

**Solutions  
of  
Optics &  
Modern Physics**

**Lesson 26<sup>th</sup> to 30<sup>th</sup>**

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# 26. Reflection of Light

## Introductory Exercise 26.1

1. Since  $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$  where  $c$  is the speed of light in vacuum hence unit of  $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$  is m/s.

2. Hence

$$B_y = 2 \times 10^{-7} \text{ T} \sin [500x - 1.5 \times 10^{11}t]$$

Comparing this equation with the standard wave equation  $B_y = B_0 \sin [kx - \omega t]$

$$k = 500 \text{ m}^{-1} \quad \omega = 1.5 \times 10^{11} \text{ rad/s}$$

$$\lambda = \frac{2\pi}{k}$$

$$\lambda = \frac{2\pi}{500} \text{ m} = \frac{\pi}{250} \text{ metre}$$

$$\omega = 1.5 \times 10^{11} \text{ rad/s}$$

$$2\pi n = \omega \quad n = \frac{\omega}{2\pi}$$

$$n = \frac{1.5}{2\pi} \times 10^{11} \text{ Hz}$$

$$\text{Speed of the wave } v = \frac{\omega}{k} = \frac{1.5 \times 10^{11}}{500} = 3 \times 10^8 \text{ m/s}$$

Let  $E_0$  be the amplitude of electric field.

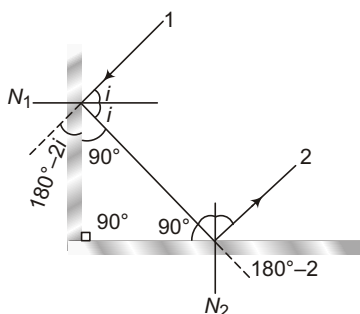
$$\text{Then } E_0 = cB_0 = 3 \times 10^8 \times 2 \times 10^{-7} = 60 \text{ V/m}$$

Since wave is propagating along  $x$ -axis and  $B$  along  $y$ -axis, hence  $E$  must be along  $z$ -axis

$$E = 60 \text{ V/m} \sin [500x - 1.5 \times 10^{11}t]$$

## Introductory Exercise 26.2

1. Total deviation produced



$$180 - 2i$$

$$360 - 2(i + i)$$

From figure

$$90 - i$$

$$360 - 2[i + (90 - i)]$$

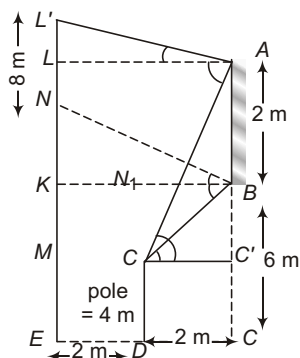
$$180$$

Hence rays 1 and 2 are parallel (anti-parallel).

2.  $v_0 = 2 \text{ m/s}$  for plane mirror  $v_i = 2 \text{ m/s}$ .

$$\text{Velocity of approach } v_0 + v_i = 4 \text{ m/s.}$$

3. In figure,  $AB$  is mirror,  $G$  is ground,  $CD$  is pole and  $M$  is the man. The minimum height to see the image of top of pole is  $EN$



$EK \quad KN \quad 6 \quad KN$

Now in  $NKB$ ,

$$\frac{NK}{KB} \tan 4 \tan NK KB \tan$$

In  $BC\ C$  we get,

$$\tan \frac{BC}{CC} = \frac{2}{2} = 1$$

45

So,  $NK = 4 \tan 45 = 4 \text{ m}$

Hence in minimum height

6 m    4 m    10 m

In  $AC \subset C$

$$\tan \frac{4}{2} = 2$$

In  $L\ LA$  we get,

$$\frac{LL}{LA} \tan$$

$$\frac{LL}{4} \quad 2$$

$LL \quad 8 \text{ m}$

Maximum height    *CA*    *LL*    8    8    16 m

## Introductory Exercise 26.3

1. Here  $f = 10$  cm (concave mirror)

(a)  $u$  25 cm

Using mirror formula,

$$\begin{array}{ccccccc} \frac{1}{v} & \frac{1}{u} & \frac{1}{f} & & & & \\ \frac{1}{v} & \frac{1}{f} & \frac{1}{u} & \frac{1}{10} & \frac{1}{25} & & \\ \frac{1}{v} & \frac{5}{50} & \frac{2}{50} & & & & \\ v & \frac{50}{3} & & 16.7 \text{ cm} & & & \end{array}$$

Hence image is real, inverted and less height of the object.

(b) Since  $u = 10 \text{ cm}$ ,

Hence object is situated on focus of the image formed at .

(c)  $u = 5, f = 10$

$$\frac{1}{v} \quad \frac{1}{f} \quad \frac{1}{u} \quad \frac{1}{10} \quad \frac{1}{5}$$

$$\frac{1}{v} = \frac{1}{10} + \frac{2}{10}$$

$v = 10 \text{ cm}$

Hence, image is virtual, erect and two time of the object.

**2.** Here  $u = 3 \text{ m}$ ,  $f = \frac{1}{2} \text{ m}$ ,

we have,

$$(a) \frac{1}{v} \quad \frac{1}{f} \quad \frac{1}{u}$$

$$\frac{1}{v} \quad 2 \quad \frac{1}{3}$$

$$v = 0.6 \text{ m}$$

As ball moves towards focus the image moves towards  $\infty$  and image is real as the distance decreases by focal length image become virtual which moves from  $\infty$  to zero.

(b) The image of the ball coincide with ball, when  $u = R = 1 \text{ m}$

Using  $h = ut + \frac{1}{2}gt^2$

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 2}{9.8}}$$

$$0.639 \text{ s}$$

Similarly again images match at  $t = 0.78 \text{ s}$ .

3. Since image is magnified, hence the mirror is concave.

$$\text{Here, } m = \frac{v}{u} = \frac{v}{5} = 5 \quad \dots(i)$$

Let distance between mirror and object is  $x$ .

Since image is formed at a distance 5 m from mirror

$$v = (5 - x) \quad \dots(ii)$$

From Eqs.(i) and (ii), we get

$$(5 - x) = 5x$$

$$4x = 5$$

$$x = 1.25$$

Hence mirror is placed at 1.25 m on right side of the object by mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

we have

$$\frac{1}{f} = \frac{1}{6.25} + \frac{1}{1.25}$$

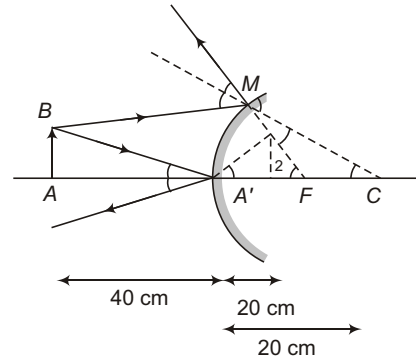
$$f = \frac{6.25}{6},$$

$$\text{Hence } R = 2f = 2 \times \frac{6.25}{6} = 2.08 \text{ m}$$

Thus mirror is concave mirror of radius of curvature 2.08 m.

4. Since the incident rays and reflected rays are parallel to each other therefore mirror is plane mirror.

5. Let us solve the first case :



By applying the geometry we can prove that,

$$PA = v = \frac{40}{3} \text{ cm}$$

Further, in triangles  $ABP$  and  $PA'B$  we have,

$$\frac{AB}{40} = \frac{A'B}{(40/3)}$$

$$A'B = \frac{AB}{3} = \frac{2}{3} \text{ cm}$$

Similarly, we can solve other parts also.

6. Simply apply :

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

and  $m = \frac{I}{O} = \frac{v}{u}$  for lateral magnification. If magnification is positive, image will be virtual. If magnification is negative, image will be real.

## AIEEE Corner

### ■ Subjective Questions (Level 1)

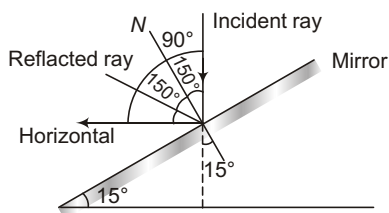
1. Here  $v = 39.2$  cm, hence  $v = 39.2$  cm

and magnification  $m = 1$

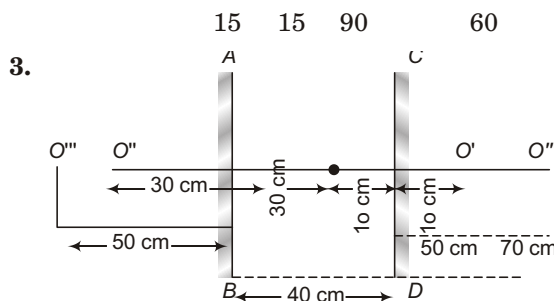
$$h_i = h_o = 4.85$$

Hence image is formed at 39.2 cm behind the mirror and height of image is 4.85 cm.

2. From figure, angle of incident  $= 15^\circ$

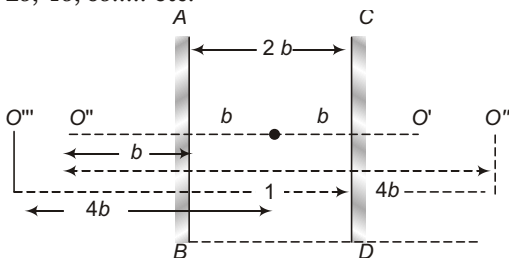


Let reflected ray makes an angle  $\theta$  with the horizontal, then



Since mirrors are parallel to each other, images are formed at distances of five closest to the object are 20 cm, 60 cm, 80 cm, 100 cm and 140 cm.

4. The distance of the object from images are  $2b, 4b, 6b, \dots$  etc.



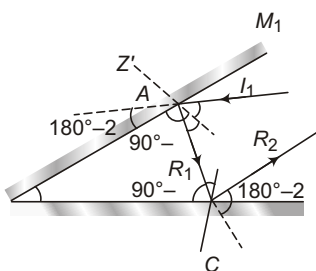
$$\frac{x}{20} \tan 30 = x = 20 \tan 30$$

$$\text{No. of reflection} = \frac{AB}{x} = \frac{160 \text{ cm}}{20 \text{ cm} \tan 30}$$

$$8\sqrt{3} \approx 14$$

Hence the reflected ray reach other end after 14 reflections.

7. The deviation produced by mirror  $M_1$  is  $180^\circ - 2\theta$



and the deviation produced by mirror  $M_2$  is  $180^\circ - 2\theta$

Hence total deviation

$$180^\circ - 2\theta + 180^\circ - 2\theta = 360^\circ - 4\theta$$

In  $\triangle ABC$  we get,

$$90^\circ + 90^\circ + 180^\circ - 4\theta = 180^\circ$$

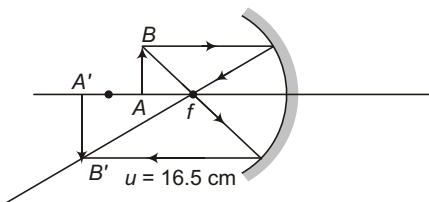
Hence deviation produces  $180^\circ - 2\theta$ .

8. Here  $f = \frac{R}{2} = \frac{22}{2} = 11 \text{ cm}$

Object height  $h_0 = 6 \text{ mm}$

$$u = 16.5 \text{ cm}$$

(a) The ray diagram is shown in figure



Using mirror formula,  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} + \frac{1}{16.5} = \frac{1}{11}$$

$$\frac{1}{v} = \frac{1}{11} - \frac{1}{16.5} = \frac{16.5 - 11}{11 \times 16.5} = \frac{5.5}{181.5}$$

$$v = \frac{181.5}{5.5} = 33 \text{ cm}$$

Hence the image is formed at 33 cm from the pole (vertex) of mirror on the object side the image is real, inverted and magnified. The absolute magnification

$$|m| = \frac{v}{u} = \frac{33}{16.5} = 2$$

Hence size of image is  $h_i = 2 \times h_0$

$$2 \times 6 = 12 \text{ mm.}$$

9. Here  $u = 12 \text{ cm}$ ,  $f = \frac{R}{2} = 10 \text{ cm}$

Using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we get

$$\frac{1}{v} + \frac{1}{12} = \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{12} = \frac{12 - 10}{120} = \frac{2}{120} = \frac{1}{60}$$

$$v = \frac{60}{1} = 60 \text{ cm}$$

The image is formed on right side of the vertex at a distance  $\frac{60}{11} \text{ cm}$ . the image is

virtual and erect the absolute magnification is given by  $|m| = \frac{v}{u}$

$$|m| = \frac{60}{11 \times \left(\frac{12}{11}\right)} = \frac{5}{11}$$

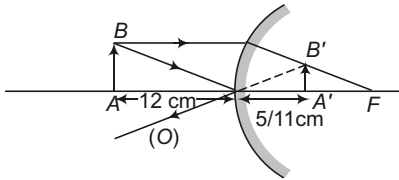
$\therefore m = 1$

Hence image is de-magnified.

