Fayoum University
Faculty of Computers & Information
Second Year
Digital Signal Processing
Year 2020 / 2021

Sheet 2

1-

a. Let $y_1(n) = 5x_1(n) + 2x_1^2(n)$, $y_2(n) = 5x_2(n) + 2x_2^2(n)$

$$y_1(n) + y_2(n) = 5x_1(n) + 2x_1^2(n) + 5x_2(n) + 2x_2^2(n)$$

For $x(n) = x_1(n) + x_2(n)$

$$y(n) = 5x(n) + 2x^2(n) = 5\left(x_1(n) + x_2(n)\right) + 2\left(x_1(n) + x_2(n)\right)^2$$
$$= 5x_1(n) + 5x_2(n) + 2x_1^2(n) + 2x_2^2(n) + 4x_1(n)x_2(n)$$

Since $y_1(n) + y_2(n) \neq y(n)$, the system is a nonlinear system.

b. Let $y_1(n) = x_1(n-1) + 4x_1(n)$, $y_2(n) = x_2(n-1) + 4x_2(n)$

$$y_1(n) + y_2(n) = x_1(n-1) + x_2(n-1) + 4x_1(n) + 4x_2(n)$$

For $x(n) = x_1(n) + x_2(n)$

$$y(n) = x(n-1) + 4x(n) = \left(x_1(n-1) + x_2(n-1)\right) + 4\left(x_1(n) + x_2(n)\right)$$
$$= x_1(n-1) + x_2(n-1) + 4x_1(n) + 4x_2(n)$$

Since $y_1(n) + y_2(n) = y(n)$, the system is a linear system.

2-

a. causal system, since the system output depends on the current input and past inputs.
b. noncausal system, since the system output depends on a future input.

Problem 3:

a) $\boldsymbol{y[n] = x^2[n-1]}$

With memory because, for example, $y[0]$ depends on a past input sample $x[-1]$.

Causal because the output at any time doesn't depend on future input samples.

$y_1[n] = x_1^2[n-1]$

$y_2[n] = x_2^2[n-1]$

$\alpha y_1[n] + \beta y_2[n] = \alpha x_1^2[n-1] + \beta x_2^2[n-1]$

$x_3[n] = \alpha x_1[n] + \beta x_2[n]$

$y_3[n] = x_3^2[n-1]$

$= (\alpha x_1[n-1] + \beta x_2[n-1])^2 = \alpha^2 x_3^2[n-1] + \beta^2 x_3^2[n-1] + 2\alpha\beta x_1[n-1]x_2[n-1]$

$y_3[n] \neq \alpha y_1[n] + \beta y_2[n]$

Non-linear.

$y_1[n] = x_1^2[n-1]$

$y_1[n-n_0] = x_1^2[(n-n_0)-1]$

$x_2[n] = x_1[n-n_0]$

$y_2[n] = x_2^2[n-1]$

$= x_1^2[(n-1)-n_0]$

$y_2[n] = y_1[n-n_0]$

Time invariant.

Stable because a bounded input $|x[n]| < B_i < \infty, \forall n$ produces a bounded output $|y[n]| < B_o < \infty \ \forall n$.

e) $\boldsymbol{y[n] = x[-n+2]}$

With memory because, for example, $y[0]$ depends on a future input sample $x[2]$.

Non-causal because, for example, $y[-1]$ depends on a future input sample $x[3]$.

$y_1[n] = x_1[-n+2]$

$y_2[n] = x_2[-n+2]$

$\alpha y_1[n] + \beta y_2[n] = \alpha x_1[-n+2] + \beta x_2[-n+2]$

$x_3[n] = \alpha x_1[n] + \beta x_2[n]$

$y_3[n] = x_3[-n+2]$

$\quad = \alpha x_1[-n+2] + \beta x_2[-n+2]$

$y_3[n] = \alpha y_1[n] + \beta y_2[n]$

Linear.

$y_1[n] = x_1[-n+2]$

$y_1[n-n_0] = x_1[-(n-n_0)+2] = x_1[-n+n_0+2]$

$x_2[n] = x_1[n-n_0]$

$y_2[n] = x_2[-n+2]$

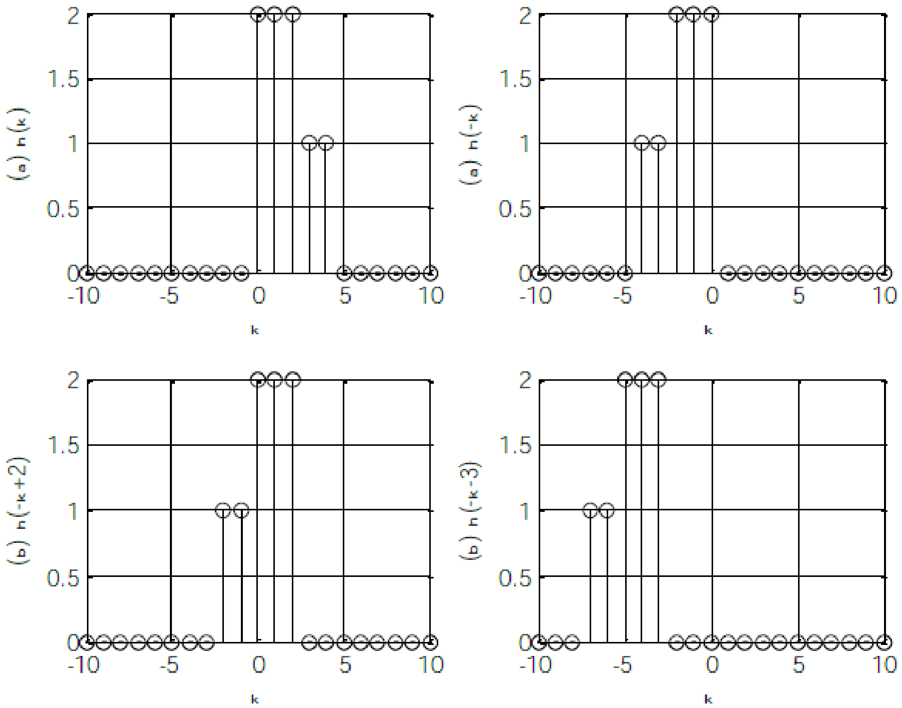
$\quad = x_1[(-n+2)-n_0] = x_1[-n+2-n_0]$

$y_2[n] \neq y_1[n-n_0]$

Time variant.

Stable because a bounded input $|x[n]| < B_i < \infty$, $\forall n$ produces a bounded output $|y[n]| < B_o < \infty \ \forall n$.

4-



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$y(0)=4$, $y(1)=6$, $y(2)=8$, $y(3)=6$, $y(4)=5$, $y(5)=2$, $y(6)=1$,
$y(n)=0$ for $n \geq 7$