

Bits, Bytes, and Integers

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Today: Bits, Bytes, and Integers

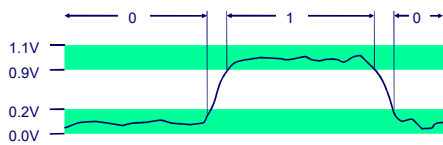
- Representing information as bits
- Bit-level manipulations
- Integers
 - Representation: unsigned and signed
 - Conversion, casting
 - Expanding, truncating
 - Addition, negation, multiplication, shifting
 - Summary
- Representations in memory, pointers, strings

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Everything is bits

- Each bit is 0 or 1
- By encoding/interpreting sets of bits in various ways
 - Computers determine what to do (instructions)
 - ... and represent and manipulate numbers, sets, strings, etc...
- Why bits? Electronic Implementation
 - Easy to store with bistable elements
 - Reliably transmitted on noisy and inaccurate wires



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For example, can count in binary

- Base 2 Number Representation
 - Represent 15213_{10} as 11101101101101_2
 - Represent 1.20_{10} as $1.0011001100110011[0011]..._2$
 - Represent 1.5213×10^4 as $1.1101101101101_2 \times 2^{13}$

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Encoding Byte Values

- Byte = 8 bits
 - Binary 00000000_2 to 11111111_2
 - Decimal: 0_{10} to 255_{10}
 - Hexadecimal 00_{16} to FF_{16}
 - Base 16 number representation
 - Use characters '0' to '9' and 'A' to 'F'
 - Write $FA1D37B_{16}$ in C as
 - `0xFA1D37B`
 - `0xfa1d37b`

| | Hex | Decimal | Binary |
|---|-----|---------|--------|
| 0 | 0 | 0000 | |
| 1 | 1 | 0001 | |
| 2 | 2 | 0010 | |
| 3 | 3 | 0011 | |
| 4 | 4 | 0100 | |
| 5 | 5 | 0101 | |
| 6 | 6 | 0110 | |
| 7 | 7 | 0111 | |
| 8 | 8 | 1000 | |
| 9 | 9 | 1001 | |
| A | 10 | 1010 | |
| B | 11 | 1011 | |
| C | 12 | 1100 | |
| D | 13 | 1101 | |
| E | 14 | 1110 | |
| F | 15 | 1111 | |

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Example Data Representations

| C Data Type | Typical 32-bit | Typical 64-bit | x86-64 |
|-------------|----------------|----------------|--------|
| char | 1 | 1 | 1 |
| short | 2 | 2 | 2 |
| int | 4 | 4 | 4 |
| long | 4 | 8 | 8 |
| float | 4 | 4 | 4 |
| double | 8 | 8 | 8 |
| long double | - | - | 10/16 |
| pointer | 4 | 8 | 8 |

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Boolean Algebra

- Developed by George Boole in 19th Century

- Algebraic representation of logic
 - Encode "True" as 1 and "False" as 0

And

- $A \& B = 1$ when both $A=1$ and $B=1$

| $\&$ | 0 | 1 |
|------|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

Or

- $A | B = 1$ when either $A=1$ or $B=1$

| $ $ | 0 | 1 |
|-----|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 1 |

Not

- $\sim A = 1$ when $A=0$

| \sim | 0 | 1 |
|--------|---|---|
| 0 | 1 | 0 |
| 1 | 0 | 1 |

Exclusive-Or (Xor)

- $A \wedge B = 1$ when either $A=1$ or $B=1$, but not both

| \wedge | 0 | 1 |
|----------|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

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General Boolean Algebras

- Operate on Bit Vectors

- Operations applied bitwise

| | | |
|---------------|--------------|-------------------|
| 01101001 | 01101001 | 01101001 |
| $\&$ 01010101 | $ $ 01010101 | \wedge 01010101 |
| 01000001 | 01111101 | 00111100 |
| | | \sim 01010101 |
| | | 10101010 |