Height of image  $h_i = |m| \times h_0$

$$h_i \quad \frac{5}{11} \quad 9 \quad \frac{45}{11} \quad 4.09 \text{ mm}$$

The ray diagram is shown in figure



10. Here  $f = 18 \text{ cm}$

Let distance of object from vertex of concave mirror is  $u$ . Since image is real hence image and object lie left side of the vertex.

$$\text{Magnification } m = \frac{v}{u} = \frac{1}{9}$$

$$v = \frac{u}{9}$$

By mirror formula,  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ , we have

$$\frac{1}{u/9} + \frac{1}{u} = \frac{1}{18} \quad \frac{10}{u} = \frac{1}{18}$$

$$u = 180 \text{ cm (left side of the vertex).}$$

11. Here  $u = 30 \text{ cm}$ , since image is inverted.

Hence the mirror is concave.

$$m = \frac{1}{2} = \frac{v}{u} \quad v = \frac{u}{2}$$

Using mirror formula,  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ , we get

$$\frac{2}{u} + \frac{1}{u} = \frac{1}{f} \quad \frac{3}{u} = \frac{1}{f} \quad f = \frac{u}{3} = \frac{30}{3} = 10 \text{ cm}$$

Hence mirror is concave of focal length 10 cm.

12. Here  $f = \frac{24}{2} \text{ cm} = 12 \text{ cm}$

(a) Since image is virtual

$$m = \frac{v}{u} \quad v = mu$$

$$v = 3u$$

$$v = 3u \text{ and } v \text{ is +ve}$$

By mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \frac{1}{3u} + \frac{1}{u} = \frac{1}{12} \quad \frac{1+3}{3u} = \frac{1}{12} \quad u = 8 \text{ cm}$$

(b) Since image is real

$$m = \frac{v}{u} = 3 \quad v = 3u$$

By using  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ , we get

$$\frac{1}{e} + \frac{1}{u} = \frac{1}{12} \quad \frac{4}{3u} = \frac{1}{12} \quad u = 16 \text{ cm}$$

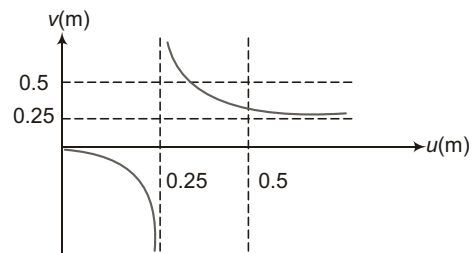
(c) Here  $m = \frac{v}{m} = \frac{1}{3} \quad v = \frac{u}{3}$

$$\frac{1}{u/3} + \frac{1}{u} = \frac{1}{12} \quad \frac{4}{u} + \frac{1}{u} = \frac{1}{12} \quad u = 48 \text{ cm}$$

13. We have  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$v = \frac{uf}{u-f} \text{ at } u = f, v$$

The variation is shown in figure



Hence focal length is asymptote of the curve.

When  $u < f$ , Image is virtual. It means  $v$  is negative.

When  $u = 2f$

$$v = 2f$$

$$u = 0, v = 0$$

14. Here  $f = 21 \text{ cm}$   $R = 2f = 42 \text{ cm}$

Since the object is placed on  $C$ . Hence its image by concave mirror is formed on  $C$ . This image acts as a virtual object for plane mirror the distance between plane mirror and virtual object  $= 21 \text{ cm}$ .

Hence plane mirror forms its real image in front of plane mirror at  $12 \text{ cm}$ .

15. Let  $u$  is the object distance from vertex,  $v$  is the image distance for vertex and  $f$  is the focal length then distance between object and focus is  $u - f$  and distance between image and focus is  $v - f$  i.e.,

$$(u - f)(v - f) = uv - (u + v)f + f^2 \quad \dots(i)$$

Using  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ , we get

$$uv - (u + v)f = f^2 \quad \dots(ii)$$

Putting the value of  $uv$  in RHS of Eq. (i), we get

$$(u - f)(u - f) - (v - u)f - (v - u)f = f^2$$

$$(u - f)(v - f) = f^2$$

Hence proved.

16. Let object is placed at a distance  $x$  from the convex mirror then for convex mirror

$$u = x \text{ and } f = \frac{R}{2}$$

Let  $v$  be the distance of the image from pole (vertex) of convex mirror.

Using  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ , we get

$$\frac{1}{v} - \frac{1}{x} = \frac{2}{R} \quad v = \frac{xR}{2x - R}$$

For concave mirror

$$u = 2R - \frac{xR}{2x - R} = \frac{2R^2 - 5xR}{2x - R}$$

$$v = (2R - x) \text{ and } f = \frac{R}{2}$$

Using  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ , we get

$$\frac{1}{(2R - x)} - \frac{(2x - R)}{(2R^2 - 5xR)} = \frac{2}{R}$$

$$4R^3 - 2x^2R - 8xR^2$$

$$8R^3 - 16xR^2 - 10x^2R$$

$$4R^3 - 8xR^2 - 8x^2R = 0$$

$$4R[R^2 - 2xR - 2x^2] = 0$$

$$2x^2 - 2xR - R^2 = 0$$

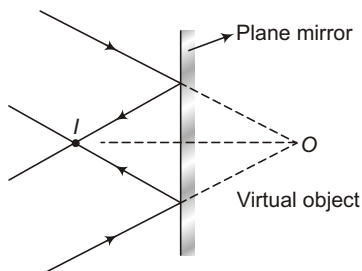
$$\therefore R = 0$$

$$x = \frac{2R \pm 2\sqrt{3}R}{4} = \frac{[1 \pm \sqrt{3}]}{2}R$$

$$x = \frac{1 \pm \sqrt{3}}{2} R$$

## ■ Objective Questions (Level 1)

1. When convergent beam incident on a plane mirror, then mirror forms real image



2. When an object lies at the focus of a concave mirror  $u = f$  focal length of a concave mirror is negative.

Using mirror formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

we get,

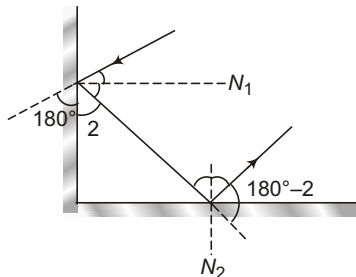
$$\frac{1}{v} - \frac{1}{f} = \frac{1}{f} \quad v =$$



also magnification  $m = \frac{v}{u}$ .

Hence, correct option is (c).

3. Total deviation,  $\theta_1 + \theta_2$



$$180^\circ - 2\theta_1 + 180^\circ - 2\theta_2$$

but  $\theta_1 + \theta_2 = 90^\circ$

$$180^\circ - 2\theta_1 + 180^\circ - 2(90^\circ - \theta_1)$$

$$180^\circ$$

Hence, option (a) is correct.

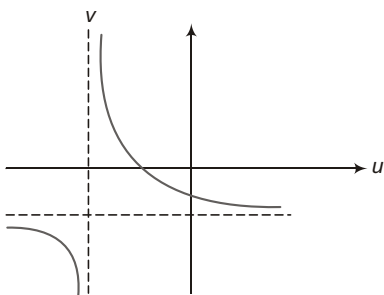
4. A concave mirror cannot form a virtual image of a virtual object.

Hence option (a) is correct.

5. For a concave mirror for normal sign convention if  $u < f$ ,  $v$  is

and  $v > u$ ,  $v$  is  $f$

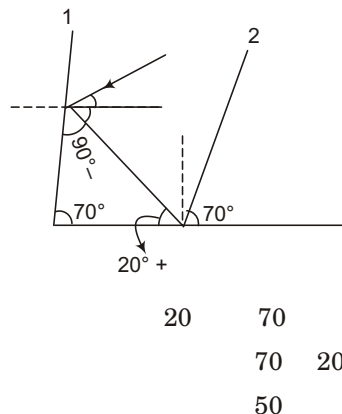
graph between  $u$  and  $v$  is



The dotted lines are the asymptotes (tangent at  $(0,0)$ ) of the curve.

Hence correct option is (b).

6. From figure



Here (1) and (2) are parallel to each other.

Hence the correct option is (a) 50.

7. The radius of curvature of convex mirror

$$R = 60 \text{ cm.}$$

Its focal length  $f = \frac{R}{2} = 30 \text{ cm}$

$$\text{Magnification } m = \frac{v}{u} = \frac{1}{2}$$

$$\text{Using mirror formula, } \frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

$$\text{we get, } \frac{1}{u/2} - \frac{1}{u} = \frac{1}{30}$$

$$\frac{3}{u} - \frac{1}{30}$$

$$u = 90 \text{ cm}$$

$$v = \frac{u}{2} = 45 \text{ cm}$$

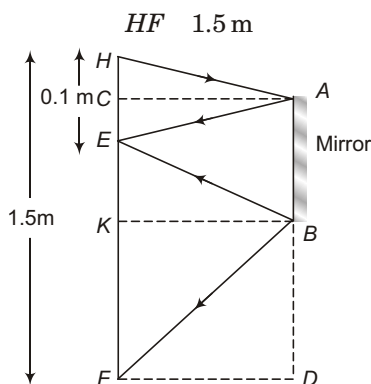
Hence distance between A and B is

$$90 - 45$$

$$45 \text{ cm}$$

Hence the correct option is (c).

8. Here it is given that height of the boy



Length of mirror  $AB = 0.75 \text{ m}$

The ray diagram is shown in above figure.

$H$  is the Head of the boy and  $F$  is the feet. It also shows the paths of the rays that leaves the head of the man enter his eyes ( $E$ ). After reflection from the mirror at point  $A$ , and the rays that leave his feet and enter his eyes after reflected at point  $B$ .

From figure  $CE = \frac{1}{2}HE = 0.05 \text{ m}$

$$\begin{array}{ccccccc} CF & HF & HC & HF & CE \\ 1.50 & 0.05 & 1.45 \text{ m} \end{array}$$

The distance of the bottom edge of mirror above the floor is

$$\begin{array}{ccccccc} BP & KF & CF & KC & CD & AB \\ 1.45 & 0.75 & 0.7 \text{ m} \end{array}$$

But according to question  $BD = 0.8 \text{ m}$  (given) which is greater than  $0.7 \text{ m}$ , the height required to see full image. Hence the boy cannot see his feet.

Option (c) is correct

9. Since the image is magnified hence mirror is concave mirror.

$$\text{Here } m = \frac{v}{u} = 3 \quad v = 3u$$

$$|v| = |3u| = 3u$$

$$\text{but } |v| = u = 80$$

$$|3u| = u = 80 \quad u = 40 \text{ cm}$$

Using mirror formula, we get

$$\begin{array}{ccc} \frac{1}{v} & \frac{1}{u} & \frac{1}{f} \\ \frac{1}{3u} & \frac{1}{u} & \frac{1}{f} \\ f & \frac{3u}{4} & \\ f & \frac{3 \cdot 40}{2} & 30 \text{ cm} \end{array}$$

Mirror is concave and focal length is  $30 \text{ cm}$ .

Correct option is (a).

$$\begin{array}{ccc} 10. \text{ Here } m & \frac{1}{n} & \frac{v}{u} \\ & v & \frac{u}{n} \end{array}$$

From mirror formula  $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ ,

we get,

$$\begin{array}{ccc} \frac{1}{f} & \frac{1}{(u/n)} & \frac{1}{u} \\ u & (n-1)f \end{array}$$

Hence the correct option is (d).

11. Differentiating mirror formula, we get

$$\frac{dv}{dt} = \frac{v^2}{u^2} \frac{du}{dt} \quad [\because \text{here } \frac{du}{dt} \text{ is } -ve]$$

Using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\text{we get } \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\text{Here } u = 60 \text{ cm}, f = 24 \text{ cm}$$

$$\text{Putting these we get, } v = 40 \text{ cm}$$

$$\text{Hence, } \frac{dv}{dt} = \frac{40^2}{60^2} \cdot 9 = 4 \text{ cm/s}$$

Hence the speed of the image is  $4 \text{ cm/s}$  away from the mirror.

