
> plot(Y.tilde,ylab="Trig process with linear trend",xlab="Time",type="o")
```

Does your $\{\tilde{Y}_t\}$ process appear to be stationary? no What is the effect of adding the linear trend term $\beta_0 + \beta_1 t$ to the model? the mean depends on t

(d) Plot the first differences of your simulated $\{\tilde{Y}_t\}$ process. To do this, use

```
> diff.Y.tilde = diff(Y.tilde)
> plot(diff.Y.tilde,ylab="First differences of Y.tilde",xlab="Time",type="o")
```

Describe the appearance of this first difference process $\{\nabla\tilde{Y}_t\}$. In particular, does it appear to be stationary in the mean level? yes it does Are you surprised? Discuss the behavior of each process: $\{Y_t\}$, $\{\tilde{Y}_t\}$, and $\{\nabla\tilde{Y}_t\}$, and how they relate to each other.

(e) In part (c), suppose that instead of the added linear trend $\beta_0 + \beta_1 t$, the added trend was quadratic, say, $\beta_0 + \beta_1 t + \beta_2 t^2$. Do you think the first differences of the quadratic trend version of the process in part (c) would be stationary? Explain your intuition.

7. **(*)** Suppose $\{e_t\}$ is a normal white noise process with mean zero and variance σ_e^2. Let $\{Y_t\}$ be a process defined as:

$$Y_t = e_t e_{t-1}.$$

Showing all your steps, find the mean function and the autocovariance function of Y_t. Is the time series $\{Y_t\}$ stationary? Explain your answer.

it wouldnt, it would have a linear trend
and a mean depending on t hence not
sationary the first diffrences reduce
one order at a time