- All of the Properties of Boolean Algebra Apply

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Example: Representing & Manipulating Sets

- Representation

- Width w bit vector represents subsets of $\{0, \dots, w-1\}$
- $a_j = 1$ if $j \in A$

- 01101001 $\{0, 3, 5, 6\}$
- 76543210

- 01010101 $\{0, 2, 4, 6\}$
- 76543210

- Operations

- $\&$ Intersection 01000001 $\{0, 6\}$
- $|$ Union 01111101 $\{0, 2, 3, 4, 5, 6\}$
- \wedge Symmetric difference 00111100 $\{2, 3, 4, 5\}$
- \sim Complement 10101010 $\{1, 3, 5, 7\}$

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Bit-Level Operations in C

- Operations $\&$, $|$, \sim , \wedge Available in C

- Apply to any "integral" data type
 - long, int, short, char, unsigned
- View arguments as bit vectors
- Arguments applied bit-wise

- Examples (Char data type)

- $\sim 0x41 \rightarrow 0xBE$
- $\sim 01000001_2 \rightarrow 10111110_2$
- $\sim 0x00 \rightarrow 0xFF$
- $\sim 00000000_2 \rightarrow 11111111_2$
- $0x69 \& 0x55 \rightarrow 0x41$
- $01101001_2 \& 01010101_2 \rightarrow 01000001_2$
- $0x69 | 0x55 \rightarrow 0x7D$
- $01101001_2 | 01010101_2 \rightarrow 01111101_2$

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Contrast: Logic Operations in C

- Contrast to Logical Operators

- $\&\&$, $||$, $!$
 - View 0 as "False"
 - Anything nonzero as "True"
 - Always return 0 or 1
 - Early termination

- Examples (char data type)

- $!0x41 \rightarrow 0x00$
- $!0x00 \rightarrow 0x01$
- $!!0x41 \rightarrow 0x01$
- $0x69 \&\& 0x55 \rightarrow 0x01$
- $0x69 || 0x55 \rightarrow 0x01$
- $p \&\& *p$ (avoids null pointer access)

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Contrast: Logic Operations in C

Contrast to Logical Operators

- `&&`, `||`, `!`
 - View 0 as "False"
 - Anything nonzero
 - Always evaluates both
 - Early exit
- Example
 - `!0x41`
 - `!0x00`
 - `!!0x41`
 - `0x69 & 0x55 → 0x01`
 - `0x69 || 0x55 → 0x01`
 - `p && *p` (avoids null pointer access)

Watch out for `&&` vs. `&` (and `||` vs. `|`)... one of the more common oopsies in C programming

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Shift Operations

Left Shift: `x << y`

- Shift bit-vector `x` left `y` positions
 - Throw away extra bits on left
 - Fill with 0's on right

| | |
|-------------|----------|
| Argument x | 01100010 |
| << 3 | 00010000 |
| Log. >> 2 | 00011000 |
| Arith. >> 2 | 00011000 |

Right Shift: `x >> y`

- Shift bit-vector `x` right `y` positions
 - Throw away extra bits on right
- Logical shift
 - Fill with 0's on left
- Arithmetic shift
 - Replicate most significant bit on left

| | |
|-------------|----------|
| Argument x | 10100010 |
| << 3 | 00010000 |
| Log. >> 2 | 00101000 |
| Arith. >> 2 | 11101000 |

Undefined Behavior

- Shift amount `< 0` or `≥` word size

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Encoding Integers

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

`short int x = 15213;`

`short int y = -15213;`

Sign Bit

C short 2 bytes long

| | Decimal | Hex | Binary |
|---|---------|-------|-------------------|
| x | 15213 | 3B 6D | 00111011 01101101 |
| y | -15213 | C4 93 | 11000100 10010011 |

Sign Bit

- For 2's complement, most significant bit indicates sign
 - 0 for nonnegative
 - 1 for negative

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Two-complement Encoding Example (Cont.)

