Bayesian-Inference

Samantha-Jo Caetano

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```
library(tidyverse)
library(patchwork)
```

Introduction to Bayesian Inference

With repeated flipping in the Bayesian setting, with our prior distribution on θ described in the slides, we would form the posterior distribution as follows:

$$p(\theta|X) = \frac{\theta^{\sum_{i=1}^{n} x_i}(1-\theta)^{n-\sum_{i=1}^{n} x_i} \times 0.2^{I(\theta=0.7)}0.8^{I(\theta=0.3)}}{\sum_{\theta=0.3,0.7} \theta^{\sum_{i=1}^{n} x_i}(1-\theta)^{n-\sum_{i=1}^{n} x_i} \times 0.2^{I(\theta=0.7)}0.8^{I(\theta=0.3)}}$$
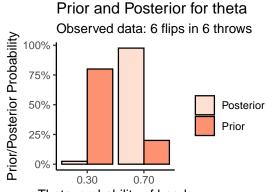
Visualization

It is helpful to visualize the prior and posterior, for the observed data. Because both prior and posterior only allow two values, we can do this using a simple bar chart:
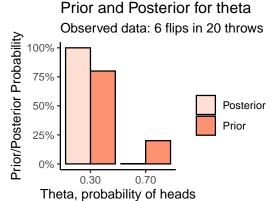
```
visualize_binomial_priorposterior <- function(sumx,n) {
  prior <- function(theta) {
    if (theta == .3) {
      return(.8)
    }
    else if (theta == .7) {
      return(.2)
    }
    0
  }
  likelihood <- function(theta) theta^sumx * (1-theta)^(n - sumx)
  marginal_likelihood <- prior(.7) * likelihood(.7) + prior(.3) * likelihood(.3)
  posterior <- function(theta) likelihood(theta) * prior(theta) / marginal_likelihood

  # Plot of the prior and posterior distributions for these observed data
  tibble(
    theta = c(.3,.7,.3,.7),
    value = c(prior(.3),prior(.7),posterior(.3),posterior(.7)),
    type = c("Prior","Prior","Posterior","Posterior")
  ) %>%
    ggplot(aes(x = theta,y = value,fill = type)) +
    theme_classic() +
```

```r
    geom_bar(stat = "identity",position = "dodge",colour = "black") +
    labs(title = "Prior and Posterior for theta",
         subtitle = str_c("Observed data: ",sumx," flips in ",n," throws"),
         x = "Theta, probability of heads",
         y = "Prior/Posterior Probability",
         fill = "") +
    scale_x_continuous(breaks = c(0.30,0.70),labels = c("0.30","0.70")) +
    scale_y_continuous(labels = scales::percent_format()) +
    scale_fill_brewer(palette = "Reds")

}
```

Plotting is nice as it lets us compare how different observed data, and different experiments (number of throws) affect the prior/posterior balance of belief:
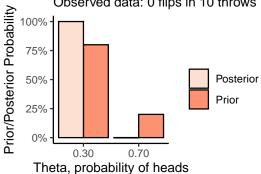
```r
# library(patchwork)
(visualize_binomial_priorposterior(6,6) | visualize_binomial_priorposterior(6,10)) /
(visualize_binomial_priorposterior(6,20) | visualize_binomial_priorposterior(6,50)) /
(visualize_binomial_priorposterior(0,10) | visualize_binomial_priorposterior(1,10)) /
(visualize_binomial_priorposterior(7,10) | visualize_binomial_priorposterior(10,10))
```
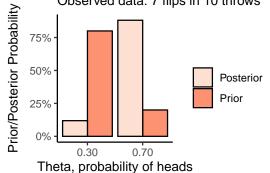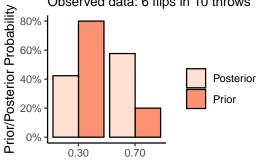
Prior and Posterior for theta
Observed data: 6 flips in 6 throws

Prior and Posterior for theta
Observed data: 6 flips in 10 throws

Prior and Posterior for theta
Observed data: 6 flips in 20 throws

Prior and Posterior for theta
Observed data: 6 flips in 50 throws

Prior and Posterior for theta
Observed data: 0 flips in 10 throws

Prior and Posterior for theta
Observed data: 1 flips in 10 throws

Prior and Posterior for theta
Observed data: 7 flips in 10 throws

Prior and Posterior for theta
Observed data: 10 flips in 10 throws