Lecture #11

- Sorting Algorithms, part II:
  - Quicksort
  - Mergesort
- Trees
  - Introduction
  - Implementation & Basic Properties
  - Traversals: The Pre-order Traversal
- On-your-own Study
  - Full binary trees

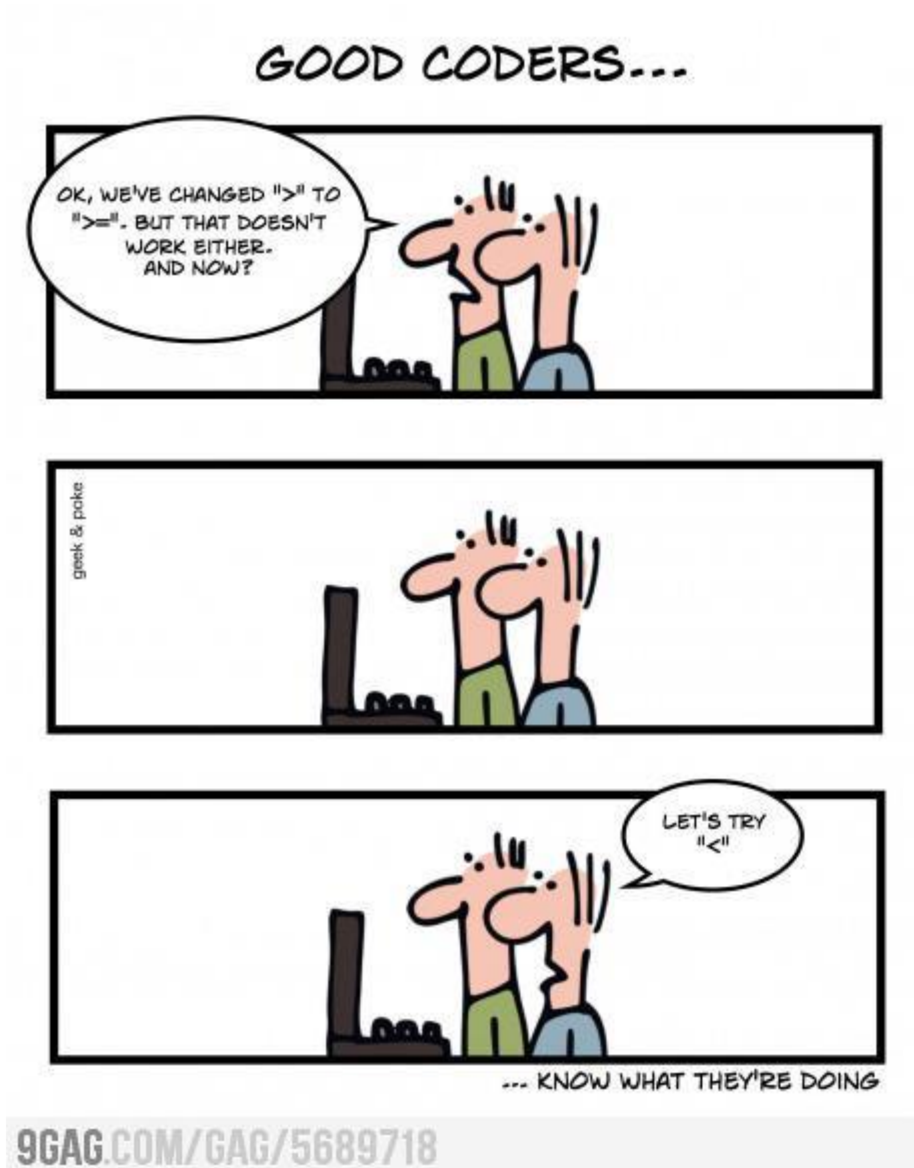
But first… STL Challenge

Give me a data structure that I can use to maintain a bunch of people's names and for each person, allows me to easily get all of the streets they lived on.

Assuming I have P total people and each person has lived on an average of E former streets…

What is the Big-Oh cost of:

A. Finding the names of all people who have lived on "Levering street"?
B. Determining if "Bill" ever lived on "Westwood blvd"?
C. Printing out every name along with each person's street addresses, in alphabetical order.
D. Printing out all of the streets that "Tala" has lived on.


GOOD CODERS...
OK, WE'VE CHANGED ">" TO ">=". BUT THAT DOESN'T WORK EITHER. AND NOW?
geek & poke
LET'S TRY "<"
... KNOW WHAT THEY'RE DOING
9GAG.COM/GAG/5689718

Advanced Sorting Algos.

Advanced Sorting Algorithms
Why should you care?

Because this is basically how folks sort stuff in real life.

You can sort billions of values in seconds. SECONDS BABY!

And because you'll be asked about them in job interviews and on exams.

So pay attention!

Divide and Conquer Sorting

The last two sorts we'll learn (for now) are Quicksort and Mergesort.

These sorts generally work as follows:

1. Divide the elements to be sorted into two groups of roughly equal size.
2. Sort each of these smaller groups of elements (conquer).
3. Combine the two sorted groups into one large sorted list.

Any time you see "divide and conquer," you should think recursion... EEK!

The Quicksort Algorithm

Divide

Conquer

1. If the array contains only 0 or 1 element, return.
2. Select an arbitrary element P from the array (typically the first element in the array).
3. Move all elements that are less than or equal to P to the left of the array and all elements greater than P to the right (this is called partitioning).
4. Recursively repeat this process on the left sub-array and then the right sub-array.

13	1	21	30	69	40	77

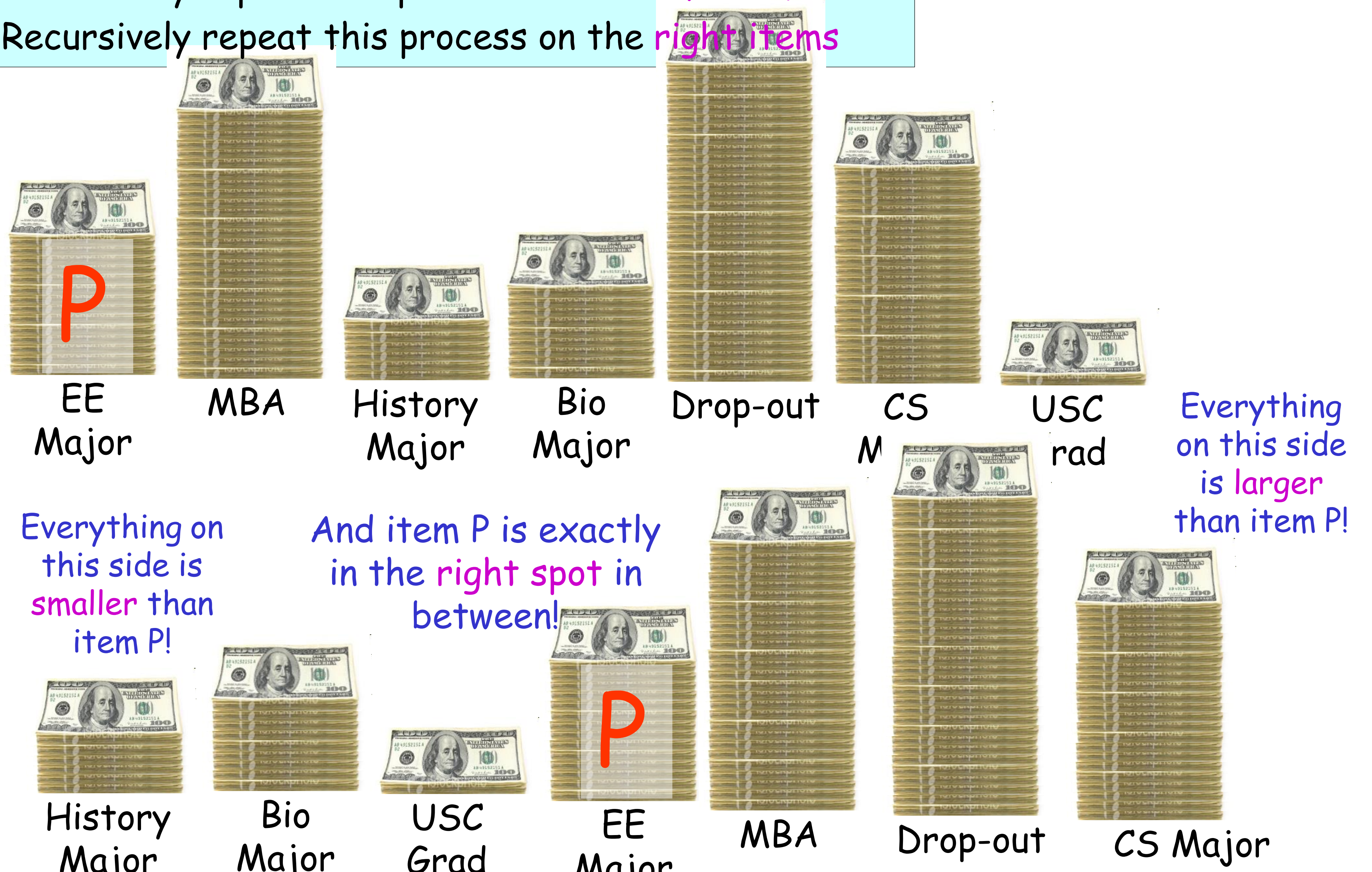
QuickSort

Select an arbitrary item P from the array.

Move items smaller than or equal to P to the left and larger items to the right; P goes in-between.

Recursively repeat this process on the left items

Recursively repeat this process on the right items



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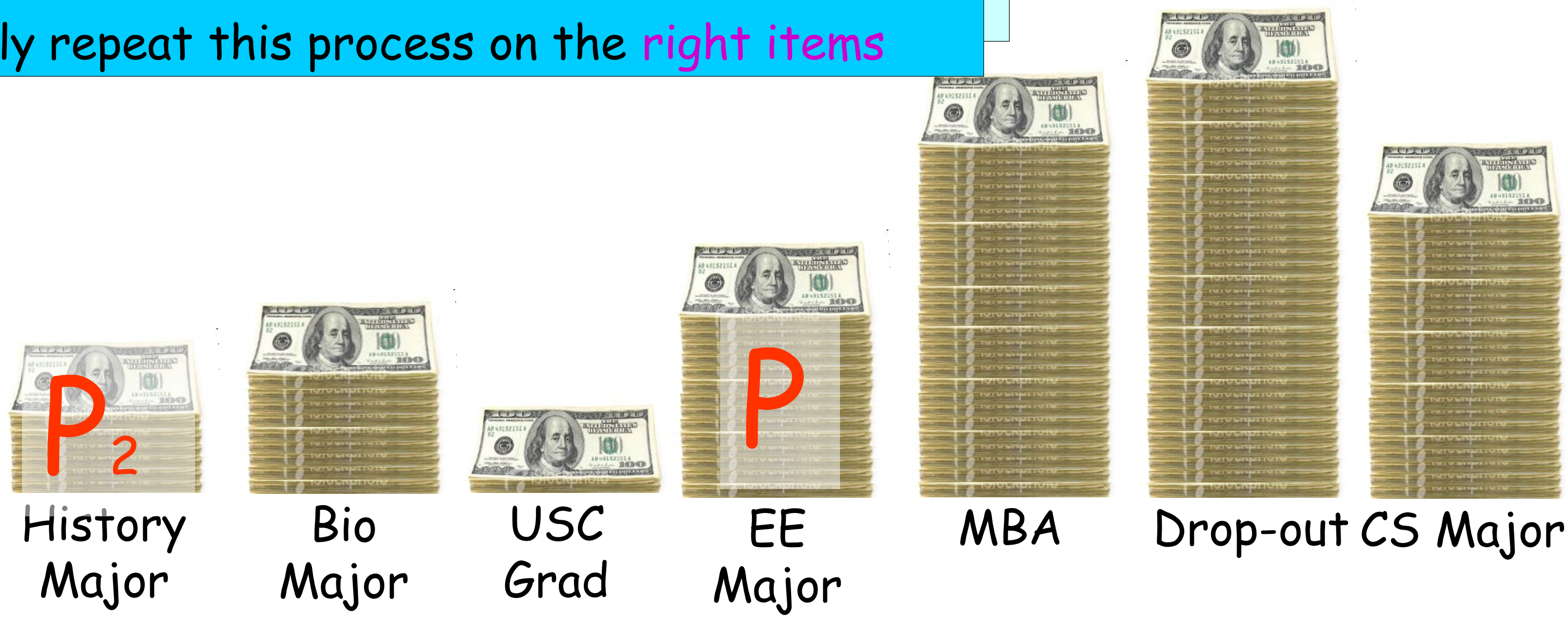


Everything left of EE Major (our first P) is now sorted!