Hence correct option is (c).

12. The wrong statement is (d)

13. Let  $v_m$  is the speed of mirror,  $v_p$  is the speed of particle and  $v_o$  is the speed of the observer, then speed of the image measured by observer is given by

$$v_{op} = 2[v_m + v_p] + v_o$$

$$v_{op} = 2[10 + 4] + 2$$

$$= 28 + 2 = 26 \text{ cm/s}$$

Hence correct option is (d).

## ■ Assertion and Reason

1. Assertion is wrong since when a virtual object is placed at a distance less than the focal length its real image is formed.

Hence answer is (d).

2. Using mirror formula  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$  we get

$$\frac{1}{v} + \frac{1}{20} = \frac{1}{20} \quad v = 10 \text{ cm}$$

ie image is virtual erect and since  $m = \frac{v}{u} = \frac{1}{2}$ .

Hence image is diminished, thus assertion is true.

If  $u = 20 \text{ cm}$  for virtual object  $v$  hence reason is true but reason is not correct explanation of assertion. Hence answer is (b).

3. Using mirror formula  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$  we get

$$\frac{1}{v} + \frac{1}{f} = \frac{1}{u}$$

If  $u$  is front of mirror  $u$  is negative and  $f$  is negative for concave mirror.

$$\frac{1}{v} + \frac{1}{f} = \frac{1}{u} \quad v = \frac{uf}{f - u}$$

$$u < f < v$$

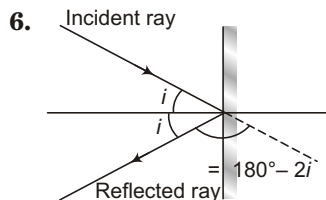
Hence assertion is true also in refractive image and object moves in opposite direction. Hence both assertion and reason are true and reason correctly explain the assertion. Correct answer is (a).

4. Real view mirror of vehicles is convex mirror, hence assertion is true.

It never makes real image of real object. Reason is also true but convex mirror is used because since its field of view is greatest. Hence both assertion and reason are true but reason is not correct explanation of assertion. Correct answer is (b).

5. Since  $m = 2$  hence it is definitely a concave mirror since only concave mirror form magnified image. Since concave mirror form only real image of real object hence reason is also true. Hence it may be true but when object is placed between  $C$  and  $F$ ,  $m < 1$ .

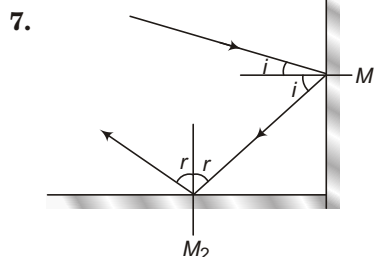
Hence correct answer be (a) or (b).



Hence assertion is true.

For normal incidence  $i = 0$  hence  $180^\circ$ .

hence assertion is true but reason is false. hence correct option is (c).



Deviation produced by  $M_1$   $180 - 2i$

Deviation produced by  $M_2$   $180 - 2r$

Total deviation produced  $360 - 2(i + r)$

But from figure  $i + r = 90^\circ$ , hence deviation  $180^\circ$  for any value of  $i$ .

Hence assertion is true but reason is false.  
Correct option is (c).

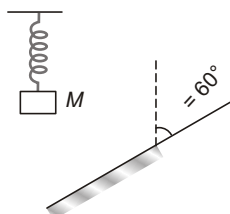
8. The correct option is (b).

9. The correct option is (a, b).

10. The correct option is (b).

## ■ Objective Questions (Level 2)

1.  $v_{\max}$  A



$$v_{\max} = \sqrt{\frac{k}{m}} A$$

$\therefore \frac{1}{2} \frac{k}{m} A^2$  for SHM

Maximum speed of insect relative to its image

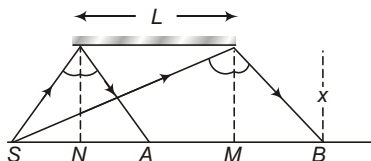
$$2v_{\max} = 2v_{\max} \sin 60^\circ$$

$$A\sqrt{3} \sqrt{\frac{k}{m}}$$

Hence correct option is (c).

2.  $au^n = g$

Height  $x$



Let after time  $t$  perpendicular distance between mirror and source is  $x$  we have from figure

$$AB = AM + MB = SM + SA + MB$$

but  $SM = MB$

$$AB = 2MB = SA + 2x \tan \theta + SA$$

$$2x \tan \theta = 2x \tan \theta$$

$$AB = 2x [\tan \theta + \tan \theta]$$

$$2x \frac{SM}{x} = \frac{SN}{x} = 2[SM + SN]$$

$$AB = 2L,$$

where  $SM = SN = L$  Length of mirror

$$\frac{d}{dt}[AB] = \frac{d}{dt}(2L) = 0$$

$\therefore$  Length of mirror is constant.

Hence the correct option is (d).

3. Here  $u = 10$  cm and  $v = 20$  cm

Using mirror formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \text{ we get } \frac{dv}{v^2} = \frac{du}{u^2} \quad 0$$

$$\frac{dv}{du} = \frac{v^2}{u^2} = \frac{20^2}{10^2} = 4$$

$$dv = 4du$$

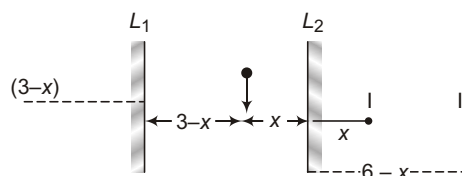
$$dv = 4(0.1), \text{ here } du = 0.1$$

$$dv = 0.4 \text{ cm,}$$

ie, 0.4 cm away from the mirror.

Hence the correct option is (a).

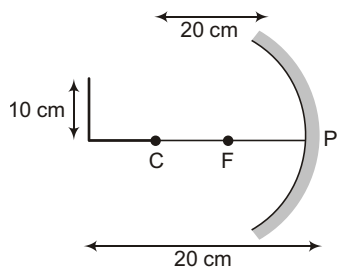
4. The first and second images are shown in figure but according to question



$$\frac{(6-x)}{2} = \frac{x}{2x} = \frac{4}{x} = \frac{1}{1m}$$

Hence the correct option is (c).

5. For vertical part  $\frac{1}{20} = \frac{1}{v} = \frac{1}{5}$



$$v = \frac{20}{3}$$

$$|m_v| = \frac{v}{u} = \frac{20/3}{20} = \frac{1}{3}$$

$$L_v = \frac{10}{3} \text{ cm}$$

For horizontal part first end is at C hence its image is also at C i.e. at  $v = 10$  cm, for other end

$$\frac{1}{20} = \frac{1}{v} = \frac{1}{5} \quad v = \frac{20}{3}$$

$$|v| = \frac{20}{3}$$

$$L_H = |v - u| = \frac{20}{3} - 10 = \frac{10}{3}$$

$$L_H = \frac{10}{3}$$

The ratio  $L_V : L_H = 1 : 1$ .

Hence correct option is (c)  $1 : 1$ .

6. Here  $u = 15$  cm,  $f = 10$  cm

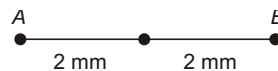
Using  $\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$

We get,  $v = 30$  cm

We have  $m = \frac{v}{u} = \frac{v_2}{u_2} = \frac{v_1}{u_1}$

$$\frac{dv}{du} = 2$$

$$dv = 2du,$$



$$AB = du = 4 \text{ mm}$$

$$dv = 2 \times 4 \text{ mm} = 8 \text{ mm}$$

Hence the correct option is (c).

7. If the mirror is rotated by an angle in anticlockwise direction about an axis to the plane mirror, the new angle of incidence becomes  $i$  and angle of reflection also  $i$ .

According to problem

$$i = i = 2 = 45$$

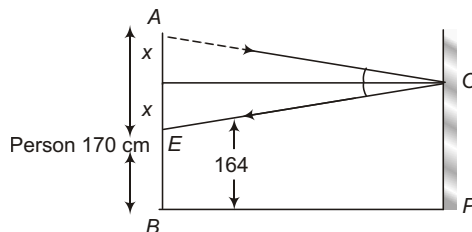
$$2i = 45 = 2 = 45 = 2 = 20 = 85$$

But angle of incidence = angle of reflection.

Hence the angle between original incident and reflected ray was  $85^\circ$ . Similarly if the mirror is rotated clockwise the angle became  $5^\circ$ .

Hence correct option is (c)  $85^\circ$  or  $5^\circ$ .

8. The person sees his hair if the incident ray starts from point A after reflection by mirror reaches his eyes. Let O is point at minimum distance  $x$  below the point A.



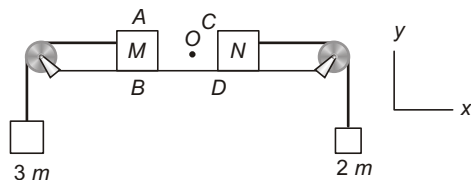
We have  $2x = 60$  cm  $x = 30$  cm

The distance of O from P is

$$170 - 30 = 140 \text{ cm}$$

Hence correct option is (a).

9. Acceleration of block



$$a_{AB} = \frac{3mg}{3m} = \frac{3g}{4}$$

Acceleration of block CD :

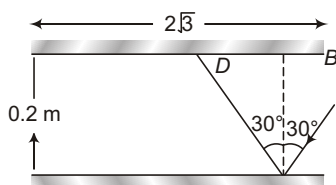
$$a_{CD} = \frac{2mg}{2m} = \frac{2g}{3}$$

Since the accelerations are in opposite directions relative acceleration of one image with respect to other is given by

$$a_{AB} + a_{CD} = \frac{3g}{4} + \frac{2g}{3} = \frac{17g}{12}$$

Hence the correct option is (c).

10. Here  $\frac{BD}{0.2} = \tan 30^\circ$

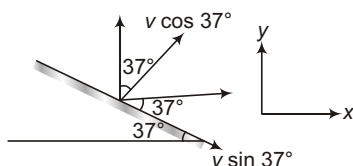


$$BD = 0.2 \cdot \frac{1}{\sqrt{3}}$$

$$\text{No. of reflections} = \frac{2\sqrt{3}}{0.2/\sqrt{3}} = 30$$

Hence, the correct option is (b).

11. Resolving velocity along parallel to mirror and perpendicular to mirror, we get



$$v_{\parallel} = v \sin 37^\circ \text{ and } v_{\perp} = v \cos 37^\circ$$

From figure, we get

$$v_x = v \cos 37^\circ \sin 37^\circ = v \sin 37^\circ \cos 37^\circ$$

$$2v \cos 37^\circ \sin 37^\circ$$

$$v_x = 2 \cdot 5 \cdot \frac{4}{5} \cdot \frac{3}{5} = \frac{24}{5} = 4.8$$

$$v_y = v \cos 37^\circ \cos 37^\circ$$

$$v \sin 37^\circ \sin 37^\circ$$

$$v_y = v [\cos^2 37^\circ - \sin^2 37^\circ]$$

$$5 \left[ \frac{4}{5} \cdot \frac{3}{5} - \frac{4}{5} \cdot \frac{3}{5} \right]$$

$$5 \left[ \frac{7}{5} \cdot \frac{1}{5} - \frac{7}{5} \cdot \frac{1}{5} \right] = 1.4$$

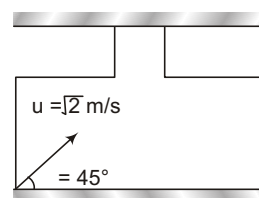
Hence velocity of image is given by

$$\mathbf{v} = v_x \hat{i} + v_y \hat{j}$$

$$\mathbf{v} = 4.8 \hat{i} + 1.4 \hat{j}$$

Hence the correct option is (c).

12. Since elevator start falling freely, the relative acceleration of the particle in elevator frame  $g - g = 0$



Hence, in elevator frame path of the particle is a straight line.

The vertical component of velocity is

$$u \sin 45^\circ = \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 1 \text{ m/s}$$

The separation between mirror and particle in 0.5 s is

$$y = v_y t = 1 \cdot 0.5 = 0.5 \text{ m}$$

The separation between image of particle and particle at this moment

$$2y = 2 \times 0.5 \text{ m} = 1 \text{ m}$$

Hence the correct option is (b).

