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Mathematical Techniques in Visualizing Data

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UAB
Universitat Autònoma
de Barcelona

- **Supervisor:** Dr.Jozsef Farkas
- **Student:** Samar Moussa

Outline

Introduction and overview

Here is an overview of our thesis and research focus:

1. Objective: To investigate the effectiveness of Radon Transform in medical image visualization and reconstruction.
2. Methods: To employ Python tools to process medical images and mathematical techniques like Fourier and Radon Transforms for image quality enhancement.
3. Importance: To improve diagnostic accuracy, efficiency, and early disease detection in the medical field.

Challenges in medical imaging:

1. Complexity of data: Medical images are difficult to interpret due to their complex structures.
2. Critical for diagnosis: Accurate and effective image visualization is key to patient treatment.
3. DICOM standard implementation: Common format used for consistency in medical images but require special software for view.

Medical imaging modalities

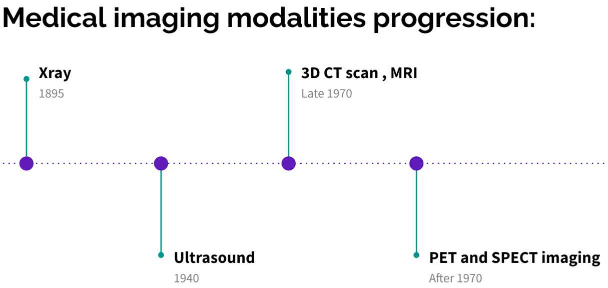


Figure: Medical imaging modalities progression

Background - Evolution of Medical Imaging Modalities

1. Origins: Inception in 1895 with Wilhelm Roentgen's discovery of X-rays. Initially used for bone structures.
2. Technological Progress: Evolved from 2D X-rays to 3D CT scans in the 1970s. Introduction of PET and SPECT for more advanced biological process visualization.
3. Modern Utility: Integral for diagnosis, treatment planning, and data-driven clinical decisions. Wide array of modalities for various medical domains.
4. Challenges: Increasing demand for effective data management systems due to high-volume, diverse image generation.

Overview of medical imaging processes

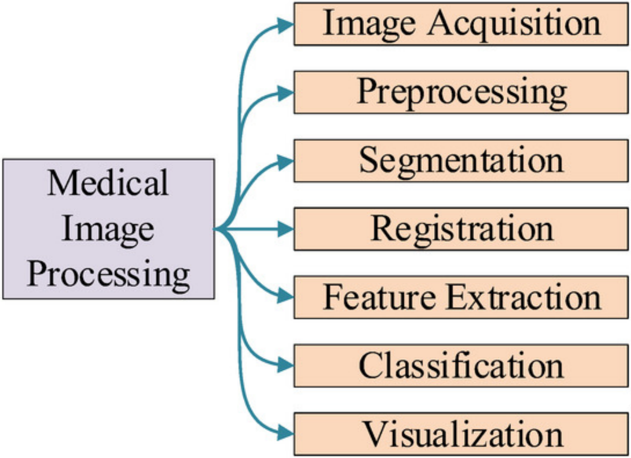


Figure: Medical imaging Process

Theoretical concepts - Mathematical transforms in medical imaging

1. Extract additional information from signals.
2. Critical for medical imaging reconstruction.
3. Fourier Transform: Widely recognized in various fields.
4. Radon Transform: Mainly used in medical imaging, introduced in 1917 by Johann Radon.
5. Applications of transforms: segmentation, classification, detection, reconstruction, and more.

Theoretical concepts - Radon transform

Definition and properties

- Maps a function f defined on a 2D plane to a function Rf on a set of lines.
- Regularity conditions ensure accuracy and reliability.
- Inverse Radon transform allows for reconstruction.

Formula representation

$$Rf(L) = \int_L f(x)\,|dx|$$

Inverse formula involves convolution with a Ramp filter.

Theoretical concepts - reconstruction using Radon transform

Medical imaging applications

- X-ray, CT scans, and SPECT scans.
- Integral to internal structure visualization.

Reconstruction process

- Utilizes inverse Radon transform.
- Filtering phase uses Ramp filter.
- Subsequent back projection recreates original image.

Role in medical imaging

- Transitions images from spatial to frequency domain.
- Reveals nuanced characteristics imperceptible in spatial representation.

Mathematical essence

$$F(\nu) = \int_{-\infty}^{\infty} f(x) \exp(-i2\pi x\nu)\, dx$$

Where $F(\nu)$ is the Fourier spectrum, $f(x)$ is the original function, and ν is the spatial frequency.