x = 15213: 00111011 01101101
y = -15213: 11000100 10010011

| Weight | 15213 | -15213 |
|--------|-------|--------|
| 1 | 1 | 1 |
| 2 | 0 | 1 |
| 4 | 1 | 0 |
| 8 | 1 | 0 |
| 16 | 0 | 1 |
| 32 | 1 | 0 |
| 64 | 1 | 0 |
| 128 | 0 | 1 |
| 256 | 1 | 0 |
| 512 | 1 | 0 |
| 1024 | 0 | 1 |
| 2048 | 1 | 0 |
| 4096 | 1 | 0 |
| 8192 | 1 | 0 |
| 16384 | 0 | 1 |
| -32768 | 0 | 1 |
| Sum | 15213 | -15213 |

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Numeric Ranges

Unsigned Values

$$UMin = 0$$

$$UMax = 2^w - 1$$

Two's Complement Values

$$TMin = -2^{w-1}$$

$$TMax = 2^{w-1} - 1$$

Other Values

Minus 1

Values for W = 16

| | Decimal | Hex | Binary |
|------|---------|-------|-------------------|
| UMax | 65535 | FF FF | 11111111 11111111 |
| TMax | 32767 | 7F FF | 01111111 11111111 |
| TMin | -32768 | 80 00 | 10000000 00000000 |
| -1 | -1 | FF FF | 11111111 11111111 |
| 0 | 0 | 00 00 | 00000000 00000000 |

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Values for Different Word Sizes

| | W | | | |
|------|------|---------|----------------|----------------------------|
| | 8 | 16 | 32 | 64 |
| UMax | 255 | 65,535 | 4,294,967,295 | 18,446,744,073,709,551,615 |
| TMax | 127 | 32,767 | 2,147,483,647 | 9,223,372,036,854,775,807 |
| TMin | -128 | -32,768 | -2,147,483,648 | -9,223,372,036,854,775,808 |

Observations

- $|TMin| = TMax + 1$
- Asymmetric range
- $UMax = 2 * TMax + 1$

C Programming

- #include <limits.h>
- Declares constants, e.g.,
 - ULONG_MAX
 - LONG_MAX
 - LONG_MIN
- Values platform specific

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Unsigned & Signed Numeric Values

| X | B2U(x) | B2T(x) |
|------|--------|--------|
| 0000 | 0 | 0 |
| 0001 | 1 | 1 |
| 0010 | 2 | 2 |
| 0011 | 3 | 3 |
| 0100 | 4 | 4 |
| 0101 | 5 | 5 |
| 0110 | 6 | 6 |
| 0111 | 7 | 7 |
| 1000 | 8 | -8 |
| 1001 | 9 | -7 |
| 1010 | 10 | -6 |
| 1011 | 11 | -5 |
| 1100 | 12 | -4 |
| 1101 | 13 | -3 |
| 1110 | 14 | -2 |
| 1111 | 15 | -1 |

Equivalence

- Same encodings for nonnegative values

Uniqueness

- Every bit pattern represents unique integer value
- Each representable integer has unique bit encoding

Can Invert Mappings

- $U2B(x) = B2U^{-1}(x)$
 - Bit pattern for unsigned integer
- $T2B(x) = B2T^{-1}(x)$
 - Bit pattern for two's comp integer

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Today: Bits, Bytes, and Integers

Representing information as bits

Bit-level manipulations

Integers

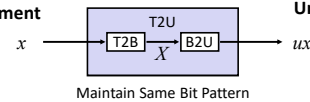
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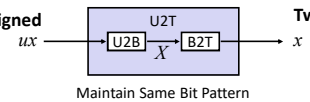
Mapping Between Signed & Unsigned

Two's Complement



Unsigned

Unsigned



Two's Complement

- Mappings between unsigned and two's complement numbers:
Keep bit representations and reinterpret