13. Here velocity of mirror

$$\mathbf{v}_m = 4\hat{i} + 4\hat{j} + 8\hat{k}$$

and velocity of object

$$\mathbf{v}_o = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

Since  $\hat{k}$  is normal to the mirror hence  $\hat{i}$  and  $\hat{j}$  components of image velocity remain unchanged i.e., velocity of image can be written as

$$\mathbf{v}_i = 3\hat{i} + 4\hat{j} + v_{iz}\hat{k}$$

but  $v_{iz} = 2u_{mz} - v_{oz} = 2 \times 8 - 5 = 11$

Hence, we get

$$\mathbf{v}_i = 3\hat{i} + 4\hat{j} + 11\hat{k} \text{ (wrt ground)}$$

Hence, the correct option is (b).

$$3\hat{i} + 4\hat{j} + 11\hat{k}$$

14. Only option (b) satisfy the given condition.

Here  $X_0 = 2, X_i = 10$

Using  $\frac{1}{X_0} + \frac{1}{X_i} = \frac{1}{f}$

we get

$$\frac{1}{10} + \frac{1}{2} = \frac{1}{f}$$

$$f = 2.5 \text{ cm}$$

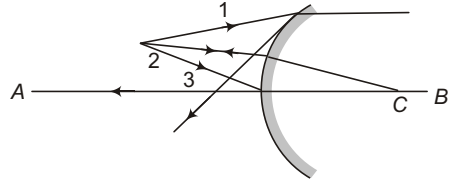
Hence, the mirror is concave.

We know that  $y_i = \frac{fy_0}{f - x_0}$

$$= \frac{2.5 \times 1}{2.5 - 2} = 5 \text{ cm}$$

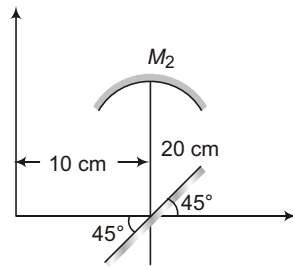
Hence, the correct option is (b).

16. There are two mistakes one in ray (1) and other in ray (3).



Hence correct option is (b).

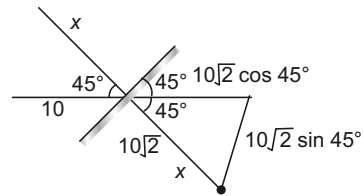
17. The image formation by plane mirror is shown as



$$\frac{10}{x} = \sin 45^\circ \quad x = 10\sqrt{2}$$

The  $x$ -coordinate is  $10\sqrt{2} \cos 45^\circ = 10$

and  $y$ -coordinate is  $10\sqrt{2} \sin 45^\circ = 10$



Hence, the correct option is (c), (10, 10).

18.  $x_i = \frac{fx_0}{f - x_0} = \frac{10 \times 10}{10 - 10} = 5 \text{ cm}$

For concave mirror  $f = 10 \text{ cm}$ .

$$y_i = \frac{fy_0}{f - x_0} = \frac{10 \times 20}{10 - 10} = 10 \text{ cm}$$

Hence the coordinates of image are (5, 10).

Therefore, the correct option is (d).

19. For convex mirror  $f = 10 \text{ cm}$

$$x_i = \frac{fx_0}{x_0 - f} = \frac{10 \cdot 10}{10 - 10}$$

$$y_i = \frac{fy_0}{f - y_0} = \frac{10 \cdot 20}{10 - 10}$$

## ■ More than one options are correct

1. Here  $f = 20 \text{ cm}$

**Case 1.** (if image is real)  $u, v$  and  $f$  all are  $-ve$ .

Here  $m = \frac{v}{u} = \frac{2u}{2u}$

using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we get,

$$\frac{1}{2u} + \frac{1}{u} = \frac{1}{20}$$

$$\frac{3}{2u} = \frac{1}{20} \quad u = 30 \text{ cm}$$

**Case 2.** (if image is virtual)

$u$  and  $f$  are  $-ve$ , while  $v$  is  $+ve$

$$\frac{1}{2u} + \frac{1}{u} = \frac{1}{20}$$

$$u = 10 \text{ cm}$$

Hence possible values of  $u$  are  $10 \text{ cm}, 30 \text{ cm}$ .

The correct options are (a) and (b).

2. Magnitude of focal length spherical mirror is  $f$  and linear magnification is  $\frac{1}{2}$

Since concave mirror forms inverted real image and magnification is less than unity, therefore  $u > 2f$ .

Hence option (a) is correct.

If image is erect then it is a convex mirror.

Let mirror is concave hence focal length  $f$ .

Here  $m = \frac{1}{2} = \frac{v}{u}$

Hence the correct option is (d).

20. If concave mirror is replaced by plane mirror the coordinates are  $(0, -40)$ .

Hence the correct option is (d).

$$v = \frac{u}{2}$$

Using mirror formula  $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ , we get

$$\frac{1}{u/2} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{3}{u} = \frac{1}{f}$$

$$u = 3f$$

Hence, if the mirror is concave the object distance will be  $3f$ .

Let mirror is convex, then

$$m = \frac{v}{u} = \frac{1}{2} \quad u = \frac{v}{2}$$

Using mirror formula, we get

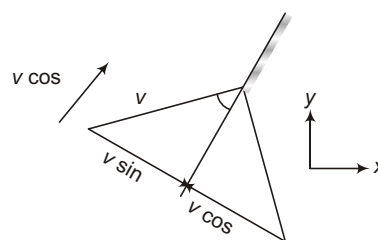
$$\frac{1}{u/2} + \frac{1}{u} = \frac{1}{f} \quad u = f$$

Hence, if mirror is convex the object distance will be  $f$ .

Hence correct options are (a), (b), (c) and (d).

3. Since by a plane mirror

speed of image = speed of object



Hence speed of image also  $v$ .



Horizontal component (along mirror)

$$v \cos$$

Vertical component (to mirror)

$$v \sin$$

Hence image velocity also make an angle with the mirror.

Resolving velocity along ( $y$ -axis *ie*, parallel to mirror) and ( $x$ -axis *ie* perpendicular to mirror).

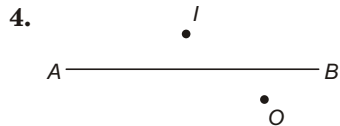
$$\mathbf{v}_0 = v \sin \hat{\mathbf{i}} + v \cos \hat{\mathbf{j}}$$

$$\mathbf{v}_i = v \sin \hat{\mathbf{i}} - v \cos \hat{\mathbf{j}}$$

Relative velocity of object w.r.t. image is

$$\mathbf{v}_{0i} = \mathbf{v}_0 - \mathbf{v}_i = 2v \sin \hat{\mathbf{i}}$$

Hence, correct options are (a), (b) and (d).



As image is on opposite side of the principle axis (inverted image) hence the mirror is concave because convex mirror always form erect image.

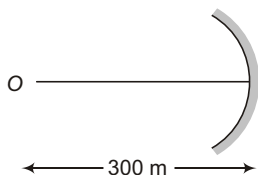
The mirror is lying to the right of  $O$  and the  $O$  lies between  $C$  and  $F$ .

If centre of curvature lies to the right hand side of  $O$  then  $v < u$ .

Hence, this option is incorrect.

Hence, the correct options are (a), (b) and (d).

5. Here  $f = 20$  cm,  $u = 30$  cm



Using mirror formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

we get,

$$v = 60$$

Different this w.r.t. time, we get

$$\frac{1}{v^2} \frac{dv}{dt} - \frac{1}{u^2} \frac{du}{dt} = 0$$

$$\frac{dv}{dt} = \frac{v^2}{u^2} \frac{du}{dt}$$

Hence in event (1),

$$\frac{du}{dt} = v$$

$$\frac{dv}{dt} = \frac{60^2}{30^2} v = 4v$$

Hence, speed of image in event (1) is  $4v$ . after time  $y$  coordinate of object  $y_0 = vt$

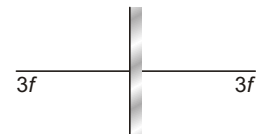
but  $x_0 = 30$

$$\text{then } y_i = \frac{fy_0}{f - x_0} = \frac{20}{20 - 30} vt$$

$$y_i = -2vt \quad \left| \frac{dy_i}{dt} \right| = 2v$$

Hence, option (b) and (c) are correct.

6. For plane mirror



$$u = 3f \quad v = 3f$$

For concave mirror

$$u = 3f$$

Using mirror formula

$$\frac{1}{v} - \frac{1}{3f} = \frac{1}{f} \quad \frac{2}{3f}$$

$$v = 1.5f$$

$$|v| = 1.5f$$

For convex mirror,

$$\frac{1}{v} - \frac{1}{3f} = \frac{1}{f} - \frac{4}{3f}$$

$$v = 0.75f$$

Hence maximum distance in event (1) if image is from plane mirror and minimum distance from convex mirror

When  $v = 1.5f$ , then  $v = 1.5f$

by plane mirror

For concave mirror

$$\frac{1}{u} - \frac{1}{1.5f} = \frac{1}{f} - \frac{2}{3f} - \frac{1}{f}$$

$$\frac{2}{3f} - \frac{3}{3f} = \frac{1}{3f}$$

$$v = 3f$$

$$|v| = 3f$$

For convex mirror

$$\frac{1}{v} - \frac{2}{3f} = \frac{1}{f} - \frac{5}{3f}$$

$$v = 0.6f$$

Hence, in event (2) maximum distance of image from the concave mirror.

Hence, correct options are (a), (b) and (c).

## ■ Match the Columns

1. (a)  $m = 2$ , since  $|m| > 1$ .

Therefore mirror is concave and  $\therefore m$  is +ve.

Hence image is real [for concave mirror  $m$  is +ve]

Therefore,

(a)  $q, r$

(b) Since  $m = \frac{1}{2}$ ,  $\therefore m$  is -ve

Hence mirror is concave and image is real.

(b)  $q, r$

(c)  $m = 2$ ,  $\therefore m > 1$

Hence mirror is concave and  $\therefore m$  is +ve

Hence image is virtual.

(c)  $q, s$

(d)  $\therefore 1/m = \frac{1}{2} < 1$  and +ve

Hence the mirror is convex and image is virtual.

(d)  $p, s$

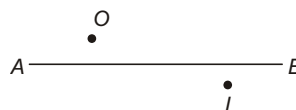
2. Plane mirror (for virtual object) only real image

(a)  $p$

(b)  $r$

(c)  $p$

3. (a) Since object and its image are on opposite side of principle axis.



Hence mirror is concave

(a)  $r$ .

(b) Similarly as for option (a).

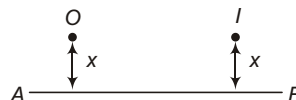
(b)  $r$

(c) Since image and object are of same height from  $AB$ .

Hence mirror is plane mirror.

(c)  $p$

(d) Since image is magnified.



Hence mirror is concave [ $D$  is. distance between  $O$  and mirror is less than the focal length].

Hence

(d) r.

4. (a) For concave mirror  $M_1$  focal length

$$20 \text{ cm}$$

When  $x = 20 \text{ cm}$ , Mirror is  $M_1$

$v$  and magnified

(a) p, s

(b) For convex mirror  $M_2$  of focal length  
20 cm if X (distance of object from pole)  
20

Using mirror formula  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

we get

$$\frac{1}{v} + \frac{1}{20} = \frac{1}{20} \quad \frac{1}{10}$$

$$v = 10 \text{ cm}$$

Hence image is virtual.

(b) r

(c)  $u = 30 \text{ cm}$ ,  $f = 20 \text{ cm}$

$$\frac{1}{v} + \frac{1}{30} = \frac{1}{20} \quad \frac{2}{60} + \frac{3}{60} = \frac{1}{60}$$

$$v = 60 \text{ cm}$$

Hence image is real.

$$m = \frac{60}{30} = 2$$

Hence image is magnified (2 times).

(c) q, s

(d) for mirror  $M_2$  (convex) at X = 30 cm

image again virtual.