Applications

- Analyzes frequency content of images.
- Separates low and high-frequency information.

PCA - Principal Component Analysis

Role in medical imaging

- Data analysis and dimensionality reduction.
- Identifies important features for efficient visualization and analysis.

Mathematical foundation

$$
\begin{aligned}
C &= \frac{1}{n-1}(X-\bar{X})^T(X-\bar{X}) \\
\lambda_i v_i &= C v_i \\
Y &= XV
\end{aligned}
$$

Applications

- Feature extraction, segmentation, and visualization in MRI, PET, CT scans.
- Reduces computational burden and noise.

Limitations

- Interpretation can be challenging.
- Assumes normal distribution of data.

SVD - Singular Value Decomposition

Role in medical imaging

- Influential in data dimensionality and reduction.
- Crucial in medical image reconstruction.

Mathematical formulation

$$A = U\Sigma V^*$$

- U is an $m \times m$ unitary matrix.
- Σ is an $m \times n$ diagonal matrix.
- V^* is the conjugate transpose of an $n \times n$ unitary matrix V.

Benefits in imaging

- Diminishes noise.
- Enables data compression.
- Enhances image clarity.

Conclusion

- Significantly improves image quality and diagnostic precision.
- Indispensable for reconstructing fragmented or degraded data.

Medical imaging - Xray

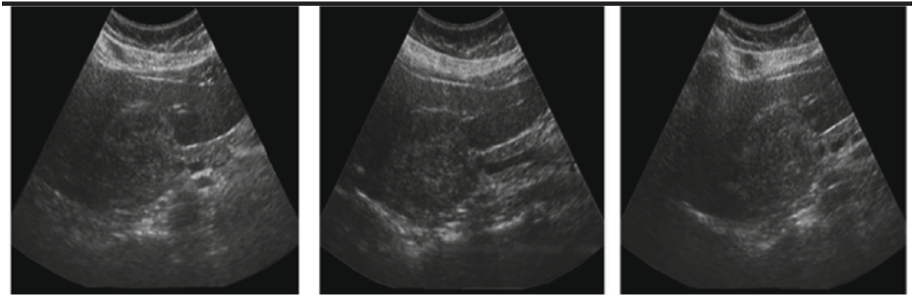
Table: X-ray imaging

Category	Description
Principle	Utilizes electromagnetic radiation.
Discovery	1895 by Wilhelm Conrad Roentgen.
Usage	Detecting bone anomalies, pulmonary conditions, dental irregularities, and some digestive system abnormalities.
Speciality	Tissue densities, bones and tumors appear in lighter shades.
Advantages	Widespread availability, cost-effective,rapid diagnostics, doesn't require hospital admission.
Limitations	Exposure to radiation, Limited resolution for discerning subtle differences in soft tissues.

Medical imaging - Ultrasound

Table: Ultrasound

Category	**Description**
Principle	High-frequency sound waves
Discovery	Implemented in the 1940s.
Usage	Obstetrics and gynecology, cardiac imaging, and abdominal applications.
Speciality	Imaging soft tissues like the uterus, ovaries, and fetus during pregnancy.
Advantages	Non-invasive,safe, versatile and quick.
Limitations	Limited to penetrate bone and air-filled organs.Operator-dependent,not suitable for all medical conditions.



Medical imaging - CT

Table: CT

Category	Description
Principle	Utilizes X-rays and computational algorithms to produce 3D images
Discovery	1970's
Usage	Diagnosing cancers, injuries, and monitoring previously treated ailments.
Speciality	Detailed cross-sectional visuals, capturing bones and tissues with higher granularity
Advantages	Detailed, sophisticated computational processing like Radon transform.
Limitations	Higher radiation dosage than X-rays, complications,anesthesia.

