Numerical Methods and Optimization

21/4/2024 – 3^{rd} Year

ENSIA — Spring 2024

Tutorial 7

- Numerical integration -

Exercise 1: Newton-Cotes formulas

We define the function $f(x) = \frac{1}{x}$.

(1) Calculate the polynomial $P_1(x)$ of degree less than or equal to 1, which approximates the function f in the sense of continuous least squares over $[\frac{1}{2}, 1]$.

(2) Calculate $\int_{\frac{1}{2}}^{1} \frac{1}{x}\, dx$ with 2 methods.

Exercise 2: Practice problems

(1) Compute the trapezoidal approximation for $\int_0^2 \sqrt{x}\ dx$ using a regular partition with $n = 4$. Compare the estimate with the exact value.

(2) Use Simpsonís rule to approximate $\int_0^2 \sqrt{x}\ dx$ using a regular partition with $n = 4$. Compare the estimate with the exact value.

Exercise 3: Generalized Trapezoidal/Simpson methods

Let the integral $I = \int_0^{\pi} \sin x\ dx$.

(1) Calculate the exact value of I.

(2) Using the generalized trapezoidal method and the generalized Simpson's method for $h = \frac{\pi}{4}$.

(a) Calculate I;

(b) Estimate the error;

(c) Evaluate the error.

(3) Provide the value of step h and the number of subdivisions of the interval $[0, \pi]$ so that the error obtained by the generalized trapezoidal method (resp. Simpson's) is less than 5×10^{-4}.

Exercise 4: A problem

The widths (in meters) of a kidney-shaped swimming pool were measured at 2-meter intervals, as indicated in the figure. Use Simpson's Rule to estimate the area of the pool.