Task 4. Algorithms for unconstrained nonlinear optimization. Stochastic and metaheuristic algorithms

Goal

The use of stochastic and metaheuristic algorithms (Simulated Annealing, Differential Evolution, Particle Swarm Optimization) in the tasks of unconstrained nonlinear optimization (in particular, for curve fitting) and the experimental comparison of them with Nelder-Mead and Levenberg-Marquardt algorithms.

Problems and methods

Generate the noisy data (x_k, y_k), where $k = 0, \dots, 1000$, according to the following rule:

$$y_k = \begin{cases} -100 + \delta_k, & f(x_k) < -100, \\ f(x_k) + \delta_k, & -100 \le f(x_k) \le 100, \\ 100 + \delta_k, & f(x_k) > 100, \end{cases} \qquad x_k = \frac{3k}{1000}, \qquad f(x) = \frac{1}{x^2 - 3x + 2},$$

where $\delta_k \sim N(0,1)$ are values of a random variable with standard normal distribution. For the data, find a solution to the rational curve fitting problem by approximate minimization of the following function:

$$D(a, b, c, d) = \sum_{k=0}^{1000} (F(x_k, a, b, c, d) - y_k)^2,$$

where $F(x, a, b, c, d) = \frac{\mathrm{ax+b}}{x^2+cx+d}$.

To solve the minimization problem, use Nelder-Mead algorithm, Levenberg-Marquardt algorithm and **one** of the methods among Simulated Annealing, Differential Evolution, Particle Swarm Optimization (you can use available implementations). If necessary, set the initial approximations and other parameters of the methods. Use $\varepsilon = 0.01$ as the precision; no more than 1000 iterations are allowed. Visualize the data and the fitting curves obtained by the numerical optimization methods in the same graph. Calculate the sum of squared residuals for each method. Analyze the results obtained in terms of the number of iterations performed and the values of sums of squared residuals.

Comments

Use any programming language you want. The findings and the plots should be informative and correct.

The report should be a pdf-document containing

- Task number and its topic, your group name, your name and surname, the report date;
- code of your programs required values and graphs, as well as analysis of the results.

Reports must be sent to chunaev@itmo.ru no later than three weeks after the task is given in English. Use the following format for the email subject: Task #, Name Surname, Group.