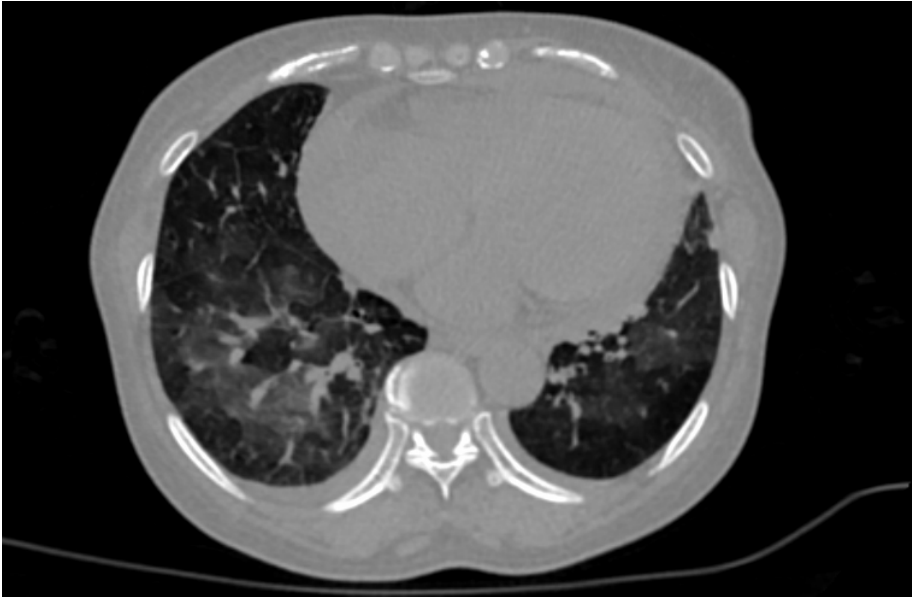


Table: Magnetic Resonance Imaging - MRI

Category	Description
Principle	Utilizes magnetic fields and radio waves.
Discovery	1970's as a non-invasive alternative in medical diagnostics.
Usage	Cerebral anomalies, musculoskeletal disorders, joint injuries, and neurological disorders.
Speciality	Precision in soft tissue imaging and does not use ionizing radiation.
Advantages	Safe, high-contrast, detailed images.
Limitations	High cost, limited availability, claustrophobic experience,restrictions due to metallic implants.

Medical imaging - PET scan

Table: PET scan

Category	Description
Principle	Uses radiotracers to capture gamma rays for metabolic chemical analysis.
Discovery	1970's
Usage	Oncology, neurology, and cardiology for diagnosing and monitoring diseases.
Speciality	Cervical cancer and lymphoma.
Advantages	Metabolic information about organs and tissues. High sensitivity and specificity in tumor progression.
Limitations	Expensive, limited spatial resolution, radiation exposure, claustrophobia, not for all medical conditions.

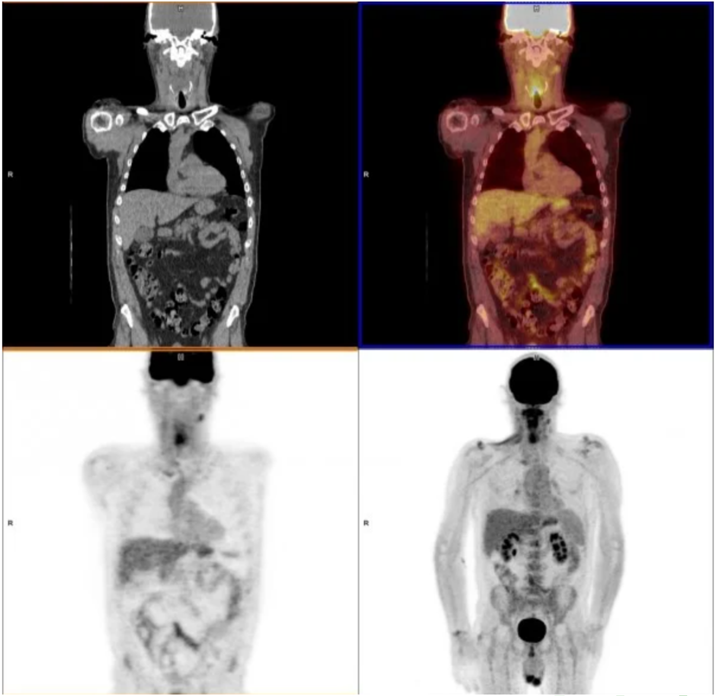
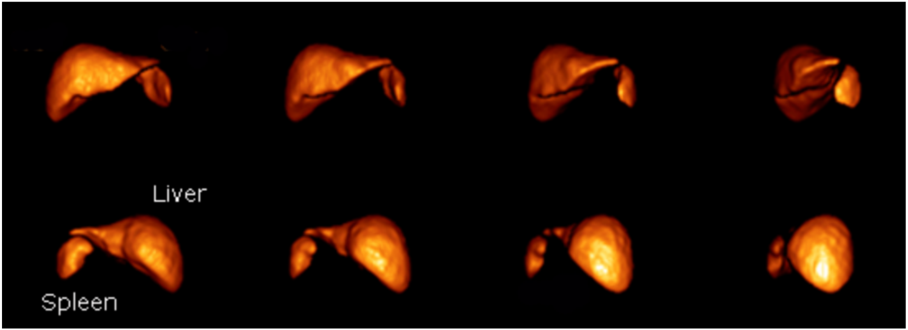
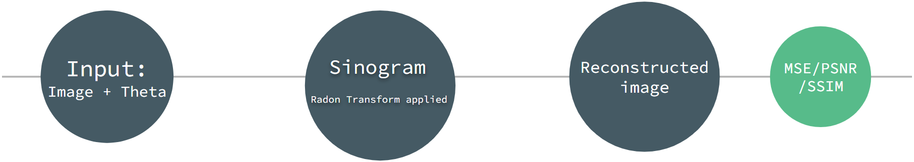