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Mapping Signed ↔ Unsigned

| Bits | Signed | Unsigned |
|------|--------|----------|
| 0000 | 0 | 0 |
| 0001 | 1 | 1 |
| 0010 | 2 | 2 |
| 0011 | 3 | 3 |
| 0100 | 4 | 4 |
| 0101 | 5 | 5 |
| 0110 | 6 | 6 |
| 0111 | 7 | 7 |
| 1000 | -8 | 8 |
| 1001 | -7 | 9 |
| 1010 | -6 | 10 |
| 1011 | -5 | 11 |
| 1100 | -4 | 12 |
| 1101 | -3 | 13 |
| 1110 | -2 | 14 |
| 1111 | -1 | 15 |

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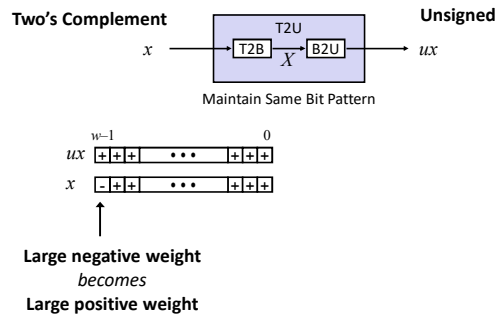
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|------|--------|----------|
| 0000 | 0 | 0 |
| 0001 | 1 | 1 |
| 0010 | 2 | 2 |
| 0011 | 3 | 3 |
| 0100 | 4 | 4 |
| 0101 | 5 | 5 |
| 0110 | 6 | 6 |
| 0111 | 7 | 7 |
| 1000 | -8 | 8 |
| 1001 | -7 | 9 |
| 1010 | -6 | 10 |
| 1011 | -5 | 11 |
| 1100 | -4 | 12 |
| 1101 | -3 | 13 |
| 1110 | -2 | 14 |
| 1111 | -1 | 15 |

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Relation between Signed & Unsigned

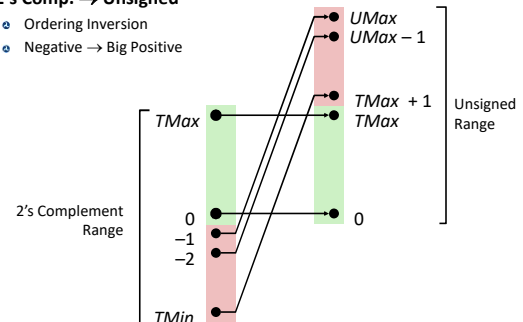


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Conversion Visualized

- 2's Comp. \rightarrow Unsigned
 - Ordering Inversion
 - Negative \rightarrow Big Positive



Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Signed vs. Unsigned in C

- Constants**
 - By default are considered to be signed integers
 - Unsigned if have "U" as suffix
 - `0U`, `4294967295U`
- Casting**
 - Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

 - Implicit casting also occurs via assignments and procedure calls

```
tx = ux;
uy = ty;
```

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Casting Surprises

- Expression Evaluation**
 - If there is a mix of unsigned and signed in single expression, **signed values implicitly cast to unsigned**
 - Including comparison operations `<`, `>`, `==`, `<=`, `>=`
 - Examples for $W = 32$: $TMin = -2,147,483,648$, $TMax = 2,147,483,647$
- | Constant ₁ | Constant ₂ | Relation | Evaluation |
|-----------------------|-----------------------|----------|------------|
| 0 | 0U | == | unsigned |
| -1 | 0 | < | signed |
| -1 | 0U | > | unsigned |
| 2147483647 | -2147483647-1 | > | signed |
| 2147483647U | -2147483647-1 | < | unsigned |
| -1 | -2 | > | signed |
| (unsigned)-1 | -2 | > | unsigned |
| 2147483647 | 2147483648U | < | unsigned |
| 2147483647 | (int) 2147483648U | > | signed |

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Summary

Casting Signed \leftrightarrow Unsigned: Basic Rules

- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting 2^w
- Expression containing signed and unsigned int
 - int is cast to unsigned!!

