

# The Memory Hierarchy



# Today

- 🌀 **Storage technologies and trends**
- 🌀 Locality of reference
- 🌀 Caching in the memory hierarchy



# Random-Access Memory (RAM)

## 🌀 Key features

- 🌀 **RAM** is traditionally packaged as a chip.
- 🌀 Basic storage unit is normally a **cell** (one bit per cell).
- 🌀 Multiple RAM chips form a memory.

## 🌀 RAM comes in two varieties:

- 🌀 SRAM (Static RAM)
- 🌀 DRAM (Dynamic RAM)



# SRAM vs DRAM Summary

|      | Trans.<br>per bit | Access<br>time | Needs<br>refresh? | Needs<br>EDC? | Cost | Applications                    |
|------|-------------------|----------------|-------------------|---------------|------|---------------------------------|
| SRAM | 4 or 6            | 1X             | No                | Maybe         | 100x | Cache memories                  |
| DRAM | 1                 | 10X            | Yes               | Yes           | 1X   | Main memories,<br>frame buffers |



# Nonvolatile Memories

## • DRAM and SRAM are volatile memories

- Lose information if powered off.

## • Nonvolatile memories retain value even if powered off

- Read-only memory (**ROM**): programmed during production
- Programmable ROM (**PROM**): can be programmed once
- Erasable PROM (**EPROM**): can be bulk erased (UV, X-Ray)
- Electrically erasable PROM (**EEPROM**): electronic erase capability
- Flash memory: EEPROMs. with partial (block-level) erase capability
  - Wears out after about 100,000 erasings

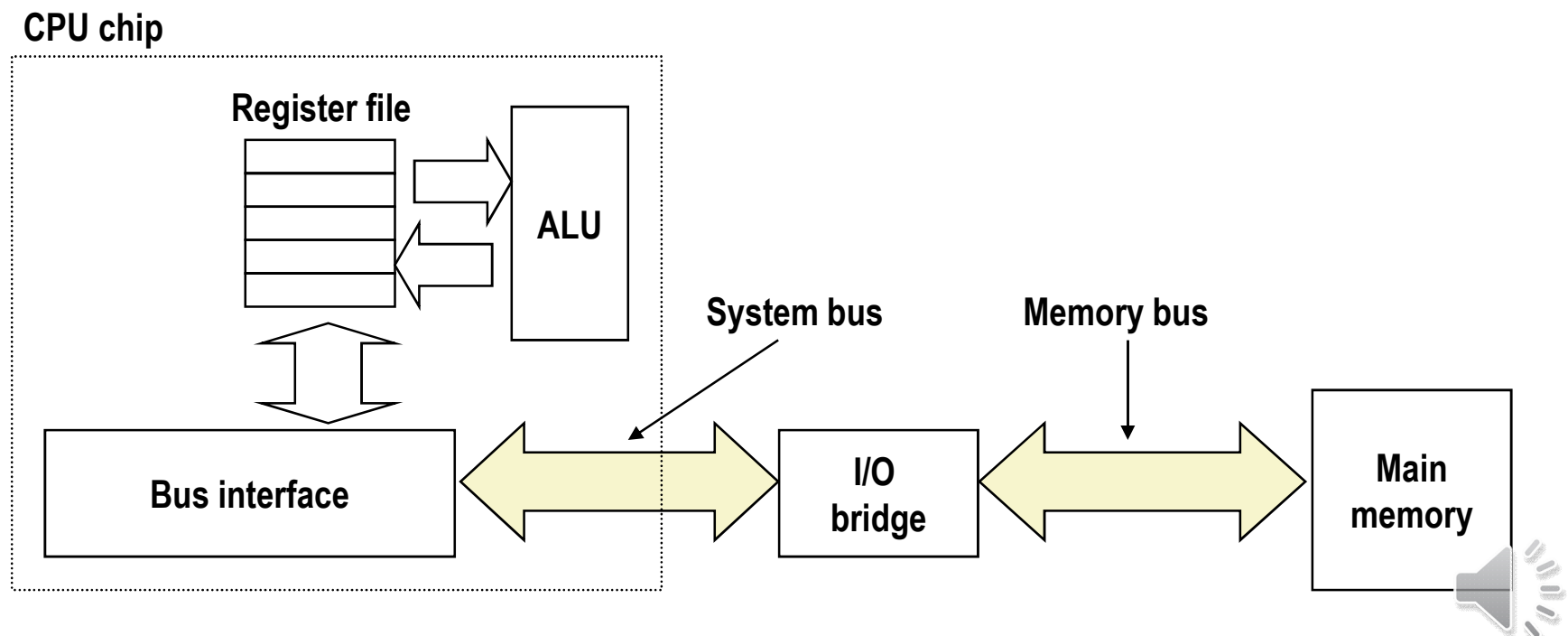
## • Uses for Nonvolatile Memories

- Firmware programs stored in a ROM (BIOS, controllers for disks, network cards, graphics accelerators, security subsystems,...)
- Solid state disks (replace rotating disks in thumb drives, smart phones, mp3 players, tablets, laptops,...)
- Disk caches



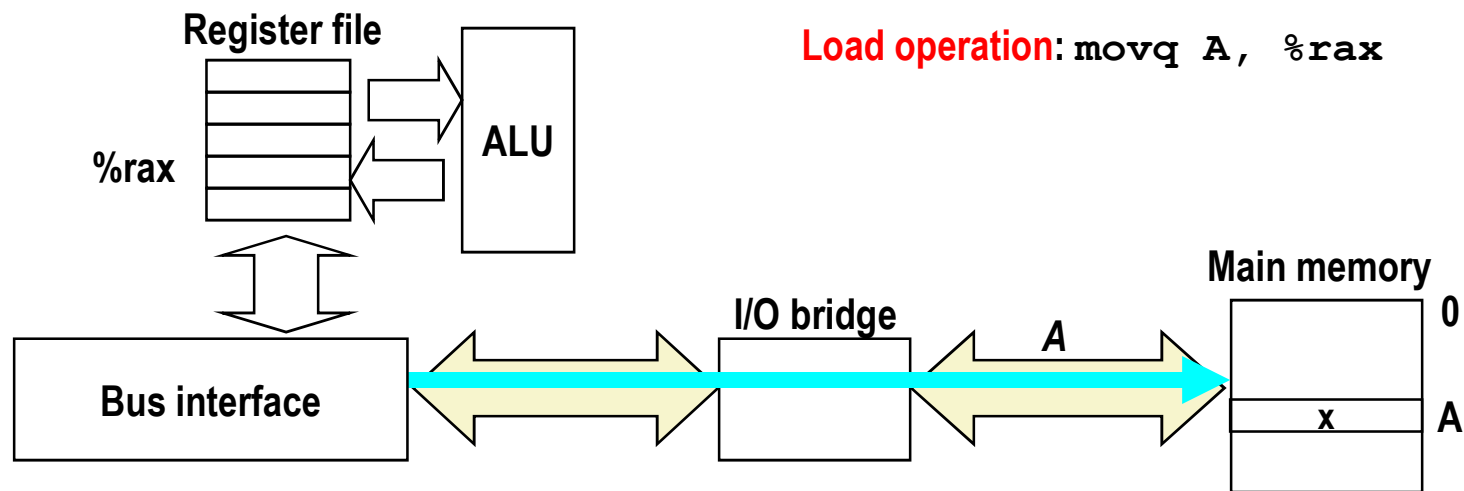
# Traditional Bus Structure Connecting CPU and Memory

- 🌀 A **bus** is a collection of parallel wires that carry address, data, and control signals.
- 🌀 Buses are typically shared by multiple devices.



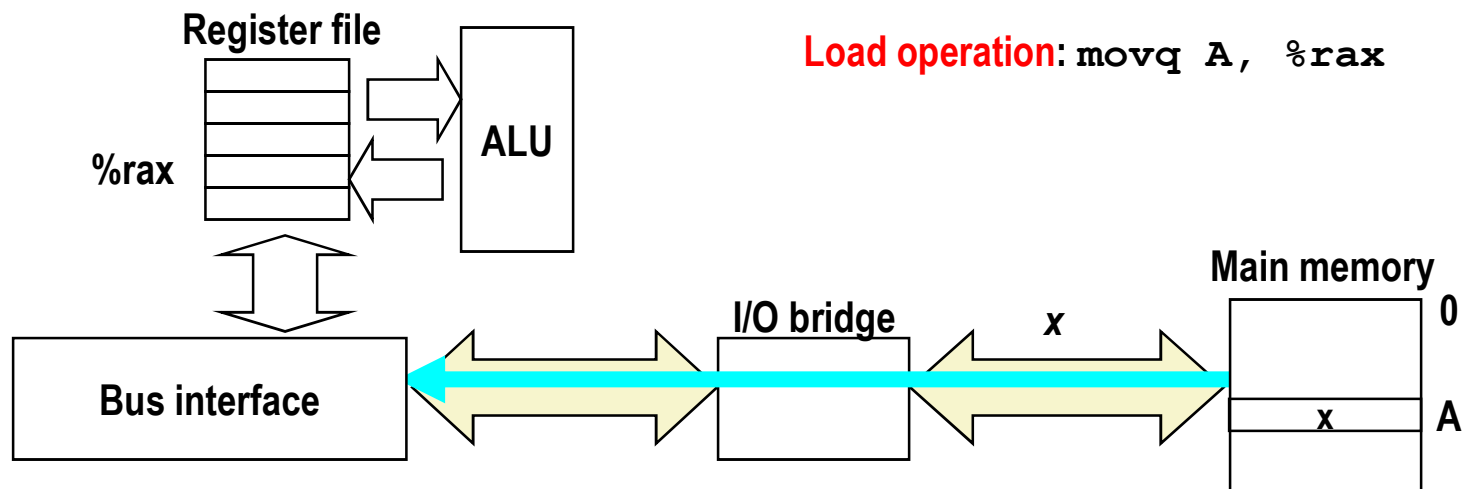
# Memory Read Transaction (1)

- 🌀 CPU places address A on the memory bus.



# Memory Read Transaction (2)

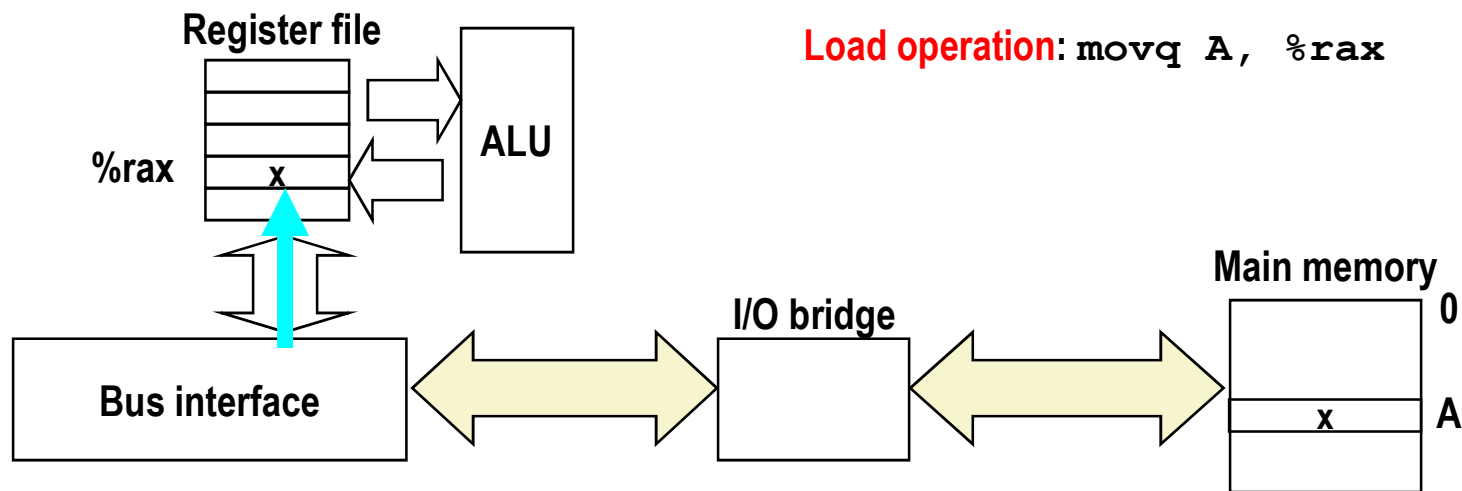
- Main memory reads  $A$  from the memory bus, retrieves word  $x$ , and places it on the bus.





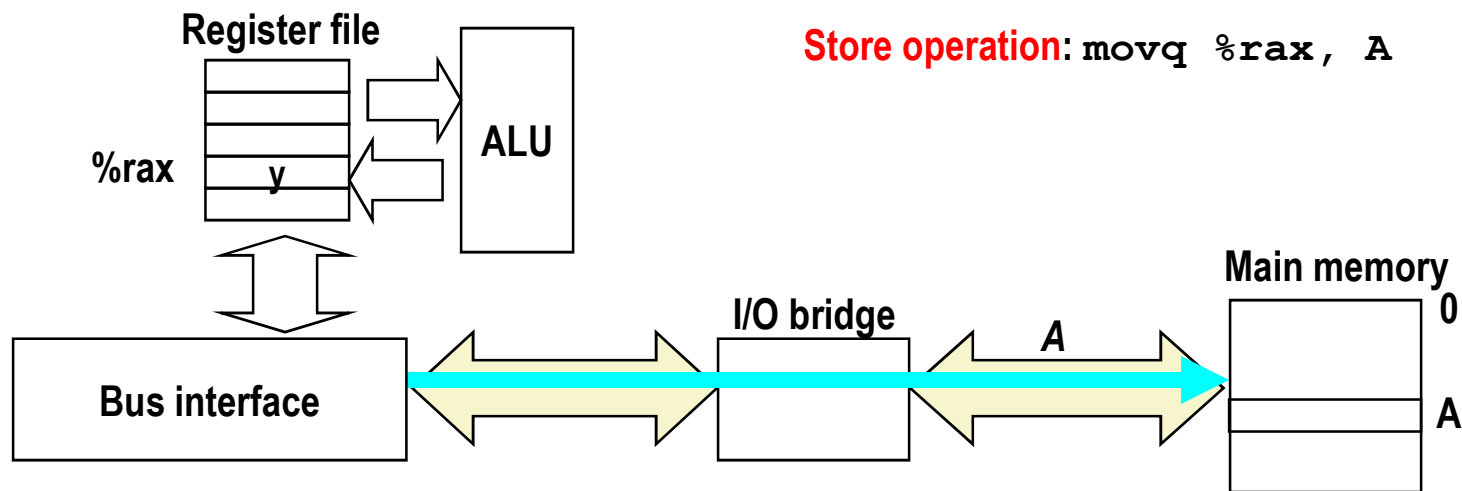
# Memory Read Transaction (3)

- 🌀 CPU read word  $x$  from the bus and copies it into register `%rax`.