Table: SPECT imaging

Category	**Description**
Principle	Radioactive tracers based on photon emission and detection.
Discovery	Similar to PET.Uses a different type of radioactive tracer.
Usage	Neurology, psychiatry for brain, cardiology, and myocardial infarction.
Speciality	Identifying ailments like Alzheimer's and coronary artery disease.
Advantages	Non-invasive, functional data about the organ,low radiation
Limitations	Limited spatial resolution, injection of a radioactive tracer. Extended scan duration,immobility.


Liver
Spleen

Reconstruction algorithm Using Radon transform and its inverse

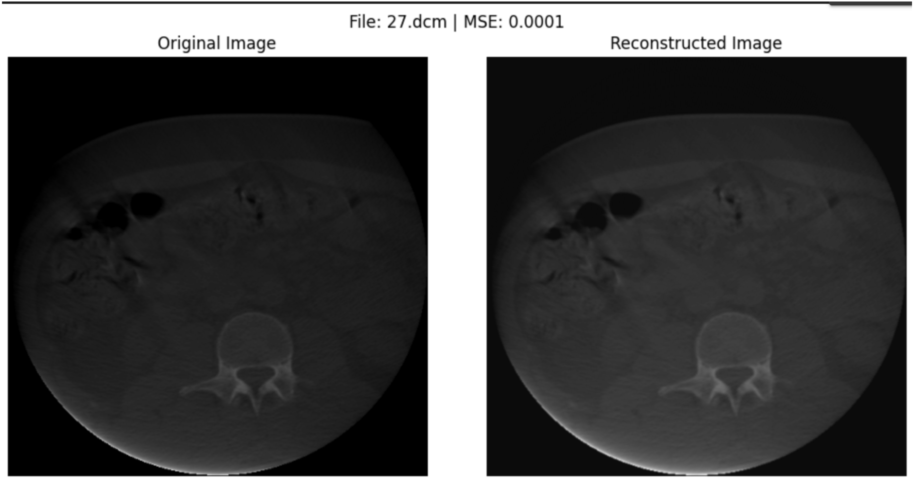


Results and discussion

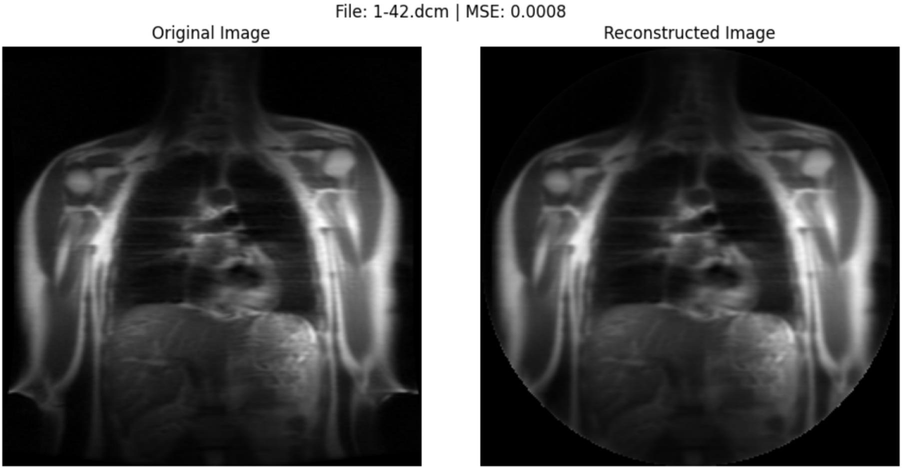
Major Contributions and Findings

1. Metrics Used: Accuracy of reconstructed images assessed using SSIM, MSE, and PSNR.
2. Types of imaging: Worked on three modalities - CT scans, MRI, and PET.
3. Results: Superior image quality in phantom medical images. PET scans showed the best image reconstruction.
4. Challenges and future Work: Time-efficiency in processing large datasets. Potential for integration with Convolutional Neural Networks (CNN) for better performance.

