6. An RC circuit consists of a voltage source, a resistor, and a capacitor (and no inductor). If Q is the charge collected on the capacitor, then the circuit satisfies the equation

$$R\frac{dQ}{dt} + \frac{Q}{C} = E(t),$$

Since $I = \frac{dQ}{dt}$, this equation can be written as

$$RI + \frac{Q}{C} = E(t).$$

Suppose that the external voltage applied to this circuit is

$$E(t) = E_0 \cos(\omega t),$$

and $Q(0) = 0$.

A) Substitute the steady periodic solution [$\mathrm{Q}_{sp}(t) = A\cos(\omega t) + B\sin(\omega t)$ in the equation, and find the expressions for A and B. The steady periodic solution may also be written in the form

$$Q_{sp}(t) = C\cos(\omega t - \beta).$$

B) Derive the expression for the amplitude C..

C) Derive the formula for the phase shift β.

D) Find the expression for the current, I, in this circuit.

E) Does the current lead or lag the external voltage? Support your answer with a formula or an argument.