Bringing Wind Energy to Market

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***Abstract*—Wind energy is a rapidly growing source of renewable energy generation. However, the current extra-market approach to its assimilation into the electric grid will not scale at deep penetration levels. In this paper, we investigate how an independent wind power producer might *optimally offer* its variable power into a competitive electricity market for energy. Starting with a stochastic model for wind power production and a model for a perfectly competitive two-settlement market, we derive explicit formulae for optimal contract offerings and the corresponding optimal expected profit. As wind is an inherently *variable* source of energy, we explore the sensitivity of optimal expected profit to uncertainty in the underlying wind process. We also examine the role of forecast information and recourse markets in this setting. We quantify the role of reserves in increasing reliability of offered contracts and obtain analytical expressions for marginal profits resulting from investments in improved forecasting and local auxiliary generation. The formulae make explicit the relationship between price signals and the value of such firming strategies.**

***Index Terms*—Electricity markets, smart grid, wind energy.**

I. Introduction

GLOBAL warming, widely regarded as one of the most critical problems facing mankind, has led to great emphasis on clean renewable energy resources such as solar, wind, and geothermal. Many nations have set ambitious goals for increasing the share of renewable energy in electric power generation—wind energy is expected to be a major contributor to the realization of these goals. However, at deep levels of penetration, the significant uncertainty and inherent variability in wind power pose major challenges to its integration into the electricity grid.

In many regions around the world, wind power receives extra-market treatment in the form of feed-in tariffs which guarantee grid access and favorable fixed feed-in prices. Specifically, in California, the *Participating Intermittent Resource Program* (PIRP) legislation compels the system operator to accept all produced wind power subject to certain contractual constraints. This amounts to a *system take-all-wind* scenario in which wind power is treated as a negative load and the burden of balancing costs falls largely on the shoulders of the load serving entities (LSEs). This socialization of added reserve costs among the LSEs will become untenable at levels of deep wind energy penetration. Hence, as penetration continues to increase, it is likely that wind power producers (WPPs) will be faced with increased exposure to market signals that incentivize reduction in output variability. For example, in the United Kingdom, large WPPs are forced to participate in conventional wholesale electricity markets where they are subject to *ex-post* financial penalties for deviations from contracts offered *ex-ante* in forward markets [1].

Motivated by this, we study the setting in which WPPs must sell their variable energy using contract mechanisms in *conventional* two-settlement electricity markets. Our goal is to formulate and solve problems of optimal contract sizing and analytically quantify the relationship between market price signals and the value of improved forecasting, value of local auxiliary generation, value of storage, and cost of increased reserves needed to accommodate the uncertainty in wind power production.

There is a considerable literature covering many important aspects of wind power ranging over comprehensive integration studies, forecasting methods, and technological challenges. Representative references include [8], [9], [14], [16], [17], [20]–[22], [25], and [30]. In the narrower context of this paper, there exist multiple contributions that study the problem of optimizing contract offering strategies in a two-settlement market structure in the face of wind power production uncertainty. A scenario-based stochastic programming approach to numerically compute DA contract offers that maximize expected profit in a two-settlement market setting is studied in [3], [29], and [31]. Two papers [7], [31] go a step further by introducing a risk-sensitive term (CVaR) to the profit function to control variability in expected profit. Additionally, Morales *et al.* [31] extend these formulations by considering a market system with an additional intra-day market that allows for contract adjustment (i.e., recourse) before the delivery time—resulting in a two-stage stochastic programming formulation, which they show reduces to linear programming (LP). The aforementioned results are primarily computational in nature. In a more general formulation, Sethi *et al.* [35] consider an analogous problem in which an LSE must incrementaly purchase a bundle of energy *ex-ante* in a sequence of N time-sequential markets (with increasing information) subject to demand uncertainty *ex-post*. Using stochastic dynamic programming, they arrive at a closed form condition for optimality of the sequential purchase amounts. Additionally, in the single forward market

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setting, Pinson *et al.* [32] show that the optimal forward contract can be analytically expressed as a probabilistic quantile on prices—a well-established result in the field of inventory theory [33]. Also, it has recently come to our attention that Dent *et al.* [15] extend the quantile result of [32] by allowing for stochastic correlation between the wind and imbalance prices. Our work is in the spirit of these latter papers. Our aim is to derive provably optimal analytical expressions to explain the interplay between production uncertainty and profitability.

Our contributions are as follows. We start with a general stochastic model for wind power production and a model for a conventional two-settlement market for energy. Using these models, we derive explicit formulae for optimal contract size and the optimal expected profit—results that make explicit the trade-off between imbalance prices and the need to *spill* some of the wind energy in order to increase the probability of meeting the contract. Our analytical characterization of the optimal contract offering is a generalization of that in [15] and [32] as it holds on the entire space of expected imbalance prices. We also provide analytical expressions for optimal contract offerings in a multi-period setting in which the WPP has a *recourse opportunity* to adjust its DA commitment in an intra-day market—offering greater analytical tractability than the LP characterization in [31]. Moreover, we show that extra information from meteorological models and data increases the expected optimal profit. We also make explicit the relationship between penalty for contract shortfall and the marginal impact of wind uncertainty on optimal expected profit. For a *uniform* characterization of wind uncertainty, we show that the optimal expected profit is *affine* in the forecast standard deviation. We also consider the scenario in which the WPP has installed a fast-acting co-located thermal generator to "hedge" against potential shortfalls on offered contracts and derive a formula for optimal contract size. In this setting, we also explore the role of local generation in managing the operational and financial risk driven by the uncertainty in generation and obtain analytical expressions for marginal profits from investing in local generation. The formulae make explicit the relationship between price signals and the value of various firming strategies. Due to space constraints, the complementary energy storage analysis is not included, but can be found in [6].

The remainder of this paper is organized as follows: In Section II, we provide some background on wind energy and electricity markets. Our problem formulation is described in Section III and our main results are contained in Sections IV–VII. We conduct an empirical study of our strategies on wind power data obtained from Bonneville Power Authority in Section VIII. Concluding comments and discussion of current and future research are contained in Section IX.

II. Background

A. Wind Energy

The inherent variability of the power output is the most significant difference between wind and traditional power generators. This variability occurs at various time scales: seconds, hours, days, months, and years. As loads are also uncertain and variable, the issue of wind power variability does not cause major problems at low wind energy penetration levels, but at deep penetration levels, it presents major engineering, economic, and societal challenges. Recently, the National Renewable Energy Laboratory (NREL) has released two major reports [17], [21] on integration of large amounts of wind power (20%–30%) into the Eastern and Western electric grid interconnections in North America. These studies show that limitations on the transmission system, increased need for reserves, impact of unpredicted large ramps, limited accuracy in wind forecasting, coordination among and conflicting objectives of independent power producers, system operators, and regulatory agencies are some of the major issues in achieving increased penetration of wind and solar energy.

