Proofs about behavior of blind bouncing robots

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Given a closed polygonal region P with boundary ∂P. There is a map $B_\theta : \partial P \rightarrow \partial P$ where a robot that is on the boundary at p will next impact the boundary at $B_\theta(p)$. The map is completely determined by the outgoing angle with respect to the wall normal, $\theta : -\pi/2 < \theta < \pi/2$.

Bounces Between Parallel Lines enter 2-cycles immediately

Bounces between "Codependent Segments" go monotonically outward from vertex

Bounces in equilateral triangle converge to triangle for certain angles

Let P be an equilateral triangle, with side length a. Let the vertices of the triangle be labelled A, B, C in counterclockwise ordering.

Start the robot at point p_0 on segment $\overline{AB}$ which has distance x from vertex A.

Let $f(x, \theta)$ be a function that returns the distance from vertex B of the point $B_\theta(p_0)$. This assumes that point $B_\theta(p_0)$ is on segment $\overline{BC}$, but this is symmetric to the other case where the bounce goes to segment $\overline{CA}$.

The points $p_0, B_\theta(p_0)$, and B form a triangle. The angle $\angle p_0 B B_\theta(p_0)$ is $\pi/3$ since this is an equilateral triangle. The angle $\angle B_\theta(p_0) p_0 B$ is $\pi/2 - \theta$, and the segment $\overline{p_0 B}$ has length $a - x$. Thus we can use the law of sines to solve for $f(x, \theta)$:

$$\frac{f(x,\theta)}{sin(\pi/2-\theta)} = \frac{a-x}{sin(\pi-\pi/3-(\pi/2-\theta))}$$

$$f(x,\theta) = \frac{(a-x)cos(\theta)}{sin(\pi/6+\theta)}$$

Let $c(\theta) = \frac{cos(\theta)}{sin(\pi/6+\theta)}$. Since θ is restricted to the range $-\pi/2 < \theta < \pi/2$, the values of $c(\theta)$ are as shown in figure 1.

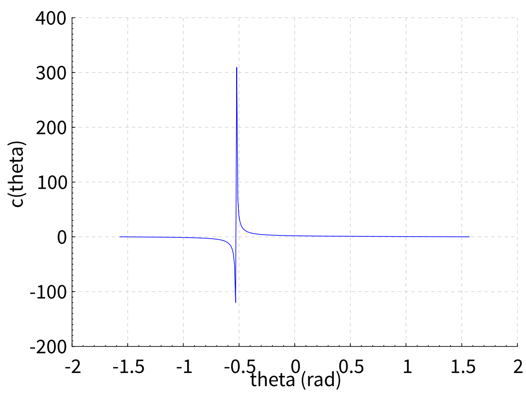

Figure 1: ctheta.pdf

This can be generalized to arbitrary regular polygons as:

$$\frac{f(x,\theta)}{sin(\pi/2-\theta)} = \frac{a-x}{sin(\pi-(n-2)\pi/n-(\pi/2-\theta))}$$

$$f(x,\theta) = \frac{(a-x)cos(\theta)}{sin(\pi/2-\pi/n+\theta)}$$