QuickSort

Select an arbitrary item P from the array.

Move items smaller than or equal to P to the left and larger items to the right; P goes in-between.

Recursively repeat this process on the left items

Recursively repeat this process on the right items

USC Grad | History Major | Bio Major | EE Major | MBA | Drop-out | CS Major

Finally, all items are sorted!

Everything right of EE Major (our first P) is now sorted!

USC Grad | History Major | Bio Major | EE Major | CS Major | MBA | Drop-out

D Quickso

First specifies the starting element of the array to sort.

Last specifies the last element of the array to sort.

Only bother sorting arrays of at least two elements!

And here's an actual Quicksort C++ function:

0 7

```
void QuickSort(int Array[],int First,int Last)
{
  if (Last - First >= 1 )
  {
    int PivotIndex;

    PivotIndex = Partition(Array,First,Last);
    QuickSort(Array,First,PivotIndex-1); // left
    QuickSort(Array,PivotIndex+1,Last);  // right
  }
}
```

3

DIVIDE
Pick an element.
Move <= items left
Move > items right

CONQUER
Apply our QS algorithm to the left half of the array.

CONQUER
Apply our QS algorithm to the right half of the array.

13	1	21	30	69	40	77	46
0	1	2	3	4	5	6	7

The QS Partition Function

The Partition function uses the first item as the pivot value and moves less-than-or-equal items to the left and larger ones to the right.

```
int Partition(int a[], int low, int high)
{
  int pi = low;
  int pivot = a[low];
  do
  {
     while ( low <= high && a[low] <= pivot )
        low++;
     while ( a[high] > pivot )
        high--;
     if ( low < high )
        swap(a[low], a[high]);
  }
  while ( low < high );
  swap(a[pi], a[high]);
  pi = high;
  return(pi);
}
```

And finally, return the pivot's index in the array (4) to the QuickSort function.

4	1	12	13	30	52	40	99	77	35	47	56
0	1	2	3	4	5	6	7	8	9	10	11

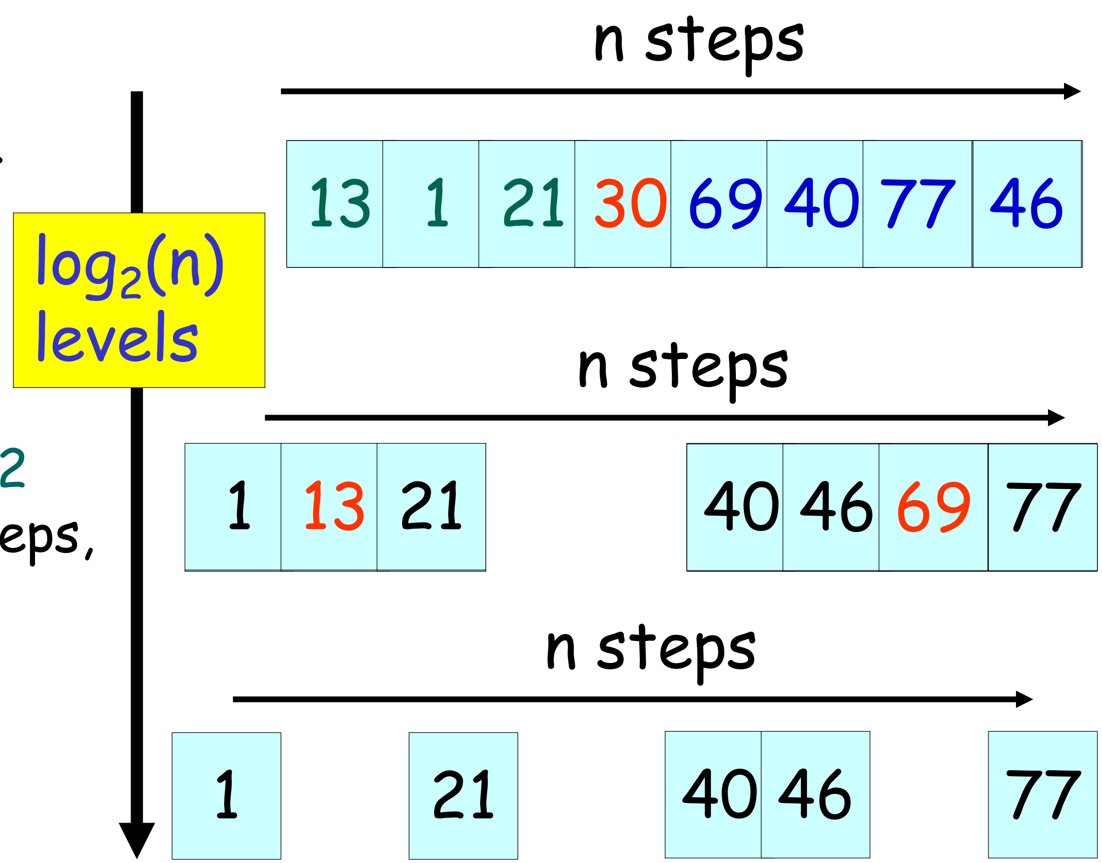
Big-oh of Quicksort

We first partition the array, at a cost of n steps.

Then we repeat the process for each half…

We partition each of the 2 halves, each taking n/2 steps, at a total cost of n steps.

Then we repeat the process for each half…



We partition each of the 4 halves, each taking n/4 steps, at a total cost of n steps.

So at each level, we do n operations, and we have $\log_2(n)$ levels, so we get: $n \log_2(n)$.

Quicksort - Is It Always Fast?

Are there any kinds of input data where Quicksort is either more or less efficient?

Yes! If our array is already sorted or mostly sorted, then quicksort becomes very slow!

1	10	20	30	40	50	60	70

Let's see why.

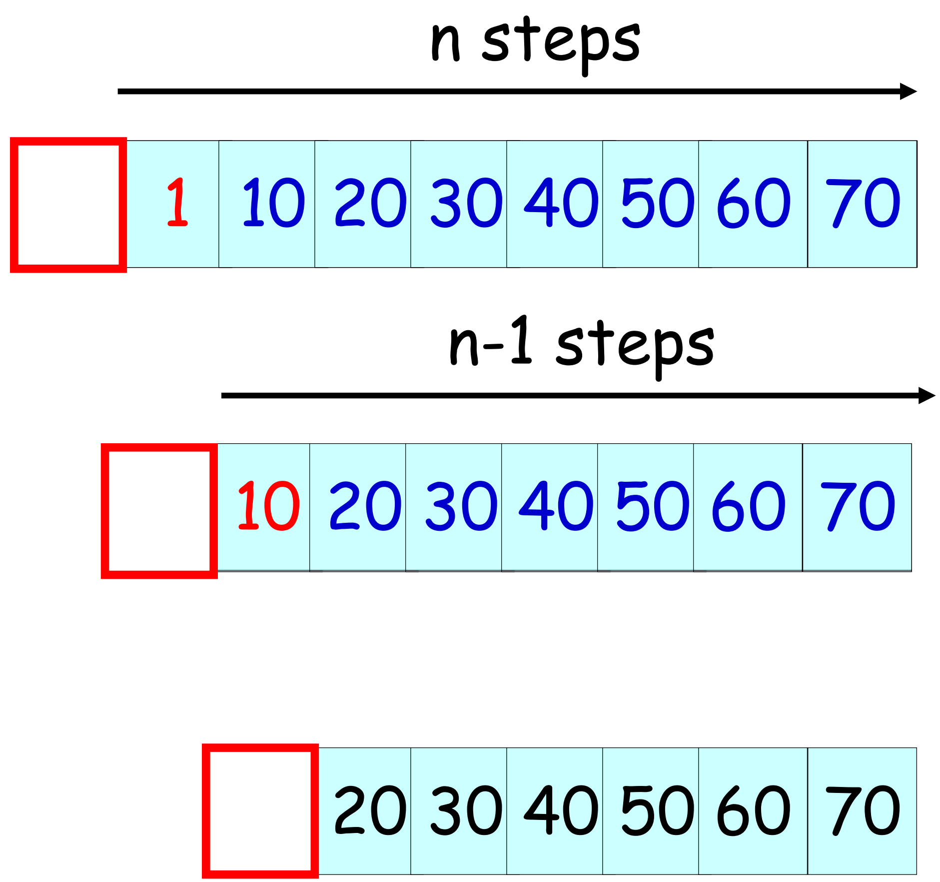
Worst-case Big-oh of Quicksort

We first partition the array, at a cost of n steps.

Then we repeat the process for the ~~left &~~ right groups…

Ok, let's partition our right group then.

Then we repeat the process for the ~~left &~~ right groups…



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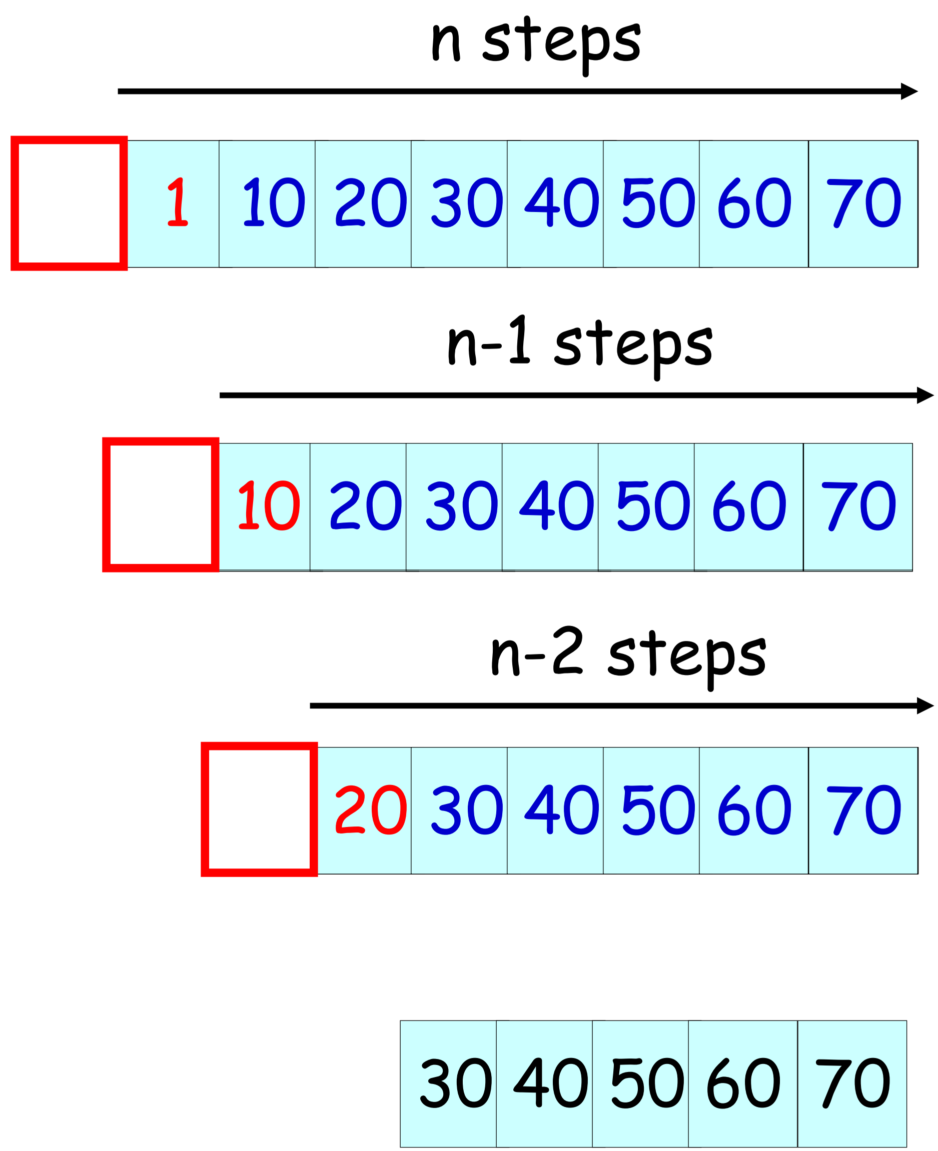