(d) r

5. (a) For concave mirror  $f = 20 \text{ cm}$

**Case I.** Image is real.

$$m = 2 \quad \frac{v}{u} \quad v = 2u$$

$$\text{Using } \frac{1}{u} + \frac{1}{u} = \frac{1}{f}$$

$$\text{we get, } \frac{1}{2u} + \frac{1}{u} = \frac{1}{20}$$

$$\frac{3}{2u} = \frac{1}{20}$$

$$u = 30 \text{ cm}$$

If image is virtual  $v = 2v$

$$\frac{1}{2u} + \frac{1}{u} = \frac{1}{20}$$

$$u = 10 \text{ cm}$$

Hence correct option are as

(a) p, q

(b) Here  $m = \frac{1}{2} = 1$

Hence image is real.

$$\frac{1}{2} = \frac{v}{u} \quad v = \frac{u}{2}$$

Using  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ , we get

$$\frac{1}{u/2} + \frac{1}{u} = \frac{1}{20}$$

$$\frac{3}{4} = \frac{1}{20} \quad u = 60 \text{ cm}$$

Hence correct option is none of these.

(b) s

(c) if  $m = 1$ , then  $u = 2f$

$$u = 40 \text{ cm}$$

Hence correct option is none of these.

(c) (s)

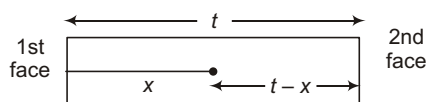
(d) Similarly as in part (b) we see that answer is none of these.

(d) (s)

# 27 Refraction of Light

## Introductory Exercise 27.1

1. Let real depth of dust particle is  $x$  and thickness of slab is  $t$



From Ist surface

$$\frac{\text{Real depth}}{\text{App. depth}} = \frac{x}{\frac{t}{1.5}} = \frac{x}{9 \text{ cm}} \quad \dots(i)$$

From other face

$$\frac{t}{4} = \frac{x}{6} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{3}{1} = \frac{4}{3} = \frac{3}{2} = 2$$

3. Frequency remain same.

Let  $v_1$  is velocity in medium (1) and  $v_2$  in Medium (2)

We have

$$\frac{v_1}{v_2} = \frac{c}{c} \text{ and } \frac{v_2}{v_1} = \frac{1}{2}$$

Similarly, wavelength  $\lambda_2 = \frac{1}{2} \lambda_1$

4. From  $v_a = n_a \lambda_a$

$$\frac{v_a}{n_a} = \frac{3 \times 10^8}{6 \times 10^{14}} = 5 \times 10^7 \text{ m}$$

50 nm

$$\frac{a}{m} = \frac{500}{300} = \frac{5}{3} = 1.67$$

## Introductory Exercise 27.2

1. Since light rays are coming from glass to air applying

$$\frac{1}{v} = \frac{1.5}{10} = \frac{1}{1.5} = \frac{1}{30} = \frac{1}{3.5} = 8.57 \text{ cm}$$

2.  $\frac{2}{v} = \frac{1}{u} = \frac{2}{R}$

$$(a) \frac{1.5}{v} = \frac{1}{(20)} = \frac{0.5}{6}$$

On solving  $v = 45 \text{ cm}$

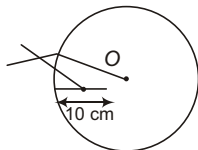
$$(b) \frac{1.5}{v} = \frac{1}{(10)} = \frac{0.5}{6}$$

On solving we get  $v = 90 \text{ cm}$

$$(c) \frac{1.5}{v} - \frac{1}{(3)} = \frac{0.5}{6}$$

On solving  $v = 6.0 \text{ cm}$

3. Light rays are coming from glass to air



$$\frac{2}{v} - \frac{1}{u} = \frac{2}{R}$$

$$\frac{1}{v} - \frac{4}{3(10)} = \frac{1}{(15)}$$

$$\frac{1}{v} - \frac{4}{30} = \frac{1}{45} \text{ on solving } v = 9 \text{ cm}$$

$$4. \text{ Applying } \frac{u_2}{v} - \frac{1}{u} = \frac{2}{R}$$

$$\frac{1.44}{v} - \frac{1}{1.25} = \frac{0.44}{R}$$

On solving  $v = 0.795 \text{ cm}$

$$5. \frac{2}{v} - \frac{1}{u} = \frac{2}{R}$$

$$\frac{1.635}{v} - \frac{1}{(9)} = \frac{0.635}{(2.50)}$$

on solving  $v = 6.993 \text{ cm}$

Lateral magnification  $m = \frac{v}{u}$

$$\frac{6.993}{9} = 0.777$$

## Introductory Exercise 27.3

1. We have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad (1) \quad \frac{1}{R_1} - \frac{1}{R_2}$$

$$\frac{1}{20} - \frac{1}{60} = (1.65 - 1) \left[ \frac{1}{R} - \frac{1}{R} \right]$$

$$\frac{3}{60} - \frac{1}{60} = 0.65 \left[ \frac{2}{R} \right]$$

$$R = 60 \times 0.65 = 39 \text{ cm}$$

2. Using  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ , we get

$$\frac{1}{50} - \frac{1}{x} = \frac{1}{30} \quad \frac{1}{x} = \frac{1}{30} - \frac{1}{50}$$

On solving  $x = 18.75 \text{ cm}$

$$m = \frac{v}{u} = \frac{50}{18.75}$$

$$\text{Height of filament image} = 2 \times \frac{50}{18.75}$$

5.3 cm

$$3. \frac{1}{f} = (1) \left[ \frac{1}{R} - \frac{1}{R} \right]$$

If lens faces becomes opposite there is no change in radius of curvature hence focal length does not change.

4. Using formula  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$  when  $u = 0, v = 0$

when  $u = f, v = \infty$  hence image moves from surface to  $\infty$ .

$$5. \frac{1}{f} = (1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$\frac{1}{f} = (1.3 - 1) \left[ \frac{1}{R} - \frac{1}{R} \right] = 0.3 \times \frac{2}{20}$$

$$f = \frac{100}{3} \text{ cm}$$

(a) When immersed in a liquid of 1.8 refractive index

$$\frac{1}{f_1} = \frac{1.3}{1.8} - 1 = \frac{2}{R} - \frac{0.5}{1.8} = \frac{2}{20}$$

$f = 36 \text{ cm}$

(b) The minimum distance is equal to the focal length  $= 36 \text{ cm}$

6. Using  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} - \frac{1}{(-20)} = \frac{1}{10}$$

On solving  $v = 20$  cm

Magnification  $\frac{v}{u} = 1$

Hence the image of same size and inverted.

Let the distance between second lens is  $x$

Since magnification is unity image distance also  $x$  using again

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

we get

$$\frac{1}{x} - \frac{1}{(-x)} = \frac{1}{f} \quad \frac{1}{10} \quad x = 20 \text{ cm}$$

Hence the distance between two lenses

$$20 \text{ cm} + 20 \text{ cm} = 40 \text{ cm}$$

7.  $\frac{1}{v} - \frac{1}{f} = \frac{1}{u}$  ... (i)

$$\frac{1}{v_1} - \frac{1}{f} = \frac{1}{u} \quad \frac{1}{du} \quad \dots (ii)$$

$$\frac{1}{v} - \frac{1}{v} = \frac{(u - du - u)}{(u - du)u} \text{ on solving, we get}$$

$$\frac{v - v}{v} = \frac{du}{u(u - du)} \text{ thickness } dv = \frac{v^2}{u^2} du$$

10. Size of image  $\sqrt{6 \frac{2}{3}} = 2 \text{ cm.}$

11. Let image distance is  $u$

$$|m| = 3 \quad v = 3u$$

$$\frac{1}{3u} - \frac{1}{u} = \frac{1}{12} \quad v = 16 \text{ cm}$$

12. Since image is upright and diminished hence lens is concave. Now

$$u - v = 20 \quad \dots (i)$$

$$m = \frac{v}{u} = \frac{1}{2}$$

$$\frac{1}{2} = \frac{u}{20}$$

$$u = 40 \text{ cm and } v = 20 \text{ cm}$$

Using  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{20} - \frac{1}{40} = \frac{1}{f} \quad f = 40 \text{ cm}$$

13. The image coincide itself if light falls normally on plane mirror hence object must be on focus i.e. 10 cm.

8.  $\frac{1}{v} - \frac{1}{u} = \frac{2(\frac{2}{3} - 1)}{R_2} = \frac{2(\frac{2}{3} - 1)}{R_1}$

$$\frac{1}{v} - \frac{1}{0.2} = \frac{2(4/3)}{0.4} = \frac{2(4/3 - 1)}{0.4}$$

On solving  $v = 12 \text{ cm}$

9. Since shift in position  $t = 0.1 \text{ m}$

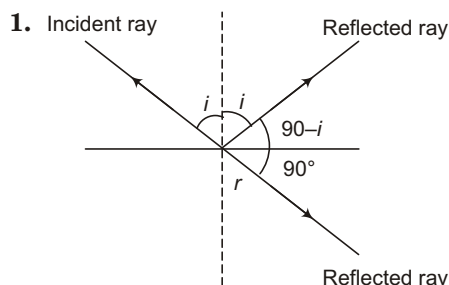
Hence real depth  $(0.1 - 0.2) \text{ m}$   
0.3 m

and apparent depth 0.2 m

$$\frac{\text{real depth}}{\text{apparent depth}}$$

$$\frac{0.3}{0.2} = 1.5$$

## AIEEE Corner



We have  $r = 90 - i$

From Snell's law  $1.5 \frac{\sin i}{\sin r} = \frac{\sin i}{\sin(90 - i)}$

$$\tan i = 1.5$$

$$i = \tan^{-1}(1.5)$$

2.  $n_w = \frac{v_{\text{air}}}{u_w} = \frac{343}{1498} = 0.229$

Critical angle  $\sin^{-1}(0.229) = 13.2^\circ$

3. Speed in glycerine  $v_g = \frac{c}{n_g} = \frac{3 \times 10^8}{1.47}$

$$t_1 = \frac{20}{v_g} = \frac{20}{3 \times 10^8} \times 1.47 = 9.8 \times 10^{-8} \text{ s}$$

Speed in glycerine  $v_g = \frac{c}{n_g} = \frac{3 \times 10^8}{1.63}$

$$t_2 = \frac{20}{v_g} = \frac{20}{3 \times 10^8} \times 1.63 \sim 10.8 \times 10^{-8}$$

$$t_2 - t_1 = (10.86 - 9.8) \times 10^{-8}$$

$$= 1.67 \times 10^{-8} \text{ s}$$

4. (a)  $t_1 = \frac{1 \times 10^{-6} \text{ m}}{v_1} = \frac{1 \times 10^{-6} \text{ m}}{3 \times 10^8 / 1.2}$

$$= \frac{1.2 \times 10^{-6}}{3 \times 10^8}$$

$$t_1 = 0.4 \times 10^{-14} \text{ s}$$

$$t_2 = \frac{1.5 \times 10^{-6}}{3 \times 10^8} = 0.5 \times 10^{-14} \text{ s}$$

$$t_3 = \frac{1 \times 10^{-6}}{3 \times 10^8 / 1.8} = \frac{1.8 \times 10^{-6}}{3 \times 10^8} = 0.6 \times 10^{-14}$$

Hence  $t_1$  is least and  $t_1 = 0.4 \times 10^{-14} \text{ s}$

(b) Total number of wavelengths

$$\frac{1 \text{ m}}{n_1} = \frac{1.5 \text{ m}}{n_2} = \frac{1 \text{ m}}{n_3}$$

$$\frac{1000}{600 \text{ nm}} = \frac{1.2 \text{ nm}}{600 \text{ nm}} = \frac{1.5}{600 \text{ nm}}$$

$$\frac{1}{600 \text{ nm}} = \frac{1.8}{600 \text{ nm}} = \frac{1000 \text{ nm}}{600 \text{ nm}}$$

$$\frac{4500}{600} = 7.5$$

5. The given wave equation is

$$E_x(y, t) = E_{ax} \sin \left( \frac{2\pi y}{5 \times 10^{-7}} - 3 \times 10^{14} t \right)$$

Comparing with standard equation

$$E_x(y, t) = E_0 \sin [ky - \omega t]$$

$$k = \frac{2\pi}{5 \times 10^{-7}} = 2 \times 3 \times 10^{14}$$

$$v = \frac{\omega}{k} = \frac{3 \times 10^{14}}{2 \times 5 \times 10^{-7}} = 1.5 \times 10^8 \text{ m/s}$$

Refractive index  $n = \frac{c}{v} = \frac{3 \times 10^8}{1.5 \times 10^8} = 2$

Wavelength in this way  $\lambda = \frac{2\pi}{k}$

$$\lambda = \frac{2}{25/5 \times 10^7} = 5 \times 10^{-7} \text{ m}$$

$$\lambda = 500 \text{ nm}$$

If vacuum, wavelength is  $\lambda_0$  then

$$n = \frac{\lambda_0}{\lambda}$$

$$\lambda_0 = n \lambda = 2 \times 500 = 1000 \text{ nm}$$

6. Refraction from plane and spherical surfaces





10.