Integration of wind power into the electric grid has been the subject of many academic and industry studies. Holttinen *et al.* [25] present a system-level perspective to coping with wind variability. Specifically, the authors recommend wind aggregation over large balancing areas as an integral strategy to variability mitigation and improved forecastability, because of the tendency of wind patterns to decorrelate with increasing spatial separation. Moreover, as prediction errors decrease with shortening of the prediction horizon, the incorporation of intra-day markets will be critical to improving the reliability of offered wind power schedules [30]. Additionally, large fast ramps in power output present serious challenges to the real-time (RT) balancing of supply and demand, as they lead to an increase in spinning reserve requirement. Prediction of the timing and magnitude of these ramp events is challenging and is the subject of our current research.

Energy storage has also been investigated as a means to firm wind power output [27]. Cavallo [12], [13] has studied compressed air energy storage for utility scale wind farms, and Greenblatt *et al.* [24] compared gas turbines and compressed air energy storage (CAES) in the context of wind as part of base-load electricity generation. Economic viability of CAES in systems with significant wind penetration, such as Denmark, has recently been investigated in [28].

B. Electrical Energy Markets

The deregulation of the electric power industry has led to the development of *open markets* for electrical energy. The two dominant modes of trading are *bilateral trading* and *competitive electricity pools*. The former involves two parties (a buyer and a seller) negotiating a price, quantity, and auxiliary conditions for the physical transfer of energy from the buyer to the seller at some future time. Such trades need to be incorporated into operational planning by the system operators. Instruments such as *contracts for difference* and *financial transmission rights* may facilitate such bilateral trading.

In contrast to bilateral trading, a competitive electricity pool consists of numerous buyers and sellers participating in a single market cleared by a third party—commonly the independent system operator (ISO). Essentially, each supplier (consumer) submits an offer (bid) for energy at a desired price to the ISO. The ISO then combines these bids and offers to construct aggregate supply and demand curves, respectively. The aggregate supply curve is constructed by stacking energy offers in order of increasing offer price. The aggregate demand curve is similarly constructed by stacking energy quantities in order of decreasing bid price. The intersection of these curves determines the market clearing price (MCP). All suppliers that submitted bids at prices below the MCP are scheduled. Conversely, all consumers that submitted offers at prices above the MCP get scheduled. All scheduled parties pay or are paid at the MCP. For a comparison between bilateral and pool trading, see [26].

For concreteness in our studies, we assume that the WPP is participating in a competitive electricity pool, although much of our analysis is portable to the bilateral trading framework. A common pool trading structure [26], [38] consists of two successive *ex-ante* markets: a DA forward market and an RT spot market. The DA market permits suppliers to offer and schedule energy transactions for the following day. Depending on the region, the DA market closes for bids and schedules by 10 AM and clears by 1 PM on the day prior to the operating day. The schedules cleared in the DA market are financially binding and are subject to deviation penalties. As the schedules submitted to the DA market are cleared well in advance of the operating day, an RT spot market is employed to ensure the balance of supply and demand in RT. This is done by allowing market participants to adjust their DA schedules based on current (and more accurate) wind and load forecasts. The RT market is cleared 5 to 15 min before the operating interval, which is on the order of 5 min.

For those market participants who deviate from their scheduled transactions agreed upon in the *ex-ante* markets, the ISO normally employs an *ex-post* deterministic settlement mechanism to compute imbalance prices for upward and downward deviations from the generator's offered schedule. This pricing scheme for penalizing schedule deviations reflects the energy imbalance of the control area as a whole and the *ex-ante* clearing prices. For example, if the overall system imbalance is negative, those power producers with a positive imbalance with respect to their particular schedules will receive a more favorable price than those producers who have negatively deviated from their schedules, and vice-versa. Depending on the region, imbalance prices may be symmetric or asymmetric. For a more detailed description of electricity market systems in different regions, we refer the reader to [10], [11], [26] and [37].

III. Models for Wind Power and Markets

A. Wind Power Model

Wind power $w(t)$ is modeled as a scalar-valued stochastic process. We normalize $w(t)$ by the nameplate capacity of the wind power plant, so $w(t) \in [0,1]$. For a fixed $t \in \mathbb{R}$, $w(t)$ is a random variable (RV) whose cumulative distribution function (CDF) is assumed known and defined as $\Phi(w;t) = \mathbb{P}\{w(t) \le w\}$. The corresponding density function is denoted by $\phi(w;t)$.

In this paper, we will work with marginal distributions defined on the time interval $[t_0, t_f]$ of width $T = t_f - t_0$. Of particular importance are the *time-averaged* density and distribution defined as

$$f(w) = \frac{1}{T}\int_{t_0}^{t_f} \phi(w;t)\,dt \tag{1}$$

$$F(w) = \frac{1}{T}\int_{t_0}^{t_f} \Phi(w;t)\,dt = \int_0^w f(x)\,dx. \tag{2}$$

Also, define $F^{-1}:[0,1]\to[0,1]$ as the *quantile function* corresponding to the CDF F. More precisely, for $\beta \in [0,1]$, the β-quantile of F is given by

$$F^{-1}(\beta) = \inf\{x \in [0,1] \mid \beta \le F(x)\}. \tag{3}$$

The quantile function corresponding to the time-averaged CDF will play a central role in our results.

B. Market Model

The two-settlement market system considered in this paper consists of an *ex-ante* DA forward market with an *ex-post* imbalance settlement mechanism to penalize uninstructed deviations from contracts scheduled *ex-ante*. Negative deviations are charged at a price $q \in \mathbb{R}$ (\$/MWh) and positive deviations are charged at price $\lambda \in \mathbb{R}$ (\$/MWh). The imbalance prices (q,λ) are assumed unknown during the DA forward market and can take on both positive and negative values depending on system conditions.

In the bulk of this paper, we analyze the problem of optimizing the offering of a constant power contract C at a price $p \in \mathbb{R}_+$ (\$/MWh) in the *DA market*, scheduled to be delivered continuously over a single time interval $[t_0, t_f]$ (typically of length 1 h). As the WPP has no energy storage capabilities for possible price arbitrage, the decision of how much constant power to offer over any individual hour-long time interval is independent of the decision for every other time interval. Hence, the problems decouple with respect to contract intervals. Market prices (\$/MWh) are denoted as follows.

p	settlement price in the forward market;
q	*ex-post* settlement price for negative imbalance ($w(t) \le C$);
λ	*ex-post* settlement price for positive imbalance ($w(t) > C$).

We make the following assumptions regarding prices and production costs.

A1: The WPP is assumed to be a *price taker* in the forward market, as the individual WPP capacity is assumed small relative to the whole market. Consequently, the forward settlement price p is assumed fixed and known.

A2: The WPP is assumed to have a *zero marginal cost of production*.

A3: As imbalance prices $(q,\lambda) \in \mathbb{R}^2$ tend to exhibit volatility and are difficult to forecast, they are modeled as *random* variables, with expectations denoted by

$$\mu_q = \mathbb{E}[q], \quad \mu_\lambda = \mathbb{E}[\lambda].$$

The imbalance prices (q,λ) are assumed to be statistically independent of the wind $w(t)$.

