3a) $l(\theta) = \sum_j \sum_n \gamma_{nj} \log(w_k) + \sum_k \sum_n \gamma_{nk} \log N(x_n | \mu_j, \Sigma_k)$

$= \sum_j \sum_n \gamma_{nj} \log(w_k) + \sum_k \sum_n \gamma_{nk} \log\left(\frac{1}{\sqrt{2\pi\Sigma_k}} e^{-\frac{(x_n-\mu_j)^2}{2\Sigma_k}}\right)$

$= \sum_j \sum_n \gamma_{nj} \log(w_k) + \sum_k \sum_n \gamma_{nk}\left(\log\left(\frac{1}{\sqrt{2\pi\Sigma_k}}\right)\right) + \left(-\frac{(x_n-\mu_j)^2}{2\Sigma_k}\right)$

So we take derivative w/ respect to μ_j to get

$$\nabla_{\mu_j} l(\theta) = \sum_n \gamma_{nj} \cdot \frac{1}{2\Sigma_k} \cdot 2(x_n - \mu_j) \Rightarrow \boxed{\sum_n \gamma_{nj} \frac{x_n-\mu_j}{\Sigma_k}}$$

3b) Solving for 0 we get:

$$0 = \sum_n \gamma_{nj} \frac{(x_n - \mu_j)}{\Sigma_j}$$

$$\Leftrightarrow 0 = \frac{1}{\Sigma_j} \sum_n \gamma_{nj} \frac{(x_n-\mu_j)}{1}$$

$$\Leftrightarrow 0 = \frac{1}{\Sigma_j} \sum_n \gamma_{nj} x_n - \sum_n \gamma_{nj} \mu_j$$

$$\Leftrightarrow \mu_j \Sigma_n \gamma_{nj} = \Sigma_n \gamma_{nj} x_n$$

$$\boxed{\mu_j = \frac{\Sigma_n \gamma_{nj} x_n}{\Sigma_n \gamma_{nj}}}$$

3c) Since we know $w_k = \frac{\Sigma_n \gamma_{nk}}{\Sigma_k \Sigma_n \gamma_{nk}}$ & $\mu_k = \frac{\Sigma_n \gamma_{nk} x_n}{\Sigma_n \gamma_{nk}}$

we get the following solutions for w_1, w_2, μ_1, μ_2

$$\boxed{w_1 = 3/5 \quad w_2 = 2/5 \quad \mu_1 = 29 \quad \mu_2 = 14}$$

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