Worst-case Big-oh of Quicksort

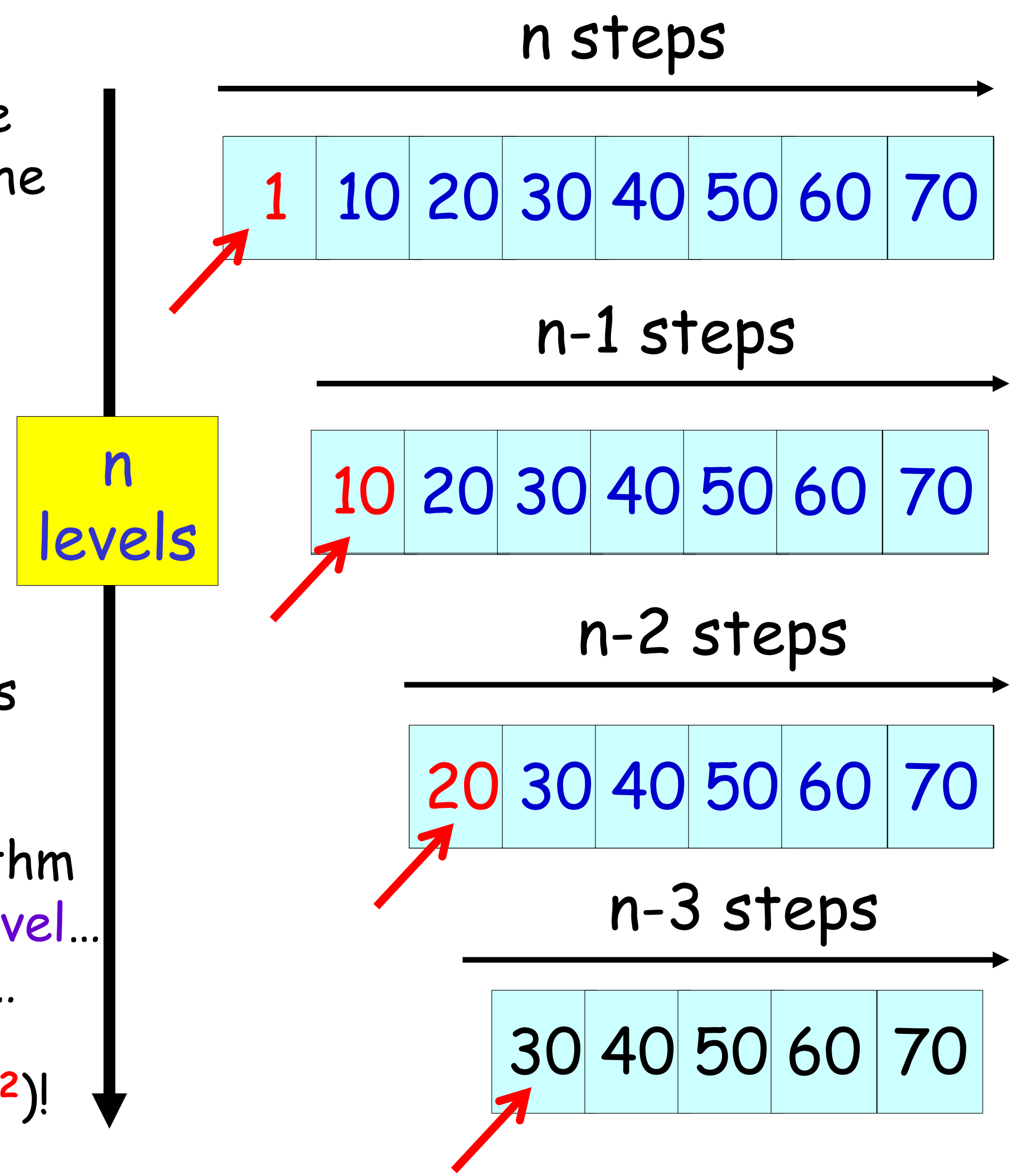
What you'll notice is that each time we partition, we remove only one item off the left side!

And if we only remove one item off the left side each time…

We're going to have to go through this partitioning process n times to process the entire array!

And if the partition algorithm requires ~n steps at each level… And we go n levels deep…

Then our algorithm is $O(n^2)$!



Other Quicksort Worst Cases?

So, as you can see, an array that's mostly in order will require an average of N^2 steps!

As you can probably guess, Quicksort also has the same problem with arrays that are in reverse order!

So if you happen to know your data will be mostly sorted (or in reverse) order, avoid Quicksort!

It's a DOG!

QuickSort Questions

Can QuickSort be applied easily to sort items within a linked list?

Is QuickSort a "stable" sort?

Does QuickSort use a fixed amount of RAM, or can it vary?

Can QuickSort be parallelized across multiple cores?

When might you use QuickSort?

Mergesort

The Mergesort is another extremely efficient sort – yet it's pretty easy to understand.

But before we learn the Mergesort, we need to learn another algorithm called "merge".

Mergesort

The basic merge algorithm takes two-presorted arrays as inputs and outputs a combined, third sorted array.

i1

i2

Merge Algorithm

Consider the left-most book in both shelves

Take the smallest of the two books

Add it to the new shelf

Repeat the whole process until all books are moved

By always selecting and moving the smallest book from either shelf we guarantee all of our books will end up sorted!

1. Initialize counter variables i1, i2 to zero
2. While there are more items to copy…
   If A1[i1] is less than A2[i2]
      Copy A1[i1] to output array B and i1++
   Else
      Copy A2[i2] to output array B and i2++
3. If either array runs out, copy the entire contents of the other array over

Merge Algorithm in C++

```
void merge(int data[],int n1, int n2)
{
  int i=0, j=0, k=0;
  int *temp = new int[n1+n2];
  int *sechalf = data + n1;

  while (i < n1 || j < n2)
  {
    if (i == n1)
       temp[k++] = sechalf[j++];
    else if (j == n2)
       temp[k++] = data[i++];
    else if (data[i] < sechalf[j])
       temp[k++] = data[i++];
    else
       temp[k++] = sechalf[j++];
  }
  for (i=0;i<n1+n2;i++)
    data[i] = temp[i];
  delete [] temp;
}
```

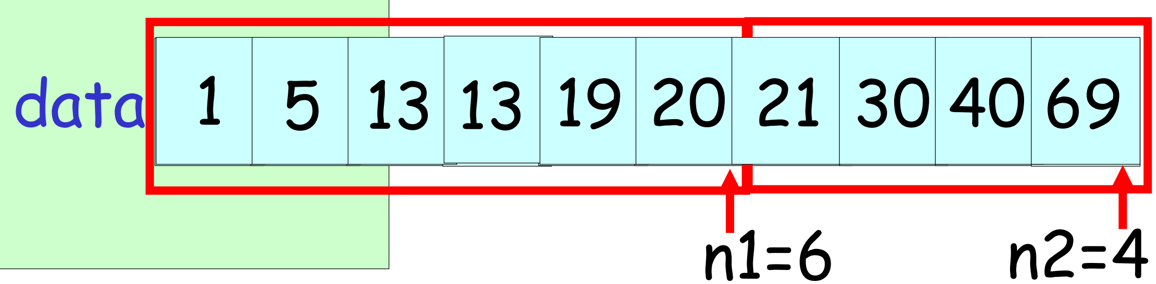
Here's the C++ version of our merge function!

Instead of passing in A1, A2 and B…

you pass in an array called data and two sizes: n1 and n2

Notice how this function uses new/delete to allocate a temporary array for merging.

data holds the merged contents at the end.



Mergesort

OK – so what's the full mergesort alogrithm:

Mergesort function :

1. If array has one element, then return (it's sorted).
2. Split up the array into two equal sections
3. Recursively call Mergesort function on the left half
4. Recursively call Mergesort function on the right half
5. Merge the two halves using our merge function

Ok, let's see how to mergesort a shelf full of books!

1. If array has 1 item, then return
2. Split array in two equal sections
3. Call Mergesort on the left half
4. Call Mergesort on the right half
5. Merge the halves back together

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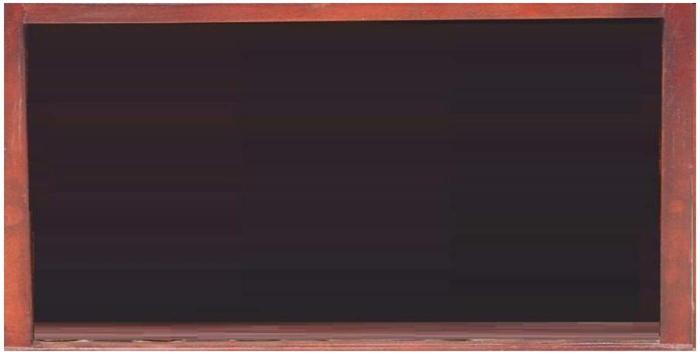

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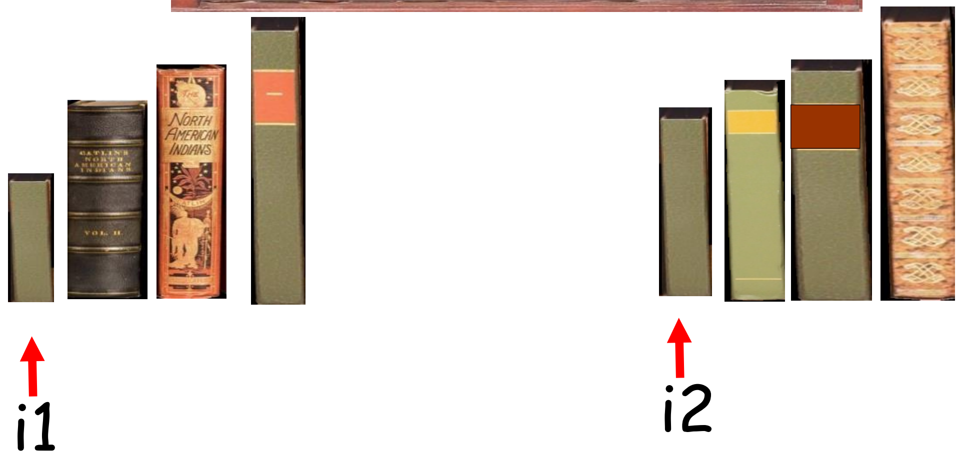
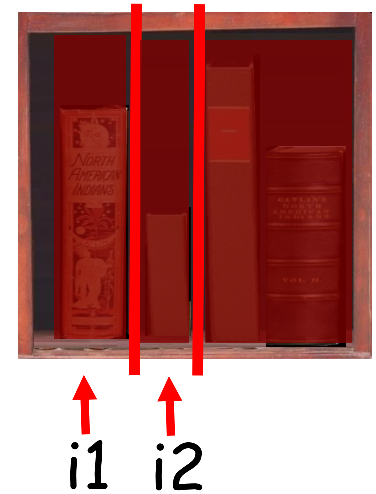


And our array is sorted!!!!