Remark III.1 (Value of Excess Wind): The positive imbalance price λ can be alternatively interpreted to represent the economic value of surplus wind power in the spot market or as a stored commodity—assuming the WPP has energy storage capabilities. □

The profit acquired, the energy shortfall, and the energy surplus associated with the contract C on the time interval $[t_0, t_f]$ are defined, respectively, as

$$\Pi(C, \mathbf{w}, q, \lambda) = pCT - q\, \Sigma_-(C, \mathbf{w}) - \lambda\, \Sigma_+(C, \mathbf{w}) \quad (4)$$

$$\Sigma_-(C, \mathbf{w}) = \int_{t_0}^{t_f} [C - w(t)]^+ \, dt \quad (5)$$

$$\Sigma_+(C, \mathbf{w}) = \int_{t_0}^{t_f} [w(t) - C]^+ \, dt \quad (6)$$

where $x^+ := \max\{x, 0\}$ for all $x \in \mathbb{R}$. As wind power $w(t)$ is modeled as a random process, we will be concerned with the *expected* profit $J(C)$, the *expected* energy shortfall $S_-(C)$, and the *expected* surplus wind energy $S_+(C)$:

$$J(C) = \mathbb{E}\, \Pi\,(C, \mathbf{w}, q, \lambda) \quad (7)$$

$$S_-(C) = \mathbb{E}\, \Sigma_-(C, \mathbf{w}) \quad (8)$$

$$S_+(C) = \mathbb{E}\, \Sigma_+(C, \mathbf{w}). \quad (9)$$

Here, expectation is taken with respect to the random prices (q, λ) and wind power process $\mathbf{w} = \{w(t) \mid t_0 \le t \le t_f\}$.

IV. Optimal Contracts

We begin by defining a profit maximizing contract C^* as

$$C^* = \arg\max_{C \ge 0} J(C). \quad (10)$$

In the following theorem, we show that C^* can be expressed analytically using a partition of the space of expected imbalance prices $\pi = (\mu_q, \mu_\lambda) \in \mathbb{R}^2$. Consider the disjoint partition of $\mathbb{R}^2$ defined by

$$\mathcal{M}_1 = \left\{(x, y) \in \mathbb{R}^2 \mid x(\mu_w - 1) + y\mu_w \le -p,\ y < -p\right\}$$
$$\mathcal{M}_2 = \left\{(x, y) \in \mathbb{R}^2 \mid x \ge p,\ y \ge -p\right\}$$
$$\mathcal{M}_3 = \left\{(x, y) \in \mathbb{R}^2 \mid x(\mu_w - 1) + y\mu_w > -p,\ x < p\right\}$$

where μ_w is the mean of the time-averaged distribution (2).

Theorem IV.1: Define the time-averaged distribution $F(w)$ as in (2). For an expected imbalance price pair $\pi = (\mu_q, \mu_\lambda)$,

a) an optimal contract C^* is given by

$$C^* = \begin{cases} 0, & \pi \in \mathcal{M}_1 \\ F^{-1}(\gamma), & \pi \in \mathcal{M}_2 \\ 1, & \pi \in \mathcal{M}_3 \end{cases} \quad \text{where } \gamma = \frac{p + \mu_\lambda}{\mu_q + \mu_\lambda}. \quad (11)$$

b) The optimal expected profit is given by

$$\frac{J(C^*)}{T} = \frac{J^*}{T} = \begin{cases} -\mu_\lambda \mu_w, & \pi \in \mathcal{M}_1 \\ \mu_q \int_0^\gamma F^{-1}(x)dx - \mu_\lambda \int_\gamma^1 F^{-1}(x)dx, & \pi \in \mathcal{M}_2 \\ p - \mu_q(1 - \mu_w), & \pi \in \mathcal{M}_3. \end{cases} \quad (12)$$

c) The optimal expected shortfall and surplus are

$$S_-(C^*) = S_-^* = T \int_0^{F(C^*)} \left[C^* - F^{-1}(x)\right] dx \quad (13)$$

$$S_+(C^*) = S_+^* = T \int_{F(C^*)}^1 \left[F^{-1}(x) - C^*\right] dx. \quad (14)$$

Proof: Part a): Using the assumption of independence between the imbalance prices (q, λ) and wind power $w(t)$, notice that $J(C)$ can be rewritten in terms of the time-averaged density $f(w)$ as defined in (1):

$$\begin{aligned} \frac{J(C)}{T} &= pC \\ &- \frac{1}{T} \int_0^1 \int_{t_0}^{t_f} \left(\mathbb{E}[q][C - w]^+ + \mathbb{E}[\lambda][w - C]^+\right) \phi(w; t)\, dt dw \\ &= pC - \mu_q \int_0^C (C - w) f(w) dw - \mu_\lambda \int_C^1 (w - C) f(w) dw. \end{aligned}$$

Clearly $J(C)$ is continuous in C on $[0, 1]$ for any probability density function $f(w)$. For technical simplicity in the proof, we additionally assume that the density $f(w)$ is continuous on $[0, 1]$—from which it follows that $J(C)$ is also differentiable in C on $[0, 1]$. Straightforward application of the Leibniz integral rule yields the first and second derivatives of $J(C)$:

$$\frac{dJ}{dC} = T\,(p + \mu_\lambda - (\mu_q + \mu_\lambda)\, F(C)) \quad (15)$$

$$\frac{d^2 J}{dC^2} = -\,T\,(\mu_q + \mu_\lambda)\, f(C). \quad (16)$$

As $f(C) \ge 0$ for all $C \in [0, 1]$, $J(C)$ is concave $\iff \mu_q + \mu_\lambda \ge 0$. Similarly, $J(C)$ is convex $\iff \mu_q + \mu_\lambda \le 0$. We now consider the optimization problem on each half-space of expected imbalance prices, separately.

1) Assume $\mu_q + \mu_\lambda > 0$. As $J(C)$ is concave on this half-space of expected imbalance prices, it follows that $C^* \in [0, 1]$ is optimal if and only if

$$(x - C^*) \left.\frac{dJ}{dC}\right|_{C = C^*} \le 0 \quad \forall\, x \in [0, 1]. \quad (17)$$

We now evaluate this optimality criterion on three subsets $\{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3\}$ that partition the half-space $\{\mu_q + \mu_\lambda > 0\}$:

$$\mathcal{B}_1 = \{\mu_q < p\} \cap \{\mu_q + \mu_\lambda > 0\}$$
$$\mathcal{B}_2 = \{\mu_q \ge p\} \cap \{\mu_\lambda \ge -p\}$$
$$\mathcal{B}_3 = \{\mu_\lambda < -p\} \cap \{\mu_q + \mu_\lambda > 0\}.$$

For all $(\mu_q, \mu_\lambda) \in \mathcal{B}_1$, it is straightforward to show that $J(C)$ is *non-decreasing* on $[0, 1]$, which yields an optimal point $C^* = 1$ satisfying (17). Conversely, $J(C)$ is *non-increasing* on $[0, 1]$ for all $(\mu_q, \mu_\lambda) \in \mathcal{B}_3$–yielding $C^* = 0$. Finally, for $(\mu_q, \mu_\lambda) \in \mathcal{B}_2$, the expected profit exhibits a *stationary point* given by $C = F^{-1}(\gamma)$. Notice that $\mathcal{B}_2 = \mathcal{M}_2$.