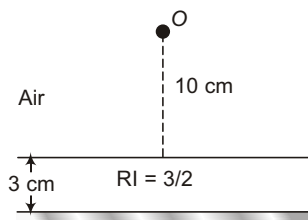


Image formed by refraction acts the virtual object for the mirror.

Here shift  $t = 1 - \frac{1}{n}$

$$3 - 1 \cdot \frac{1}{3/2} = 1 \text{ cm}$$

Hence object appear to the mirror

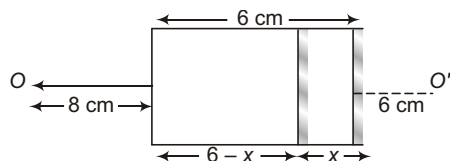
$$(10 - 1) \text{ cm}$$

$$11 \text{ cm}$$

The image formed by mirror  $11 \text{ cm}$

Hence image formed by the mirror at  $11 \text{ cm}$  behind the mirror.

11.



**Step.** Let shift in mirror is  $x$  then the distance of object.

From the mirror is  $8 - (6 - x)$ .

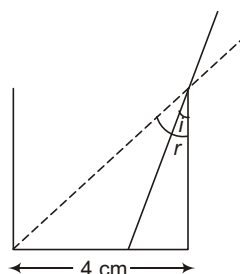
**Step II.** Plane mirror form image behind the mirror at same distance as the distance object from mirror hence

$$8 - (6 - x) = x \Rightarrow 6 - x = 4 \text{ cm}$$

**Step III.**  $\frac{\text{real depth}}{\text{app. depth}} = \frac{6}{4} = 1.5$

hence real position of the bubble inside sphere is  $1.2 \text{ cm}$  from the surface.

12.



$$\text{Here } \sin r = \frac{4}{\sqrt{4^2 + n^2}}$$

$$\sin i = \frac{2}{\sqrt{2^2 + n^2}}$$

$$u = \frac{\sin i}{\sin r} = \frac{4}{3} = \frac{4\sqrt{2^2 + n^2}}{2\sqrt{4^2 + n^2}}$$

$$\frac{4}{9} = \frac{4}{16} \cdot \frac{n^2}{n^2} \Rightarrow 5n^2 = 28$$

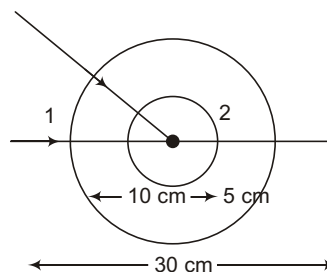
$$n^2 = \frac{\sqrt{28}}{5} \Rightarrow n = 2.4 \text{ cm}$$

13. Using  $\frac{2}{v} = \frac{1}{u} + \frac{2}{R}$

$$\frac{1}{v} = \frac{1.5}{1.5} + \frac{1}{5}$$

$$\frac{1}{v} = \frac{1}{10} \Rightarrow v = 10 \text{ cm}$$

14. For first surface  $\frac{2}{v} = \frac{1}{u} + \frac{2}{R}$



$$\frac{1.5}{v} = \frac{1}{10} + \frac{0.5}{10} \Rightarrow v = 30 \text{ cm}$$

For 2nd surface

$$\frac{1}{v} - \frac{1.5}{(15)} = \frac{0.5}{5}$$

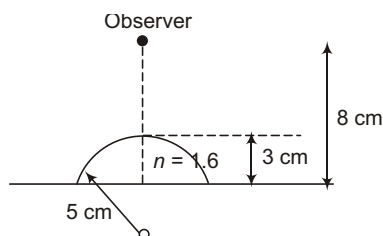
$$\frac{1}{v} - \frac{1}{10} = \frac{1}{10} \quad v = 5 \text{ cm}$$

Hence the distance from first face is

$$(10 - 5) \text{ cm}$$

$$15 \text{ cm}$$

15. Since rays goes from paper weight ( $n = 1.6$ ) to air hence



$$\frac{1}{v} - \frac{1.6}{(3)} = \frac{1}{5}$$

$$\frac{1}{v} - \frac{0.6}{5} = \frac{1.6}{3}$$

On solving we get  $v = 0.58$   $|v| = 0.58 \text{ cm}$

Hence the distance between observer and table top is  $(8 - 0.58) \text{ cm} = 7.42 \text{ cm}$ .

16. Let real velocity of bird  $v_B \text{ cm/s}$

Velocity of bird w.r.t. fish  $16 \text{ cm/s}$

Velocity of bird w.r.t. water  $v_B$

But  $v_B = v_f = 16 \text{ cm/s}$

Here  $v_f = 4 \text{ cm/s}$

$$\frac{4}{3} v_B = 4 + 16$$

$$\frac{4}{3} v_B = 20 \text{ cm/s}$$

$$v_B = 15 \text{ cm/s}$$

17. Let the distance between the object and screen is  $d$  and let distance between object and lens is  $x$

Using lens formula  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$  we get

$$\frac{1}{d - x} - \frac{1}{x} = \frac{1}{16}$$

$$x^2 - xd - 16d = 0$$

$$x = \frac{d \pm \sqrt{d^2 + 64d}}{2}$$

Let  $x_1 = \frac{d + \sqrt{d^2 + 64d}}{2}$

and  $x_2 = \frac{d - \sqrt{d^2 + 64d}}{2}$

$$x_1 - x_2 = \sqrt{d^2 + 64d}$$

But  $x_1 - x_2 = 60$   $d^2 + 64d = 3600$   $0$

$$(d - 100)(d - 36) = 0$$

$$d = 100 \therefore d = 36$$

Hence the distance between object and screen is  $100 \text{ cm}$ .

18.  $\therefore \frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$

For identical double convex lens

$$|R_1| = |R_2| = R$$

but  $R_1 = R$  and  $R_2 = -R$

$$\frac{1}{f} = (n - 1) \left( \frac{1}{R} - \frac{1}{-R} \right) = (n - 1) \frac{2}{R}$$

$$f = \frac{R}{2(n - 1)} \text{ hence } f_1 = \frac{R}{2(1.5 - 1)} = R$$

and  $f_2 = \frac{R}{2(1.7 - 1)} = \frac{R}{1.4}$

(a)  $\frac{f_1}{f_2} = 1.4$   $f_1 : f_2 = 1.4 : 1$

(b) For first lens

$$f_i = \frac{R}{2 \left( \frac{1.5}{1.6} - 1 \right)} = \frac{1.6R}{2 \cdot 1} = 0.8R$$

hence first lens become concave (diverging)

For 2nd lens

$$f_2 = \frac{R}{2 \frac{1.7}{1.6} - 1} = \frac{1.6R}{2 \cdot 0.1} = 8R$$

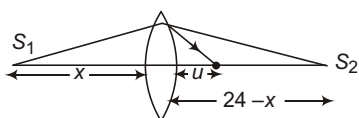
Hence 2nd lens remain **convex**.

19. Here  $u = 10$  cm acts the virtual object for the lens  $v = 15$  cm using  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{15} - \frac{1}{10} = \frac{1}{f}$$

On solving we get  $f = 30$  cm.

20. Situation is shown in figure.



For 1st source direction left to right is  $\rightarrow$  ve

Using  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ , we get

$$\frac{1}{v} - \frac{1}{(x)} = \frac{1}{9} \quad \frac{1}{v} - \frac{1}{x} = \frac{1}{9} \quad \dots(i)$$

For 2nd source direction right to left is  $\leftarrow$  ve

hence  $\frac{1}{v} - \frac{1}{24-x} = \frac{1}{9} \quad \dots(ii)$

On adding Eqs. (i) and (ii) we set

$$\frac{1}{x} - \frac{1}{24-x} = \frac{2}{9} \quad x^2 - 24x + 108 = 0$$

$$x = 6, 18$$

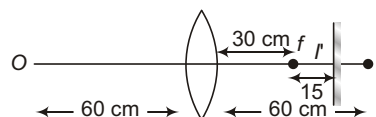
hence lens can be placed at a distance of 6 cm from any source.

21. Since the object is placed at  $c$  of first lens hence image also form at  $c$  and of same magnification *i.e.*  $v = 2f$ . Since two lens are separated by distance  $f$  hence the distance between 2nd lens and image is  $f$ . This image acts a virtual object for this lens using  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ , we get  $\frac{1}{v} - \frac{1}{f} = \frac{1}{v} - \frac{f}{2}$

$$\text{and magnification } m_2 = \frac{v}{u} = \frac{f/2}{f} = \frac{1}{2}$$

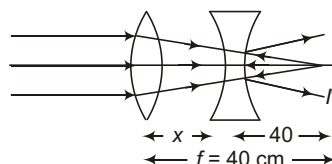
Hence image is formed at  $\frac{f}{2}$  right of 2nd lens.

22. Since the object is placed at  $2f$  hence image also form  $2f$  by lens *i.e.*, at 60 cm. The mirror must be placed at that place that it made the final image at focus of lens. The difference is shown below.



Hence the distance between lens and mirror  
40 cm 15 cm 45 cm

23. The diagram is shown in figure.

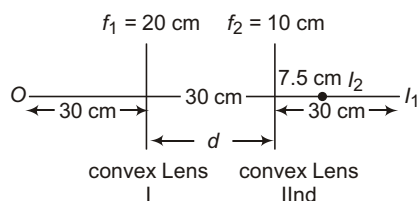


The parallel ray after refraction on convergent lens meet at focus 40 cm. Let distance between two lenses is  $x$  then using  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$  for diverging lens

$$\frac{1}{v} - \frac{1}{(40-x)} = \frac{1}{15}$$

$$x = 25 \text{ cm}$$

24. Here  $f_1 = 20$  cm  $f_2 = 10$  cm and  $d = 30$  cm



For first lens using  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$  we get

$$\frac{1}{v} - \frac{1}{30} = \frac{1}{20} \quad \frac{1}{v} = \frac{1}{20} + \frac{1}{30} \quad v = 60 \text{ cm}$$

For 2nd lens

$$\frac{1}{v} = \frac{1}{30} + \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{30} + \frac{1}{10}$$

$$\frac{1}{v} = \frac{4}{30}$$

$$v = 7.5 \text{ cm}$$

Hence image formed at 7.5 cm from 2nd lens.

25. For lens  $u = 40 \text{ cm}$   $f = 20 \text{ cm}$  using

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

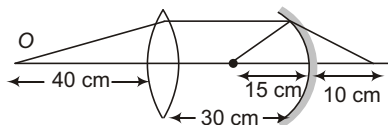
$$\frac{1}{v} = \frac{1}{20} + \frac{1}{40} = \frac{2}{40} + \frac{1}{40}$$

$$v = 40 \text{ cm}$$

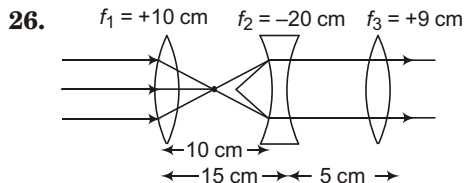
Hence the image is at 40 cm right from the lens. Since distance between mirror and lens is 30 cm. Hence for mirror  $v = 10 \text{ cm}$ ,  $f = 10 \text{ cm}$ .

Using  $\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$   $\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$

$$\frac{1}{v} = \frac{1}{10} + \frac{1}{10} \quad v = 5 \text{ cm}$$



Hence image is formed at 5 cm from the mirror toward lens.



For first lens  $v = 10 \text{ cm}$

For 2nd lens  $u = 5 \text{ cm}$ ,  $f = 20 \text{ cm}$

Using  $\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$

$$\frac{1}{v} = \frac{1}{20} + \frac{1}{5}$$

$$v = 4 \text{ cm}$$

For third lens  $v = 9 \text{ cm}$  and  $f = 9 \text{ cm}$

$$\frac{1}{v} = \frac{1}{(-9)} + \frac{1}{9}$$

$$v = \infty$$

Hence the image formed at  $\infty$  or rays become parallel.