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Today: Bits, Bytes, and Integers

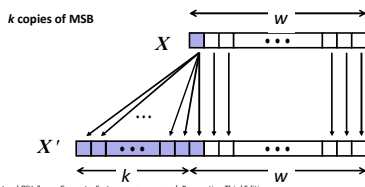
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Sign Extension

- Task:
 - Given w -bit signed integer x
 - Convert it to $w+k$ -bit integer with same value
- Rule:
 - Make k copies of sign bit:
 - $X' = X_{w-1}, \dots, X_{w-1}, X_{w-1}, X_{w-2}, \dots, X_0$



Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Sign Extension Example

```
short int x = 15213;
int ix = (int) x;
short int y = -15213;
int iy = (int) y;
```

| | Decimal | Hex | Binary |
|------|---------|-------------|-------------------------------------|
| x | 15213 | 3B 6D | 00111011 01101101 |
| ix | 15213 | 00 00 3B 6D | 00000000 00000000 00111011 01101101 |
| y | -15213 | C4 93 | 11000100 10010011 |
| iy | -15213 | FF FF C4 93 | 11111111 11111111 11000100 10010011 |

- Converting from smaller to larger integer data type
- C automatically performs sign extension

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Summary: Expanding, Truncating: Basic Rules

- Expanding (e.g., short int to int)
 - Unsigned: zeros added
 - Signed: sign extension
 - Both yield expected result
- Truncating (e.g., unsigned to unsigned short)
 - Unsigned/signed: bits are truncated
 - Result reinterpreted
 - Unsigned: mod operation
 - Signed: similar to mod
 - For small numbers yields expected behavior

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Unsigned Addition

Operands: w bits

$$\begin{array}{r} u \\ + v \\ \hline \end{array}$$

True Sum: $w+1$ bits

$$u + v$$

Discard Carry: w bits

$$UAdd_w(u, v)$$

- Standard Addition Function
 - Ignores carry output
- Implements Modular Arithmetic

$$s = UAdd_w(u, v) = u + v \bmod 2^w$$

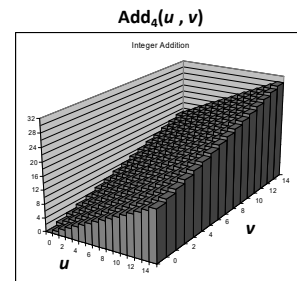
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Visualizing (Mathematical) Integer Addition

Integer Addition

- 4-bit integers u, v
- Compute true sum $Add_4(u, v)$
- Values increase linearly with u and v
- Forms planar surface



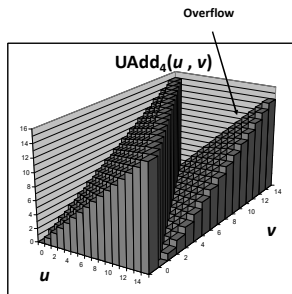
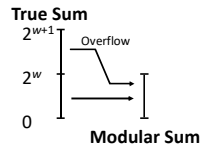
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Visualizing Unsigned Addition

Wraps Around

- If true sum $\geq 2^w$
- At most once

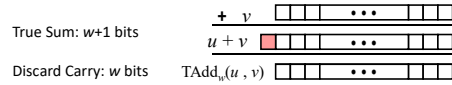


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Two's Complement Addition

Operands: w bits



TAdd and UAdd have Identical Bit-Level Behavior

- Signed vs. unsigned addition in C:


```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v;
// Will give s == t
```

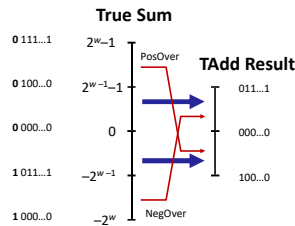
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TAdd Overflow

Functionality

- True sum requires $w+1$ bits
- Drop off MSB
- Treat remaining bits as 2's comp. integer



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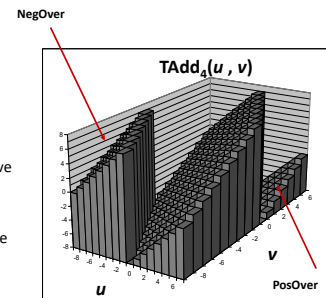
Visualizing 2's Complement Addition