Mergesort - One Final Detail

While I showed the Mergesort moving books into a bunch of small piles…

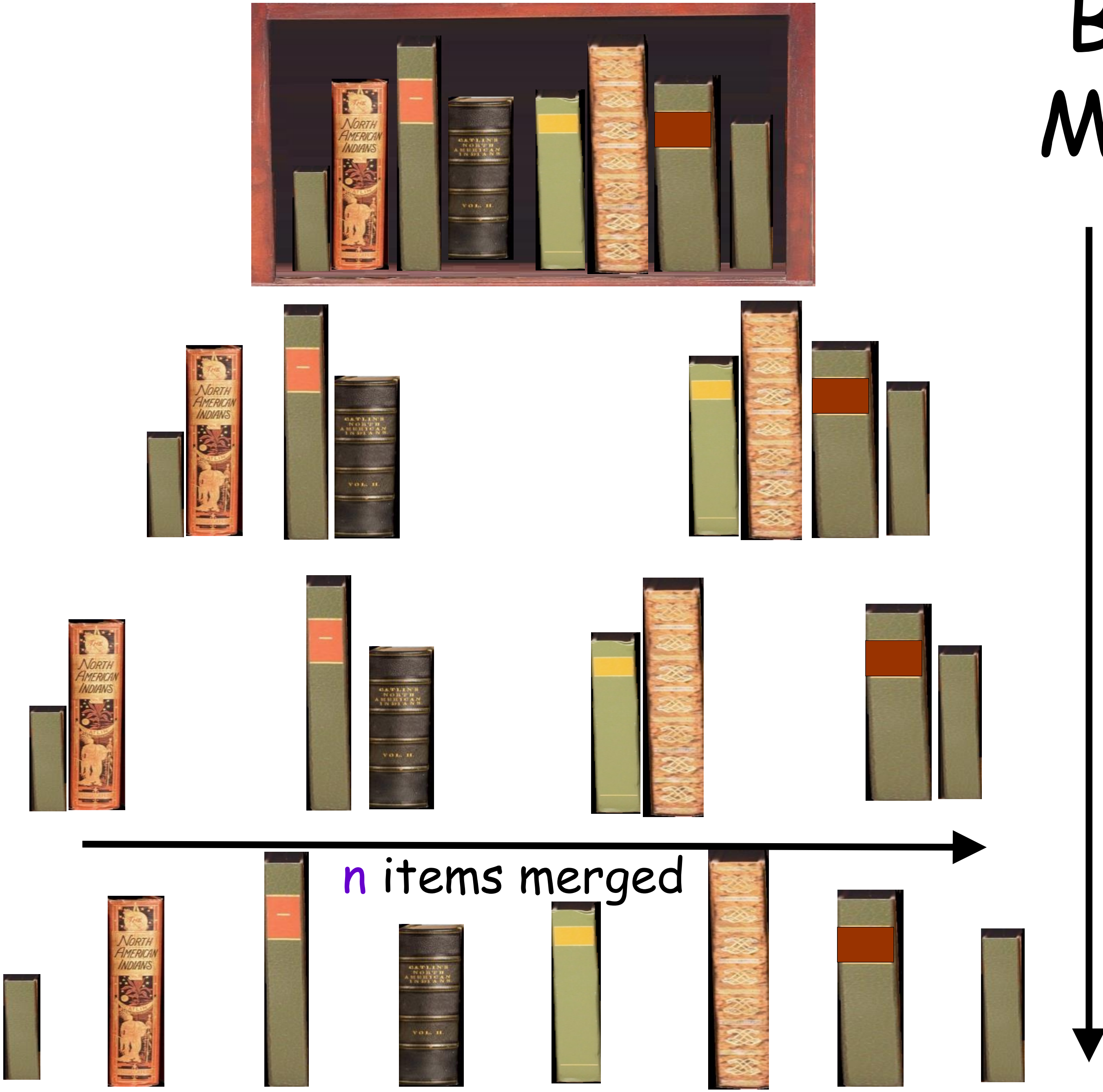


The real algorithm sorts the data in-place in the array…

and only uses a separate array for merging.

Let's see how it really works!

Big-oh of Mergesort



$\log_2 n$ levels deep

Why? Because we keep dividing our piles in half…

until our piles are just 1 book!

Big-oh of Mergesort

n items merged

n items merged

$\log_2 n$ levels deep

Why? Because we keep dividing our piles in half…

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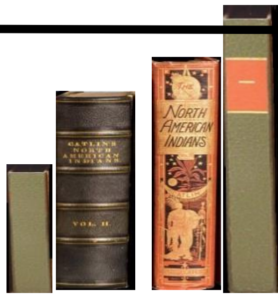
Big-oh of Mergesort



n items merged

n items merged

n items merged

$\log_2 n$ levels deep

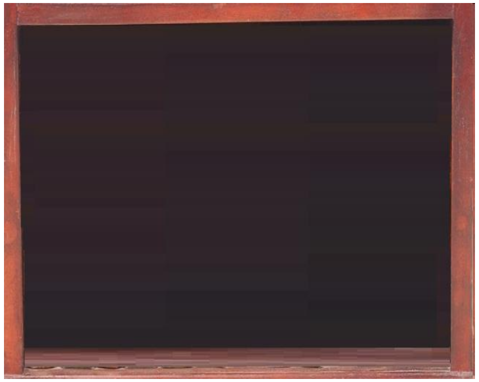
Why? Because we keep dividing our piles in half…

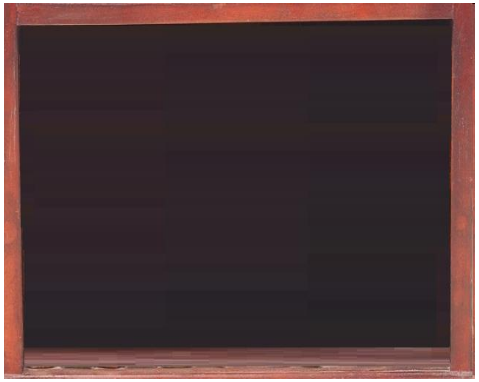
until our piles are just 1 book!

Overall, this gives us $n \cdot \log_2(n)$ steps to sort n items of data. Not bad! ☺

Mergesort – Any Problem Cases

So, are there any cases where mergesort is less efficient?





No! Mergesort works equally well regardless of the ordering of the data…

However, because the merge function needs secondary arrays to merge, this can slow things down a bit…

In contrast, quicksort doesn't need to allocate any new arrays to work.

MergeSort Questions

Can MergeSort be applied easily to sort items within a linked list?

Is MergeSort a "stable" sort?

Are there any special uses for MergeSort that other sorts can't handle?

Can MergeSort be parallelized across multiple cores?

Sorting Overview

Sort Name	Stable/ Non-stable	Notes
Selection Sort	Unstable	Always $O(n^2)$, but simple to implement. Can be used with linked lists. Minimizes the number of item-swaps (important if swaps are slow)
Insertion Sort	Stable	$O(n)$ for already or nearly-ordered arrays. $O(n^2)$ otherwise. Can be used with linked lists. Easy to implement.
Bubble Sort	Stable	$O(n)$ for already or nearly-ordered arrays (with a good implementation). $O(n^2)$ otherwise. Can be used with linked lists. Easy to implement. Rarely a good answer on an interview!
Shell Sort	Unstable	$O(n^{1.25})$ approx. OK for linked lists. Used in some embedded systems (eg, in a car) instead of quicksort due to fixed RAM usage.
Quick Sort	Unstable	$O(n \log_2 n)$ average, $O(n^2)$ for already/mostly/reverse ordered arrays or arrays with the same value repeated many times. Can be used with linked lists. Can be parallelized across multiple cores. Can require up $O(n)$ slots of extra RAM (for recursion) in the worst case, $O(\log_2 n)$ avg.
Merge Sort	Stable	$O(n \log_2 n)$ always. Used for sorting large amounts of data on disk (aka "external sorting"). Can be used to sort linked lists. Can be parallelized across multiple cores. Downside: Requires n slots of extra memory/disk for merging – other sorts don't need xtra RAM.
Heap Sort	Unstable	$O(n \log_2 n)$ always. Sometimes used in low-RAM embedded systems because of its performance/low memory req'ts.

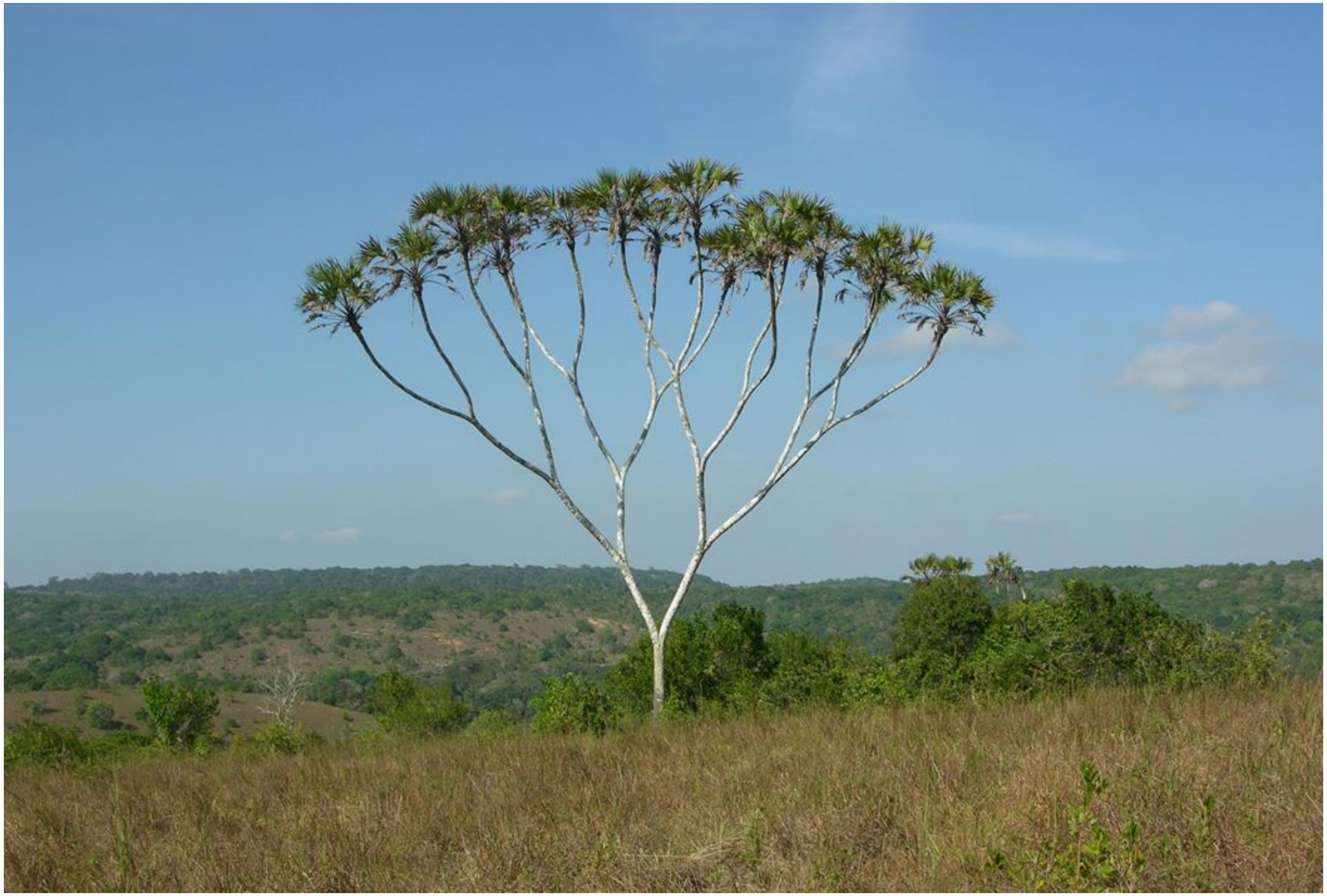
Challenge Problems

1. Give an algorithm to efficiently determine which element occurs the largest number of times in the array.

2. What's the best algorithm to sort 1,000,000 random numbers that are all between 1 and 5?


when your code is meant to be O(NlogN) but it's been 30 minutes and it still hasn't finished
N=3
breath in
boi

Trees



Trees
Why should you care?

