

فصل هجتم

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۴۰۲۱۲۰۲۴۳۳

x_i	۱,۱	۱,۲	۱,۳	۱,۴	۱,۵
$P(x_i)$	۰,۴۱	۰,۷۸	۰,۹۷	۱,۱۸	۱,۴۱

$$f' = f'(1,3)$$

الف) تفاضل پیشرو و نقطه ای

ب) تفاضل پسرو و نقطه ای

ج) تفاضل مرکزی و نقطه ای

$$f'(x) = \frac{f(x+h) - f(x)}{h} \xrightarrow[h=0,1]{x=1,3} f'(1,3) \approx \frac{f(1,4) - f(1,3)}{0,1}$$

$$= \frac{1,18 - 0,97}{0,1} = \frac{0,21}{0,1} = 2,1 \checkmark$$

$$f'(x) \sim \frac{f(x) - f(x-h)}{h} \xrightarrow[h=0,1]{x=1,3} \frac{f(1,3) - f(1,2)}{0,1} = \frac{0,97 - 0,78}{0,1} = 1,9$$

$$f'(x) \sim \frac{f(x+h) - f(x-h)}{2h} \xrightarrow[h=0,1]{x=1,3} \frac{f(1,4) - f(1,2)}{2 \times 0,1} =$$

$$\frac{1,18 - 0,78}{0,2} = 2$$

(۲)

$$\Sigma \times \Sigma =$$

$$\Sigma \wedge \Sigma \wedge \Sigma$$

$$\Sigma \vee \Sigma \vee \Sigma = \Sigma$$

$$1 \vee \Sigma =$$

$$f(x+h) = f(x) + f'(x)h + \frac{f''(x)}{2!}h^2 + \frac{f'''(x)}{3!}h^3 + \frac{f^{(4)}(x)}{4!}h^4 + o(h^4) \quad (*)$$

$$f(x-h) = f(x) - f'(x)h + \frac{f''(x)}{2!}h^2 - \frac{f'''(x)}{3!}h^3 + \frac{f^{(4)}(x)}{4!}h^4 + o(h^4)$$

$$f(x+h) = f(x) + f'(x)h + \frac{f''(x)}{2!}h^2 + \frac{f'''(x)}{3!}h^3 + \frac{f^{(4)}(x)}{4!}h^4 + o(h^4) \quad x, x+h, x+2h$$

$$+ \quad f(x+2h) = f(x) + f'(x)2h + \frac{f''(x)}{2!}(2h)^2 + \frac{f'''(x)}{3!}(2h)^3 + \frac{f^{(4)}(x)}{4!}(2h)^4 + o(h^4)$$

$$f(x+h) = f(x) + f'(x)h + \frac{f''(x)}{2!}h^2 + \frac{f'''(x)}{3!}h^3 + \frac{f^{(4)}(x)}{4!}h^4 + o(h^4)$$

$$+ \quad f(x+3h) = f(x) + f'(x)3h + \frac{f''(x)}{2!}(3h)^2 + \frac{f'''(x)}{3!}(3h)^3 + \frac{f^{(4)}(x)}{4!}(3h)^4 + o(h^4)$$

$$f(x+h) - 3f(x+2h) + 3f(x+h) = (1 - 3 + 3)f(x) +$$

$$f'(x)h - 4f'(x)h + 9f'(x)h + o(h^4)$$

$$\Rightarrow f'(x)h - 4f'(x)h + 9f'(x)h = 0$$

$$\text{Ans } \boxed{f''(x)} = -f(x-2h) + 3f(x-h) - 4f(x) +$$

$$+ \frac{f(x+h)}{2h^2} - f(x+2h)$$

(۳) تقریب از عدد π ، با استفاده از انتگرال زید درست آید

$$\pi = \int_0^1 \frac{x}{x^2+1} dx$$

الف)

$$T_n = \frac{b-a}{n} \left(f(a) + \sum_{i=1}^{n-1} f(x_i) + f(b) \right)$$

$$n=10$$

$$b=1$$

$$a=0$$

$$h = \frac{b-a}{n} = \frac{1}{10}$$

$$h=0.1$$

$$f(x) = \frac{x}{x^2+1} \quad f(0) = 0, \quad f(0.1) = \frac{0.1}{0.1+1} = 0.0909$$