Values

- 4-bit two's comp.
- Range from -8 to +7

Wraps Around

- If sum $\geq 2^{w-1}$
 - Becomes negative
 - At most once
- If sum $< -2^{w-1}$
 - Becomes positive
 - At most once



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Multiplication

Goal: Computing Product of w -bit numbers x, y

- Either signed or unsigned

But, exact results can be bigger than w bits

- Unsigned: up to $2w$ bits
 - Result range: $0 \leq x * y \leq (2^w - 1)^2 = 2^{2w} - 2^{w+1} + 1$
- Two's complement min (negative): Up to $2w-1$ bits
 - Result range: $x * y \geq (-2^{w-1}) * (2^{w-1} - 1) = -2^{2w-2} + 2^{w-1}$
- Two's complement max (positive): Up to $2w$ bits, but only for $(TMin_w)^2$
 - Result range: $x * y \leq (-2^{w-1})^2 = 2^{2w-2}$

So, maintaining exact results...

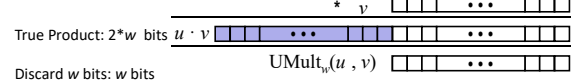
- would need to keep expanding word size with each product computed
- is done in software, if needed
 - e.g., by "arbitrary precision" arithmetic packages

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Unsigned Multiplication in C

Operands: w bits



Standard Multiplication Function

- Ignores high order w bits

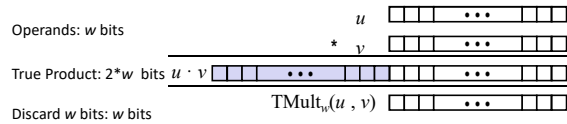
Implements Modular Arithmetic

$$\text{UMult}_w(u, v) = u \cdot v \bmod 2^w$$

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Signed Multiplication in C



Standard Multiplication Function

- Ignores high order w bits
- Some of which are different for signed vs. unsigned multiplication
- Lower bits are the same

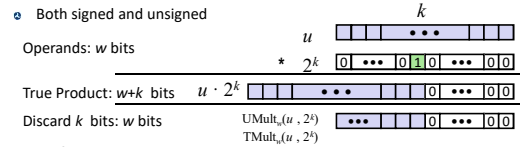
Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Power-of-2 Multiply with Shift

Operation

- $u \ll k$ gives $u * 2^k$
- Both signed and unsigned



Examples

- $u \ll 3 == u * 8$
- $(u \ll 5) - (u \ll 3) == u * 24$
- Most machines shift and add faster than multiply
 - Compiler generates this code automatically

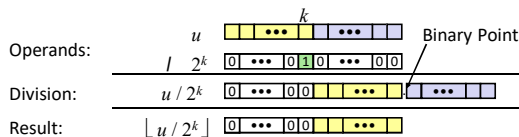
Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Unsigned Power-of-2 Divide with Shift

Quotient of Unsigned by Power of 2

- $u \gg k$ gives $\lfloor u / 2^k \rfloor$
- Uses logical shift



| | Division | Computed | Hex | Binary |
|-----------|------------|----------|-------|-------------------|
| x | 15213 | 15213 | 3B 6D | 00111011 01101101 |
| $x \gg 1$ | 7606.5 | 7606 | 1D B6 | 00011101 10110110 |
| $x \gg 4$ | 950.8125 | 950 | 03 B6 | 00000011 10110110 |
| $x \gg 8$ | 59.4257813 | 59 | 00 3B | 00000000 00111011 |

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Today: Bits, Bytes, and Integers

Representing information as bits

Bit-level manipulations

Integers

- Representation: unsigned and signed
- Conversion, casting
- Expanding, truncating
- Addition, negation, multiplication, shifting
- Summary

Representations in memory, pointers, strings

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Arithmetic: Basic Rules

Addition:

- Unsigned/signed: Normal addition followed by truncate, same operation on bit level
- Unsigned: addition mod 2^w
 - Mathematical addition + possible subtraction of 2^w
- Signed: modified addition mod 2^w (result in proper range)
 - Mathematical addition + possible addition or subtraction of 2^w

Multiplication:

- Unsigned/signed: Normal multiplication followed by truncate, same operation on bit level
- Unsigned: multiplication mod 2^w
- Signed: modified multiplication mod 2^w (result in proper range)

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Why Should I Use Unsigned?

