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Modélisation de la diffusion 3D d'ondes élastiques par des structures complexes pour le calcul des échos de géométrie. Application à la simulation des CND par ultrasons.

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Acronyms

G | K | L | N | P | S | U

G

GE Geometrical Elastodynamics. 2, 5
GTD Geometrical Theory of Diffraction. 2, 5, 6

K

KA Kirchhoff Approximation. 2, 5

L

LT Laplace Transform. 2, 3, 5, 6

N

NDE Non Destructive Evaluation. 1
NDT Non Destructive Testing. 1

P

PTD Physical Theory of Diffraction. 2, 5

S

SF Spectral Functions. 2, 3, 5–7
SI Sommerfeld Integral. 2, 3, 5, 7

U

UAT Uniform Asymptotic Theory. 2, 5
UTD Uniform Theory of Diffraction. 2, 5, 7

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Introduction

The term Non Destructive Testing (NDT), also known as Non Destructive Evaluation (NDE), regroups all the inspection techniques that preserve the inspected specimen's integrity. This is particularly useful in areas such as civil engineering, aeronautics, nuclear energy or the automobile and railway industries, as it enables users to test industrial components during their production phase or during the course of their lifetime. There exists a wide range of NDT methods, the most frequently used being visual testing, eddy-current, magnetic-particle, liquid penetrant, radiographic and finally ultrasonic testing.

This thesis focuses on ultrasonic testing, an approach in which ultrasounds are emitted into a specimen and the waves scattered inside the specimen are analyzed in order to detect anomalies. These waves, which propagate through solid mediums without causing structural damage or changes, are elastic waves. The signal collected by the receiving transducer, which corresponds to the wave scattered by the specimen's boundaries and inhomogenities, contains information about the component's state and must therefore be analyzed.

The feasibility of ultrasonic inspections is predicted using numerical modeling. Numerical modeling also helps with the analysis of the received signals. To this end, CEA-LIST (Commissariat à l'Énergie Atomique et aux Énergies nouvelles - Laboratoire d'Intégration des Systèmes et Technologies) offers an NDT simulation tool via the CIVA software platform. These inspections deal with high frequency ($f \approx 2-5$ MHz) ultrasonic waves therefore simulation of realistic inspections by finite elements and finite differences can be time consuming because such methods require a small mesh step for a precise description of the scattered wave (on the order of $1/5^{th}$ of the wavelength). Semi-analytical methods are consequently preferred for high frequency problems, in order to reduce computation time.

Ultrasonic inspection of a specimen generates echoes from the entry and back-wall surfaces of this specimen. If these surfaces contain dihedral corners, it is then necessary to provide a correct model of the interaction between the ultrasonic beam and these wedges. These interactions may be linked to two different phenomena : reflection from the wedge faces and diffraction of the incident rays by the wedge edge. Both must be correctly taken into account by the model.

During the course of a previous thesis (Audrey Kamta-Djakou's PhD thesis), the specular model, which models reflection but not diffraction and therefore is not spatially continuous, was combined to an edge-diffraction model. The resulting model is called the Uniform Theory of Diffraction (UTD) and provides a spatially uniform high frequency representation of the scattered field. In order for the UTD model to be valid, a preexisting trustworthy ~~edge diffraction~~ model is necessary.

The aim of this thesis is to propose and validate a generic and reliable elastodynamic wedge-diffraction model, valid for all wedge angles and for 3D incidences. So far, this has not yet been done in elastodynamics. This is done by extending a method called the Spectral Functions (SF) method and proposing the corresponding numerical resolution schemes.

This manuscript is divided into 4 chapters.