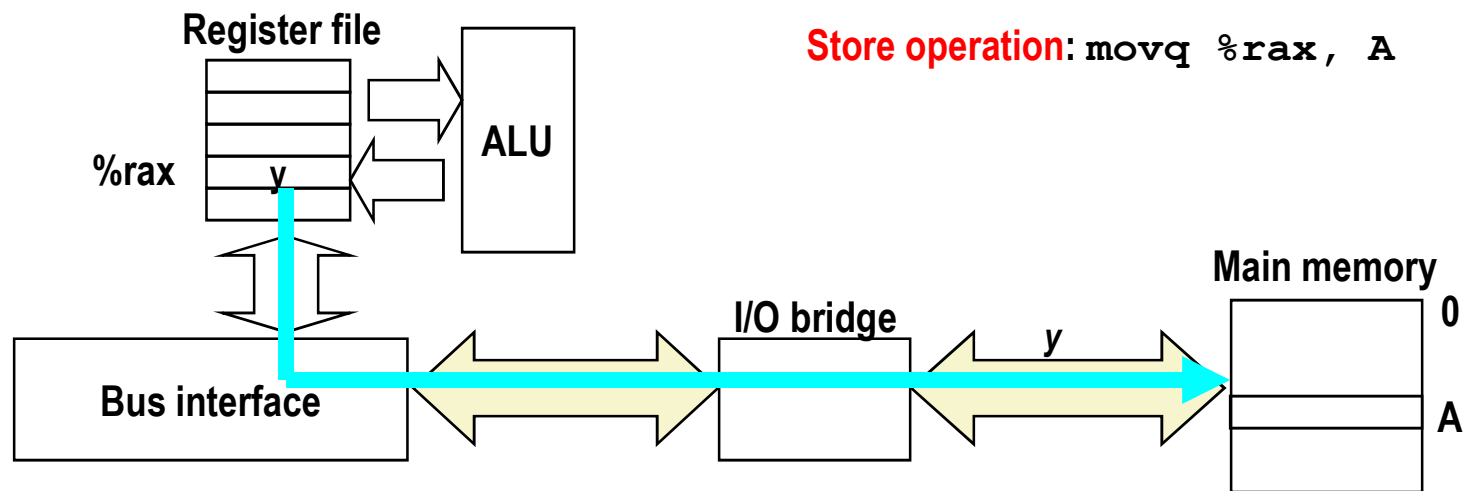
# Memory Write Transaction (1)

- 🌀 CPU places address A on bus. Main memory reads it and waits for the corresponding data word to arrive.



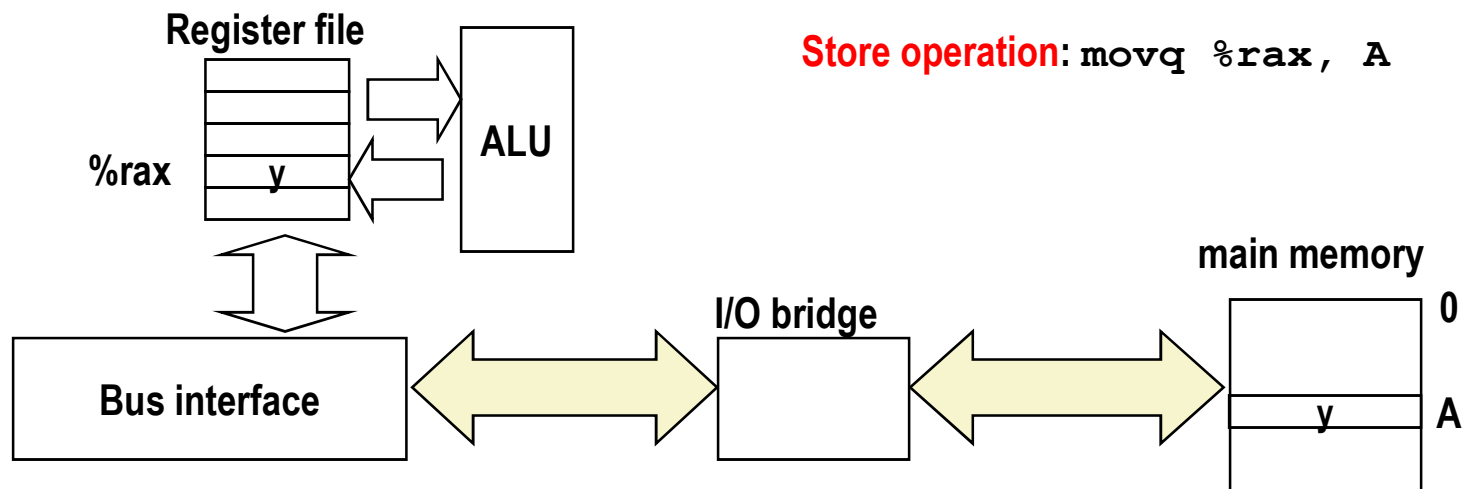
# Memory Write Transaction (2)

- 🔄 CPU places data word  $y$  on the bus.

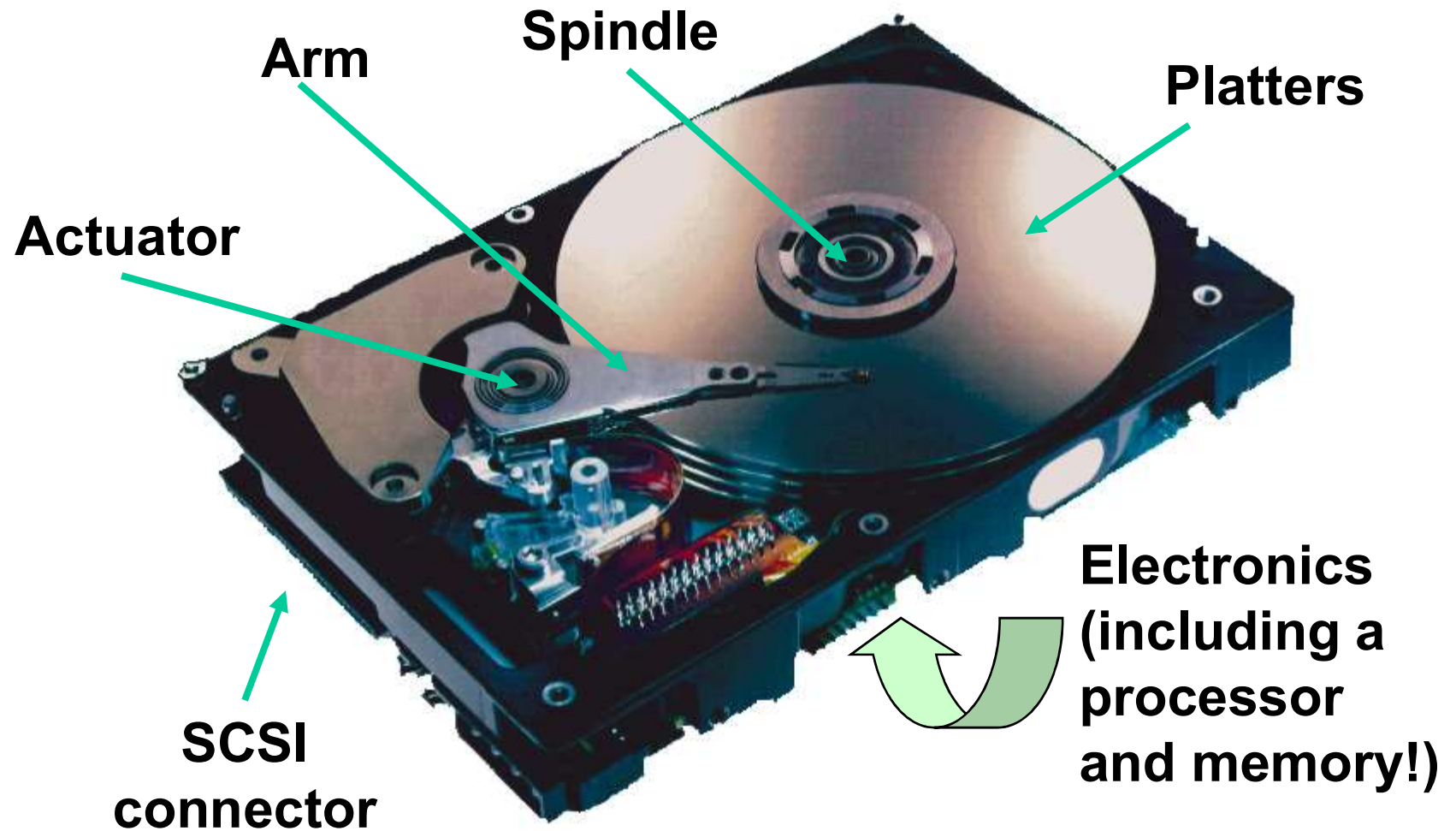


# Memory Write Transaction (3)

- Main memory reads data word  $y$  from the bus and stores it at address  $A$ .