Don't use without understanding implications

- Easy to make mistakes


```
unsigned i;
for (i = cnt-2; i >= 0; i--)
    a[i] += a[i+1];
```
- Can be very subtle


```
#define DELTA sizeof(int)
int i;
for (i = CNT; i-DELTA >= 0; i-= DELTA)
    ...
```

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Counting Down with Unsigned

- Proper way to use unsigned as loop index


```
unsigned i;
for (i = cnt-2; i < cnt; i--)
    a[i] += a[i+1];
```
- See Robert Seacord, *Secure Coding in C and C++*
 - C Standard guarantees that unsigned addition will behave like modular arithmetic
 - $0 - 1 \rightarrow UMax$
- Even better


```
size_t i;
for (i = cnt-2; i < cnt; i--)
    a[i] += a[i+1];
```

 - Data type `size_t` defined as unsigned value with length = word size
 - Code will work even if `cnt = UMax`
 - What if `cnt` is signed and `< 0`?

Notes adapted from Bryant and O'Hallaron, *Computer Systems: A Programmer's Perspective*, Third Edition

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Why Should I Use Unsigned? (cont.)

- Do Use When Performing Modular Arithmetic
 - Multiprecision arithmetic
- Do Use When Using Bits to Represent Sets
 - Logical right shift, no sign extension

Notes adapted from Bryant and O'Hallaron, *Computer Systems: A Programmer's Perspective*, Third Edition

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Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
 - Representation: unsigned and signed
 - Conversion, casting
 - Expanding, truncating
 - Addition, negation, multiplication, shifting
 - Summary
- Representations in memory, pointers, strings

Notes adapted from Bryant and O'Hallaron, *Computer Systems: A Programmer's Perspective*, Third Edition

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Byte-Oriented Memory Organization



- Programs refer to data by address
 - Conceptually, envision it as a very large array of bytes
 - In reality, it's not, but can think of it that way
 - An address is like an index into that array
 - and, a pointer variable stores an address
- Note: system provides private address spaces to each "process"
 - Think of a process as a program being executed
 - So, a program can clobber its own data, but not that of others

Notes adapted from Bryant and O'Hallaron, *Computer Systems: A Programmer's Perspective*, Third Edition

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Machine Words

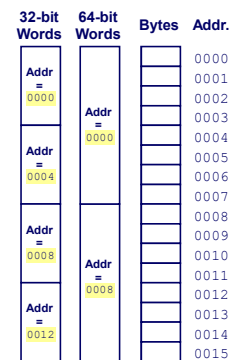
- Any given computer has a "Word Size"
 - Nominal size of integer-valued data
 - and of addresses
 - Until recently, most machines used 32 bits (4 bytes) as word size
 - Limits addresses to 4GB (2^{32} bytes)
 - Increasingly, machines have 64-bit word size
 - Potentially, could have 18 EB (exabytes) of addressable memory
 - That's 18.4×10^{18}
 - Machines still support multiple data formats
 - Fractions or multiples of word size
 - Always integral number of bytes

Notes adapted from Bryant and O'Hallaron, *Computer Systems: A Programmer's Perspective*, Third Edition

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Word-Oriented Memory Organization

- Addresses Specify Byte Locations
 - Address of first byte in word
 - Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Notes adapted from Bryant and O'Hallaron, *Computer Systems: A Programmer's Perspective*, Third Edition

