The National Higher School of Artificial Intelligence ENSIA

Stochastic Modeling and Simulation Semester 5 2023/2024

Worksheet 2

Exercice 1 *Calculate the n-th arrival time law of a Poisson process in two ways:*

a) *Using* $\{S_n \leq t\} = \{N_t \geq n\}$;

b) *Using* $S_n = T_1 + T_2 + ... + T_n$.

Reminder: X follows the Gamma distribution with parameters $\alpha > 0$ *and* $\beta > 0, x \rightsquigarrow \Gamma(\alpha, \beta))$ *if its density of probability is*

$$f(x) = \frac{x^{\alpha-1}\beta^{\alpha}e^{-\beta x}}{\Gamma(x)}, x > 0.$$

Exercice 2 *On a one-way road, the flow of cars can be described by a Poisson process of intensity* $\lambda = \frac{1}{6}s^{-1}$. *A pedestrian wishing to cross the road needs an interval of at least* $4s$ *between* 2 *successive cars. Calculate :*

1. *the probability that he will have to wait ;*
2. *the mean duration of the intervals that allow it to cross the road.*

Exercice 3 *A medical hotline has only one doctor on duty. Patients arrive at the reception desk according to a Poisson process, separated by intervals of* 20 *minutes (the unit of time being the hour). The mean time taken to treat a patient is fifteen minutes, distributed according to an exponential law.*

1. *What is the mean number of patients in the waiting room?*
2. *What is the mean waiting time?*
3. *Calculate the state probabilities (limiting them to those greater than* 0.05*).*
4. *What is the probability of a patient waiting more than one hour in the waiting room?*
5. *Management is considering putting a second doctor on duty if only one is occupied for more than 75% of the duty time. Should a second doctor be on duty?*

Exercice 4 *A public agency is open every working day from* 9*am to* 5*pm without interruption. It welcomes a mean of* 64 *users a day; one counter is used solely to process each user's file. A statistical study has shown that the random duration of services follows an exponential distribution with a mean of* 2.5 *minutes, and that user arrivals form a Poisson process. It is assumed that steady state is rapidly reached.*

1. *Give the Kendall notation for this queuing system; the mean time spent waiting; the mean time spent in the organization for each user.*
2. *What are the probabilities that no customers will arrive between* 3*pm and* 4*pm? That* 6 *customers arrive between* 4*pm and* 5*pm?*

3. *What is the probability of observing a queue of* 4 *customers, behind the current customer?*

4. *What is the probability that a customer will spend more than* 15 *minutes in the organization?*

Exercice 5 *A bank has two counters, one for commercial accounts and the second for personal accounts. The arrival and service rates at the commercial counter are* 6 *and* 12 *per hour respectively. The corresponding rates at the personal counter are* 12 *and* 24 *per hour respectively. Let's assume that arrivals follow a Poisson process and that service times are exponentially distributed.*

1. *Assuming that the two counters operate independently of each other, determine the mean number of customers on the queue and the mean waiting time for each counter.*

2. *What is the effect of running the two queues as a single two-server queue with an arrival rate of* 18 *per hour and a service rate of* 18 *per hour? What do you conclude in this case?*