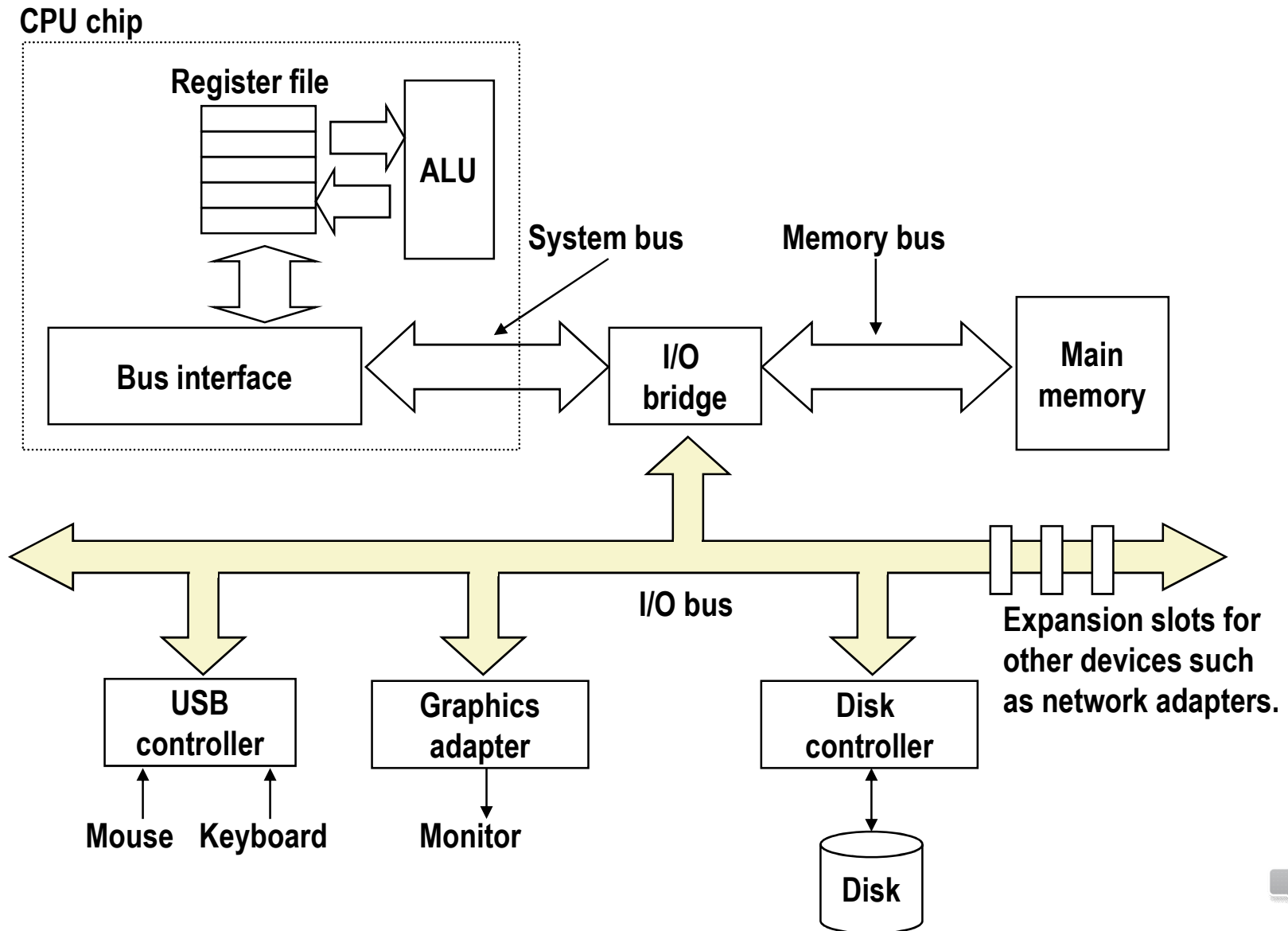


# What's Inside A Disk Drive?

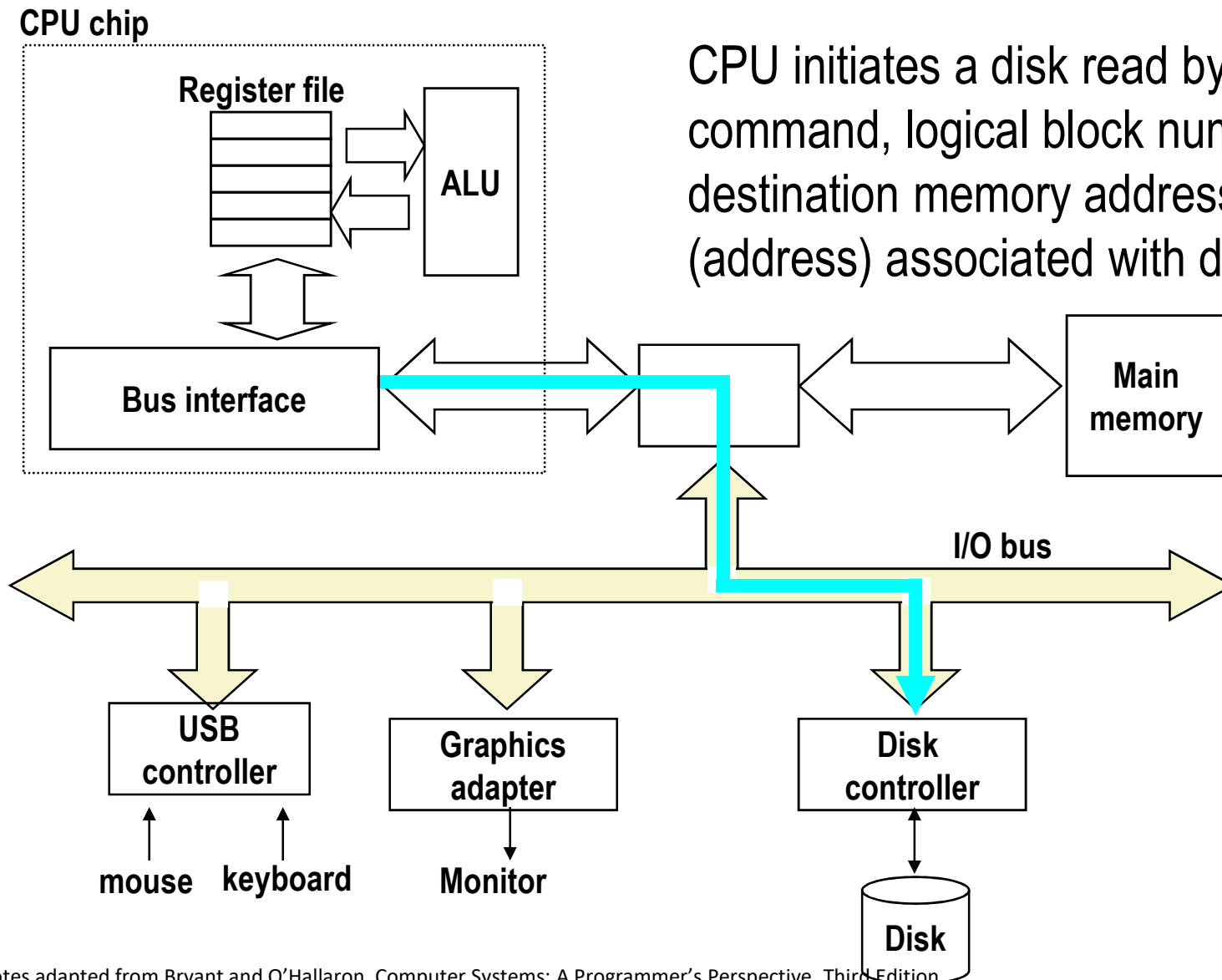


*Image courtesy of Seagate Technology*

# I/O Bus



# Reading a Disk Sector (1)

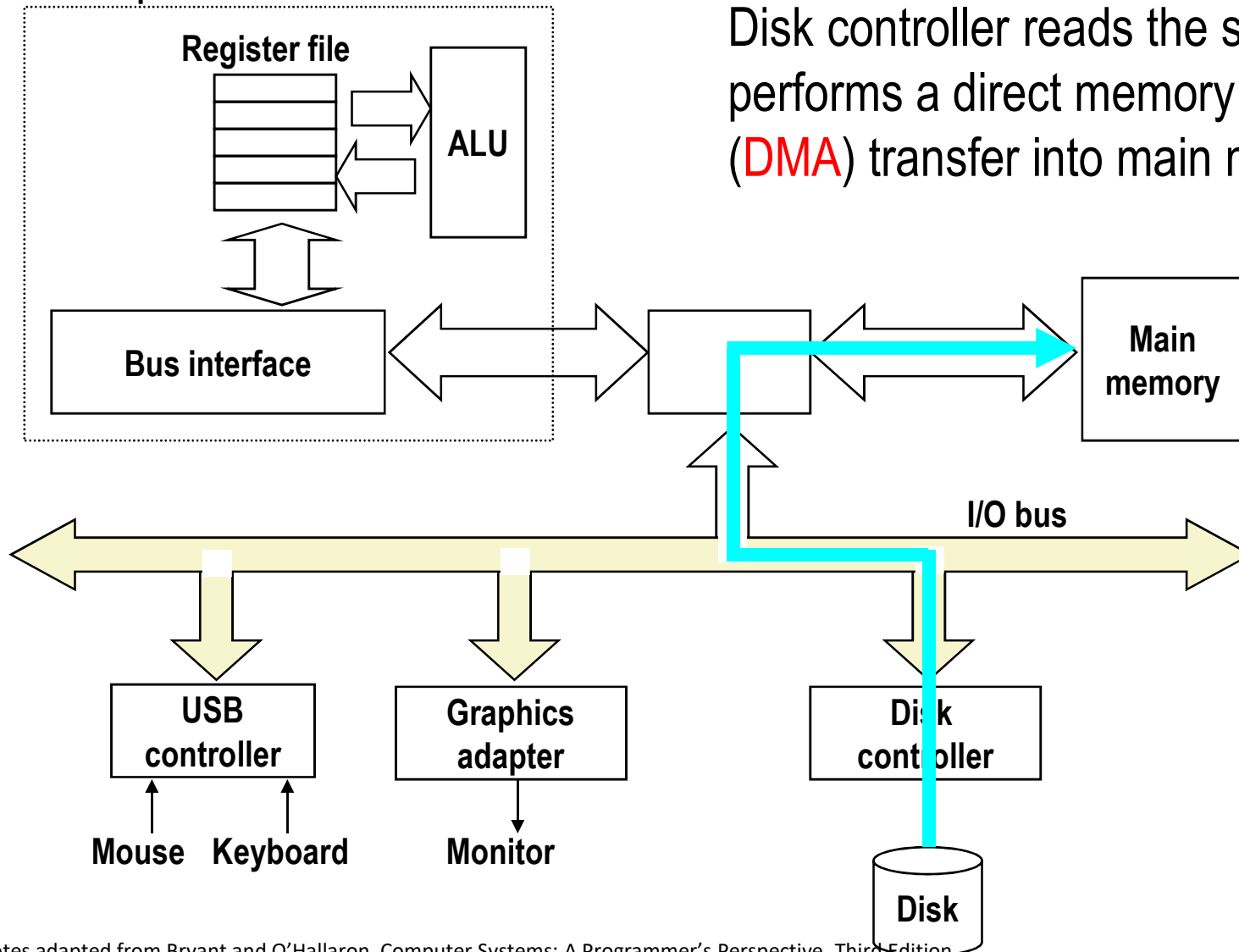


CPU initiates a disk read by writing a command, logical block number, and destination memory address to a **port** (address) associated with disk controller.



# Reading a Disk Sector (2)

CPU chip

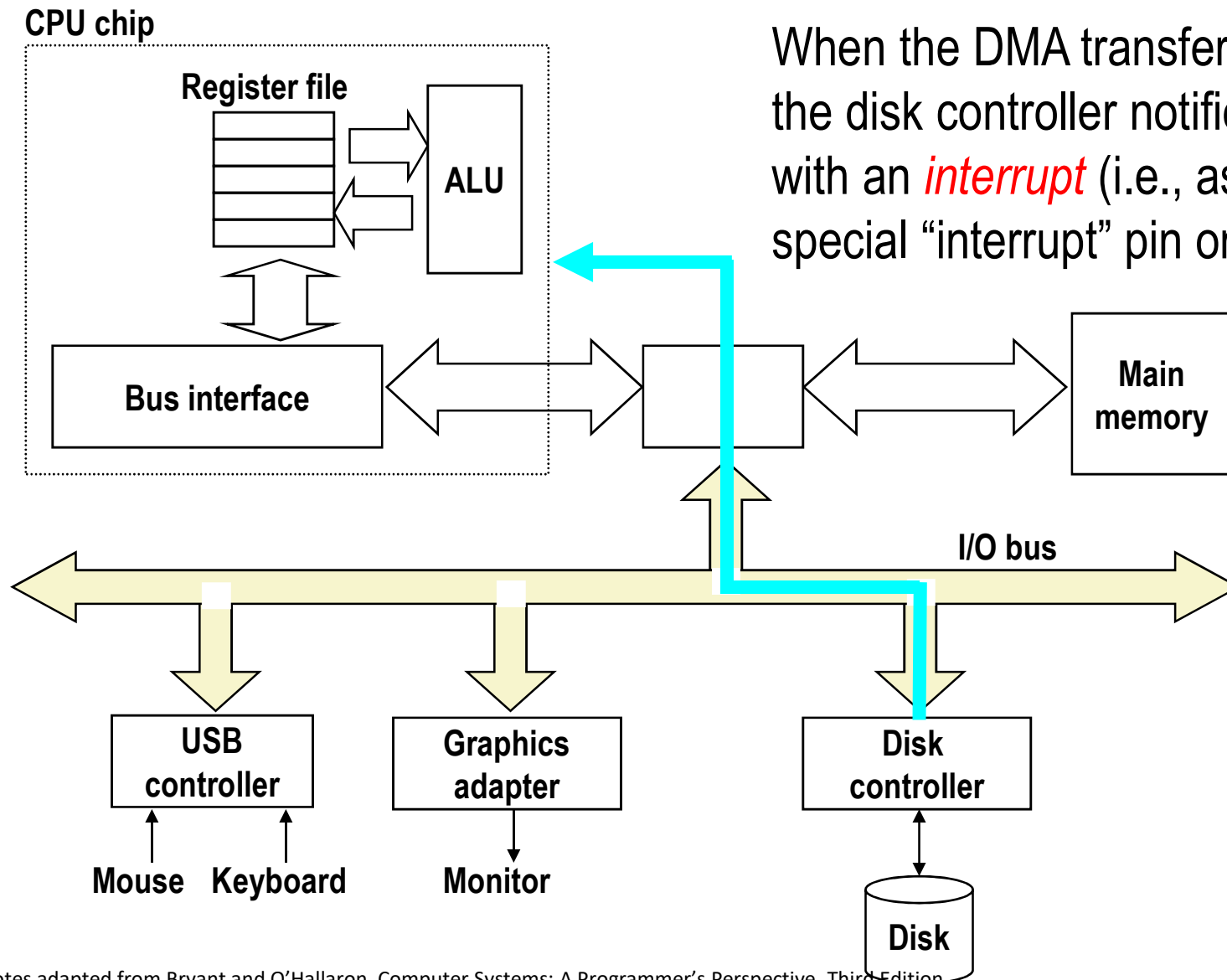


Disk controller reads the sector and performs a direct memory access (**DMA**) transfer into main memory.





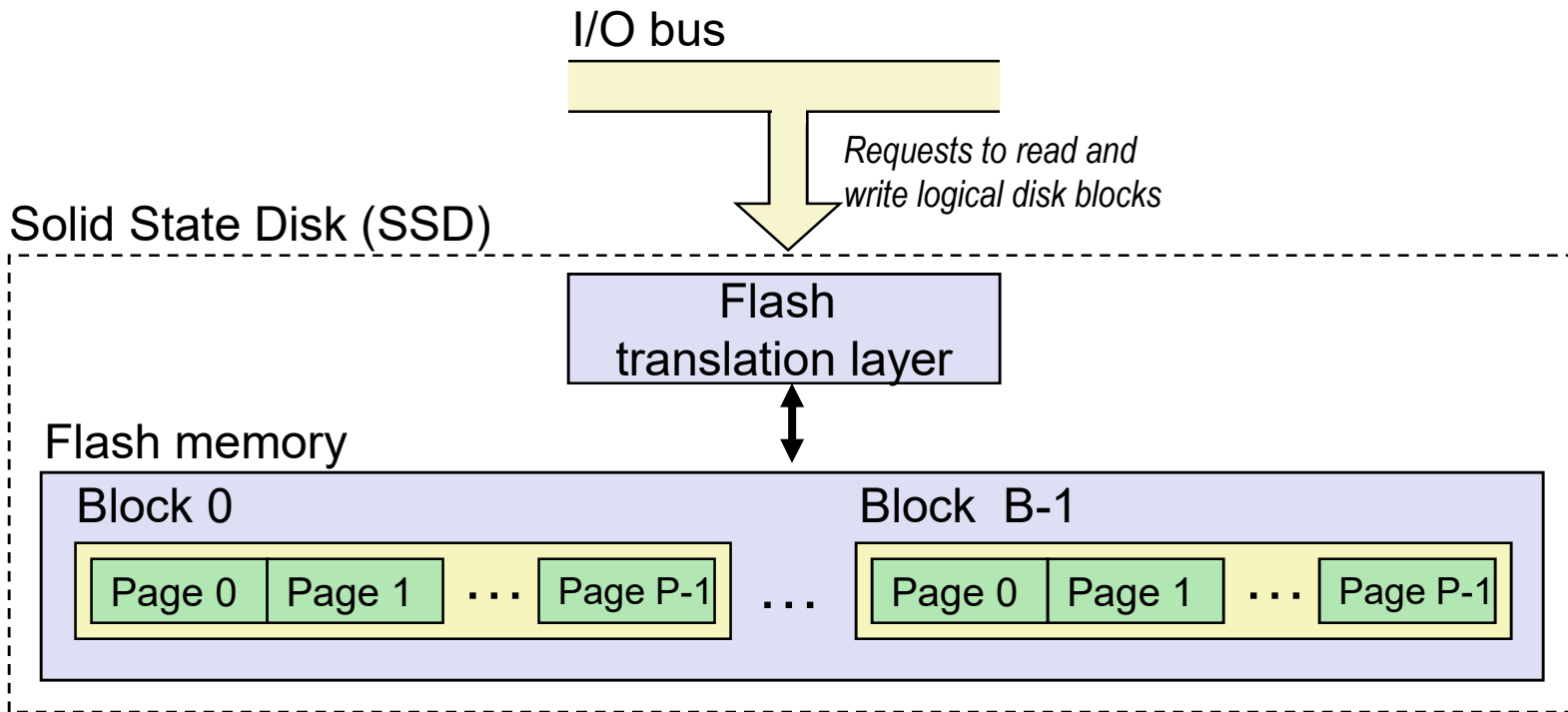
# Reading a Disk Sector (3)



When the DMA transfer completes, the disk controller notifies the CPU with an *interrupt* (i.e., asserts a special “interrupt” pin on the CPU)



# Solid State Disks (SSDs)



- 🌀 **Pages: 512KB to 4KB, Blocks: 32 to 128 pages**
- 🌀 **Data read/written in units of pages.**
- 🌀 **Page can be written only after its block has been erased**
- 🌀 **A block wears out after about 100,000 repeated writes.**



# SSD Performance Characteristics

|                      |          |                       |          |
|----------------------|----------|-----------------------|----------|
| Sequential read tput | 550 MB/s | Sequential write tput | 470 MB/s |
| Random read tput     | 365 MB/s | Random write tput     | 303 MB/s |
| Avg seq read time    | 50 us    | Avg seq write time    | 60 us    |

## ⌚ Sequential access faster than random access

- ⌚ Common theme in the memory hierarchy

## ⌚ Random writes are somewhat slower

- ⌚ Erasing a block takes a long time (~1 ms)
- ⌚ Modifying a block page requires all other pages to be copied to new block
- ⌚ In earlier SSDs, the read/write gap was much larger.