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Example Data Representations

| C Data Type | Typical 32-bit | Typical 64-bit | x86-64 |
|-------------|----------------|----------------|--------|
| char | 1 | 1 | 1 |
| short | 2 | 2 | 2 |
| int | 4 | 4 | 4 |
| long | 4 | 8 | 8 |
| float | 4 | 4 | 4 |
| double | 8 | 8 | 8 |
| long double | - | - | 10/16 |
| pointer | 4 | 8 | 8 |

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Byte Ordering

- So, how are the bytes within a multi-byte word ordered in memory?
- Conventions
 - Big Endian: Sun, PPC Mac, Internet
 - Least significant byte has highest address
 - Little Endian: x86, ARM processors running Android, iOS, and Windows
 - Least significant byte has lowest address

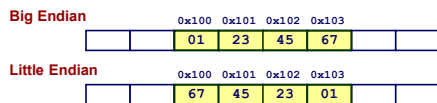
Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Byte Ordering Example

Example

- Variable x has 4-byte value of 0x01234567
- Address given by &x is 0x100



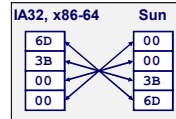
Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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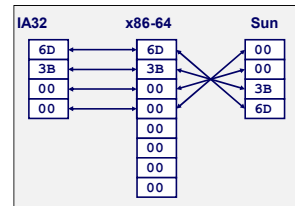
Representing Integers

Decimal: 15213
Binary: 0011 1011 0110 1101
Hex: 3 B 6 D

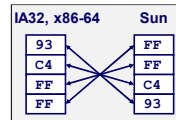
int A = 15213;



long int C = 15213;



int B = -15213;



Two's complement representation

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Examining Data Representations

Code to Print Byte Representation of Data

- Casting pointer to unsigned char * allows treatment as a byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, size_t len) {
    size_t i;
    for (i = 0; i < len; i++)
        printf("%p\t0x%.2x\n", start+i, start[i]);
    printf("\n");
}
```

Printf directives:
%p: Print pointer
%x: Print Hexadecimal

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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show_bytes Execution Example

```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

Result (Linux x86-64):

```
int a = 15213;
0x7fffb7f71dbc 6d
0x7fffb7f71dbd 3b
0x7fffb7f71dbe 00
0x7fffb7f71dbf 00
```

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Representing Pointers

```
int B = -15213;
int *P = &B;
```

| Sun | IA32 | x86-64 |
|-----|------|--------|
| EF | AC | 3C |
| FF | 28 | 1B |
| FB | F5 | FE |
| 2C | FF | 82 |
| | | FD |
| | | 7F |
| | | 00 |
| | | 00 |

Different compilers & machines assign different locations to objects
Even get different results each time run program

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Representing Strings

```
char S[6] = "18213";
```

Strings in C

- Represented by array of characters
- Each character encoded in ASCII format
 - Standard 7-bit encoding of character set
 - Character "0" has code 0x30
 - Digit i has code $0x30+i$
- String should be null-terminated
 - Final character = 0

Compatibility

- Byte ordering not an issue

| IA32 | Sun |
|------|-----|
| 31 | 31 |
| 38 | 38 |
| 32 | 32 |
| 31 | 31 |
| 33 | 33 |
| 00 | 00 |

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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Integer C Puzzles

Initialization

```
int x = foo();
int y = bar();
unsigned ux = x;
unsigned uy = y;
```

- $x < 0 \rightarrow ((x*2) < 0)$
- $ux \geq 0$
- $x \& 7 == 7 \rightarrow (x \ll 30) < 0$
- $ux > -1$
- $x > y \rightarrow -x < -y$
- $x * x \geq 0$
- $x > 0 \&\& y > 0 \rightarrow x + y > 0$
- $x \geq 0 \rightarrow -x \leq 0$
- $x \leq 0 \rightarrow -x \geq 0$
- $(x|-x) \gg 31 == -1$
- $ux \gg 3 == ux/8$
- $x \gg 3 == x/8$
- $x \& (x-1) != 0$

Notes adapted from Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition

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