Trees are used to organize data in many software applications, including:

Databases (B-TREES)
Data Compression (Huffman Trees)
Bitcoin (Merkle Trees)
Medical Diagnosis (Decision Trees)

And because you'll be asked about them in job interviews and on exams.

So pay attention!

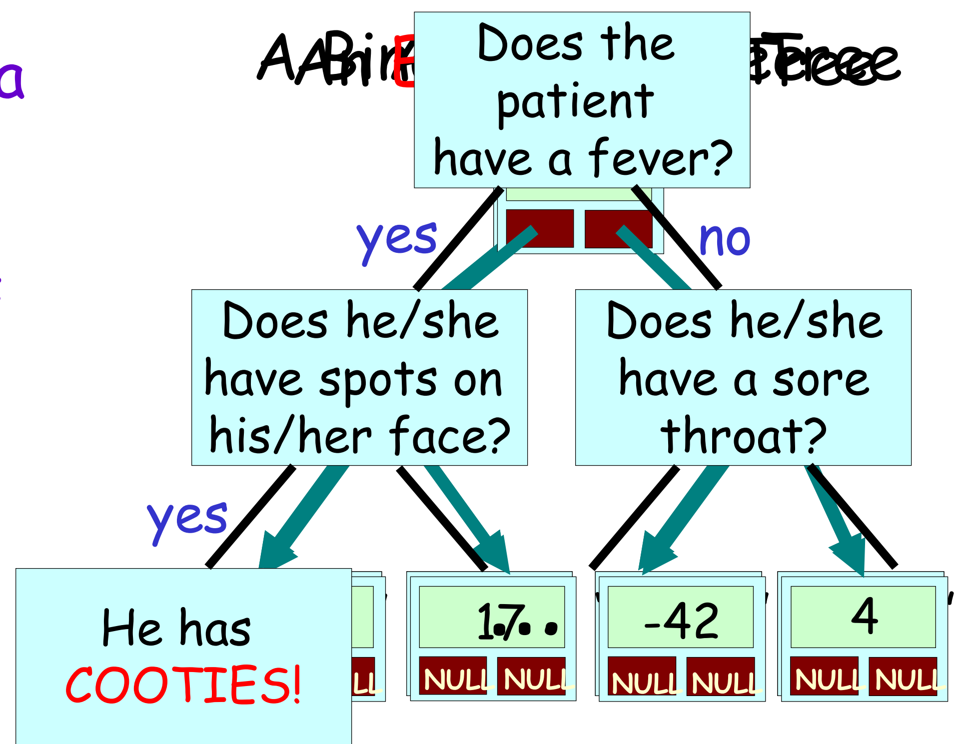
Trees

"I think that I shall never see a data structure as lovely as a tree." - Carey Nachenberg

A Tree is a special linked list-based data structure that has many uses in Computer Science:

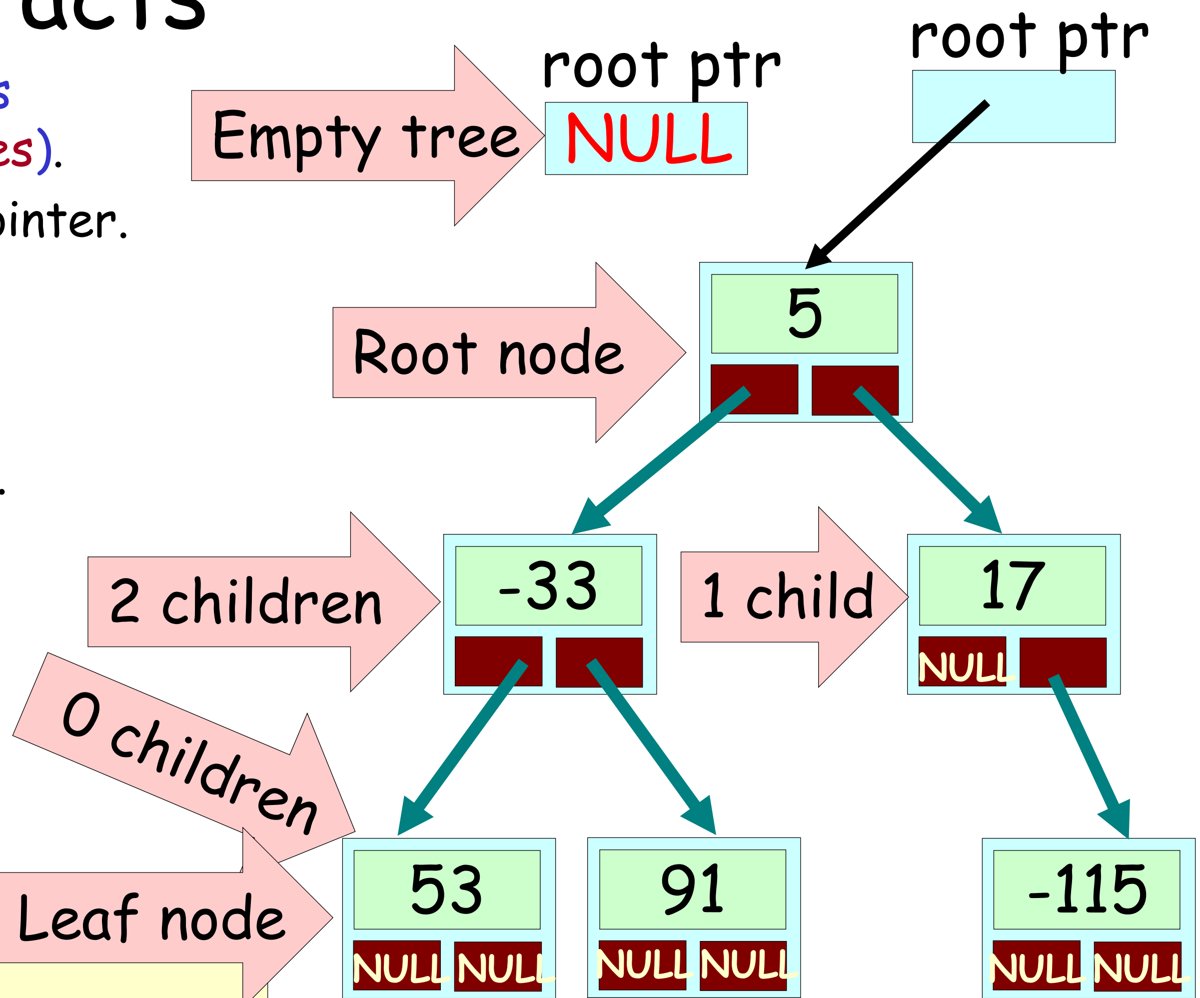
- To organize hierarchical data
- To make information easily searchable
- To simplify the evaluation of mathematical expressions
- To make decisions

A Decision Tree



Basic Tree Facts

1. Trees are made of nodes (just like linked list nodes).
2. Every tree has a "root" pointer.
3. The top node of a tree is called its "root" node.
4. Every node may have zero or more "children" nodes.
5. A node with 0 children is called a "leaf" node.
6. A tree with no nodes is called an "empty tree."




```
struct node
{
  int value;  // some value

  node *left, *right;
};

node *rootPtr;
```

But instead of just one next pointer, a tree node can have two or more next pointers!

The tree's root pointer is like a linked list's head pointer!

Tree Nodes Can Have Many Children

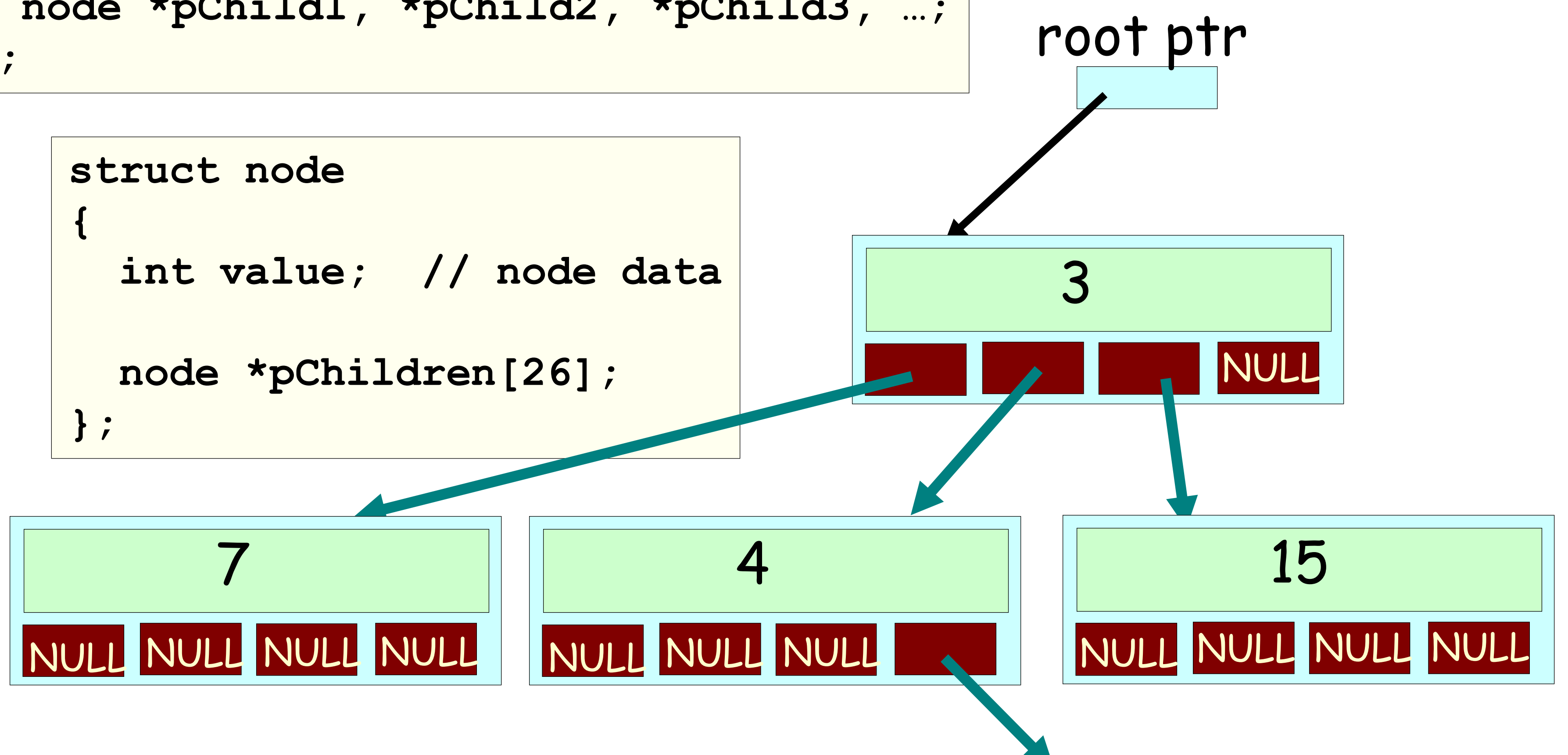
A tree node can have more than just two children:

```
struct node
{
  int value;  // node data

  node *pChild1, *pChild2, *pChild3, …;
};
```

```
struct node
{
  int value;  // node data

  node *pChildren[26];
};
```