**Source: Intel SSD 730 product specification.**



# SSD Tradeoffs vs Rotating Disks

## ⌚ Advantages

- ⌚ No moving parts → faster, less power, more rugged

## ⌚ Disadvantages

- ⌚ Have the potential to wear out
  - ⌚ Mitigated by “wear leveling logic” in flash translation layer
  - ⌚ E.g. Intel SSD 730 guarantees 128 petabyte ( $128 \times 10^{15}$  bytes) of writes before they wear out
- ⌚ In 2015, about 30 times more expensive per byte

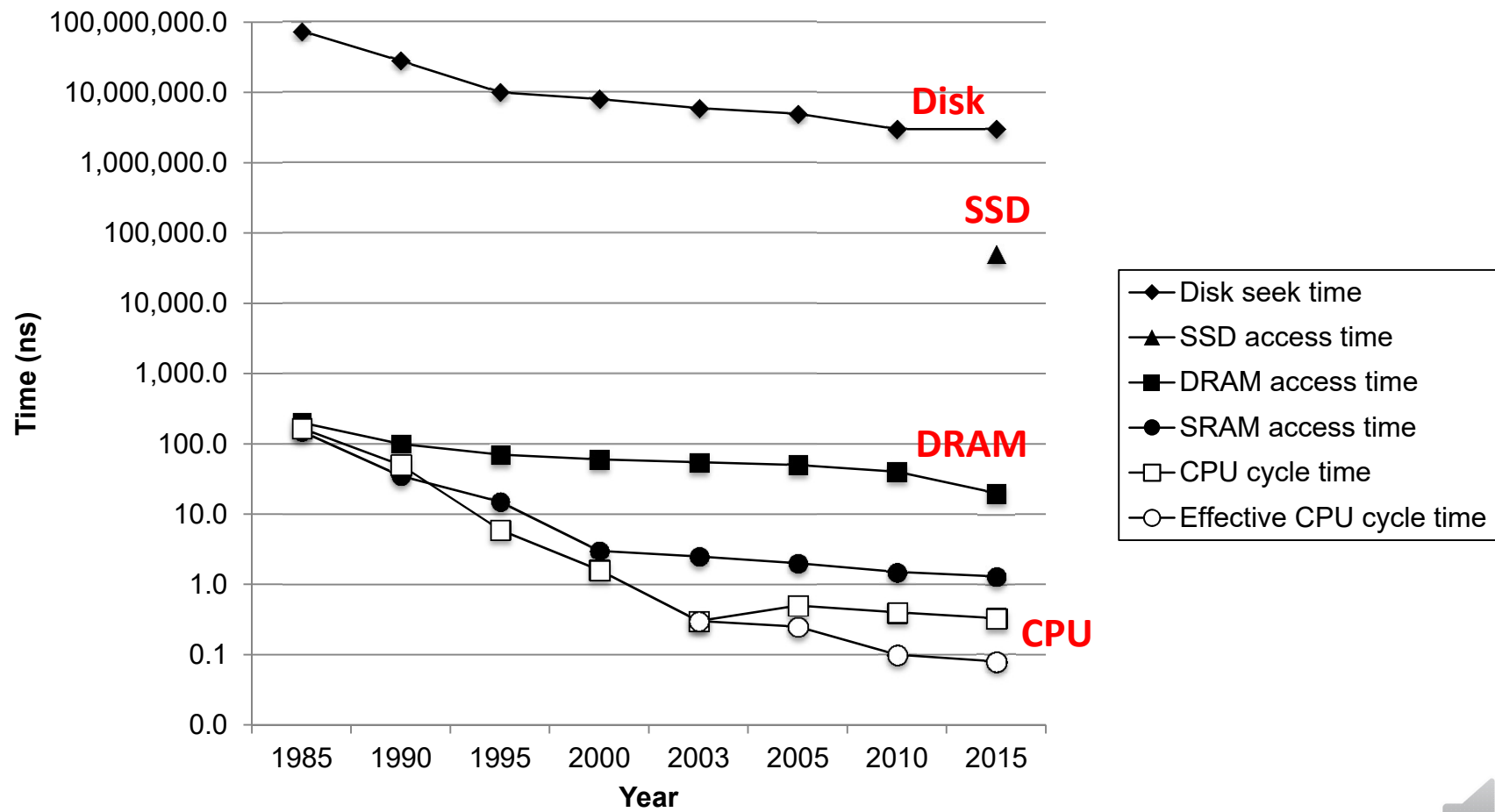
## ⌚ Applications

- ⌚ MP3 players, smart phones, laptops
- ⌚ Beginning to appear in desktops and servers



# The CPU-Memory Gap

The gap widens between DRAM, disk, and CPU speeds.



# Locality to the Rescue!

The key to bridging this CPU-Memory gap is a fundamental property of computer programs known as **locality**



# Today

- 🌀 Storage technologies and trends
- 🌀 **Locality of reference**
- 🌀 Caching in the memory hierarchy

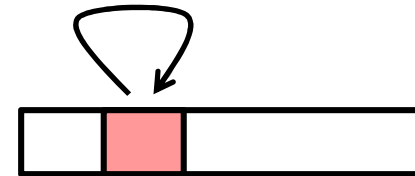


# Locality

- 🌀 **Principle of Locality:** Programs tend to use data and instructions with addresses near or equal to those they have used recently

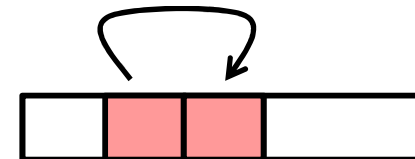
- 🌀 **Temporal locality:**

- 🌀 Recently referenced items are likely to be referenced again in the near future



- 🌀 **Spatial locality:**

- 🌀 Items with nearby addresses tend to be referenced close together in time





# Locality Example

```
sum = 0;  
for (i = 0; i < n; i++)  
    sum += a[i];  
return sum;
```

## 🌀 Data references

- 🌀 Reference array elements in succession (stride-1 reference pattern).
- 🌀 Reference variable `sum` each iteration.

**Spatial locality**

**Temporal locality**

## 🌀 Instruction references

- 🌀 Reference instructions in sequence.
- 🌀 Cycle through loop repeatedly.

**Spatial locality**

**Temporal locality**



# Qualitative Estimates of Locality

- 🔊 **Claim:** Being able to look at code and get a qualitative sense of its locality is a key skill for a professional programmer.
- 🔊 **Question:** Does this function have good locality with respect to array *a*?

```
int sum_array_rows(int a[M][N])
{
    int i, j, sum = 0;

    for (i = 0; i < M; i++)
        for (j = 0; j < N; j++)
            sum += a[i][j];
    return sum;
}
```



# Locality Example

🔊 **Question:** Does this function have good locality with respect to array *a*?

```
int sum_array_cols(int a[M][N])
{
    int i, j, sum = 0;

    for (j = 0; j < N; j++)
        for (i = 0; i < M; i++)
            sum += a[i][j];
    return sum;
}
```



# Memory Hierarchies

- **Some fundamental and enduring properties of hardware and software:**
  - Fast storage technologies cost more per byte, have less capacity, and require more power (heat!).
  - The gap between CPU and main memory speed is widening.
  - Well-written programs tend to exhibit good locality.
- **These fundamental properties complement each other beautifully.**
- **They suggest an approach for organizing memory and storage systems known as a **memory hierarchy**.**

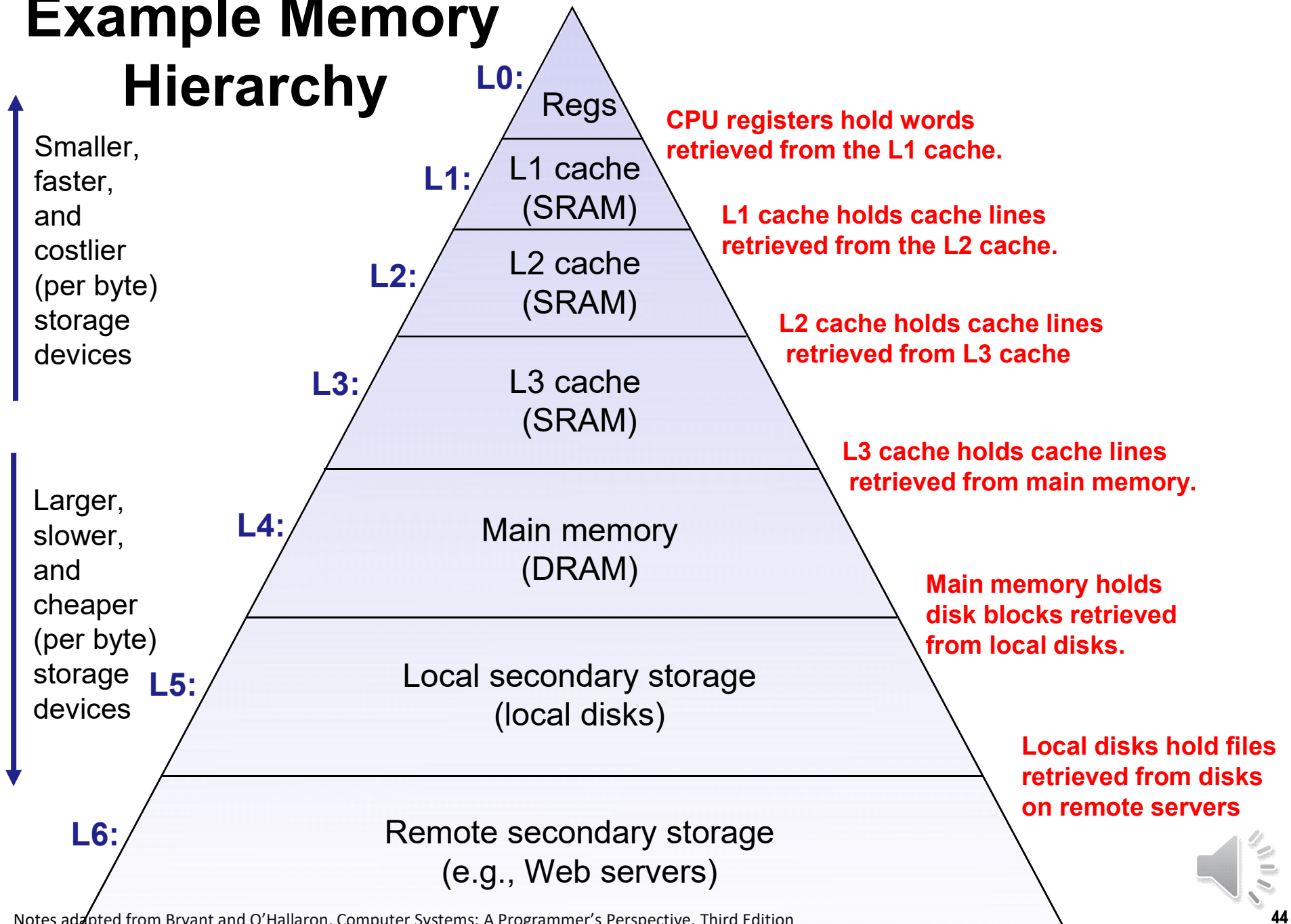


# Today

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- 🌀 Locality of reference
- 🌀 **Caching in the memory hierarchy**



# Example Memory Hierarchy

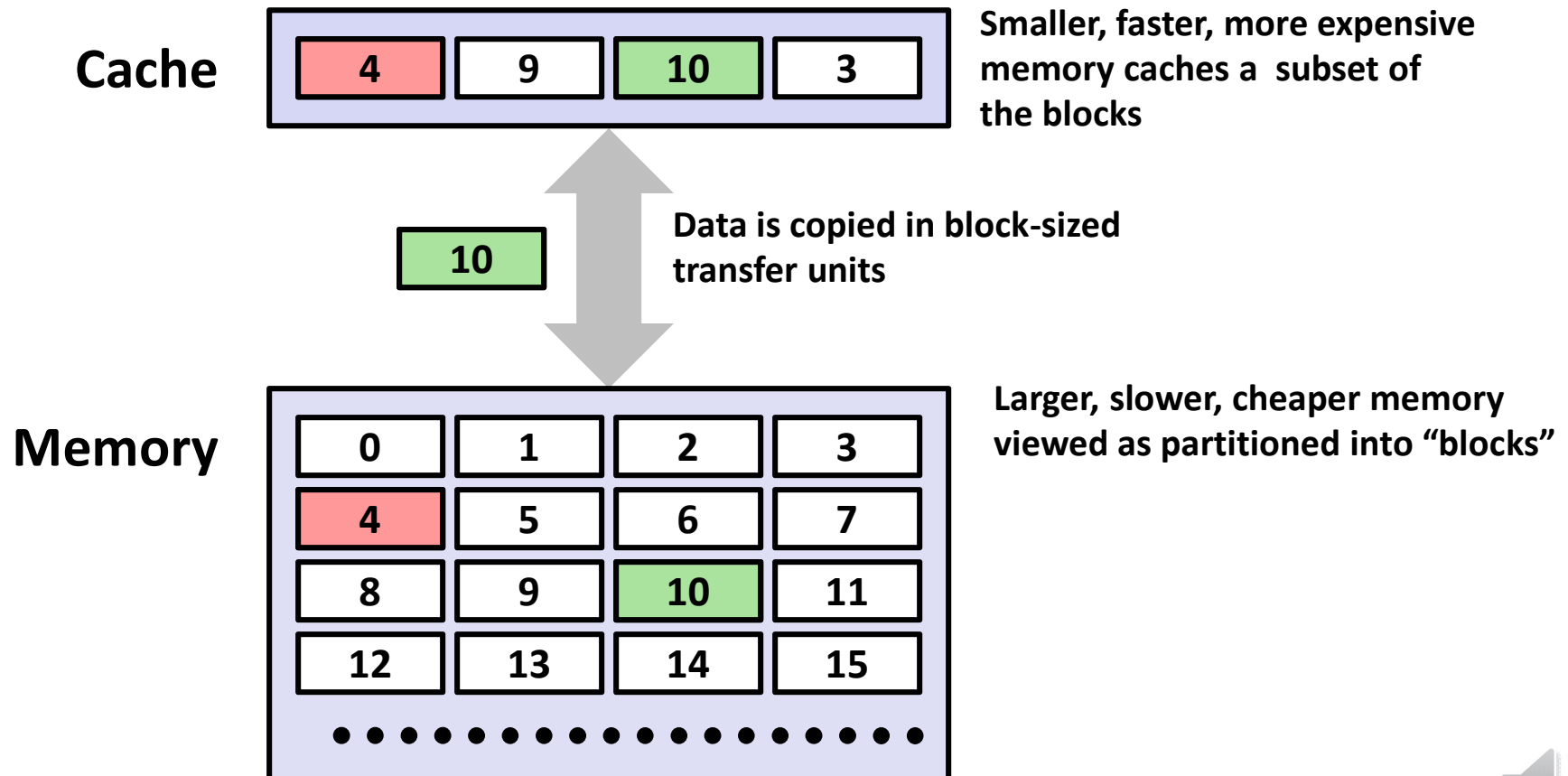


# Caches

- 🌀 **Cache:** A smaller, faster storage device that acts as a staging area for a subset of the data in a larger, slower device.
- 🌀 **Fundamental idea of a memory hierarchy:**
  - 🌀 For each  $k$ , the faster, smaller device at level  $k$  serves as a cache for the larger, slower device at level  $k+1$ .
- 🌀 **Why do memory hierarchies work?**
  - 🌀 Because of locality, programs tend to access the data at level  $k$  more often than they access the data at level  $k+1$ .
  - 🌀 Thus, the storage at level  $k+1$  can be slower, and thus larger and cheaper per bit.
- 🌀 **Big Idea:** The memory hierarchy creates a large pool of storage that costs as much as the cheap storage near the bottom, but that serves data to programs at the rate of the fast storage near the top.

