Q1) $\theta_1 = \mu$, $\theta_2 = \sigma^2$ MLE?

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$f(x,\theta_1,\theta_2) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_1)}{2\theta_2}}$$

$\theta_1 \in (-\infty, \infty)$

$\theta_2 \in [0, \infty)$ $\quad L(\theta_1,\theta_2) = \prod_{i=1}^{n} f(x_i,\theta_1,\theta_2) = \theta_2^{-n/2} \cdot (2\pi)^{-n/2} \cdot e^{-\frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2}$

$$\log L(\theta_1,\theta_2) = -\frac{n}{2}\log\theta_2 - \frac{n}{2}\log(2\pi) - \frac{\Sigma(x_i-\theta_1)^2}{2\theta_2}$$

taking Partial derivative, we get (wrt θ_1)

$$\frac{d\log L(\theta_1,\theta_2)}{d\theta_1} = \frac{-2\,\Sigma(x_i-\theta_1)(-1)}{2\theta_2} = 0$$

$$\Sigma(x_i-\theta_1) = 0$$

$$\hat{\theta}_1 = \mu = \frac{\Sigma x_i}{n} = \bar{x}.$$

for θ_2

$$\frac{d\log L(\theta_1,\theta_2)}{d\theta_2} = \frac{-n}{2\theta_2} + \frac{\Sigma(x-\theta_1)^2}{2\theta_2^2} = 0$$

$-n\theta_2 + \Sigma(x - \theta_1)^2 = 0$

~~As~~ $\theta_2^2 \neq 0$ as in denominator

$$\hat{\theta}_2 = \hat{\sigma}_2 = \frac{\Sigma(x_i - \bar{x})^2}{n}$$

$$\therefore \hat{\mu} = \frac{\Sigma x_i}{n} \quad \& \quad \hat{\sigma}^2 = \frac{\Sigma(x_i - \bar{x})^2}{n}$$

Q2) $\binom{m}{x_i} \theta^{x_i} (1-\theta)^{m - x_i} \quad B(m, \theta)$

$$L(\theta \mid x_1 \dots x_n) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

Log likelihood

$$\log L(\theta \mid x \dots x_n) = \sum_{i=1}^{n} \log \binom{m}{x_i} + x_i \log \theta + (m + x_i)(\log 1 - \theta_i)$$

$$\frac{\partial L}{\partial \theta} = \frac{x_i}{\theta} \cancel{+} \left(\cancel{(m + x_i)} - \frac{m - x_i}{1-\theta} \right.$$

$$\frac{\partial L}{\partial \theta} = 0 \quad ; \quad \sum_{i=1}^{n} \frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} = 0$$

$$\sum_{i=1}^{n} \frac{x_i - \theta m}{\theta(1-\theta)} = 0$$

$$\sum_{i=1}^{n} \left[\frac{x_i}{\theta(1-\theta)} \right] - \sum_{i=1}^{n} \frac{\cancel{\theta} m}{\cancel{\theta}(1-\theta)} = 0$$

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PAGE

$$\frac{1}{\theta(1-\theta)} \sum_{i=1}^{n} x_i - \frac{m}{1-\theta} \sum_{i=1}^{n} 1 = 0$$

$$\frac{\sum_{i=1}^{n} x_i}{\theta(1-\theta)} = \frac{m \cdot n}{(1-\theta)}$$

$$\theta = \frac{\sum_{i=1}^{n} x_i}{m \cdot n}$$

$\therefore$ MLE of θ for Binomial dist.

$B(m, \theta)$ is $\frac{\sum_{i=1}^{n} x_i}{m \cdot n}$