Binary Trees

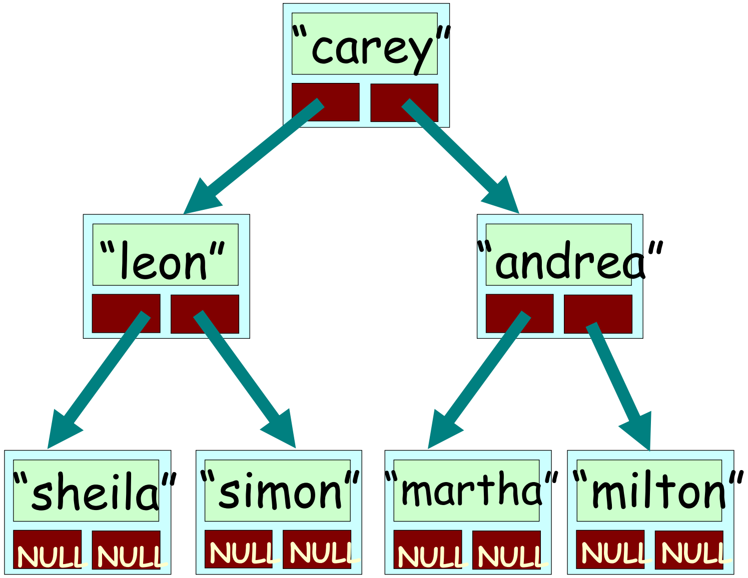
A binary tree is a special form of tree. In a binary tree, every node has at most two children nodes:

A left child and a right child.

```
struct BTNODE  // binary tree node
{
  string value;  // node data

  BTNODE *pLeft, *pRight;
};
```

A Binary Tree



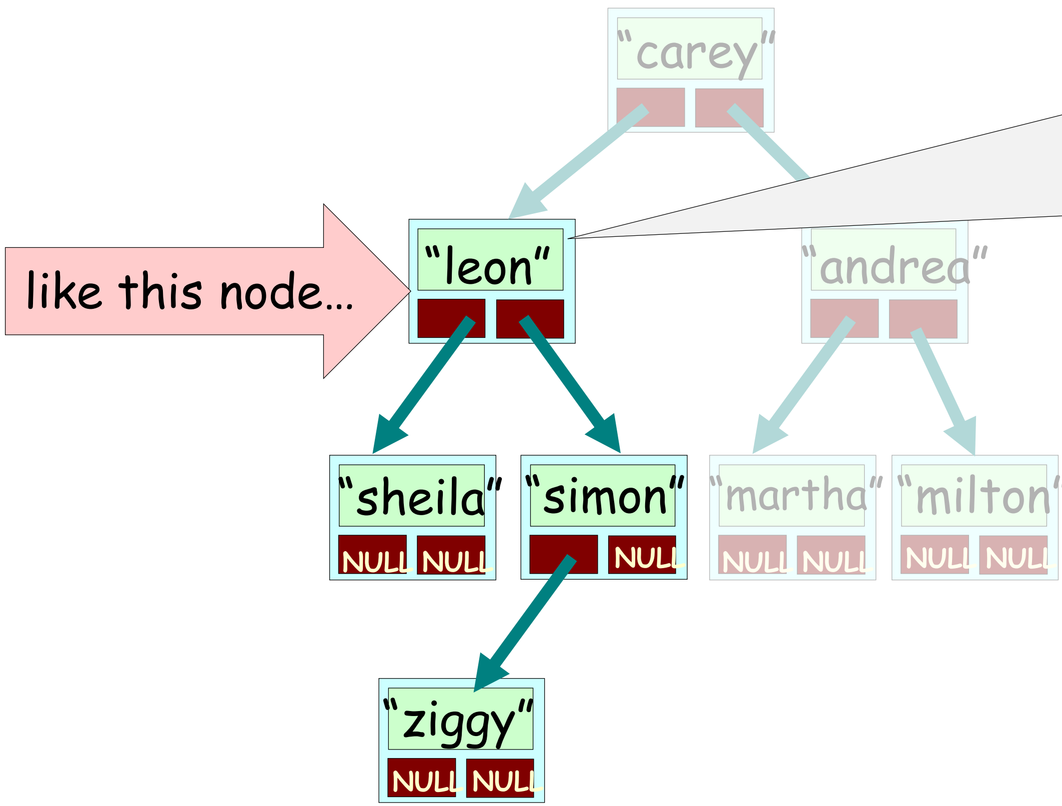
Binary Tree Subtrees

We can pick any node in the tree…

And then focus on its "subtree" - which includes it and all of nodes below it.

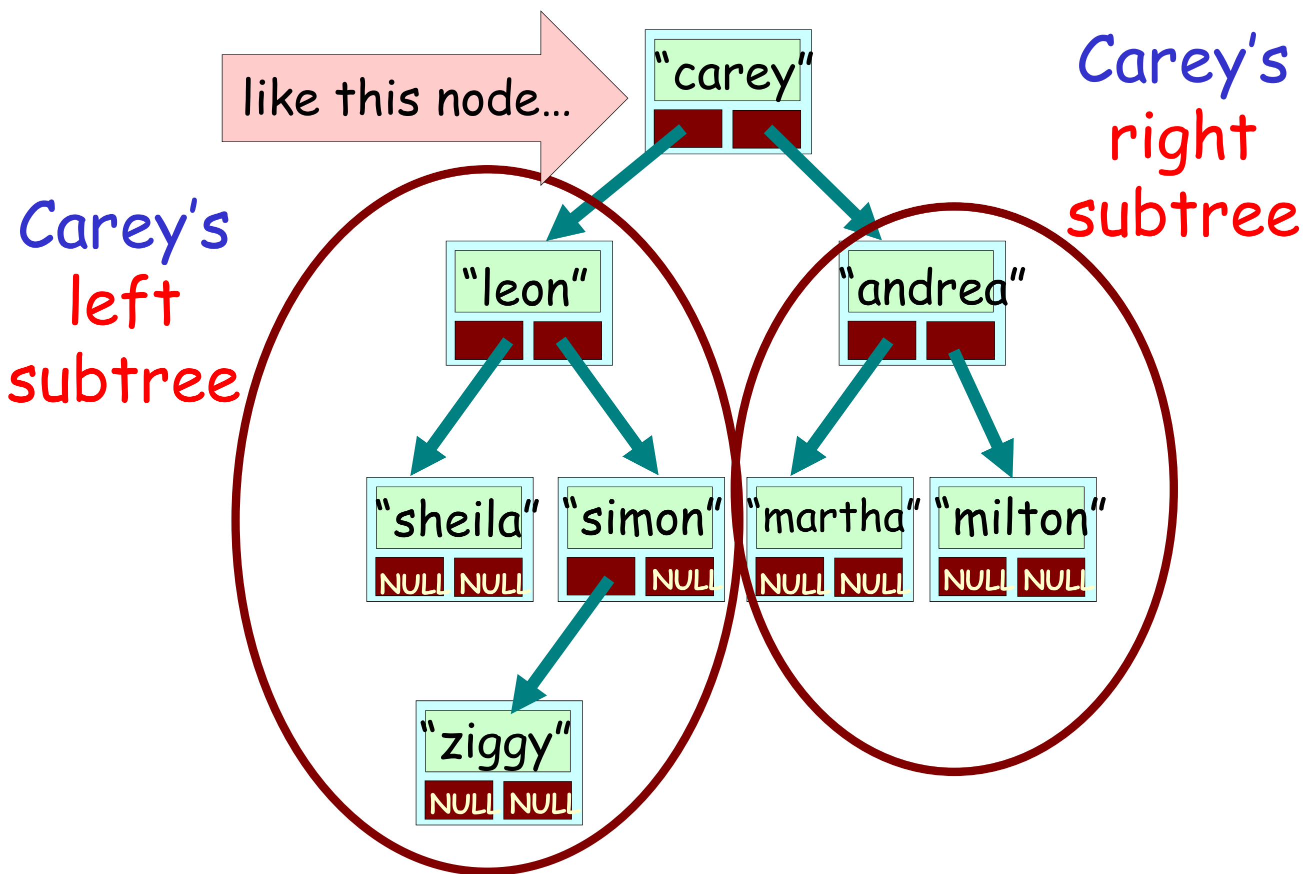
This subtree includes four different nodes…

It has the "leon" node as its root.



Binary Tree Subtrees

If we pick a node from our tree...
we can also identify *its* left and right sub-trees.



Operations on Binary Trees

The following are common operations that we might perform on a Binary Tree:

- enumerating all the items
- searching for an item
- adding a new item at a certain position on the tree
- deleting an item
- deleting the entire tree (destruction)
- removing a whole section of a tree (called **pruning**)
- adding a whole section to a tree (called **grafting**)

We'll learn about many of these operations over the next two classes.

A Simple Tree Example

```
struct BTNODE  // node
{
  int value;   // data
  BTNODE *left, *right;
};
```

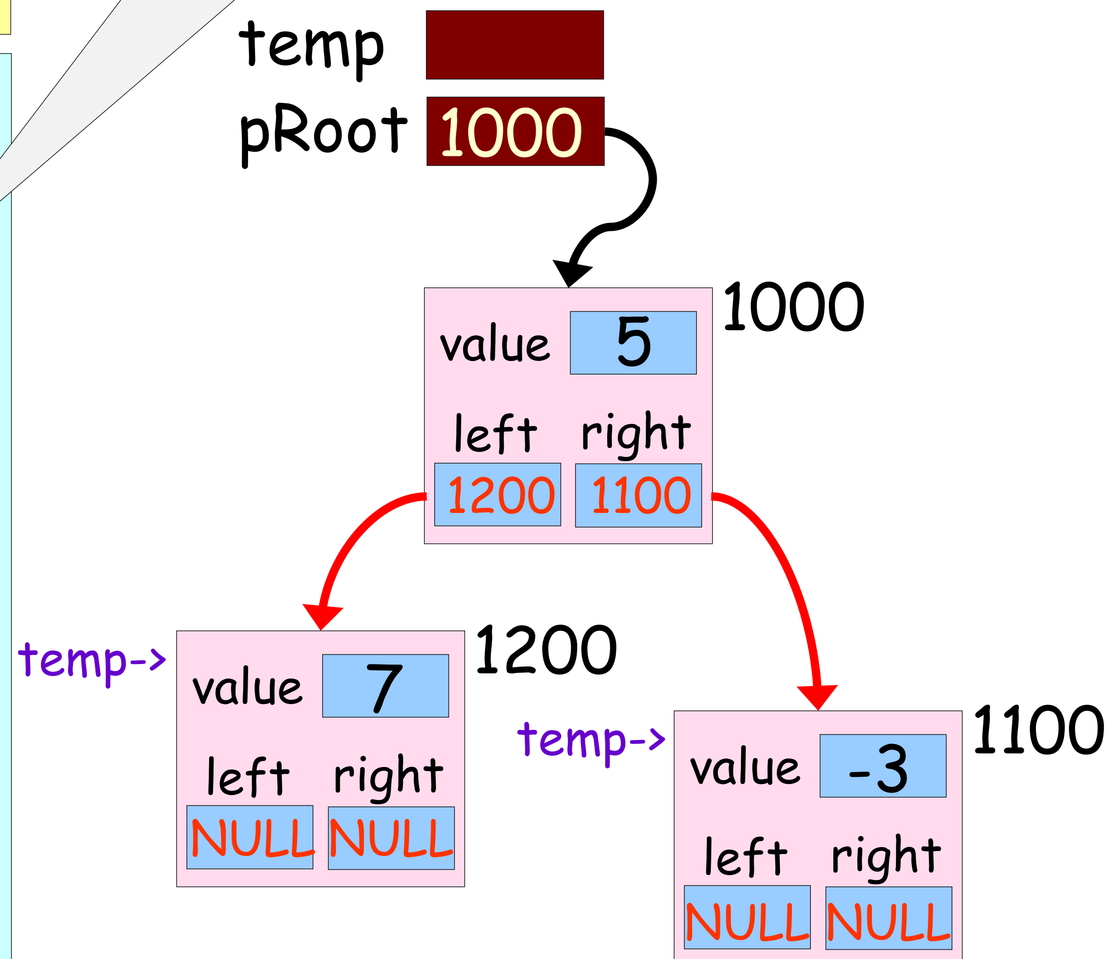
```
main()
{
   BTNODE *temp, *pRoot;

   pRoot = new BTNODE;
   pRoot->value = 5;

   temp = new BTNODE;
   temp->value = 7;
   temp->left = NULL;
   temp->right = NULL;
   pRoot->left = temp;

   temp = new BTNODE;
   temp->value = -3;
   temp->left = NULL;
   temp->right = NULL;
   pRoot->right = temp;
   // etc…
```

As with linked lists, we use dynamic memory to allocate our nodes.



