Proofs about behavior of blind bouncing robots

Alli Nilles

Given a closed polygonal region P with boundary ∂P. There is a map $B_\theta : \partial P \to \partial P$ where a robot that is on the boundary at p will next impact the boundary at $B_\theta(p)$. The map is completely determined by the outgoing angle with respect to the wall normal, $\theta : -\pi/2 < \theta < \pi/2$.

Bounces Between Parallel Lines enter 2-cycles immediately

See Erickson, LaValle et al

Bounces between "Codependent Segments" go monotonically outward from vertex

See Erickson, LaValle et al

Bounces in equilateral triangle converge to triangle for certain angles

Let P be an equilateral triangle, with side length a. Let the vertices of the triangle be labelled A, B, C in counterclockwise ordering.

Start the robot at point p_0 on segment $\overline{AB}$ which has distance x from vertex A.

Let $f(x, \theta)$ be a function that returns the distance from vertex B of the point $B_\theta(p_0)$. This assumes that point $B_\theta(p_0)$ is on segment $\overline{BC}$, but this is symmetric to the other case where the bounce goes to segment $\overline{CA}$.

The points $p_0, B_\theta(p_0)$, and B form a triangle. The angle $\angle p_0 B B_\theta(p_0)$ is $\pi/3$ since this is an equilateral triangle. The angle $\angle B_\theta(p_0) p_0 B$ is $\pi/2 - \theta$, and the segment $\overline{p_0 B}$ has length $a - x$. Thus we can use the law of sines to solve for $f(x, \theta)$:

$$\frac{f(x,\theta)}{sin(\pi/2-\theta)} = \frac{a-x}{sin(\pi - \pi/3 - (\pi/2-\theta))}$$

$$f(x,\theta) = \frac{(a-x)cos(\theta)}{sin(\pi/6+\theta)}$$

Let $c(\theta) = \frac{cos(\theta)}{sin(\pi/6+\theta)}$. Fix θ and let the infinite iteration of $f(f(...(x))) = F$. Then:

$F = a\sum_{i=1}^{\infty}(-1)^{i+1}c^i$

which converges when $|c| < 1$. This means that $f(x, \theta)$ has a fixed point when $c(\theta)$ is between -1 and 1.