2) Now, assume that $\mu_q + \mu_\lambda \le 0$. As $J(C)$ is convex on this half-space of expected imbalance prices, the optimum will necessarily occur at the boundary of the feasible set $[0, 1]$. This leads to a simple test for optimality:

$C^* = 1$ if $J(1) \geq J(0)$, and $C^* = 0$ otherwise. Evaluation of the expected profit criterion $J(C)$ at the boundary points yields

$$J(1) - J(0) = T\left(p - \mu_q(1-\mu_w) + \mu_\lambda \mu_w\right).$$

Hence, for $\mu_q + \mu_\lambda \leq 0$, we have

$$C^* = \begin{cases} 1, & \mu_q(\mu_w - 1) + \mu_\lambda \mu_w \geq -p \\ 0, & \mu_q(\mu_w - 1) + \mu_\lambda \mu_w < -p. \end{cases}$$

Combining this threshold result with those of part 1), we recover the desired result in Theorem IV.1-a).

Part b): For $(\mu_q, \mu_\lambda) \in \mathcal{M}_1, \mathcal{M}_3$, the result is easily proven by direct substitution of C^* into the expected profit criterion (6). For $(\mu_q, \mu_\lambda) \in \mathcal{M}_2$, consider the change of variables $\theta = F(w)$:

$$\begin{aligned}\frac{J^*}{T} &= pC^* - \mu_q \int_0^{C^*} (C^* - w) f(w) dw \\ &\quad - \mu_\lambda \int_{C^*}^1 (w - C^*) f(w) dw \\ &= pC^* - \mu_q \int_0^\gamma (C^* - F^{-1}(\theta)) d\theta \\ &\quad - \mu_\lambda \int_\gamma^1 (F^{-1}(\theta) - C^*) d\theta \\ &= \underbrace{(p + \mu_\lambda - (\mu_q + \mu_\lambda)\gamma)}_{=0} C^* \\ &\quad + \mu_q \int_0^\gamma F^{-1}(\theta) d\theta \\ &\quad - \mu_\lambda \int_\gamma^1 F^{-1}(\theta) d\theta\end{aligned}$$

which gives us the desired result.

Part c): The proof is analogous to that of part b). ■

Remark IV.2 (Newsvendor): The profit criterion and quantile structure of the optimal policy (11) are closely related to the classical *Newsvendor* problem in operations research [33]. □

Remark IV.3 (Graphical Interpretation): Parts b) and c) of Theorem IV.1 provide explicit formulae for the optimal expected profit J^*, energy shortfall S_-^*, and energy surplus S_+^*. These three quantities can be graphically represented as areas bounded by the time-averaged CDF $F(w)$ as illustrated in Fig. 2 for $F(C^*) = \gamma = 0.5$:

$$\begin{aligned} J^* &= T \begin{cases} \mu_q A_1 - \mu_\lambda (A_3 + A_4), & \pi \in \mathcal{M}_1, \mathcal{M}_2 \\ \mu_q A_1 + p - \mu_q, & \pi \in \mathcal{M}_3 \end{cases} \\ S_-^* &= T\, A_2 \quad S_+^* = T\, A_3. \end{aligned}$$

From Fig. 2, it is apparent that a reduction of "statistical dispersion" in the time-averaged distribution $F(w)$ will generally result in an increase in area (A_1) and a decrease in areas $(A_2,\ A_3)$–all of which are favorable consequences that result in an increase in optimal expected profit and decrease in expected energy shortfall and surplus. This intuition will be made more precise in Section V where we derive an analytical expression for the marginal value of uncertainty reduction (i.e., improved forecasting). □

Remark IV.4 (Price Elasticity of Supply): Under certain assumptions, the quantile rule (11) in Theorem IV.1 can be interpreted as the *supply curve* for the WPP. Of primary importance is the assumption that the WPP is a price taker in the forward market, ensuring that it wields no influence over the market price p. For a fixed a pair of expected imbalance prices (μ_q, μ_λ), one can interpret the optimal quantile rule (11) as indicating the amount of energy that the WPP is willing to supply at a price p. Specifically, for imbalance prices $\mu_q + \mu_\lambda \geq 0$, the WPP's supply curve is given by

$$C(p) = \begin{cases} 0, & p < -\mu_\lambda \\ F^{-1}(\gamma), & \mu_\lambda \leq p \leq \mu_q \\ 1, & p > \mu_q \end{cases} \text{ where } \gamma = \frac{p + \mu_\lambda}{\mu_q + \mu_\lambda}.$$

With this explicit characterization of the WPP's supply curve, it is straightforward to see that the WPP is perfectly inelastic for prices $p \notin [-\mu_\lambda, \mu_q]$. Conversely, for prices $p \in [-\mu_\lambda, \mu_q]$, the price elasticity of supply, E_C, can be readily derived as

$$E_C := \frac{d \ln C(p)}{d \ln p} = \frac{\gamma}{F^{-1}(\gamma)} \frac{dF^{-1}(\gamma)}{d\gamma} = \frac{\gamma}{C f(C)}.$$

□

Remark IV.5 (Role of γ): Consider the set of expected imbalance prices $(\mu_q, \mu_\lambda) \in \mathcal{M}_2$. It follows from the quantile structure (11) that C^* is chosen to be the largest contract C such that the probability of *shorting* on said contract—with respect to the time averaged distribution $F(w)$—is less than or equal to $\gamma = (p + \mu_\lambda)/(\mu_q + \mu_\lambda)$. $1 - \gamma$ is interpreted as a confidence level.

Clearly then, the imbalance price ratio γ plays a critical role in implicitly controlling the probability of shortfall associated with optimal contracts $C^* = F^{-1}(\gamma)$. Consider the scenario in which the ISO has direct control over the shortfall imbalance price q. As the expected short price μ_q becomes more *harsh* (i.e., larger), the price ratio γ decreases—resulting in smaller offered contracts C^*. This follows from the fact that the quantile function $F^{-1}(\gamma)$ is non-decreasing in γ (non-increasing in μ_q). Consequently, the probability of shortfall $\Phi(C^*; t)$ with respect to the optimal contract C^* is non-increasing in μ_q. □

Remark IV.6 (Curtailment): The expected optimal shortfall S_-^* can be further interpreted as the expected amount of energy that must be supplied by the ISO to balance the shortfall in the WPP's contractual obligation. A straightforward corollary of Theorem IV.1 is that the expected optimal shortfall S_-^* and surplus S_+^* are monotonically non-decreasing and non-increasing in γ, respectively. This makes explicit the claim that some wind energy must be curtailed in order to reduce the amount of operational reserve capacity needed to hedge against uncertainty in the wind power. □

Remark IV.7 (Market Simplification): We have thus far considered all possible combinations of forward prices p and expected imbalance prices (μ_q, μ_λ), as depicted in Fig. 1. In the remainder of this paper, we assume that

A4: the WPP has *curtailment capability* and restrict our attention to the case $\lambda = 0$.