And of course, later we'd have to delete our tree's nodes.

We've created a binary tree… now what?

Now that we've created a binary tree, what can we do with it?

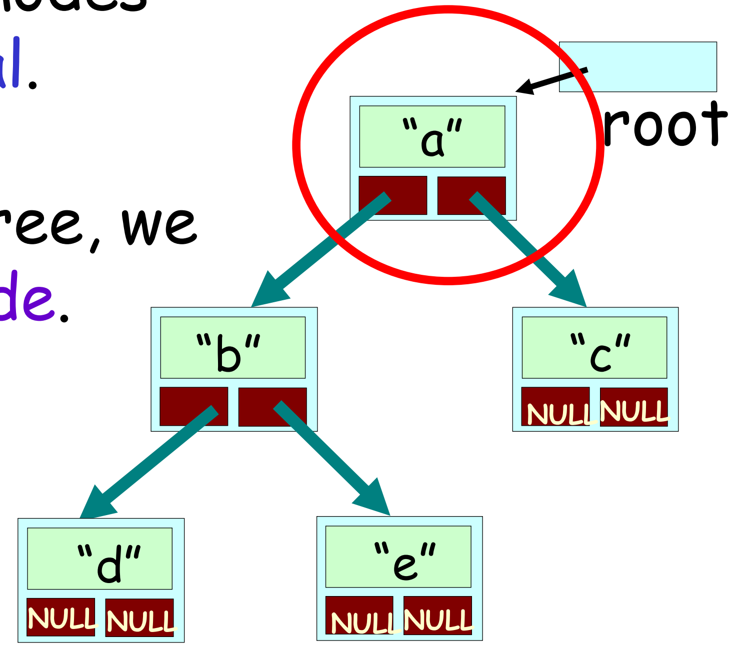
Well, next class we'll learn how to use the binary tree to speed up searching for data.

But for now, let's learn how to iterate through each item in a tree, one at a time.

This is called "traversing" the tree, and there are several ways to do it.

Binary Tree Traversals

When we iterate through all the nodes in a tree, it's called a traversal.



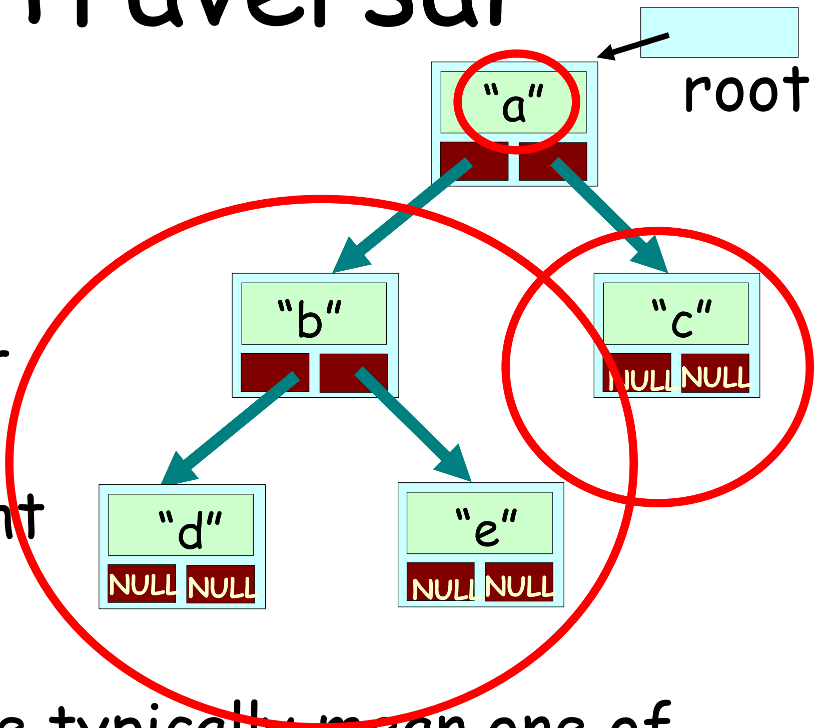
Any time we traverse through a tree, we always start with the root node.

There are four common ways to traverse a tree.

Each technique differs in the order that each node is visited during the traversal:

1. Pre-order traversal
2. In-order traversal
3. Post-order traversal
4. Level-order traversal

The Preorder Traversal



Preorder:

1. Process the current node.
2. Process the nodes in the left sub-tree.
3. Process the nodes in the right sub-tree.

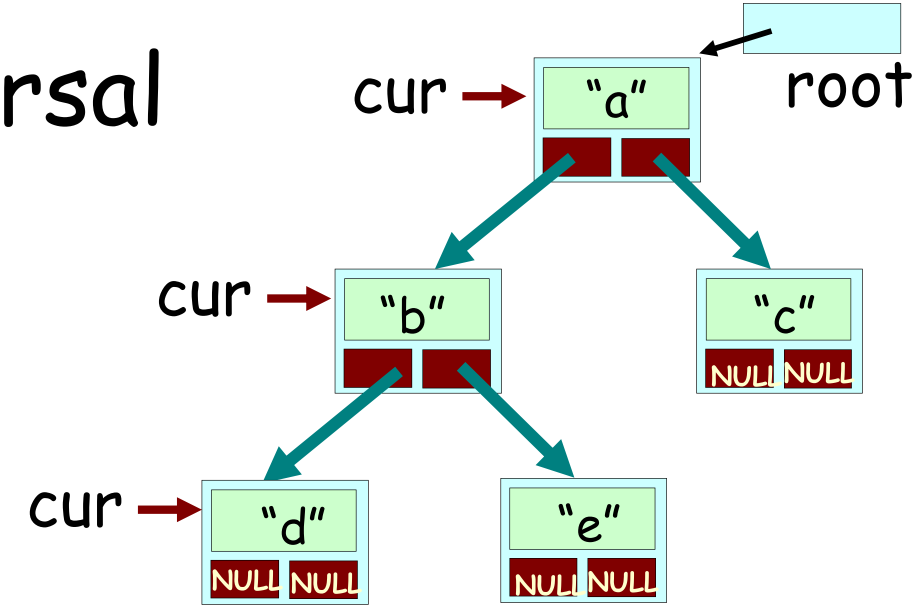
By "process the current node" we typically mean one of the following:

1. Print the current node's value out.
2. Search the current node to see if its value matches the one you're searching for.
3. Add the current node's value to a total for the tree
4. Etc…

The Pre-order Traversal

Output:

a b d




```
void PreOrder(Node *cur)
{
   if (cur == NULL)              // if empty, return...
        return;

   cout << cur->value;        // Process the current node.

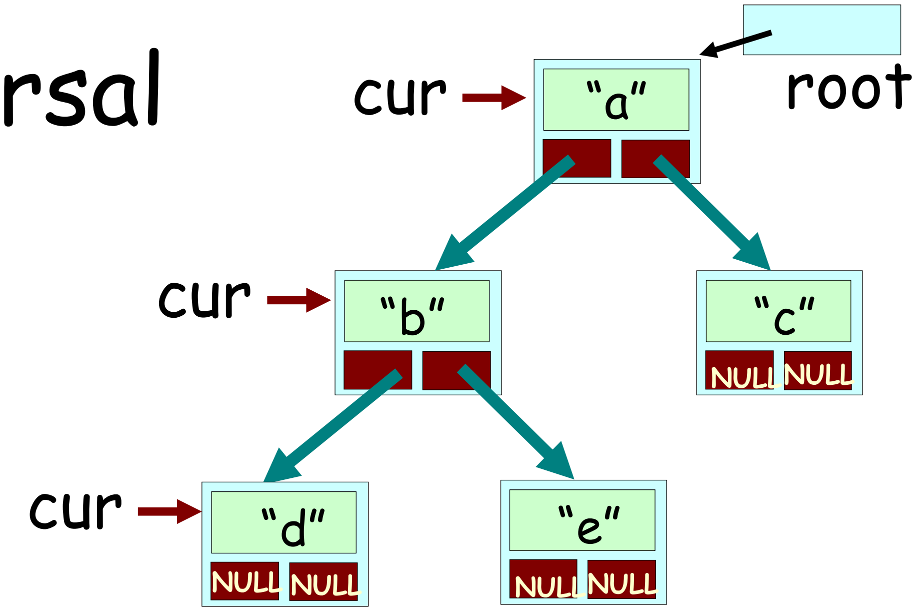
   PreOrder(cur->left);      // Process nodes in left sub-tree.
   PreOrder(cur-> right);  // Process nodes in left sub-tree.
}
```

```
main()
{
   Node *root;

   ...

   PreOrder(root);
}
```

The Pre-order Traversal



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a b d

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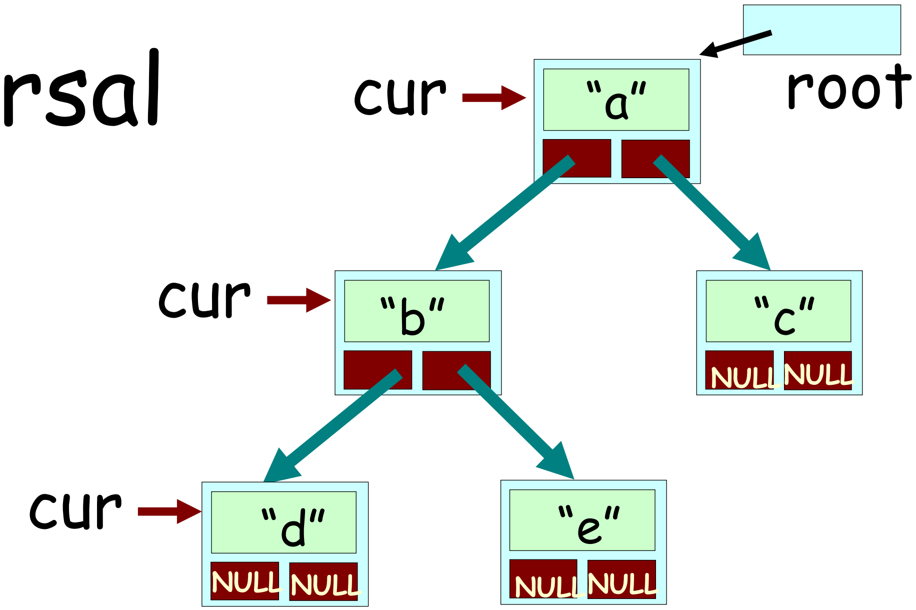
    PreOrder(cur-> right);     // Process nodes in left sub-tree.
}
```

```
n()

lode *root;

 PreOrder(root);
}
```

The Pre-order Traversal



Output:

a b d

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```

```
ain()

  Node *root;

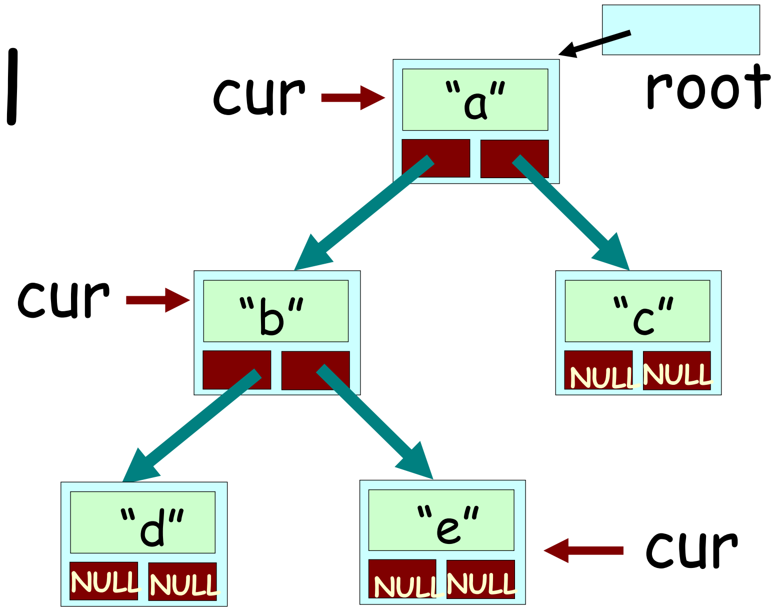
  ...