In chapter 1, a review of high frequency wedge scattering models is presented. The first section of this chapter describes non-uniform asymptotic methods (non-uniform in the sense that the resulting scattered field is not spatially continuous), namely Geometrical Elastodynamics (GE), which models specular reflection but not diffraction and the Geometrical Theory of Diffraction (GTD), which models reflection and diffraction but diverges in certain directions. The second section presents uniform corrections of these non-uniform models : the Kirchhoff Approximation (KA), the Physical Theory of Diffraction (PTD), the Uniform Asymptotic Theory (UAT) and finally the Uniform Theory of Diffraction (UTD). All these models require a reliable preexisting wedge-diffraction GTD solution in order to be accurate. The third and final section therefore presents the two main existing GTD wedge diffraction models : the Sommerfeld Integral (SI) method and the Laplace Transform (LT) method.

In chapter 2, the Spectral Functions (SF) method is developed as a first step for the simpler case of an acoustic wave scattered by a soft (Dirichlet boundary conditions) or hard (Neumann boundary conditions) wedge. This is done by first, determining an integral formulation of the scattering problem. This formulation is given with respect to two unknown functions called the spectral functions. A system of functional equations of which these spectral functions are the solution is then determined using the problem's boundary conditions. This system permits the decomposition of the spectral functions into two terms : a singular function, determined analytically using a recursive algorithm and a regular function which is approached numerically using a Galerkin collocation method. The accuracy of this numerical approximation is improved by a method called "propagation of the solution". Finally, the solution computed using the Spectral Functions method is validated by comparison to the exact solution given by Sommerfeld.

Chapter 3 deals with extension of the Spectral Functions method to the 2D case of an elastic wave scattered by a stress-free wedge. The outline of the method is similar, but the unknown spectral functions are now two-dimensional vectors and the incident, reflected and diffracted waves can be longitudinal or transversal and mode conversion can occur. All of these phenomena are accounted for by the Spectral Functions method, which has the advantage of being valid for all wedge angles (as opposed to the previously existing Laplace Transform (LT) and Sommerfeld Integral (SI) methods). The resulting code is validated for wedge angles lower than π by comparison to the LT code and for wedge angles higher than π by comparison to a finite elements code. In addition, experimental validation is also carried out, thanks to previously made experimental measurements.

Finally, in chapter 4, the Spectral Functions method is developed for 3D cases, meaning for cases where the incident wave is no longer in the plane normal to the wedge edge. The spectral functions are now three-dimensional vectors and the possible wave polarizations are longitudinal, transverse horizontal and transverse vertical. An additional difficulty is created in the case of an incident transversal wave, when the skew angle (the angle between the incident ray and the plane normal to the edge) is higher than a certain angle called the critical angle (which depends on the propagation medium), extra care must be taken to deal with the branch points of the spectral functions, as some of them now lie on the imaginary axis (whereas up till now, the branch points were all real numbers). Nevertheless, the method is developed in detail and for all wedge angles and tested numerically. In cases where imaginary branch points appear, an additional numerical approximation method is proposed.

Conclusion

The aim of this thesis is to propose and validate a generic and reliable elastodynamic wedge-diffraction model valid for all wedge angles and for 3D incidences. This is done by extending a method called the Spectral Functions (SF) method and proposing the corresponding numerical resolution schemes. The principal results of this thesis are summarized in the following.

In the first chapter of this manuscript, a review of high-frequency wedge diffraction models is done. First, the two main non-uniform asymptotic methods are described : Geometrical Elastodynamics (GE), which only model reflected and refracted rays and the Geometrical Theory of Diffraction (GTD), which accounts for diffraction but diverges at observation directions close to specular reflections. Then, some uniform corrections of these models are presented. The Kirchhoff Approximation (KA), which produces a uniform scattered field but models diffraction inaccurately, the Physical Theory of Diffraction (PTD) which provides a good description of the scattered field in all directions but is computationally expensive, the Uniform Asymptotic Theory (UAT) which also provides a good description of the scattered field but is difficult to implement and finally the Uniform Theory of Diffraction (UTD) which is accurate, simple to implement and computationally cheap. For these reasons, UTD is the preferred uniform asymptotic model for wedge diffraction. Its accuracy relies on the existence of a reliable GTD wedge diffraction model. With that in mind, the two main existing wedge diffraction models, the Laplace Transform (LT) method and the Sommerfeld Integral (SI) method, are presented briefly. Neither of them has been developed for an elastic wave incident on a wedge of angle higher than π.