We of course realize that in some circumstances, surplus wind has economic value ($\lambda < 0$), while in cases of systemic overproduction, excess wind must be curtailed or has negative value ($\lambda > 0$). Nevertheless, in the remaining sections, we assume

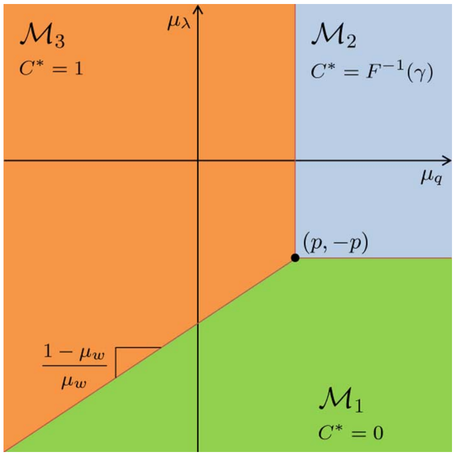


Fig. 1. Graphical illustration of the optimal offer policy C^* as a function of the expected imbalance prices (μ_q, μ_λ).

$\lambda = 0$. We make this choice to make our exposition more transparent, and to isolate our studies to one issue at a time. We remark that all of our results generalize to the case $\lambda \neq 0$ at the expense of clarity in exposition. Note that under our assumption $\lambda = 0$, the price penalty ratio simplifies to

$$\gamma = \frac{p}{\mu_q}$$

–a quantity that will play a central role in the interpretation of the results to follow. □

V. Role of Information

Intuitively, an increase in uncertainty in future wind power production will increase contract sensitivity to imbalance prices. Hence, it is of vital importance to understand the effect of information (such as available implicitly through forecasts) on expected optimal profit. More explicitly, consider a simple scenario in which the WPP observes a random variable Y that is statistically correlated to the wind process $w(t)$. The random variable Y can be interpreted as an observation of a meteorological variable relevant to the wind. As Y is observed prior to the contract offering, the WPP will naturally base its contract on the distribution of the wind $F(w|y)$ conditional on the observation $Y = y$, which is defined as

$$F(w|y) = \frac{1}{T}\int_{t_0}^{t_f} \Phi(w; t|y)\, dt \tag{18}$$

where $\Phi(w; t|y)$ is the CDF of $w(t)$ conditioned on the realization $Y = y$. It follows then from Theorem IV.1 that the optimal contract offering and corresponding expected profit are given by

$$C^*(y) = F^{-1}(\gamma|y), \quad \text{where } \gamma = \frac{p}{\mu_q}$$
$$J^*(y) = \mu_q T \int_0^\gamma F^{-1}(w|y) dw.$$

Using this construction, it is straightforward to show that information has economic value:

$$\mathbb{E}[J^*(Y)] \geq J^* \tag{19}$$

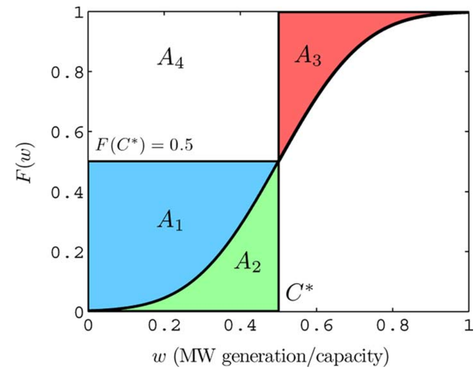


Fig. 2. Graphical interpretation of optimal profit J^* (proportional to A_1), deficit $S_-^*(A_2)$, and surplus $S_+^*(A_3)$ for $F(C^*) = \gamma = 0.5$.

where J^* denotes the unconditional expected profit. Clearly then, information helps in the metric of expected profit. Moreover, Fig. 2 offers some intuition as to how a reduction in "statistical dispersion" of the CDF F results in increased expected optimal profit. In the limit as the time-averaged distribution $F(w)$ approaches the Heaviside function $H(w - \overline{w})$ (i.e., the associated random variable takes on the realization $\overline{w}$ with probability one, which is equivalent to no uncertainty), we have that the optimal expected profit goes to $J^* = \mu_q T(\gamma\overline{w}) = pT\overline{w}$.

A. *Quantifying the Effect of Uncertainty*

It is of interest to more generally quantify the marginal improvement of expected optimal profit with respect to information increase in various metrics of dispersion. In practice, there are numerous *deviation measures* of dispersion of probability distributions (e.g., standard deviation, mean absolute deviation). In [34], the authors take an axiomatic approach to construct a class of *deviation measures* for which there is a one-to-one correspondence with a well-known class of functionals known as *expectation-bounded risk measures*. We refer the reader to [2] and [34] for a detailed exposition. For our purposes, it suffices to realize that

$$\mathcal{D}_\gamma(X) = \mathbb{E}[X] - \mathbb{E}\left[X \mid X \leq F^{-1}(\gamma)\right] \tag{20}$$
$$= \mathbb{E}[X] - \frac{1}{\gamma}\int_0^\gamma F^{-1}(x)dx \tag{21}$$

is a *valid* deviation measure [34] for all square-integrable random variables X with CDF $F(x)$ and $\gamma \in (0,1)$. It is sometimes referred to as the *conditional value-at-risk* (CVaR) deviation measure. It measures the distance between the unconditional mean and the mean in the γ probability tail of the distribution.

The particular choice of the CVaR deviation measure $\mathcal{D}_\gamma$ is special in that it permits the analytical characterization of the marginal improvement of optimal expected profit J^* with respect to the wind uncertainty, as measured by $\mathcal{D}_\gamma$. Simple algebraic manipulation of the formula for optimal expected profit (12) reveals J^* to be an affine function in $\mathcal{D}_\gamma(W)$, where W is distributed according to the time averaged distribution $F(w)$:

$$J^* = \underbrace{pT\,\mathbb{E}[W]}_{\text{expected revenue}} - \underbrace{pT\,\mathcal{D}_\gamma(W)}_{\substack{\text{loss due to uncertainty}\\ \text{and deviation penalty}}}. \tag{22}$$

This result quantifies the increase in expected profit that results from a reduction in $\mathcal{D}_\gamma(W)$ using sensors and forecasts. Further, it makes explicit the joint sensitivity of optimal expected profit J^* to uncertainty and prices. Essentially, the loss term $pT\,\mathcal{D}_\gamma(W)$ can be interpreted as the *price of uncertainty*.