27. Situation is shown in figure when space between two convex lenses is filled with refractive index 1.3 it become a concave lens of radii  $R_1 = 30 \text{ cm}$  and  $R_2 = 70 \text{ cm}$  hence its focal length is



$$\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f} = (1.3 - 1) \left( \frac{1}{30} - \frac{1}{70} \right)$$

$$f = 70 \text{ cm}$$

Hence the equivalent focal length of combination  $\frac{1}{F} = \frac{1}{30} + \frac{1}{70} + \frac{1}{70}$

$F = 30 \text{ cm}$  if

$u = 90 \text{ cm}$  then using

$$\frac{1}{F} = \frac{1}{v} + \frac{1}{u}$$

we get  $\frac{1}{30} = \frac{1}{v} + \frac{1}{90}$

$$\frac{1}{v} = \frac{1}{30} - \frac{1}{90} = \frac{2}{90} - \frac{1}{45}$$

$$v = 45 \text{ cm}$$

$$28. n_{\text{ice}} = \frac{c}{v} = \frac{3 \times 10^8}{2.3 \times 10^8} = 1.30$$

$$\sin^i c = \sin^i \frac{1}{n_{\text{ice}}} = \sin^i \frac{1}{1.30}$$

$$\sin^i (0.77)$$

29. (a) Let angle of refraction in material 2 is  $r$  then

$$\frac{\sin^i}{\sin r} = \frac{1.8}{1.6} = \frac{18}{16} \quad \dots(i)$$

For (2) to (3) interface

$$\frac{\sin r}{\sin 90} = \frac{1.3}{1.8} = \frac{13}{18}$$

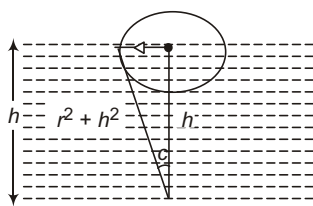
$$\sin r = \frac{13}{18} \quad \dots(ii)$$

From (i) and (ii)  $\sin^i = \frac{18}{16} = \frac{13}{18}$

$$\sin^i = \frac{13}{16}$$

(b) Yes, if  $\sin^i$  decreases  $r$  also decreases and become less than the critical angle and hence light goes into material 3.

30. Let maximum height of liquid is  $h$ . From figure for critical angle  $C$



$$\frac{1}{\sin C} = \frac{\sqrt{r^2 + h^2}}{r} \quad \dots(i)$$

Here  $r = 1$  cm and  $\frac{4}{3}$  putting these values in Eq. (i). Solving we get  $h = \frac{4}{3}$  cm

Here  $\sin^i c = \frac{1}{g} = \frac{2}{3}$

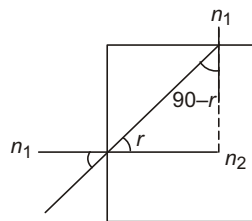
Now if water film is poured on the glass air surface. Let emergent angle at glass water surface is  $r$ , then

$$\frac{\sin^i c}{\sin r} = \frac{w}{g} = \frac{4}{3} = \frac{2}{3}$$

$$\sin r = \frac{9}{8} \sin^i c = \frac{9}{8} \times \frac{2}{3} = \frac{3}{4}$$

$$r = \sin^i \frac{3}{4}$$

32. For total internal reflection at top surface



$$\frac{\sin(90 - r)}{\sin 90} = \frac{n_1}{n_2}$$

$$\cos r = \frac{n_1}{n_2}$$

and

$$\frac{\sin r}{\sin^i} = \frac{n_2}{n_1}$$

$$\sin r = \frac{n_2}{n_1} \sin^i$$

$$\sin r = \frac{n_2}{n_1} \sqrt{1 - \cos^2 r}$$

$$\frac{n_2}{n_1} \sqrt{1 - \frac{n_1^2}{n_2^2}}$$

$$\sin r = \sqrt{\frac{n_2^2 - n_1^2}{n_1^2}}$$

$$\sin^i r = \sqrt{\frac{n_2^2 - n_1^2}{n_1^2}}$$

33. The deviation angle vary from  $0^\circ$  to  $90^\circ - C$  where  $C$  is the critical angle

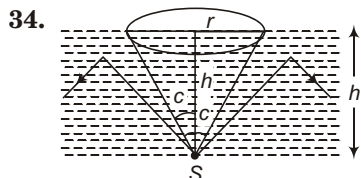
where  $C$  is the critical angle

Now,  $\sin^i c = \frac{w}{g} = \frac{4/3}{3/2} = \frac{8}{9}$

From Eq. (i)  $\cos^i = \frac{\sin C}{\cos} = \frac{8}{9}$

$$\cos^{-1} \frac{8}{9}$$

Hence deviation angle vary from  $0^\circ$  to  $\cos^{-1} \frac{8}{9}$ .



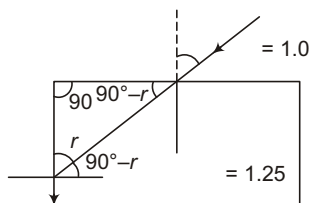
(a) Only circular patch light escapes because only those rays which are incident within a cone of semivertex angle  $C$  [Critical angle] are refracted out of the water surface. All other rays are totally internally reflected as shown in figures

(b) Now  $\frac{1}{\sin C} = \frac{\sqrt{r^2 + h^2}}{r}$

or  $C = \sin^{-1} \frac{r}{\sqrt{r^2 + h^2}}$

$$\sin^{-1} \frac{r}{\sqrt{r^2 + h^2}}$$

35.



For maximum angle the angle  $90 - r$  at left surface must be equal to critical angle

$$\sin(90 - r) = \frac{1}{1.25} = \frac{100}{125} = \frac{4}{5}$$

$$\cos r = \frac{4}{5}$$

$$\sin r = \frac{3}{5}$$

Now,  $\frac{\sin}{\sin r} = 1.25 = \frac{5}{4}$

$$\sin^{-1} \frac{5}{4} \sin r = \frac{5}{4} \frac{3}{5}$$

$$\sin^{-1} \frac{3}{4}$$

$$\sin \frac{A}{2} = \frac{m}{2}$$

36.

$$\frac{\sin \frac{A}{2}}{\sin \frac{A}{2}}$$

$$\sqrt{3} \frac{\sin \frac{A}{2} \sin \frac{A}{2}}{\sin \frac{A}{2}} = \frac{2 \sin \frac{A}{2} \cos \frac{A}{2}}{\sin \frac{A}{2}}$$

$$\cos \frac{A}{2} = \frac{\sqrt{3}}{2} \quad \frac{A}{2} = 30^\circ$$

$$A = 60^\circ$$

37. Here  $i_1 = r_1 = 0^\circ$ .

Now, let other face angle of incidence is  $r_2$

$$\therefore r_1 = r_2 = A = 0^\circ \quad r_2 = A$$

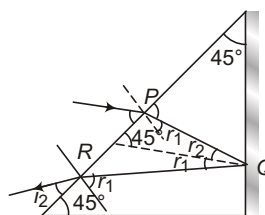
$$r_2 = A = 30^\circ$$

$$\frac{\sin r_2}{\sin i_2} = \frac{1}{1.5} \quad \sin i_2 = 1.4 \sin r_2$$

$$\sin i_2 = 1.5 \sin 30^\circ$$

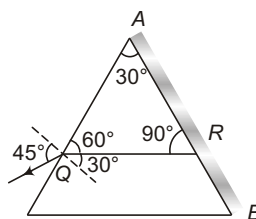
$$i_2 = \sin^{-1}(0.75) = 48.6^\circ$$

38. From figure  $\sin \angle OQP = \sin \angle OQR$



Hence the ray retrace its path.

39.



The ray retrace its path from ref. by surface AB hence  $\angle AR = 90^\circ$  from geometry it is clear that  $r = 30^\circ$

$$\frac{\sin i}{\sin r} = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}$$

40. Depends on formula.

41. The maximum angle will be  $A = 2C$  where  $C$  is the critical angle

$$\text{Now, } C = \sin^{-1} \frac{1}{1.5} = 41.81^\circ$$

$$\text{Hence } A = 2C = 2 \times 41.81 = 83.62^\circ$$

$$42. \quad \frac{\sin \frac{A}{2}}{\sin \frac{A}{2}} \text{ here } A = 60^\circ$$

$$1.5 = \frac{\sin \frac{60^\circ}{2}}{\sin 30^\circ}$$

$$0.75 = \sin \frac{60^\circ}{2}$$

$$60^\circ = \sin^{-1}(0.75) = 48.19^\circ$$

$$\text{and not deviation } = 180^\circ - 2 \times 48.19^\circ = 83.62^\circ$$

(b) If the system is placed in water

$$\frac{1.5}{4/3} = \frac{4.5}{4}$$

$$60^\circ = \sin^{-1}(1.125 \sin 30^\circ)$$

$$= \sin^{-1} \frac{1.125}{2} = 33.75^\circ$$

$$\text{Net deviation } = 180^\circ - 2 \times 33.75^\circ = 112.5^\circ$$

$$43. \quad \frac{V}{Y} = \frac{R}{1} = 1$$

$$0.0305 = \frac{1.665 - 1.645}{Y} = 1$$

On solving we get  $Y = 1.656$

$$44. \quad \frac{1}{f_1} + \frac{2}{f_2} = 0$$

$$\frac{0.18}{20} + \frac{2}{30} = 0 \quad \frac{0.18}{20} = \frac{30}{20}$$

$$\text{Now, } \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F} + \frac{1}{20} = \frac{1}{30}$$

$$F = 60 \text{ cm}$$

$$45. \quad \frac{1}{f_1} + \frac{2}{f_2} = 0$$

$$\frac{1}{2} + \frac{f_1}{f_2} = 0$$

$$\frac{3}{2} = \frac{f_1}{f_2}$$

$$f_1 = \frac{3}{2} f_2$$

$$\text{Now, } \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{150} = \frac{1}{f_2} + \frac{2}{3f_2}$$

$$f_2 = 50 \text{ cm and } f_1 = 75 \text{ cm}$$

$$46. \quad \text{Applying } \frac{\sin i_1}{\sin r_1}$$

Find angle  $r_1$  for two different refraction indices. Because  $i_1 = 65^\circ$  from both the cases.

Then again apply

$$\frac{\sin i_2}{\sin r_2} \text{ and find } i_2. \text{ Because } r_2 = A - r_1$$

Then apply :

$$(i_1 - i_2) = A$$

for two refraction indices. Then difference in deviations is :

$$= 1 - 2$$

## Objective Questions (Level-1)

1. Endoscope is based on total internal reflection. Hence, correct option is (c)

2. Here  $A = \frac{B}{2}$

$\therefore$  is dimensionless.

$$\frac{B}{2} \text{ dimension of}$$

$$B^2 \quad B \text{ has dimension of Area}$$

Hence, correct option is (d).

3. Shift  $= 1 \frac{1}{2}$

$\therefore R$  is minimum. than other visible colour.  
Red colour least raised.

correct option is (c)

4. Critical angle  $C = \sin^{-1} \frac{1}{2}$

$\therefore \theta_0$  is maximum for violet colour hence  $C$  for violet colour is least.

Hence correct option is (d)

5. We have  $P = \frac{1}{f(\text{metre})}$

$$\frac{100}{f(\text{cm})} = 100 (n-1) \frac{1}{R} = \frac{1}{r}$$

$$P = \frac{100 \times 0.6 \times 2}{10} = 12$$

Hence, correct option is (a).

6. Speed of light in water  $= \frac{c}{w}$

$$v_w = \frac{3 \times 10^8}{4/3} = 2.25 \times 10^8 \text{ m/s}$$

Hence correct option is (c).

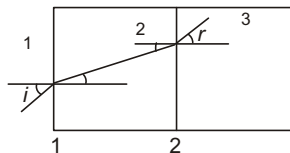
7. Due to TIR emergent beam will turn into black.

Hence correct option is (c).

8.  $\therefore v \propto n$  but frequency  $n$  remain constant and  $v$  decreases hence  $\lambda$  decreases.

Hence correct option is (b).

9. Using Snell's law



On first and 2nd interface

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} \quad \dots(i)$$

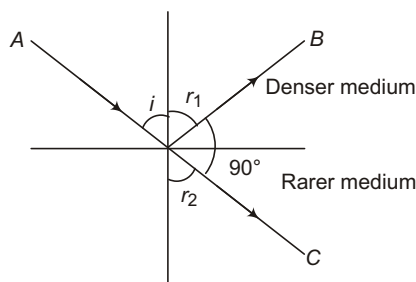
and  $\frac{\sin r}{\sin r'} = \frac{n_3}{n_2} \quad \dots(ii)$

Multiplying (i) and (ii), we get

$$\frac{\sin i}{\sin r'} = \frac{n_3}{n_1}$$

Hence correct option is (b).