In the second chapter of this manuscript, the spectral functions method is developed as a first step in the simpler case of an acoustic wave scattered by a soft (Dirichlet boundary conditions) or hard (Neumann boundary conditions) wedge of arbitrary angle. To do so, an integral formulation of the solution to the scattering problem is derived. This formulation is given with respect to two unknown functions called the spectral functions. A far-field asymptotic evaluation of this integral formulation leads to an expression of the GTD diffraction coefficient as a function of the spectral functions. The integral formulation is then injected into the problem's boundary conditions, yielding an integral system of functional equations of which the spectral functions are the solution. This system is then solved semi-analytically. This means that the spectral functions are decomposed as the sum of two terms : a singular function, which is determined analytically thanks to a recursive algorithm, and a regular function, which is approached numerically thanks to a Galerkin collocation method. Finally, the accuracy of the numerical approximation of the regular part is improved using a technique called the "propagation of the solution". The method is validated by comparing the GTD diffraction coefficients obtained using the semi-analytical spectral functions method to the GTD diffraction coefficients derived from the exact solution given by Sommerfeld.

In the third chapter of the manuscript, the spectral functions method is applied to the more complex problem of elastic wave diffraction by a stress-free wedge of arbitrary angle. The main steps of the method are the same as in the previous chapter but the corresponding computations are more complex, since the spectral functions are now two-dimensional vectors and the incident, reflected and edge-diffracted waves can be polarized longitudinally and transversally. These two propagation modes are coupled by the wedge boundary conditions, meaning that mode conversion occurs. For each given configuration, two diffraction coefficients are therefore computed : one for longitudinal diffracted waves and one for transversal diffracted waves. The code developed using the Spectral Functions (SF) method is validated for wedge angles lower than π by comparison to the Laplace Transform (LT) code. However, the existing LT code is only valid for wedge angles lower than π. For wedge angles higher than π, the SF code is validated by comparison to a finite elements code. Finally, the results are also validated experimentally using the same measurements that were made to validate the LT code.

In the fourth and final chapter of the manuscript, the spectral functions method is applied to the 3D case of elastic wave diffraction by a stress-free wedge, where the incident wave vector is not necessarily in the the plane normal to the wedge edge. In this case, the incident ray on the wedge edge produces a cone of diffracted rays called Keller's cone of diffraction. The angle of this cone is determined by Snell's law of diffraction. According to Snell's law of diffraction, when the incident wave is transversal and the incident skew angle (i.e. the angle between the incident wave vector and the plane normal to the wedge edge) is higher than a certain angle called the critical angle, there is no diffracted longitudinal wave. The spectral functions then have imaginary branch points and extra care must be taken in dealing with these. The spectral functions method is developed in detail for the 3D case, for all types of incidences and for wedge angles higher and lower than π. An additional numerical approximation is proposed in order to compute the regular part of the spectral functions in the case of a transversal incident wave with a skew angle higher than the critical angle. The corresponding code is tested successfully in the particular cases of 2D incidences (the skew angle is set to 0), of the "acoustic limit" (the longitudinal and transversal wave velocities are set to mimic acoustic wave propagation) and of an infinite plane (the wedge angle is equal to π and there is no diffracted wave).

The results obtained during this thesis led to two publications in peer-reviewed journals [1, 2] as well to two communications in international conferences with conference proceedings [3, 4].