  PreOrder(root);
}
```

The Pre-order Traversal

Output:

a b d e

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{
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        return;

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}
```

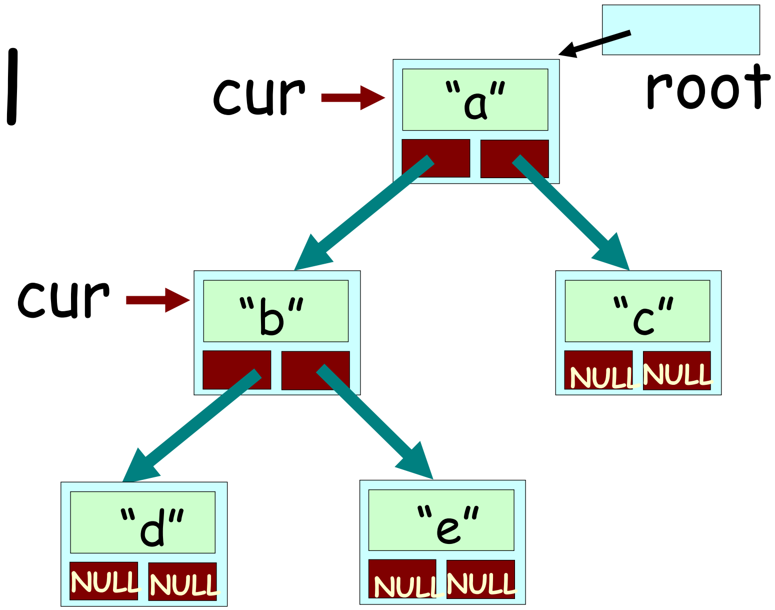
```
ain()

Node *root;

...

PreOrder(root);
}
```

The Pre-order Traversal



Output:

a b d e

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void PreOrder(Node *cur)
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         return;

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```

```
main()
{
   Node *root;

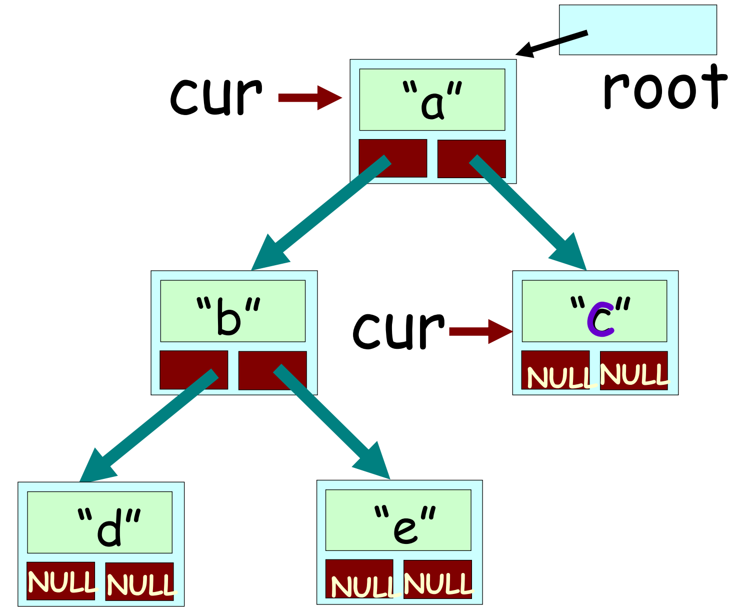
   ...

   PreOrder(root);
}
```

The Pre-order Traversal

Output:

a b d e c

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}
```

```
nain()

    Node *root;

    ...

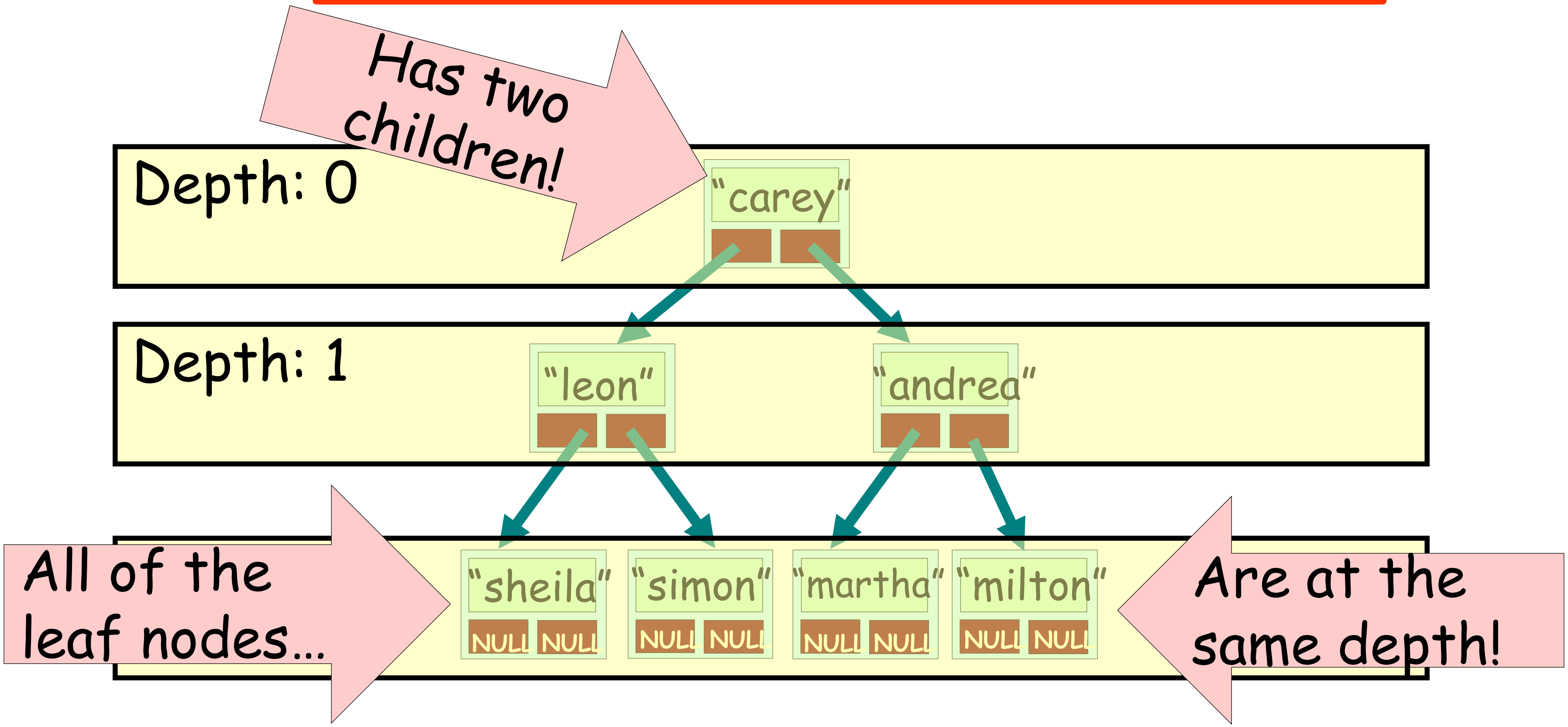
    PreOrder(root);
}
```

Appendix – On Your Own Study

- Full Binary Trees

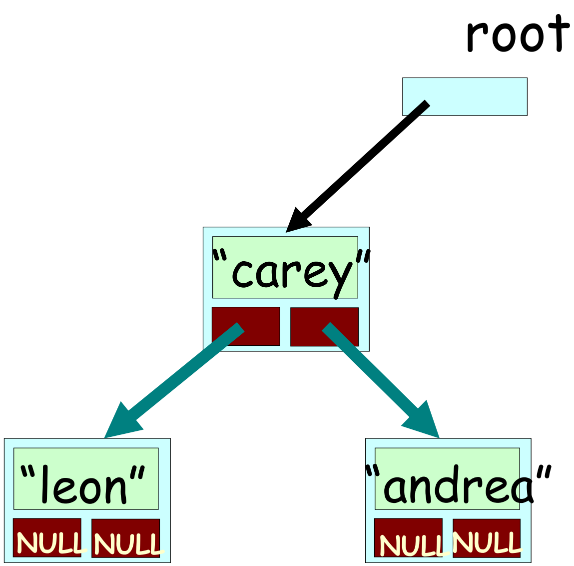
Full Binary Trees

A full binary tree is one in which every leaf node has the same depth, and every non-leaf has exactly two children.

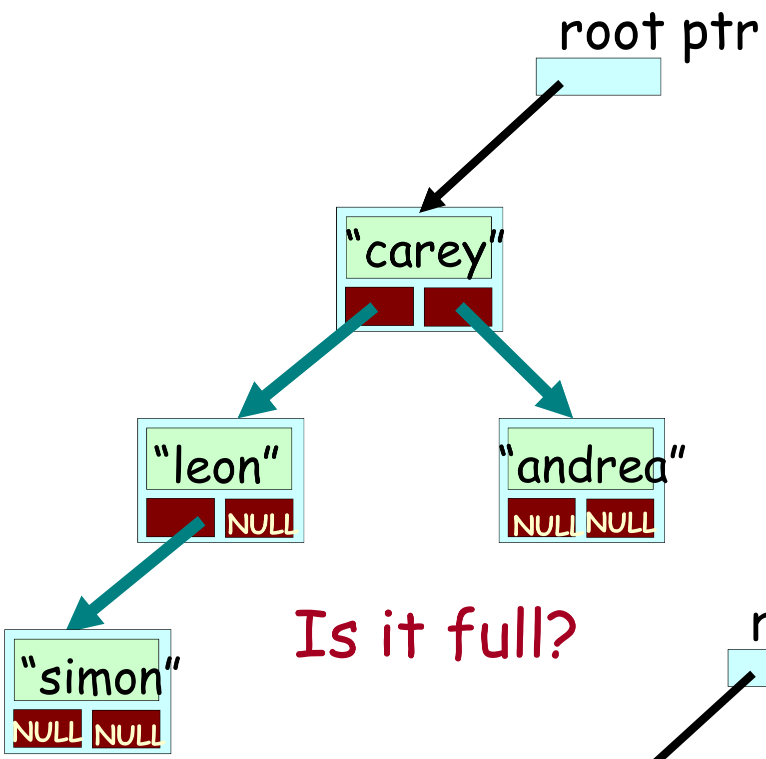


Full Binary Trees

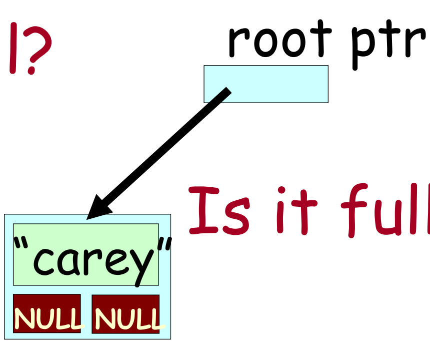
A full binary tree is one in which every leaf node has the same depth, and every non-leaf has exactly two children.



Is it full?



Is it full?



Is it full?