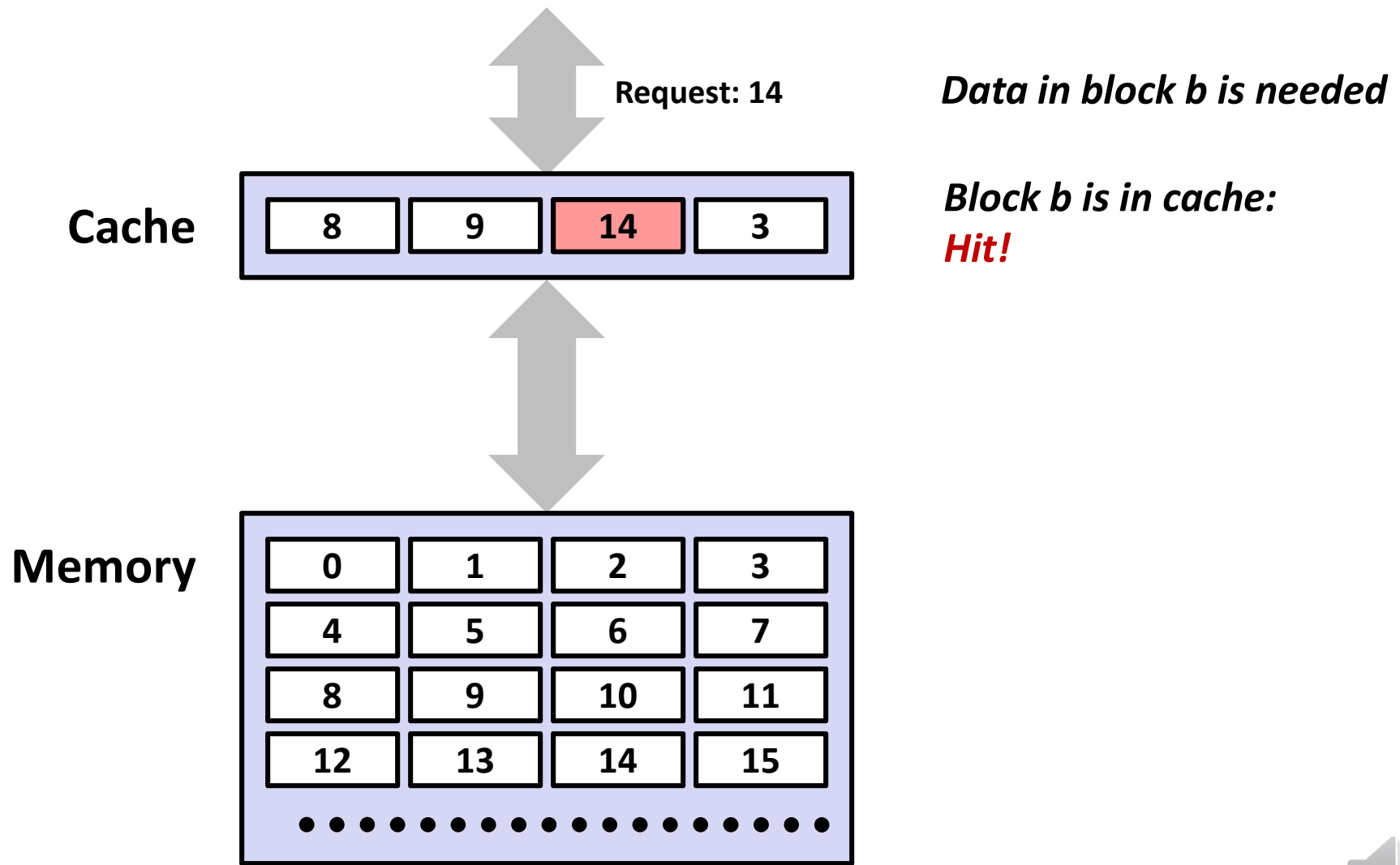


# General Cache Concepts

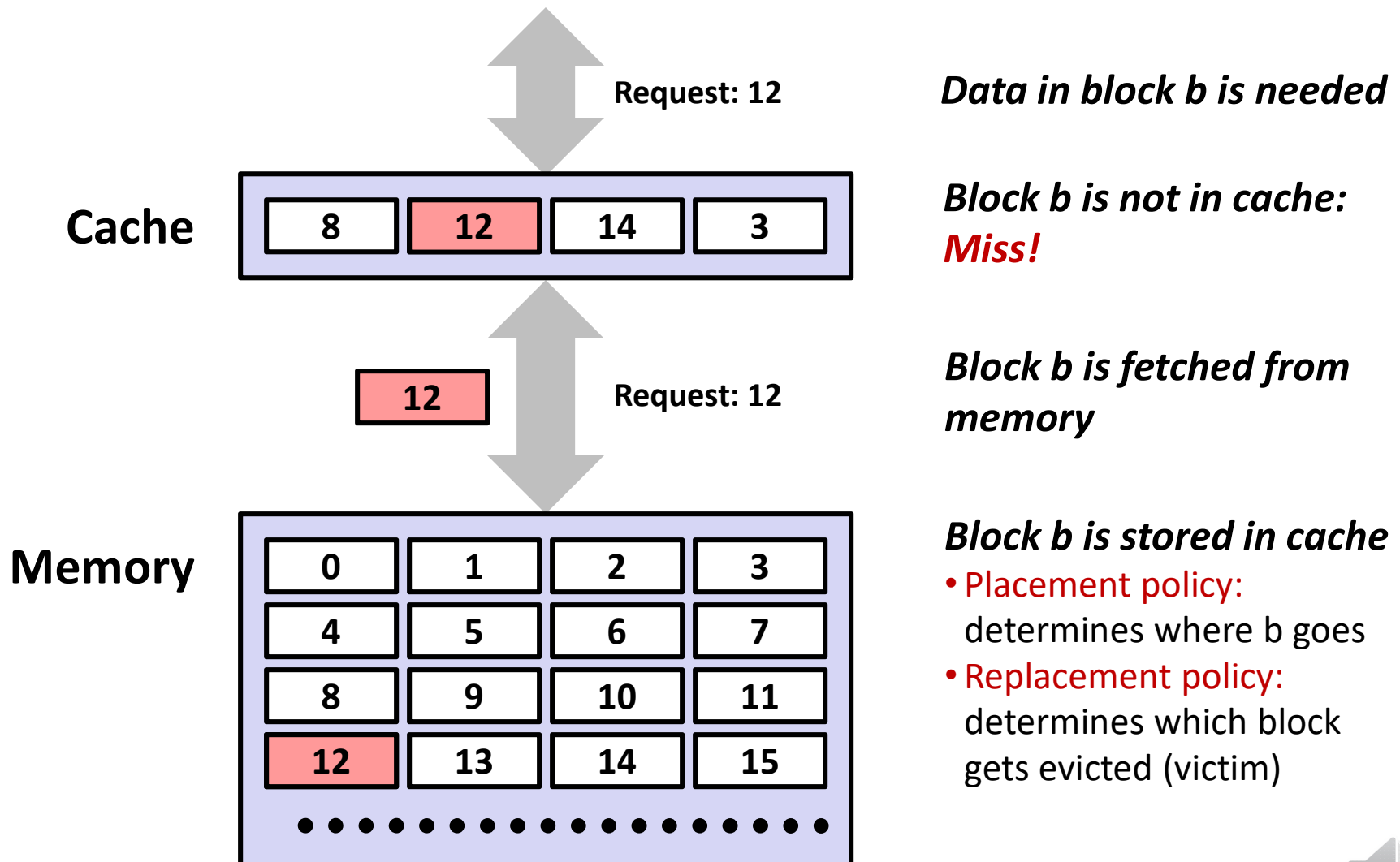




# General Cache Concepts: Hit



# General Cache Concepts: Miss



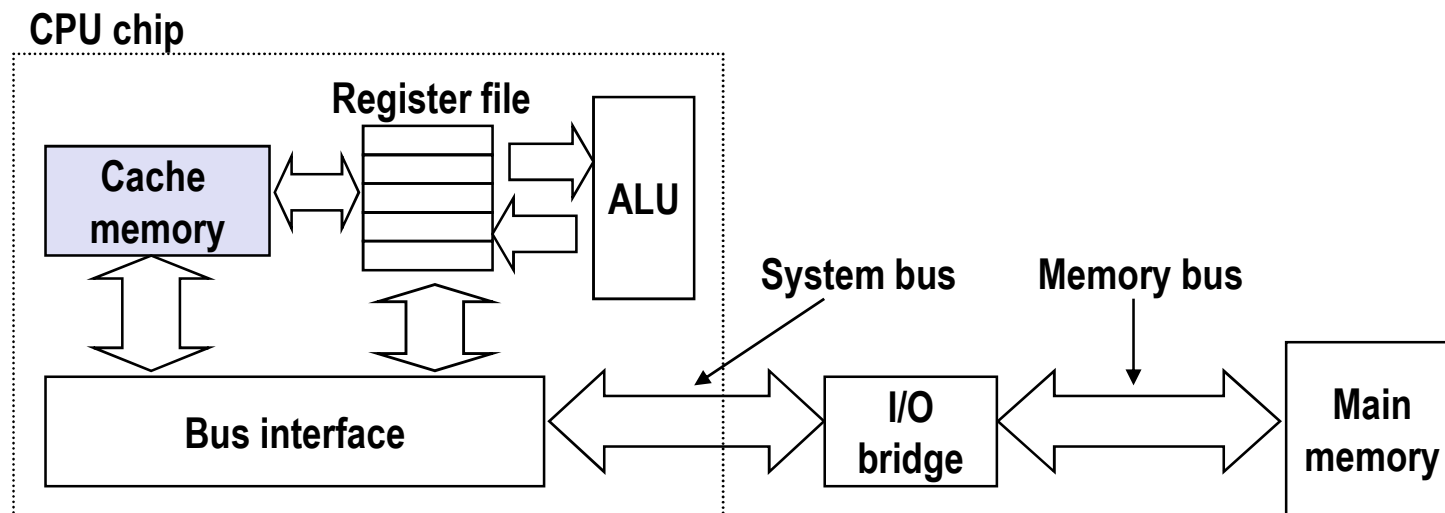
# Examples of Caching in the Mem. Hierarchy

| Cache Type           | What is Cached?      | Where is it Cached? | Latency (cycles) | Managed By       |
|----------------------|----------------------|---------------------|------------------|------------------|
| Registers            | 4-8 bytes words      | CPU core            | 0                | Compiler         |
| TLB                  | Address translations | On-Chip TLB         | 0                | Hardware MMU     |
| L1 cache             | 64-byte blocks       | On-Chip L1          | 4                | Hardware         |
| L2 cache             | 64-byte blocks       | On-Chip L2          | 10               | Hardware         |
| Virtual Memory       | 4-KB pages           | Main memory         | 100              | Hardware + OS    |
| Buffer cache         | Parts of files       | Main memory         | 100              | OS               |
| Disk cache           | Disk sectors         | Disk controller     | 100,000          | Disk firmware    |
| Network buffer cache | Parts of files       | Local disk          | 10,000,000       | NFS client       |
| Browser cache        | Web pages            | Local disk          | 10,000,000       | Web browser      |
| Web cache            | Web pages            | Remote server disks | 1,000,000,000    | Web proxy server |