Some suggestions for future work are given below :

- In the final chapter of this thesis, the regular part of the spectral functions diverges in certain cases. Further investigations need to be made in order to find the cause of these divergences and a new method of computation must be proposed.
- A thorough numerical and/or experimental validation of the code implemented to treat the case of 3D diffraction of an elastic wave must be conducted.
- The spectral functions method could be further extended to treat dihedral interfaces between two elastic materials. This would be the continuity of Lucien Rochery's internship.
- The UTD model was developed by Audrey Kamta-Djakou [5] using the Sommerfeld Integral (SI) pole propagation algorithm. It should be adapted to the SF method so it can be applied to 3D incidences.
- The elastodynamic diffraction coefficients present a slight discontinuity at critical angles, due to the presence of head waves. In the continuity of Fradkin et al. [6], further investigations must be made in order to model the contribution of these waves correctly.
- In the final chapter, it was shown that for transversal incident waves with a skew angle higher than the critical angle, an evanescent longitudinal wave is produced. The asymptotic contribution of this wave should be determined.
- Following the ideas of Kamotskii [7], the Spectral Functions method could also be adapted to treat scattering by multiple adjacent wedges.

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ÉCOLE DOCTORALE
Physique et ingénierie : électrons, photons, sciences du vivant (EOBE)

Titre : Modélisation de la diffusion 3D d'ondes élastiques par des structures complexes pour le calcul des échos de géométrie. Application à la simulation des CND par ultrasons.

Mots clés : Elastodynamique, Diffraction, Méthodes asymptotiques

Résumé : Le sujet de la thèse s'inscrit dans le cadre du développement de modèles pour la simulation du contrôle non-destructif (CND) par ultrasons. L'objectif à long terme est la mise au point, par une méthode de rayons, d'un outil complet de simulation des échos issus de la géométrie (surfaces d'entrée, de fond. . .) ou des structures internes des pièces inspectées. La thèse vise plus précisément à intégrer le phénomène de diffraction par les dièdres à un modèle existant dérivant de l'acoustique géométrique et qui prend uniquement en compte les réflexions sur les faces.
Pour cela, la méthode dite des fonctions spectrales, développée initialement pour le cas d'un dièdre immergé, est développée et validée dans un premier temps dans le cas des ondes acoustiques pour des conditions aux limites de type Dirichlet ou Neumann. La méthode est ensuite étendue à la diffraction des ondes élastiques par des dièdres infinis à faces libres et d'angles quelconques, pour une incidence 2D puis pour une incidence 3D. Cette méthode est semi-analytique puisque les solutions recherchées s'écrivent sous la forme d'une somme d'une fonction singulière, qui est déterminée analytiquement à l'aide d'un algorithme récursif, et d'une fonction régulière, qui est approchée numériquement.
Les codes correspondants sont validés par comparaison à une solution exacte dans le cas acoustique et par comparaison à d'autres codes (semi-analytiques et numériques) dans le cas élastique. Des validations expérimentales du modèle élastodynamique sont également proposées.

Title : Modelling of the 3D scattering of elastic waves by complex structures for specimen echoes calculation. Application to ultrasonic NDT simulation.

Keywords : Elastodynamics, Diffraction, Asymptotic Methods

Abstract : This thesis falls into the framework of model development for simulation of ultrasonic non-destructive testing (NDT). The long-term goal is to develop, using ray methods, a complete simulation tool of specimen echoes (input, back-wall surfaces...) or echoes of inner structures of inspected parts. The thesis aims more specifically to integrate the phenomenon of diffraction by wedges to an existing model derived from geometrical acoustics, which only accounts for reflections on the wedge faces.
To this end, a method called the spectral functions method, which was initially developed for immersed wedges, is developed and validated as a first step in the case of acoustic waves with Dirichlet or Neumann boundary conditions. The method is then extended to elastic wave diffraction by infinite stress-free wedges of arbitrary angles, for 2D and 3D incidences. This method is semi-analytic since the unknown solutions are expressed as the sum of a singular function, determined analytically using a recursive algorithm, and a regular function which is approached numerically.
The corresponding codes are validated by comparison to an exact solution in the acoustic case and by comparison to other codes (semi-analytic and numerical) in the elastic case. Experimental validations of the elastodynamic model are also proposed.

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