Remark V.1 (Role of γ): As we discovered earlier, the price-penalty ratio, $\gamma = p/\mu_q$, plays a central role in controlling the shortfall probability associated with optimal contracts $C^* = F^{-1}(\gamma)$. In a related capacity, the price-penalty ratio γ also acts to discount the impact of uncertainty in the underlying wind process, $w(t)$, on optimal expected profit J^*. This assertion is made rigorous by the fact that $\mathcal{D}_\gamma(W)$ is monotone non-increasing in γ. Its limiting values are given by

$$\lim_{\gamma \longrightarrow 0} \mathcal{D}_\gamma(W) = \mathbb{E}[W]$$
$$\lim_{\gamma \longrightarrow 1} \mathcal{D}_\gamma(W) = 0.$$

Hence, as the expected short price μ_q approaches the forward price p from above, we have that $\gamma \longrightarrow 1$, which attenuates the sensitivity of expected profit to uncertainty in the underlying wind process, as measured by $\mathcal{D}_\gamma(W)$. □

1) Example (Uniform Distribution): It is informative to consider the case in which $F(w)$ is taken be a uniform distribution having support on a subset of $[0,1]$. Under this assumption, it is straightforward to compute the optimal expected profit as

$$J^* = pT\left(\mathbb{E}[W] - \sigma\sqrt{3}(1-\gamma)\right)$$

where σ is the *standard deviation* of W–the most commonly used measure of statistical dispersion. Note that the expected profit is affine in the standard deviation σ. The marginal expected profit with respect to wind uncertainty, as measured by σ, is

$$\frac{dJ^*}{d\sigma} = -pT\sqrt{3}(1-\gamma).$$

A direct consequence is that the expected profit's sensitivity to uncertainty, σ, increases as the expected short price μ_q becomes more harsh—or equivalently, as $\gamma \longrightarrow 0$.

VI. Role of Reserve Margins and Local Generation

In order to maintain reliable operation of the electric grid, the ISO is responsible for procuring *ancillary services* (AS) to balance potential deviations between generation and load. The various underlying phenomena responsible for these deviations result in system imbalances with varying degrees of uncertainty on differing time scales. In order to absorb this variability on the different time scales, multiple ancillary services must be procured. Broadly, these services consist of *regulation, load-following, reserve (spinning and non-spinning), voltage control, and reactive power compensation*.

Based on the scheduled energy, the ISO first determines the total reserve requirement for the entire control area needed to satisfy pre-specified reliability criteria. The ISO then assigns to each participating LSE a share of the total reserve requirement based roughly on its scheduled demand, because of the uncertainty in load [23]. Each LSE has the option to procure all or a portion of its reserve requirement through bilateral contracts or forward markets. The remaining portion of the reserve requirement not provided by the LSE is procured by the ISO through ancillary service markets. A detailed exposition on ancillary services can be found in [26].

Wind power is inherently difficult to forecast. Moreover, it exhibits variability on multiple time scales ranging from single-minute to hourly. It follows then that regulation, load-following, and reserve services will be necessary to compensate imbalances resulting from fluctuations in wind [20]. Several wind integration studies have computed detailed estimates of the increase in additional reserves of various types needed to compensate the added variability due to wind [17]. To simplify our analysis, we will lump all of these ancillary services into a single service that we refer to as “reserve margin”. Under the current *low* capacity penetration levels of wind power ($\sim 1\%$), the added variability of wind is largely absorbed by existing reserve margins used to cover fluctuations in the load. As the capacity penetration of wind increases, its affect on operating reserve margins will become more pronounced [20], [21]. Moreover, it will become economically infeasible to continue the socialization of the added reserve costs, stemming from wind variability, among participating LSEs. Hence, it is likely that the WPP will have to bear the added cost of reserve margins [18].

This transfer of financial burden to the WPP has already begun to emerge in the Pacific Northwest. The Bonneville Power Administration (BPA) in cooperation with Iberdrola Renewables has deployed a pilot program in which the WPP is responsible for *self-supplying* its own balancing services—from owned and/or contracted dispatchable generation capacity—to satisfy certain reliability criteria (on imbalances) imposed by the BPA [41].

Motivated by this change, consider now the scenario in which the WPP can procure its reserve margin from a small fast-acting power generator co-located with its wind farm. In addition to assumptions A1-A3 in Section III-B, we also assume the following.

A5: The co-located generator has fixed power capacity L (MW) and fixed and known operational cost $q_L > 0$ (\$/MWh);

A6 The co-located generator operational cost is greater than the forward price (i.e., $q_L \geq p$), to avoid trivial solutions;

A7 *Ex-ante*, at the time of the contract offering, the *sign of the random shortfall imbalance price* q (relative to q_L) is revealed to the WPP. The corresponding mean of the shortfall price q conditioned on either event is denoted by

$$\mu_q^+ = \mathbb{E}\left[q \mid q > q_L\right]$$
$$\mu_q^- = \mathbb{E}\left[q \mid q \leq q_L\right].$$

In the event that $\{q \leq q_L\}$, the WPP derives no financial benefit from the co-located generator, and we revert back to our original market setting without local generation support—as outlined in Section IV. It follows from Theorem IV.1 that an optimal contract offering conditioned on the information $\{q \leq q_L\}$ is given by

$$C^* = \begin{cases} F^{-1}\left(\frac{p}{\mu_q^-}\right), & \mu_q^- > p \\ 1, & \mu_q^- \leq p. \end{cases}$$

In the complementary event that $\{q > q_L\}$, it follows that the co-located generator can be used to mitigate shortfall risk by covering contract shortfalls $[C - w(t)]^+$ up to a limit L at a cost q_L. For incremental shortfalls larger than L, the WPP pays at the shorfall imbalance price q. This *augmented penalty mechanism* corresponding to the event $\{q > q_L\}$ is captured by the following penalty function $\phi : \mathbb{R} \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$:

$$\phi(x, L) = \begin{cases} qx - (q - q_L)L & x \in (L, \infty) \\ q_L x & x \in [0, L] \\ 0 & x \in (-\infty, 0). \end{cases} \tag{23}$$

It follows that the fiscal cost and benefit of local generation capacity to the WPP can be explicitly accounted for in the following expected profit criterion:

$$J_L(C) = \mathbb{E} \int_{t_0}^{t_f} pC - \phi\left(C - w(t),\ L\right) dt \tag{24}$$

where expectation is taken with respect to $(\mathbf{w}, q)$ conditioned on the event $\{q > q_L\}$. As before, we define a profit maximizing contract C_L^* as

$$C_L^* = \arg\max_{C \geq 0} J_L(C). \tag{25}$$

Because of the *significant capital costs* associated with the investment in local generation, it is important to quantify the marginal improvement in profit resulting from the investment in a generator with *small* power capacity L. The following theorem distills this notion.

Theorem VI.1: Define the time-averaged distribution $F(w)$ as in (2).

a) An optimal contract C_L^* is given by any solution C of

$$p = q_L F(C) + \left(\mu_q^+ - q_L\right) F(C - L).$$

b) The marginal expected optimal profit with respect to power capacity L is given by

$$\left.\frac{dJ_L^*}{dL}\right|_{L=0} = \left(1 - \frac{q_L}{\mu_q^+}\right) pT.$$

Proof: The proof for part a) follows from direct application of the proof technique for Theorem IV.1-a).