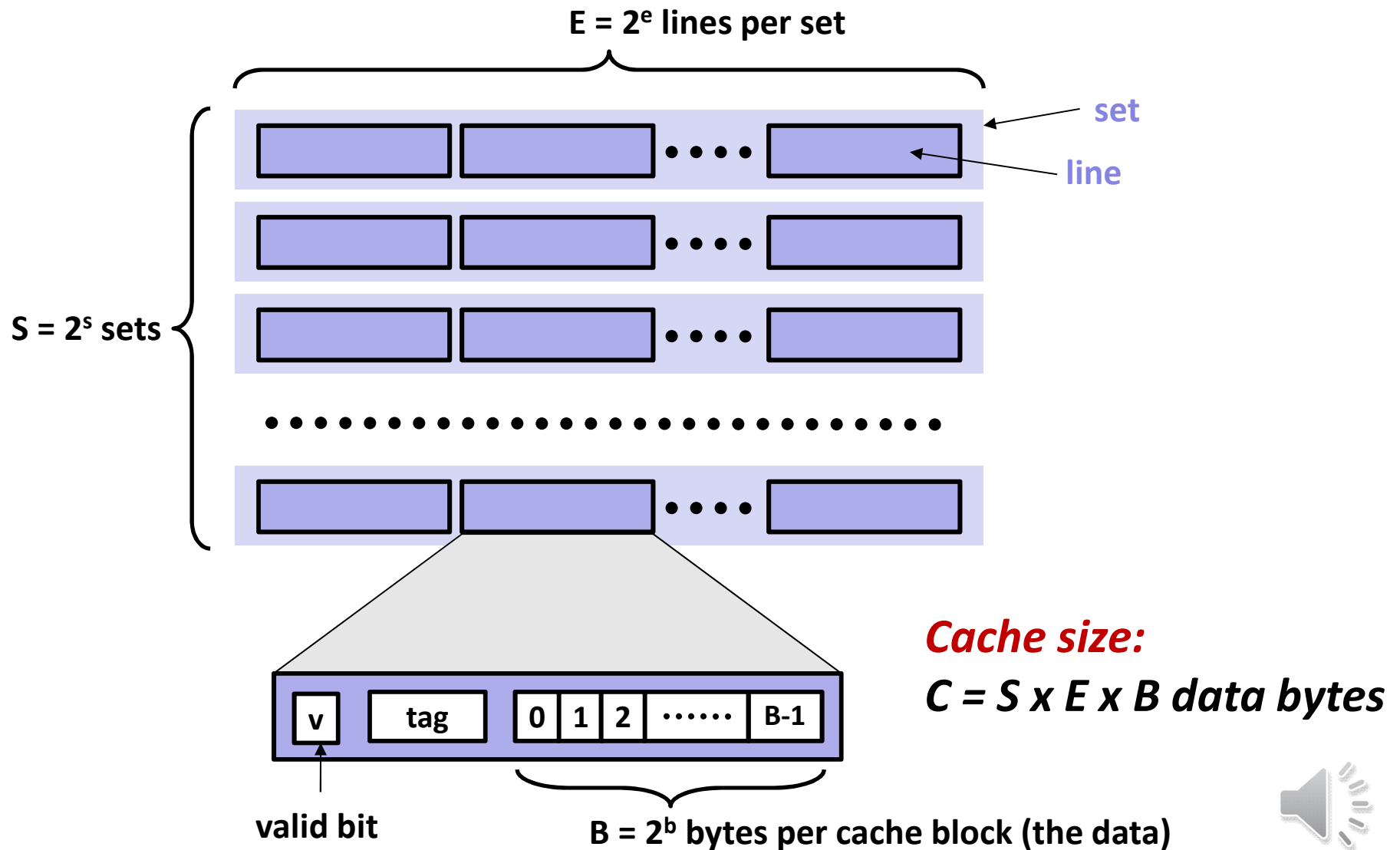


# Cache Memories

- 🌀 **Cache memories** are small, fast SRAM-based memories managed automatically in hardware
  - 🌀 Hold frequently accessed blocks of main memory
- 🌀 **CPU looks first for data in cache**
- 🌀 **Typical system structure:**

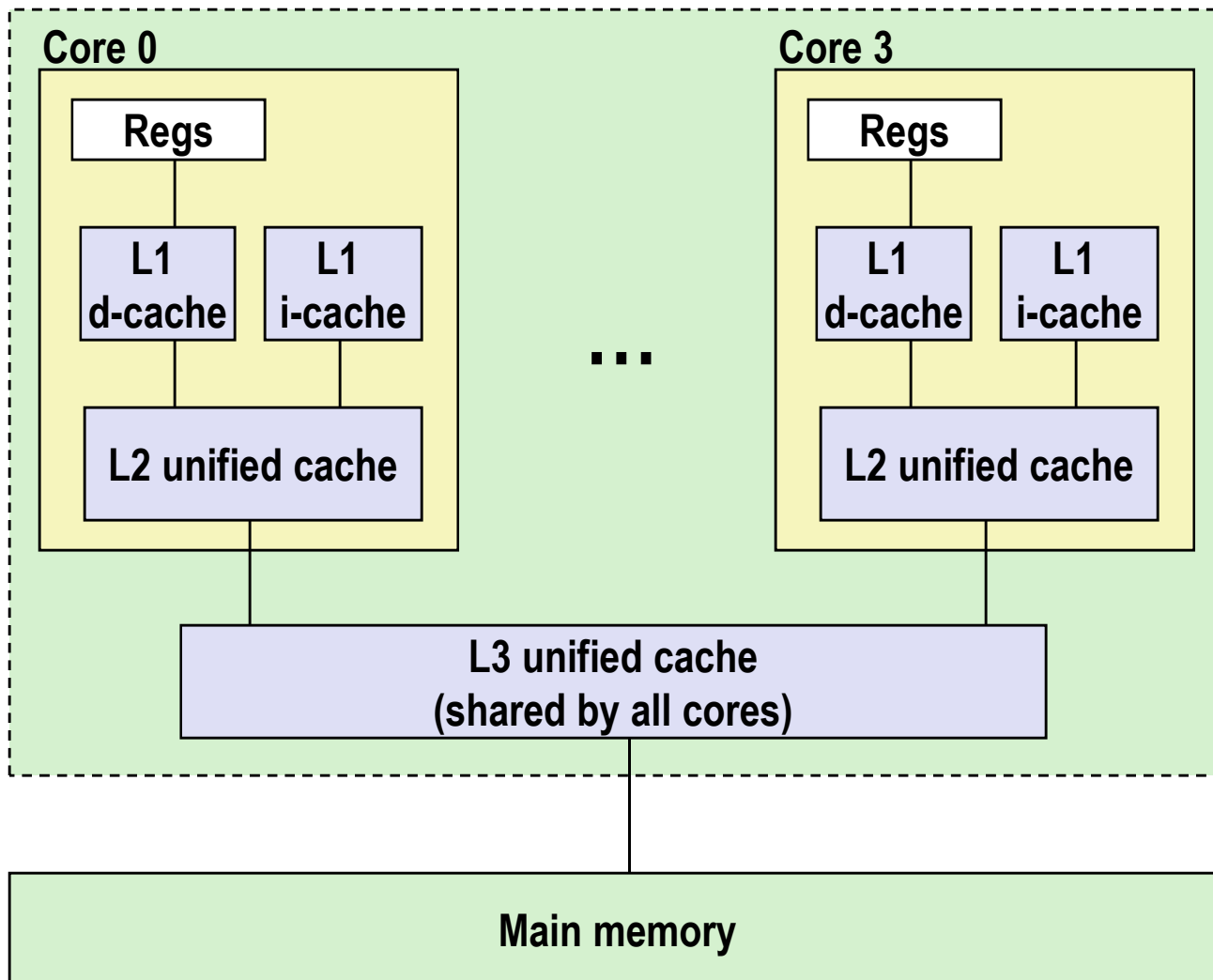


# General Cache Organization (S, E, B)



# Intel Core i7 Cache Hierarchy

Processor package



**L1 i-cache and d-cache:**  
32 KB, 8-way,  
Access: 4 cycles

**L2 unified cache:**  
256 KB, 8-way,  
Access: 10 cycles

**L3 unified cache:**  
8 MB, 16-way,  
Access: 40-75 cycles

**Block size:** 64 bytes for  
all caches.



# Cache Performance Metrics

## • Miss Rate

- Fraction of memory references not found in cache (misses / accesses)  
=  $1 - \text{hit rate}$
- Typical numbers (in percentages):
  - 3-10% for L1
  - can be quite small (e.g.,  $< 1\%$ ) for L2, depending on size, etc.

## • Hit Time

- Time to deliver a line in the cache to the processor
  - includes time to determine whether the line is in the cache
- Typical numbers:
  - 4 clock cycle for L1
  - 10 clock cycles for L2

## • Miss Penalty

- Additional time required because of a miss
  - typically 50-200 cycles for main memory (Trend: increasing!)



# Let's think about those numbers

## 🌀 Huge difference between a hit and a miss

- 🌀 Could be 100x, if just L1 and main memory

## 🌀 Would you believe 99% hits is twice as good as 97%?

- 🌀 Consider:  
cache hit time of 1 cycle  
miss penalty of 100 cycles

- 🌀 Average access time:

97% hits:  $1 \text{ cycle} + 0.03 * 100 \text{ cycles} = 4 \text{ cycles}$

99% hits:  $1 \text{ cycle} + 0.01 * 100 \text{ cycles} = 2 \text{ cycles}$

## 🌀 This is why “miss rate” is used instead of “hit rate”





# Writing Cache Friendly Code

- 🌀 **Make the common case go fast**

- 🌀 Focus on the inner loops of the core functions

- 🌀 **Minimize the misses in the inner loops**

- 🌀 Repeated references to variables are good (**temporal locality**)
  - 🌀 Stride-1 reference patterns are good (**spatial locality**)

**Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories**



# The Memory Mountain

- 🌀 **Read throughput** (read bandwidth)

- 🌀 Number of bytes read from memory per second (MB/s)

- 🌀 **Memory mountain:** Measured read throughput as a function of spatial and temporal locality.

- 🌀 Compact way to characterize memory system performance.



# Memory Mountain Test Function

```
long data[MAXELEMS]; /* Global array to traverse */
```

```
/* test - Iterate over first "elems" elements of  
 *   array "data" with stride of "stride", using  
 *   using 4x4 loop unrolling.  
 */
```

```
int test(int elems, int stride) {  
    long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;  
    long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;  
    long length = elems, limit = length - sx4;
```

```
/* Combine 4 elements at a time */
```

```
for (i = 0; i < limit; i += sx4) {  
    acc0 = acc0 + data[i];  
    acc1 = acc1 + data[i+stride];  
    acc2 = acc2 + data[i+sx2];  
    acc3 = acc3 + data[i+sx3];  
}
```

```
/* Finish any remaining elements */
```

```
for (; i < length; i++) {  
    acc0 = acc0 + data[i];  
}  
return ((acc0 + acc1) + (acc2 + acc3));
```

```
}
```

*mountain/mountain.c*

Call test () with many combinations of elems and stride.

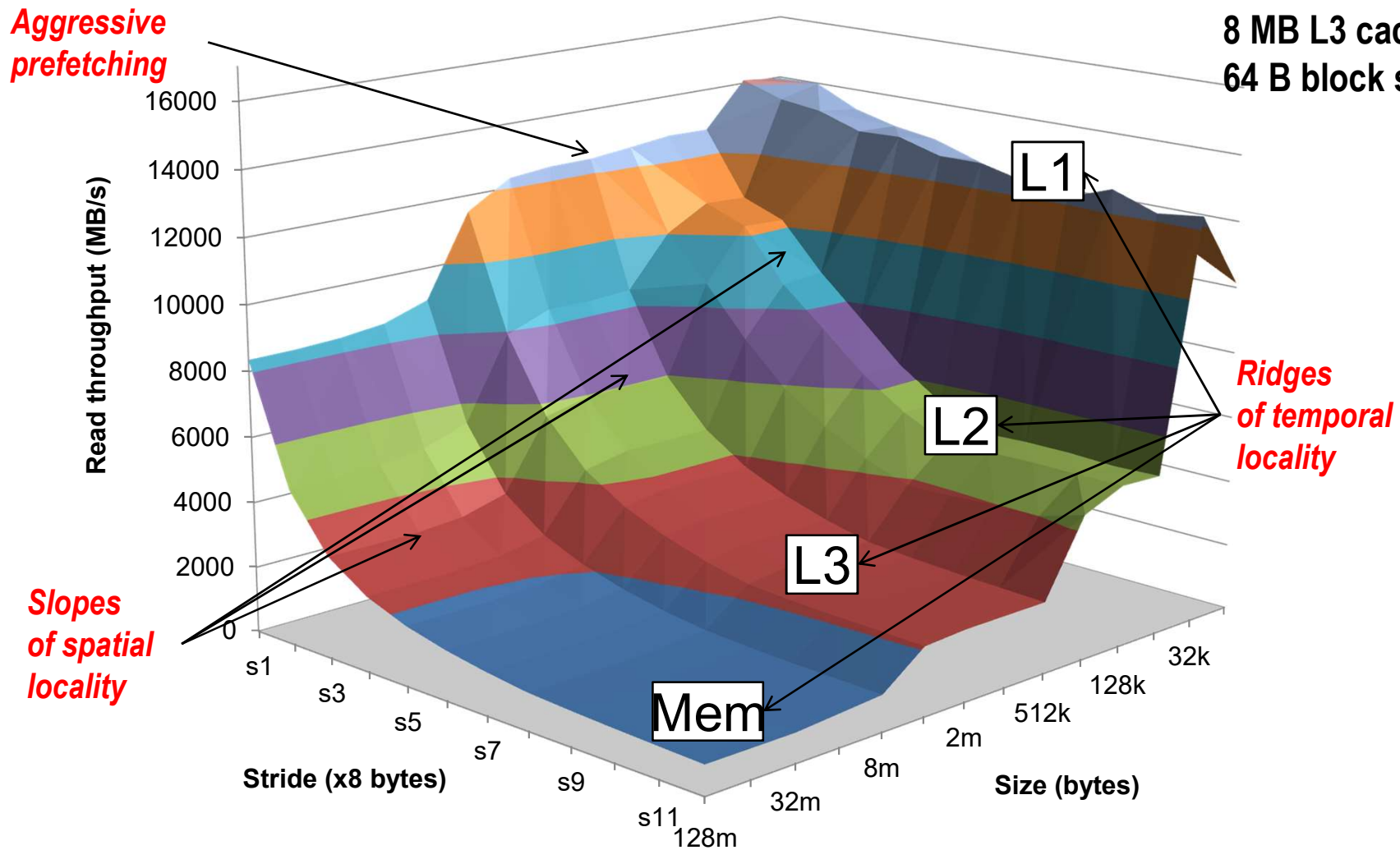
For each elems and stride:

1. Call test () once to warm up the caches.
2. Call test () again and measure the read throughput (MB/s)



# The Memory Mountain

Core i7 Haswell  
2.1 GHz  
32 KB L1 d-cache  
256 KB L2 cache  
8 MB L3 cache  
64 B block size



