MCMC_PDF

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1 MCMC (Metropolis-Hesting) on Supernova-la dataset

1.1 Cosmology with Supernovae Ia: theory

- The flux of supernova of luminosity L is given by $f = \frac{L}{4\pi {D_L}^2}$, Where D_L is the *Luminosity distance*.
- D_L as a function of *redshift* Z $D_L = \frac{(1+z)c}{H_0\sqrt{|1-\Omega|}} S_k(r)$
- Where $r(z) = \sqrt{|\ 1 - \Omega\ |} \int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3+\Omega_v+(1-\Omega)(1+z')^2}}$, $S_k(r)$ = Sin r , r, Sinh r depending upon , Ω > 1, =1, <1, here $\Omega \equiv \Omega_m + \Omega_v$
- For flat universe $\Omega = 1$, $D_L(z) = 3000h^{-1}(1+z)\int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3+1-\Omega_m}}$Mpc, Where $H_0 = 100.hKmS^{-1}Mpc^{-1}$
- For flat only universe , another way is to use fitting formula without any integration $D_L(z) = \frac{c}{H_0}(1+z)[\eta(1,\Omega_m) - \eta(\frac{1}{1+z},\Omega_m)]$
- Where , $\eta(a,\Omega_m) = 2\sqrt{s^3+1}[\frac{1}{a^4} - 0.1540\frac{s}{a^3} + 0.4304\frac{s^2}{a^2} + 0.19097\frac{s^3}{a} + 0.066941\frac{s^4}{a^0}]^{-1/8}$, and $s^3 \equiv \frac{(1-\Omega_m)}{\Omega_m}$,has accurecy of 0.4% for $0.2 \leq \Omega_m \leq 1$
- Fluxes are usually expressed in magnitudes, where $ m=−2.5 log_{10}F+constant$. The distance modulus is $\mu = m - M$, where M is the absolute magnitude, which is the value of m if the source is at a distance 10pc:
- $\mu = 25 - 5log_{10}h + 5log_{10}(\frac{D_L*}{Mpc})$, The Hubble constant has been factored out of D_L : $D_L* \equiv D_L(h=1)$. If we have measurements of μ, then we can use Bayesian arguments to estimate the parameters Ω_m, Ω_v, h.

Model

defining η : $\eta(a,\Omega_m) = 2\sqrt{s^3+1}[\frac{1}{a^4} - 0.1540\frac{s}{a^3} + 0.4304\frac{s^2}{a^2} + 0.19097\frac{s^3}{a} + 0.066941\frac{s^4}{a^0}]^{-1/8}$, where $s^3 \equiv \frac{(1-\Omega_m)}{\Omega_m}$

defining the function for luminosity distance D_l : $D_L(z) = \frac{c}{H_0}(1+z)[\eta(1,\Omega_m) - \eta(\frac{1}{1+z},\Omega_m)]$

cleverly we have modified this function coefficent as 3000.(1+z) as we are choosing D_L*, $h \equiv 1$
$D_L*(z) = 3000.0.(1+z)[\eta(1,\Omega_m) - \eta(\frac{1}{1+z},\Omega_m)]$

function for distance modulus μ : $\mu = 25 - 5log_{10}h + 5log_{10}(\frac{D_L*}{Mpc})$

1.1.1 Above was the model for which we have to find out the best fit parameters values for Ω_m and h using bayesian methods.

2 Baye's theorem : $P(\theta|D) = \frac{P(\theta).P(D|\theta)}{\Sigma P(D)}$

where $\theta's$ are the parameter , D is the data

* $P(\theta|D)$ **is called 'Posterior Probability'.**

* $P(\theta)$ **is called 'Prior'.**

* $P(D|\theta)$ **is the 'Likelyhood'.**

* $\Sigma P(D)$ **is our 'Normalization constant'.**

- Here, in this problem we have chosen a "flat prior", which means that the distribution of posterior does not depends on the prior's distribution.
- Also, We have a stationary model. So We don't need to bother about the "Normalizing term" as well.
- In this case, the baye's formula can replaced into : # $P(\theta|D) \propto P(D|\theta)$

3 MH algorithm:

- looping from (1,N=10000)

In Marcov's chain parts we basically do like this:

- Next values of parameters are chosen from a normal distribution centered around the previous values of parameter with some standard deviation σ(here 0.01).

  next,

- we calculate the log likelyhood for the new randomly generated parameter values.
- if next value of likelyhood is grater than previous 'or' ratio of likelyhoods is greater than random uniform (0,1). Then accept the parameter values, else repeat the previous step.