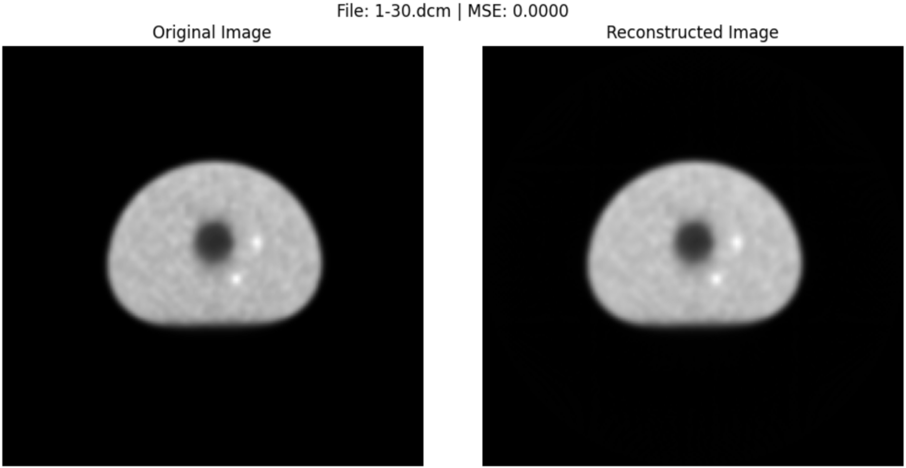
Results - Reconstructed CT



Results - Reconstructed MRI



Results - Reconstructed phantom PET



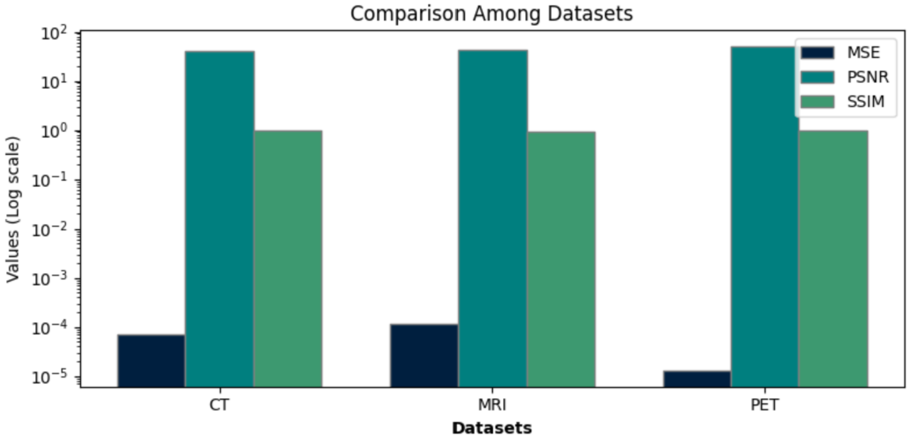
Summary of results

- **MSE (Mean Squared Error)**: Lower values indicate better reconstruction accuracy.
- **PSNR (Peak Signal-to-Noise Ratio)**: Higher values denote better image quality.
- **SSIM (Structural Similarity Index Measure)**: Values close to 1 indicate high structural integrity.

Modality	MSE	PSNR	SSIM
CT	7.14×10^{-5}	41.47	0.994
MRI	1.14×10^{-4}	43.49	0.946
PET	1.27×10^{-5}	49.97	0.996

Table: Summary of measurements for used modalities

Comparison between modalities accuracy



Interpretation of Results

- Radon and inverse Radon transforms are effective across CT, MRI, and PET.
- Lowest MSE in PET indicates highest reconstruction accuracy.
- Consideration of clinical relevance and acceptable error margins is essential.

Conclusion

- **Accuracy and Quality**: Metrics (MSE, PSNR, SSIM) confirm Radon-based reconstruction's efficacy, especially in PET scans.
- **Versatility**: Effective across CT, MRI, and PET, highlighting broad-spectrum utility.
- **Clinical Implications**: Potential for improved diagnostics and patient care.
- **Visualization**: Advanced techniques aid interpretation and interdisciplinary collaboration.

Future Directions

Optimization of Radon parameters and integration with machine learning algorithms.

Summary: Radon methods show promise in enhancing image quality across modalities, with potential for significant impact in diagnostic medicine.

Questions

Thank You!