# Matrix Multiplication Example

## 🌀 Description:

- 🌀 Multiply  $N \times N$  matrices
- 🌀 Matrix elements are doubles (8 bytes)
- 🌀  $O(N^3)$  total operations
- 🌀  $N$  reads per source element
- 🌀  $N$  values summed per destination
  - 🌀 but may be able to hold in register

```
/* ijk */
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

*Variable **sum**  
held in register*

*matmult/mm.c*



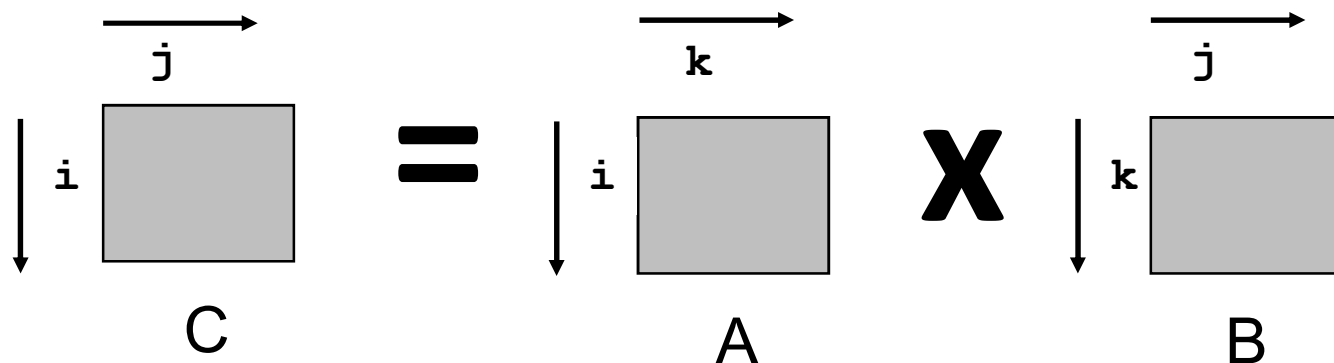
# Miss Rate Analysis for Matrix Multiply

## Assume:

- Block size = 32B (big enough for four doubles)
- Matrix dimension (N) is very large
  - Approximate  $1/N$  as 0.0
- Cache is not even big enough to hold multiple rows

## Analysis Method:

- Look at access pattern of inner loop



# Layout of C Arrays in Memory (review)

- **C arrays allocated in row-major order**

- each row in contiguous memory locations

- **Stepping through columns in one row:**

- ```
for (i = 0; i < N; i++)  
    sum += a[0][i];
```

- accesses successive elements

- if block size (B) > sizeof(a<sub>ij</sub>) bytes, exploit spatial locality

- miss rate = sizeof(a<sub>ij</sub>) / B

- **Stepping through rows in one column:**

- ```
for (i = 0; i < n; i++)  
    sum += a[i][0];
```

- accesses distant elements

- no spatial locality!

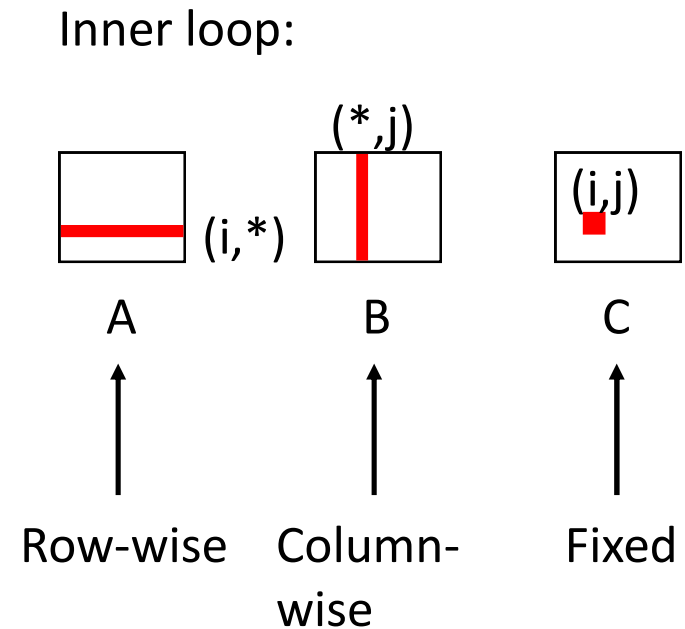
- miss rate = 1 (i.e. 100%)



# Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

*matmult/mm.c*



Misses per inner loop iteration:

| <u>A</u> | <u>B</u> | <u>C</u> |
|----------|----------|----------|
| 0.25     | 1.0      | 0.0      |



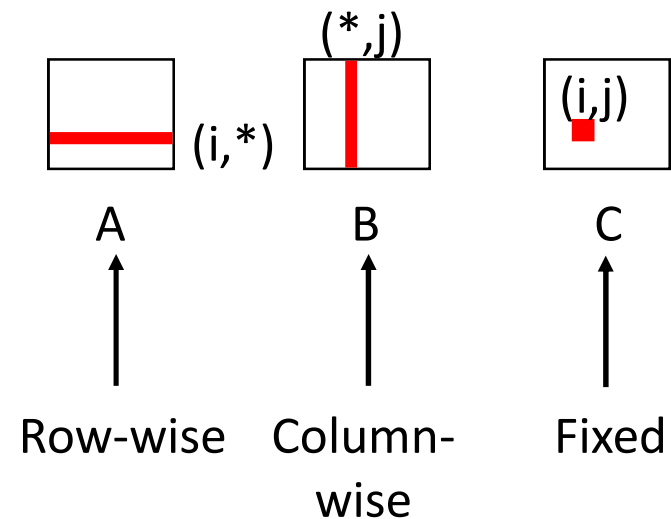


# Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
    for (i=0; i<n; i++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum
    }
}
```

*matmult/mm.c*

Inner loop:



Misses per inner loop iteration:

| <u>A</u> | <u>B</u> | <u>C</u> |
|----------|----------|----------|
| 0.25     | 1.0      | 0.0      |

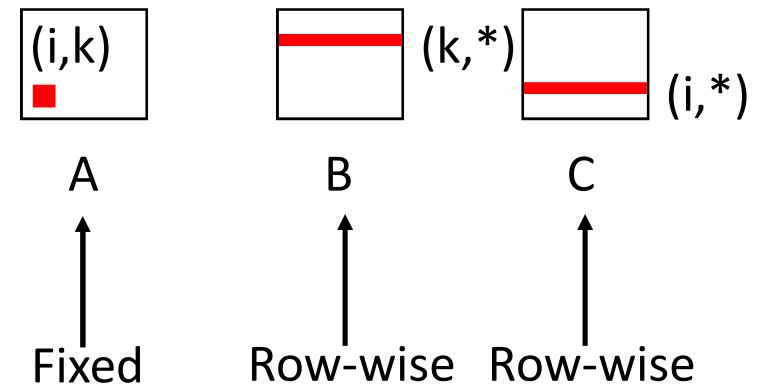


# Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
    for (i=0; i<n; i++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

*matmult/mm.c*

Inner loop:



Misses per inner loop iteration:

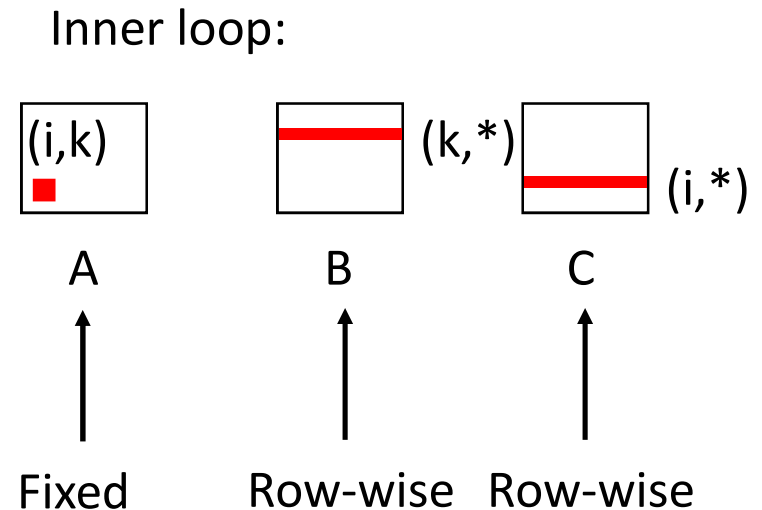
| <u>A</u> | <u>B</u> | <u>C</u> |
|----------|----------|----------|
| 0.0      | 0.25     | 0.25     |



# Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
    for (k=0; k<n; k++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

*matmult/mm.c*



Misses per inner loop iteration:

| <u>A</u> | <u>B</u> | <u>C</u> |
|----------|----------|----------|
| 0.0      | 0.25     | 0.25     |

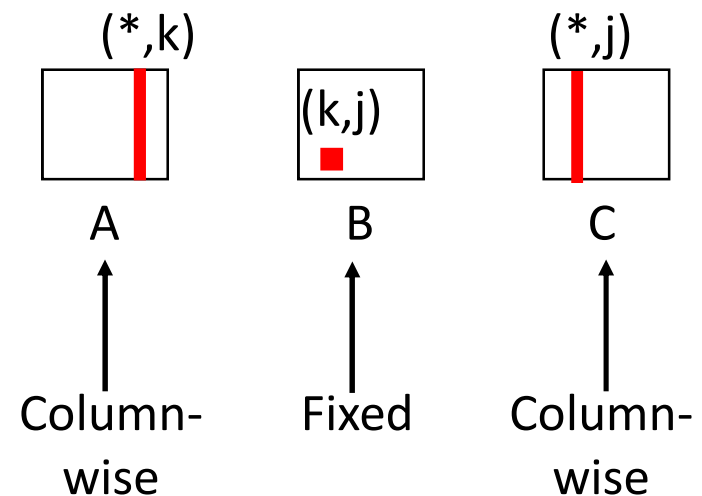


# Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
    for (k=0; k<n; k++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}
```

*matmult/mm.c*

Inner loop:



Misses per inner loop iteration:

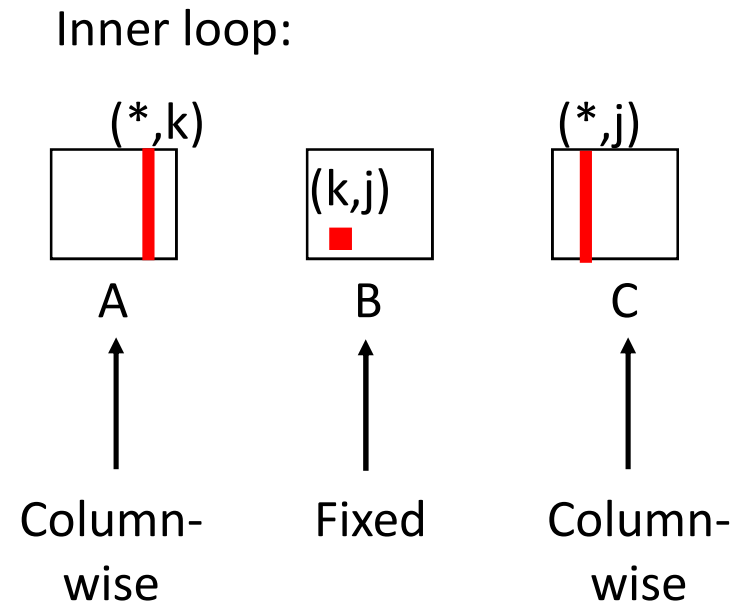
| <u>A</u> | <u>B</u> | <u>C</u> |
|----------|----------|----------|
| 1.0      | 0.0      | 1.0      |



# Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
    for (j=0; j<n; j++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}
```

*matmult/mm.c*



Misses per inner loop iteration:

| <u>A</u> | <u>B</u> | <u>C</u> |
|----------|----------|----------|
| 1.0      | 0.0      | 1.0      |



# Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {  
    for (j=0; j<n; j++) {  
        sum = 0.0;  
        for (k=0; k<n; k++)  
            sum += a[i][k] * b[k][j];  
        c[i][j] = sum;  
    }  
}
```

**ijk (& jik):**

- 2 loads, 0 stores
- misses/iter = **1.25**

```
for (k=0; k<n; k++) {  
    for (i=0; i<n; i++) {  
        r = a[i][k];  
        for (j=0; j<n; j++)  
            c[i][j] += r * b[k][j];  
    }  
}
```

**kij (& ikj):**

- 2 loads, 1 store
- misses/iter = **0.5**

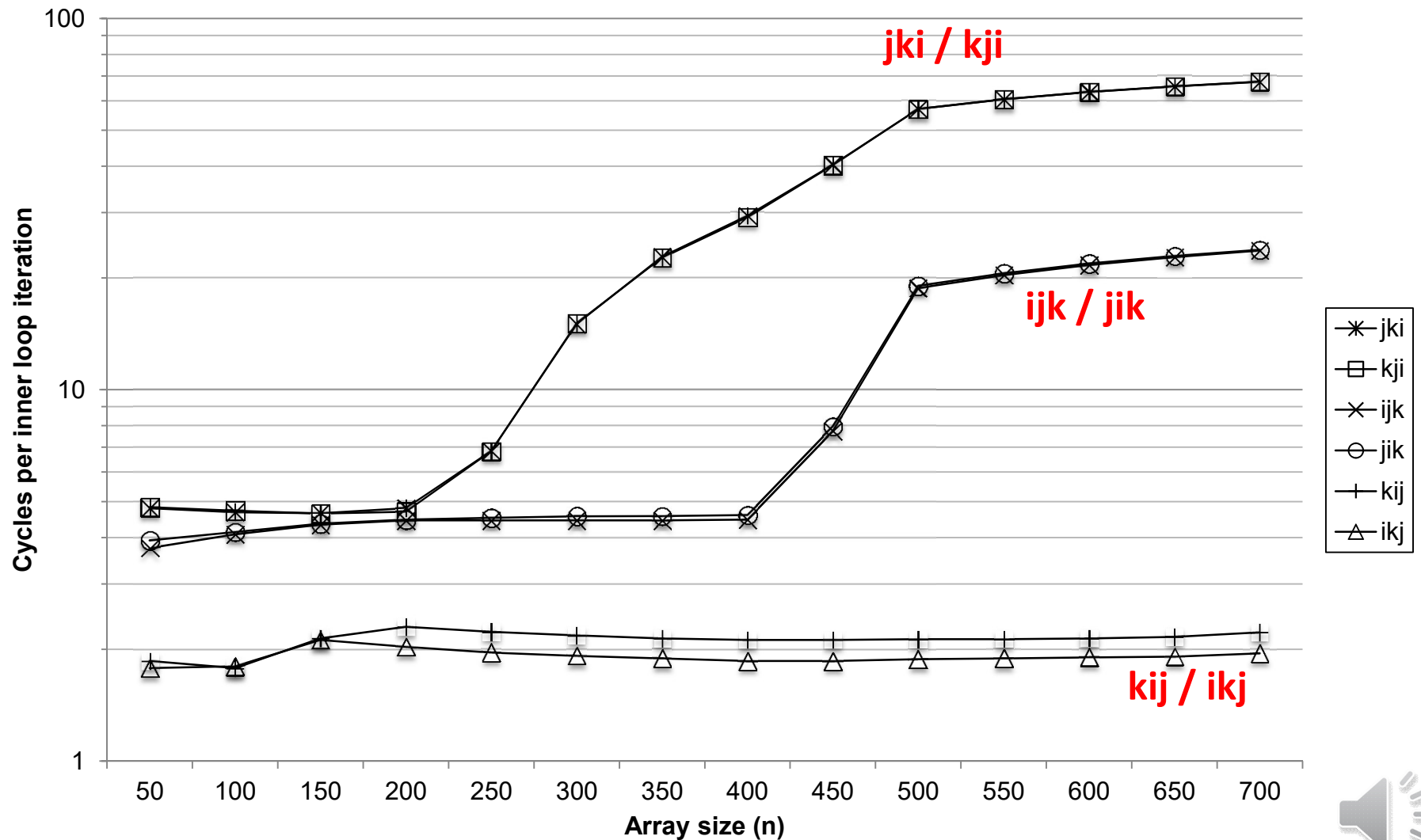
```
for (j=0; j<n; j++) {  
    for (k=0; k<n; k++) {  
        r = b[k][j];  
        for (i=0; i<n; i++)  
            c[i][j] += a[i][k] * r;  
    }  
}
```

**jki (& kji):**

- 2 loads, 1 store
- misses/iter = **2.0**



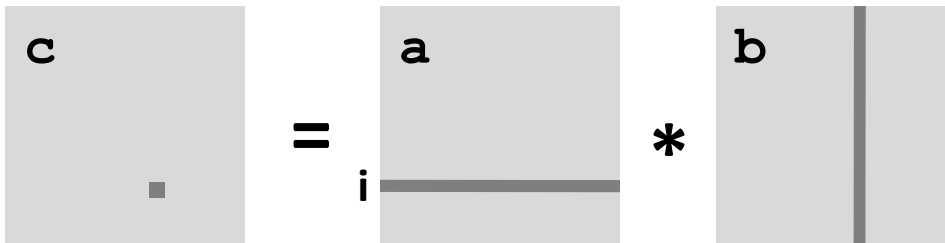
# Core i7 Matrix Multiply Performance



# Example: Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i++)
        for (j = 0; j < n; j++)
            for (k = 0; k < n; k++)
                c[i*n + j] += a[i*n + k] * b[k*n + j];
}
```