Part b) is proven as follows. It is straightforward to show that the expected profit is given by

$$\frac{J_L(C)}{T} = pC - \int_0^{C-L} \left[\mu_q^+(C - w) - (\mu_q^+ - q_L)L\right] f(w)dw - \int_{C-L}^{C} q_L(C - w)f(w)dw$$

for any choice of C and L. Taking the derivative with respect to L yields

$$\frac{dJ_L(C)}{dL} = \left(\mu_q^+ - q_L\right) F(C - L)T.$$

Taking the limit as L goes to zero and substituting for the first order optimality condition a) yields the desired result. ■

Remark VI.2: According to the optimality condition in Theorem VI.1-a), as the expected shortfall price $\mu_q^+ \rightarrow \infty$, we have that the optimal $C_L^* \rightarrow 0$, as is consistent with intuition. Moreover, as $\mu_q^+ \rightarrow q_L$, we have that $C_L^* \rightarrow F^{-1}(p/q_L)$–recovering the optimal policy in Theorem IV.1. □

Remark VI.3 (Capacity Reservation): Note that this framework is easily extendable to model the setting in which the WPP does not physically posses a co-located thermal generator, but rather can purchase *ex-ante*, reserve generation of any power capacity L at a capacity price q_c (\$/MW). If the reserve capacity is called on, the WPP must pay at the energy cost q_e (\$/MWh), which is analogous to the operational cost q_L in the co-located generation setting. □

VII. Markets With Recourse

Until now, we have operated under the assumption that the WPP has no market recourse—i.e, the WPP has no opportunity to use improved forecasts to modify its initial DA offer. We now relax this assumption by augmenting our market model to include an *intra-day* market permitting contract recourse. More specifically, the market system consists of two sequential *ex-ante* markets in which the WPP can incrementally offer a contract $C = C_1 + C_2$ to be delivered on some future time interval $[t_0, t_f]$. The WPP initially offers a contract C_1 at a price $p_1 \geq 0$ in the DA market (stage-1). In a successive intra-day market (stage-2), the WPP observes a random variable Y (e.g., weather conditions, wind speed, and direction) that is statistically correlated to the wind power process $\mathbf{w}$. Using this information Y, the WPP has the option to make an additional offer C_2 at a price p_2 in the intra-day market. Note that the recourse contract is taken to be a function of Y–i.e., $C_2 = C_2(Y)$. We maintain the price taking assumption, as before. Moreover, to avoid trivial solutions, we assume that $p_1 \geq p_2 \geq 0$. If it were the case that $p_2 > p_1$, there would be no incentive for the WPP to make an offer in the DA market.

Ex-post, the WPP is penalized at a price q for shortfalls with respect to the cumulative offered contract $C = C_1 + C_2$. As before, the shortfall price q is assumed unknown *ex-ante* and is modeled as random variable that is statistically uncorrelated to the wind, with mean $\mathbb{E}[q] = \mu_q$. We additionally assume that $\lambda = 0$ to isolate the effect of an additional intra-day market on DA contract offerings. Our objective is to find explicit characterizations of contracts C_1^* and $C_2^*(Y)$ that optimize the expected profit criterion:

$$J(C) = \mathbb{E}\int_{t_0}^{t_f} p_1 C_1 + p_2 C_2(Y) - q\left[C_1 + C_2(Y) - w(t)\right]^+ dt \tag{26}$$

where expectation is taken with respect to $(\mathbf{w}, Y, q)$. For brevity and clarity in exposition, we focus our analysis on the important case of $\mu_q \geq p_1$. Optimal contract offerings under the alternative assumption are easily derived. Related work can be found in [35].

Theorem VII.1: Let $\gamma_1 = p_1/\mu_q$ and $\gamma_2 = p_2/\mu_q$. Define the random variable W to be distributed according to the time-averaged distribution $F(w)$ as in (2). Correspondingly, define the conditional distribution $F(w|y) = \mathbb{P}\{W \leq w \mid Y = y\}$,

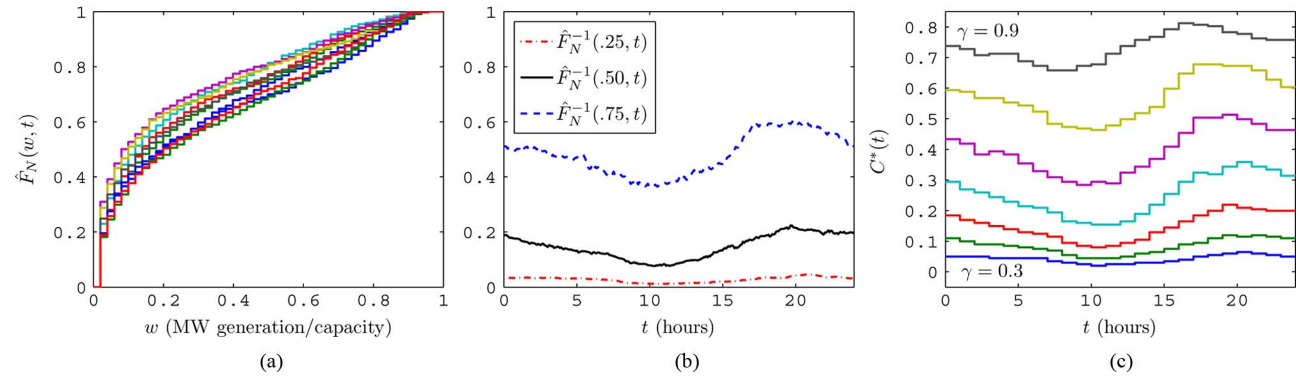


Fig. 3. (a) Empirical CDFs $\hat{\Phi}_N(w;\tau)$ for nine equally spaced times throughout the day. (b) Trajectory of the empirical median $\hat{\Phi}_N^{-1}(.5,t)$ and its corresponding interquartile range $[\hat{\Phi}_N^{-1}(.25;t), \hat{\Phi}_N^{-1}(.75;t)]$. (c) Optimal contracts offered in the DA market for various values of $\gamma = 0.3, 0.4, \ldots, 0.9$.

from which we can define the derived random variable Z given by its γ_2 quantile

$$Z = F^{-1}(\gamma_2 | Y).$$

The portfolio of profit maximizing contracts $\{C_1^*, C_2^*(Y)\}$ is given by the following. The stage-1 optimal contract C_1^* is given by a solution to

$$\gamma_1 - \gamma_2 \mathbb{P}\{Z \geq C_1^*\} - \mathbb{P}\{Z \leq C_1^*, W \leq C_1^*\} = 0. \quad (27)$$

The stage-2 optimal contract $C_2^*(Y)$ is given by the threshold

$$C_2^*(Y) = [Z - C_1^*]^+. \quad (28)$$

Proof: Proof is analogous to that in Theorem IV.1. ■

VIII. Empirical Studies

Using a wind power time series data set provided by the BPA, we are in a position to illustrate the utility and impact of the theory developed in this paper.