$$f(0.2) = \frac{0.2}{0.2+1} = 0.1667$$

$$f(0.3) = \frac{0.3}{0.3+1} = 0.2308$$

$$f(0.4) = \frac{0.4}{0.4+1} = 0.2857$$

$$f(0.5) = \frac{0.5}{0.5+1} = 0.3333$$

$$f(0.6) = \frac{0.6}{0.6+1} = 0.3846$$

$$f(0.7) = \frac{0.7}{0.7+1} = 0.4375$$

$$f(0.8) = \frac{0.8}{0.8+1} = 0.4762$$

$$f(0.9) = \frac{0.9}{0.9+1} = 0.5143$$

$$f(1) = 0.5$$

$$T_{10} = 0.1 \left(0 + 2(0.0909 + 0.1667 + 0.2308 + 0.2857 + 0.3333 + 0.3846 + 0.4375 + 0.4762 + 0.5143) + 0.5 \right)$$

$$= 0.1 \left(0 + 2(2.5) + 0.5 \right) = 0.1(5.5) = 0.55$$

ب)

$$S_n = \frac{b-a}{n} \left(f(a) + f(x_1) + f(x_2) + \dots + f(x_{n-1}) + f(b) \right)$$

$$S_{10} = \frac{1-0}{10} \left(f(0) + f(0.1) + f(0.2) + f(0.3) + f(0.4) + f(0.5) + f(0.6) + f(0.7) + f(0.8) + f(0.9) + f(1) \right)$$

$$= \frac{1}{10} (0 + 0.0909 + 0.1667 + 0.2308 + 0.2857 + 0.3333 + 0.3846 + 0.4375 + 0.4762 + 0.5143 + 0.5)$$

$$= \frac{1}{10} (3.1098) = 0.31098$$

$$\frac{3.1098 - 3.1416}{0.11329} = -0.284$$

$$u_n = h \sum_{n=1}^n f(a + (i-1)h) \quad h=0.1 \quad c$$

$$h = \frac{b-a}{n} = 0.1$$

$$x_1 = a + (1-0.1) \times 0.1 = 0.1 \quad f(0.1) = 1.914$$

$$x_2 = a + (2-0.1) \times 0.1 = 0.2 \quad f(0.2) = 1.914$$

$$x_3 = a + (3-0.1) \times 0.1 = 0.3 \quad f(0.3) = 1.949$$

$$x_4 = a + (4-0.1) \times 0.1 = 0.4 \quad f(0.4) = 1.949$$

$$x_5 = a + (5-0.1) \times 0.1 = 0.5 \quad f(0.5) = 1.949$$

$$x_6 = a + (6-0.1) \times 0.1 = 0.6 \quad f(0.6) = 1.949$$

$$x_7 = a + (7-0.1) \times 0.1 = 0.7 \quad f(0.7) = 1.949$$

$$x_8 = a + (8-0.1) \times 0.1 = 0.8 \quad f(0.8) = 1.949$$

$$x_9 = a + (9-0.1) \times 0.1 = 0.9 \quad f(0.9) = 1.949$$

$$x_{10} = a + (10-0.1) \times 0.1 = 1.0 \quad f(1.0) = 1.949$$

$$S \approx 1.949 \quad I_n = hS = 0.1 \times 1.949 = 1.949$$

$$\int_0^{0.1} (0.1 + 1.949x - 1.0x^2 + 4.949x^3 - 9.00x^4 + 1.00x^5) dx \quad (f)$$

(c) $a=0$
 $b=0.1$
 $n=1$

$$h = \frac{b-a}{n} = \frac{0.1-0}{1} = 0.1 \quad x_0 = a \quad x_1 = 0.1 \quad x_2 = 0.2$$

$$f(0) = 0.1 \quad f(0.1) = -1.9 \quad f(0.2) = -2.94$$

$$I_n = \frac{h}{p} [f(x_0) + 2f(x_1) + f(x_2)]$$

$$I_2 = 0.1(0.1) - 3.8 - 2.94 = -1.74$$

$$h = \frac{b-a}{n} = 0,1$$

$$n = 2$$

$$b = 0,1$$

$$a = 0$$



$$x_0 = a \quad x_1 = 0,1 \quad x_2 = 0,2$$

$$x_3 = 0,3 \quad x_4 = 0,4$$

$$f(x_0) = 0,1$$

$$f(x_1) = -1,9$$

$$f(x_2) = -29,4$$

$$f(x_3) = -1,4$$

$$f(x_4) = -19,4$$

$$I_n = \frac{h}{n} [f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4)]$$

~~$$I_n = \frac{h}{n} [f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4)]$$~~

$$I_n = \frac{0,1}{2} (0,1 + (-1,9) + (-29,4) + (-1,4) + (-19,4))$$

$$I_n = -\frac{49,94}{2} \approx -24,97$$

~~$$I_n = -24,97$$~~

$$I_1 = -14,97$$

$$I_2 = -14,97$$