# Cache Miss Analysis

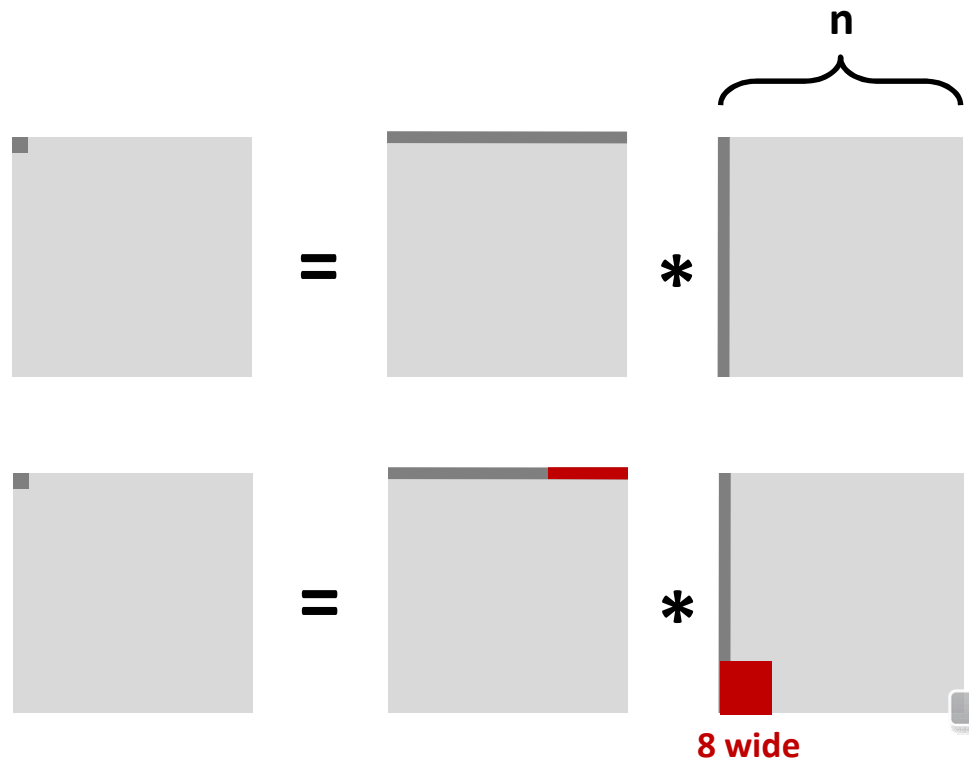
## Assume:

- Matrix elements are doubles
- Cache block = 8 doubles
- Cache size  $C \ll n$  (much smaller than  $n$ )

## First iteration:

- $n/8 + n = 9n/8$  misses

- Afterwards **in cache**:  
(schematic)



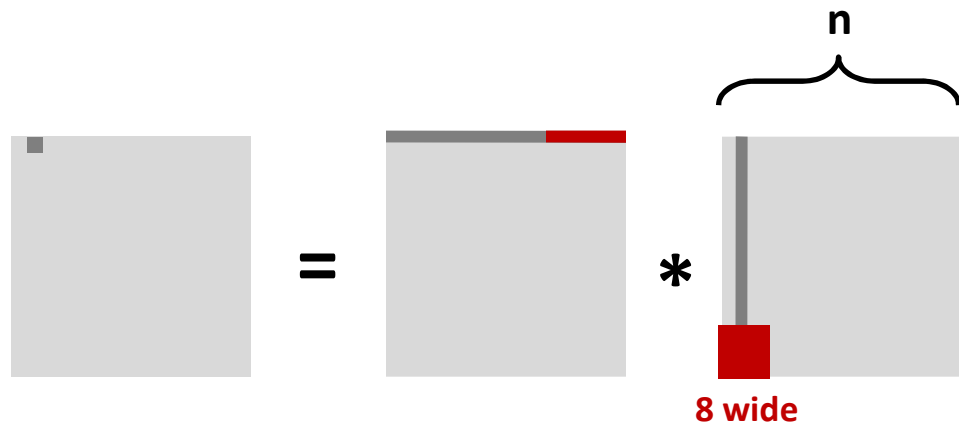
# Cache Miss Analysis

## Assume:

- Matrix elements are doubles
- Cache block = 8 doubles
- Cache size  $C \ll n$  (much smaller than  $n$ )

## Second iteration:

- Again:  
 $n/8 + n = 9n/8$  misses



## Total misses:

- $9n/8 * n^2 = (9/8) * n^3$

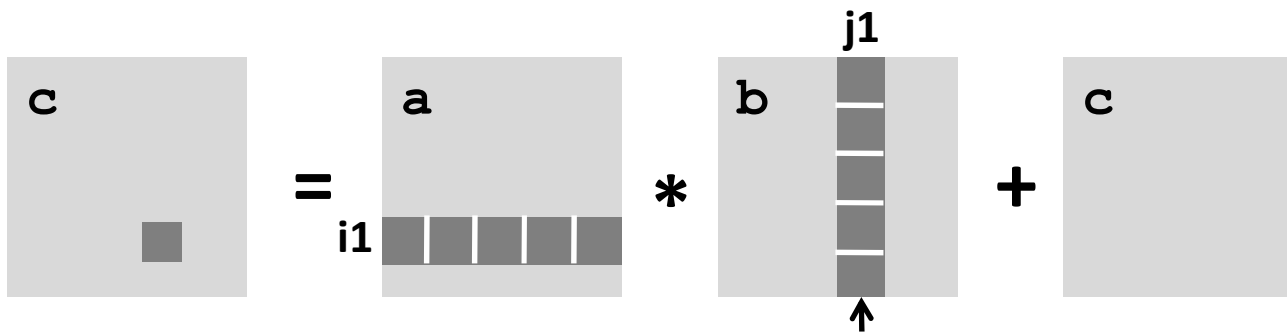


# Blocked Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);


/* Multiply n x n matrices a and b */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
            for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                for (i1 = i; i1 < i+B; i1++)
                    for (j1 = j; j1 < j+B; j1++)
                        for (k1 = k; k1 < k+B; k1++)
                            c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}
```

*matmult/bmm.c*



# Cache Miss Analysis

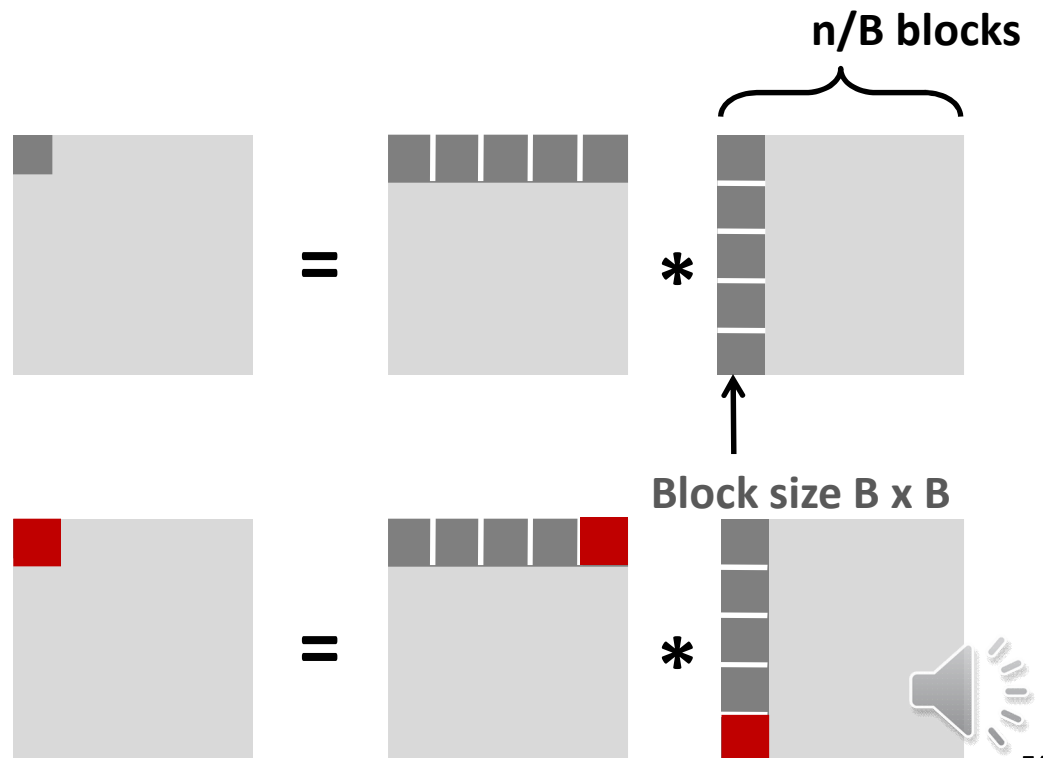
## Assume:

- Cache block = 8 doubles
- Cache size  $C \ll n$  (much smaller than  $n$ )
- Three blocks  fit into cache:  $3B^2 < C$

## First (block) iteration:


- $B^2/8$  misses for each block
- $2n/B * B^2/8 = nB/4$   
(omitting matrix  $c$ )

- Afterwards in cache  
(schematic)



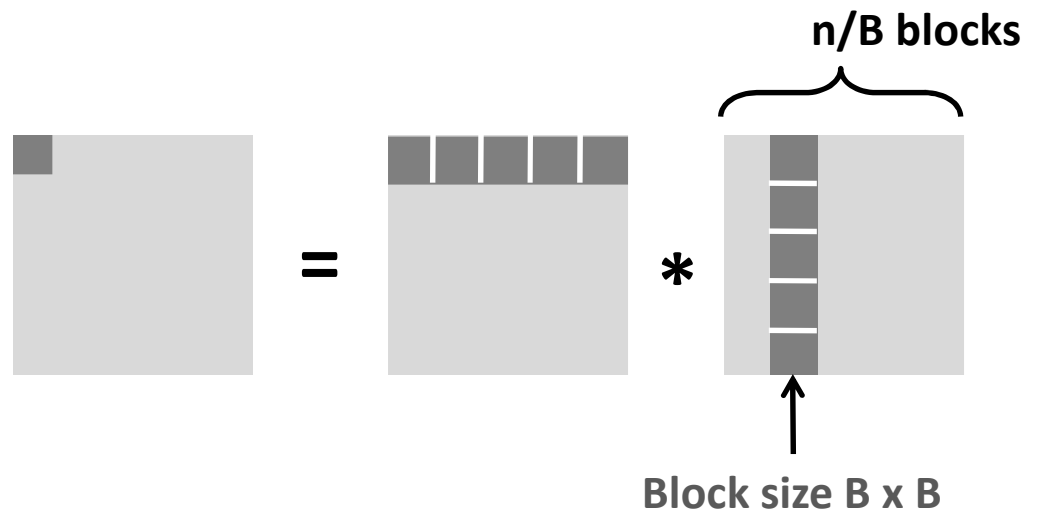
# Cache Miss Analysis

## Assume:

- Cache block = 8 doubles
- Cache size  $C \ll n$  (much smaller than  $n$ )
- Three blocks  fit into cache:  $3B^2 < C$

## Second (block) iteration:

- Same as first iteration
- $2n/B * B^2/8 = nB/4$



## Total misses:

- $nB/4 * (n/B)^2 = n^3/(4B)$



# Blocking Summary

- No blocking:  $(9/8) * n^3$
- Blocking:  $1/(4B) * n^3$
- Suggest largest possible block size  $B$ , but limit  $3B^2 < C$ !
- Reason for dramatic difference:
  - Matrix multiplication has inherent temporal locality:
    - Input data:  $3n^2$ , computation  $2n^3$
    - Every array elements used  $O(n)$  times!
  - But program has to be written properly



# Summary

- The speed gap between CPU, memory and mass storage continues to widen.
- Well-written programs exhibit a property called *locality*.
- Memory hierarchies based on *caching* close the gap by exploiting locality.
  - Cache memories can have significant performance impact
- **You can write your programs to exploit this!**
  - Focus on the inner loops, where bulk of computations and memory accesses occur.
  - Try to maximize spatial locality by reading data objects with sequentially with stride 1.
  - Try to maximize temporal locality by using a data object as often as possible once it's read from memory.