A. Data Description

The data set consists of a time series of measured wind power aggregated over the 14 wind power generation sites in the BPA control area [4]. The wind power is sampled every 5 min and covers the 2008 and 2009 calendar years. To account for additional wind power capacity coming online at various points in time over the 2-year horizon, all of the data are normalized by the aggregate nameplate wind power capacity as a function of time.

B. Empirical Probability Model

As stated earlier, wind power is modeled as a continuous time stochastic process whose marginal cumulative distribution is denoted by $\Phi(w;t)$. While the identification of stochastic models that accurately capture the non-stationarity and statistical variability in wind power is of critical importance, this is not the focus of our paper. We will make some simplifying assumptions on the underlying physical wind process to facilitate our analysis.

A8: The wind process $w(t)$ is assumed to be *first-order cyclostationary in the strict sense* with period $T_0 = 24$ hours—i.e., $\Phi(w;t) = \Phi(w;t+T_0)$ for all t [19], [39]. Note that we are ignoring the effect of seasonal variability.

A9: For a fixed time τ, the discrete time stochastic process $\{w(\tau + nT_0) \mid n \in \mathbb{N}\}$ is *independent across days* indexed by (n).

Fix a time $\tau \in [0, T_0]$ and consider a finite length sample realization of the discrete time process $z_\tau(n) := w(\tau + nT_0)$ for $n = 1, \ldots, N$. Using this data set, we take the empirical distribution $\hat{\Phi}_N(w;\tau)$ as an approximation of the underlying distribution $\Phi(w;\tau)$:

$$\hat{\Phi}_N(w;\tau) = \frac{1}{N}\sum_{n=1}^{N} \mathbf{1}\{z_\tau(n) \leq w\}. \quad (29)$$

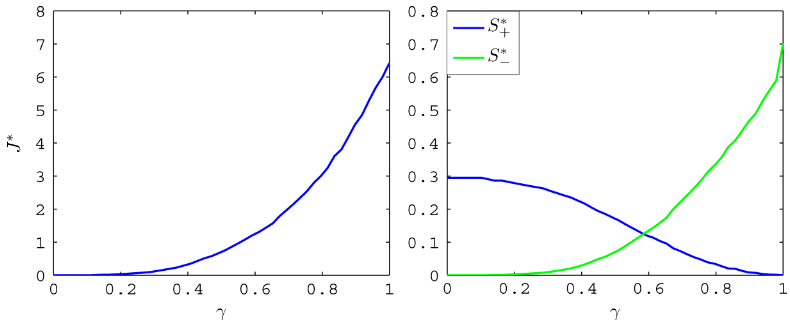
Invoking the strong law of large numbers under the working assumptions A8-A9, it can be shown that the $\hat{\Phi}_N(w;\tau)$ is consistent with respect to $\Phi(w;\tau)$[5]. Fig. 3(a) depicts nine representative marginal empirical distributions identified from the BPA data set described earlier. Note that the times corresponding to the nine distributions are equally spaced throughout the day to provide a representative sample. Fig. 3(b) depicts the trajectory of the empirical median $\hat{\Phi}_N^{-1}(0.5;t)$ and its corresponding interquartile range $[\hat{\Phi}_N^{-1}(0.25;t), \hat{\Phi}_N^{-1}(0.75;t)]$.

C. Optimal Contracts in Two-Settlement Markets

Using the empirical wind power distributions identified from the BPA wind power data set, we are now in a position to compute and appraise optimal DA contracts offered by a representative Oregon WPP participating in the idealized market system described in Sections III and IV. For the set of expected imbalance prices $(\mu_q, \mu_\lambda) \in \mathcal{M}_2$, we are also able to examine the effect of the *price-penalty ratio* $\gamma = (p+\mu_\lambda)/(\mu_q+\mu_\lambda)$ on J^*, S_-^*, and S_+^*. The following empirical studies assume a contract structure $\{[t_{i-1}, t_i), C_i\}_{i=1}^{24}$, where $[t_{i-1}, t_i)$ is of length 1 h for all i.

Remark VIII.1 (Optimal DA Contracts): Fig. 3(c) depicts optimal contracts $(C_1^*, \ldots C_{24}^*)$ for various ratios $\gamma = 0.3, 0.4, \ldots, 0.9$. As expected, as the price-penalty ratio γ decreases, the optimal contract C^* decreases. From Fig. 3(c), it is evident that WPPs will tend to offer larger contracts during morning/night periods when wind speed is typically higher than during mid-day [as indicated by Fig. 3(b)]. □

Remark VIII.2 (Profit, Shortfall, and Surplus): Fig. 4(a) and (b) demonstrates the effect of the price-penalty ratio γ on the



(a) (b)

Fig. 4. (a) Optimal expected profit J^* as a function of γ. (b) Optimal expected energy shortfall and surplus for the 12th hour interval, as a function of γ.

optimal expected profit, energy shortfall, and energy surplus. The units of S_-^* and S_+^* are (MWh)/(nameplate capacity). In computing the optimal expected profit, we assumed the WPP to have curtailment capability, which is equivalent to $\mu_\lambda = 0$. The units of J^* are in \$/($\mu_q\cdot$ nameplate capacity). When $\mu_q = p$, we have that $\gamma = 1$ and the WPP sells all of its energy production at price $p = \mu_q$. This is equivalent to the current policy of system-take-all-wind. In this situation, the expected profit per hour (see Fig. 4 at $\gamma = 1$) of approximately $6.4/24$ equals the ratio of average production to nameplate capacity. This number is consistent with typical values for wind production *capacity factor* ($\approx 25\%$). The energy surplus S_+^* and shortfall S_-^* are relatively insensitive to variations in γ (for $\gamma \in [0, 0.1]$), because the marginal empirical distributions are steep there. □

IX. Conclusion

In this paper, we have formulated and solved a variety of problems on optimal contract sizing for a wind power producer offering power in a two-settlement electricity market. Our results have the merit of providing key insights into the trade-offs between a variety of factors such as expected imbalance penalties, cost of local generation, value of information, etc. In our current and future work, we will investigate a number of intimately connected research directions: improved forecasting of wind power, development of probabilistic reliability criteria for required reserve margins, dynamic optimization of reserve capacity procurement, improved dispatchability of wind power, network aspects of renewable energy aggregation and profit sharing, and the development of novel market systems that price-differentiate *quality of supply* to facilitate the integration of renewable sources. We are also studying the important case of markets with recourse where the producer has opportunities to adjust bids in successive intra-day markets. We are also developing large-scale computational simulations which can be used to test the behavior of simplified analytically tractable models and suggest new avenues for research applicable to real-world grid-scale problems.

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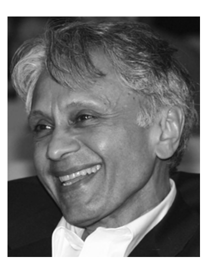

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