10. We have  $i = r_1$  and  $r_2 = 90^\circ - i$



Now  $\frac{\sin i}{\sin r_2} = \frac{1}{n}$

$$\frac{\sin i}{\sin (90^\circ - i)} = \frac{1}{n}$$

$$\tan i = \frac{1}{n} \quad \dots(i)$$

If  $C$  is the critical angle then  $C = \sin^{-1} \frac{1}{n}$

$$C = \sin^{-1}(\tan i)$$

Hence correct option is (a).



11. Let angle of minimum deviation is  $m$   
we know that

$$\frac{\sin \frac{A+m}{2}}{\sin \frac{A}{2}} = \frac{\sin \frac{(60+m)}{2}}{\sin 30}$$

$$\frac{1}{\sqrt{2}} \sin \frac{60+m}{2} = \frac{60+m}{2} \quad 45 \quad m \quad 30$$

Hence correct option is (a).

12. We know that

$$\frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{a^2} = (1.5-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{0.2} = (0.5-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(i)$$

Let refractive index of the liquid is  $n_l$

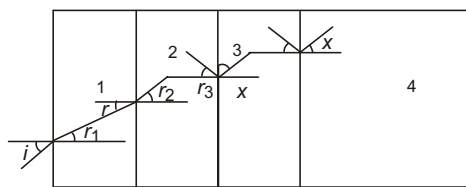
$$\frac{1}{f_{cm}} = \frac{n}{n_l} - 1 \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{0.5} = \frac{1.5}{n_l} - 1 \left( \frac{1}{5} - \frac{1.5}{n_l} \right)$$

$$\frac{1.5}{n_l} = \frac{4}{5} \quad \frac{5}{4} = \frac{1.5}{n_l} \quad n_l = \frac{15}{8}$$

Hence correct option is (b).

- 13.



We have,  $\frac{\sin i}{\sin r_1} = \frac{1}{n_l} \quad \dots(i)$

$$\frac{\sin r_1}{\sin r_2} = \frac{2}{1} \quad \dots(ii)$$

and  $\frac{\sin r_2}{\sin r_3} = \frac{3}{2} \quad \dots(iii)$

$$\frac{\sin r_3}{\sin x} = \frac{4}{3} \quad \dots(iv)$$

Multiplying (i), (ii), (iii) and (iv), we get

$$\frac{\sin i}{\sin x} = \frac{4}{3} \quad \sin x = \frac{3}{4} \sin i$$

Hence correct option is (b)

14. Let radius of curvature of the lens is  $R$

then  $\frac{1}{f} = (n-1) \left( \frac{1}{R} - \frac{1}{R} \right) = f = \frac{R}{2(x-1)}$

Let focal length of one part is  $f$

then  $\frac{1}{f} = (n-1) \left( \frac{1}{R} - \frac{1}{R} \right)$

$$f = \frac{R}{(n-1) \cdot 2f}$$

The focal length of the combination is

$$\frac{1}{F} = \frac{1}{2f} + \frac{1}{2f} = \frac{1}{f} \quad F = \frac{f}{2}$$

Hence correct option is (b).

15. Here  $P = 5D$   $f = 20 \text{ cm}$

$$\frac{1}{20} = (1.5-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(i)$$

and  $\frac{1}{100} = \frac{1.5}{n_e} - 1 \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(ii)$

Dividing Eq. (ii) by (i)

$$\frac{\frac{1}{100}}{\frac{1}{20}} = \frac{1.5}{n_l} - 1 \quad \frac{1.5}{n_l} = \frac{1}{5}$$

$$n_l = \frac{5}{3}$$

Hence correct option is (b).

16. We know that

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{d}{f_1 f_2}$$

$\therefore$  whole system is concave  $F < 0$

$$\frac{d}{f_1 f_2} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow d = f_1 + f_2$$

$$d = (10 + 20) \text{ cm} = 30 \text{ cm}$$

Hence only possible value of the given values is 40 cm.

Hence correct option is (d).

17. Since when  $A = 2c$  total light is reflect (TIR) take place hence maximum value of  $A$  is  $2c$ .

Hence correct option is (c).

18.  $\therefore$  emergent ray is to the surface

$$i_2 = r_2 = 0$$

$$\therefore r_1 = r_2 = A \Rightarrow r_1 = A$$

$$\text{Now } \frac{\sin i}{\sin A}$$

Since  $i$  and  $A$  are small

$$\sin i \sim i \text{ and } \sin A \sim A$$

$$\frac{i}{A} = i = A$$

Hence correct option is (c).

$$19. \frac{\sin \frac{A}{2}}{\sin \frac{A}{2}}$$

$$\cot \frac{A}{2} = \frac{\sin \frac{A}{2}}{\sin \frac{A}{2}}$$

$$\frac{A}{2} = 90 \Rightarrow \frac{A}{2}$$

$$A = 180 \Rightarrow A$$

$$m = 180 - 2A$$

Hence correct option is (d).

20. Minimum deviation condition

$$r_1 = r_2 = r = 90^\circ \Rightarrow r = A = 180^\circ$$

$$A = 2r$$

$$r = \frac{A}{2} = 30^\circ$$

Now

$$\frac{\sin i}{\sin r} = \frac{\sin i}{\sin 30^\circ}$$

$$\sin i = \frac{1}{\sqrt{2}}$$

$$i = 45^\circ$$

Hence correct option is (a).

$$21. \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{d}{f_1 f_2} \quad \dots(i)$$

$$\frac{1}{2F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{2d}{f_1 f_2} \quad \dots(ii)$$

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{4d}{f_1 f_2} \quad \dots(iii)$$

On solving (i), (ii) and (iii) we get  $F = 2F$ .

Correct option is (a).

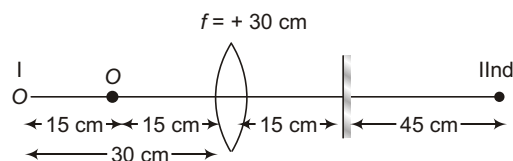
$$22. \left| \frac{1}{f} \right| = (n - 1) \left| \frac{1}{R_1} - \frac{1}{R_2} \right|$$

$$\frac{1}{24} = (1.5 - 1) \left| \frac{1}{R} - \frac{1}{2R} \right|$$

$$\frac{1}{24} = \frac{0.5}{2R} \Rightarrow R = 6 \text{ and } 2R = 12$$

Hence correct option is (a).

23. The system is shown in figure.



$$\text{Using } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \left( \frac{1}{-15} \right) = \frac{1}{30} \Rightarrow v = 30 \text{ cm}$$

This image  $I_1$  act as a virtual object for mirror since plane mirror form image at same distance as object. Hence the distance between object and image is  $(30 + 45) \text{ cm} = 75 \text{ cm}$ .

Hence correct option is (c).

$$24. \frac{\sin 45^\circ}{\sin r_1} = \frac{\sqrt{3}}{\sin r_2} \quad \dots(i)$$

$$\frac{\sin r_1}{\sin r_2} = \frac{\sqrt{2}}{\sqrt{3}}$$

Multiplying Eq. (i) and (ii)

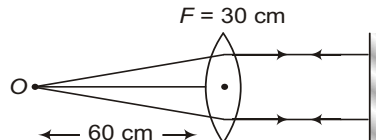
$$\frac{\sin 45^\circ}{\sin r_2} = \sqrt{2}$$

$$\sin r_2 = \frac{1}{2}$$

$$r_2 = 30^\circ$$

Hence correct option is (a).

25. For small angle prism



min (1) A if increases  
min increases

Hence correct option is (a)

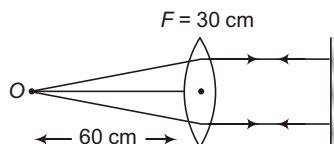
$$26. \frac{\sin i}{\sin r} = \frac{n_2}{n_1} = \frac{\sin i}{\sin \frac{i}{2}} = n$$

$$\therefore r = \frac{i}{2}$$

$$2 \cos \frac{i}{2} = n \quad i = 2 \cos^{-1} \frac{n}{2}$$

Hence correct option is (c).

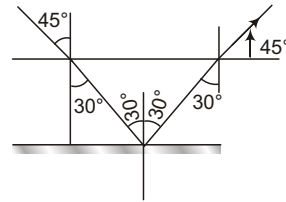
27. Situation is shown in figure.



Since image after reflection form on object itself hence the object must be placed at focus of the lens. The rays after refraction by lens becomes parallel to optic axis. Hence reflection rays follow the same path and final image form on  $x$  itself. Hence  $x = 30 \text{ cm}$ .

Correct option is (b).

$$28. \frac{\sin 45^\circ}{\sin r} = \frac{\sqrt{2}}{\sin r} = \frac{1}{2} \quad r = 30^\circ$$



hence total deflection  $45^\circ + (45^\circ - 90^\circ) = 0^\circ$

Hence correct option is (a).

$$29. \frac{\sin \frac{A}{2}}{\sin \frac{A}{2}} = \frac{\sin \frac{90^\circ}{2}}{\sin 45^\circ} = \frac{\sin 45^\circ}{\sin 45^\circ} = 1$$

Hence correct option is (c)

$$30. \frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\text{here } n = \frac{n_{\text{air}}}{n_{\text{glass}}} = \frac{1}{1.5}$$

$$\frac{1}{f} = \frac{1}{1.5} - \frac{1}{10} = \frac{1}{10}$$

$f = 15 \text{ cm}$ . Hence lens is concave

Correct option is (a).

31. For lens  $u = 12$  cm and  $F = 10$  cm

We have  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} - \frac{1}{12} = \frac{1}{10}$$

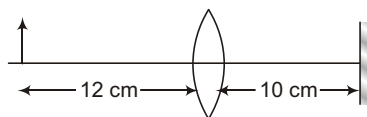
$$v = 60 \text{ cm}$$

Since the distance between lens and mirror is 10 cm. Hence the image formed 50 cm from convex mirror. The rays retrace its path if image is formed at the centre of curvature of the mirror i.e.,

$$R_{\text{convex}} = 50 \text{ cm} \quad F = \frac{R}{2} = 25 \text{ cm}$$

Hence the correct option is (b)

32.



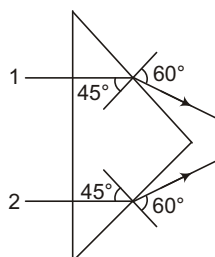
Using  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$   $\frac{1}{v} - \frac{1}{12} = \frac{1}{20}$

$$v = 30 \text{ cm}$$

Hence the distance between mirror and this image is 40 cm. Therefore second image formed 40 cm behind the mirror.

Hence correct option is (c).

33. We have  $\frac{\sin i}{\sin r}$



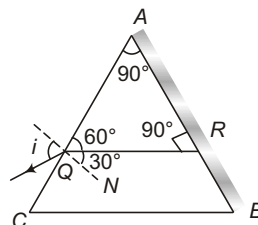
$$\frac{\sin r}{\sin i} = \frac{\sin 45^\circ}{\sin 60^\circ} \quad r = 60^\circ \text{ (on both faces)}$$

Hence the angle between emergent ray

$$90^\circ - (30^\circ - 30^\circ) = 30^\circ$$

Hence correct option is (b).

34.



Since the ray retraces its path hence

$$\angle ARQ = 90^\circ \quad \angle RQN = r = 30^\circ$$

$$\frac{\sin i}{\sin r} = \frac{\sin i}{\sin 30^\circ} = \frac{\sin r}{\sin 30^\circ}$$

$$\sin i = \frac{1}{\sqrt{2}} \quad i = 45^\circ$$

Hence correct option is (c).

35.  $f = 10$  cm

the focal length of  $\frac{1}{F} = \frac{1}{10} + \frac{1}{10}$

$$F = 5 \text{ cm}$$

Here  $u = 7.5$  cm,  $F = 5$  cm

$$\frac{1}{v} - \frac{1}{7.5} = \frac{1}{5} \quad \frac{1}{v} = \frac{1}{7.5} + \frac{1}{5} = \frac{7.5 + 5}{7.5 \times 5}$$

$$v = 15 \text{ cm}$$

Hence  $|m| = \left| \frac{v}{u} \right| = \frac{\text{height of image}}{\text{height of object}} = \frac{15}{7.5}$

Height of image = 2 cm

Hence correct option is (a).

36.  $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

$$\frac{1}{v} - \frac{3}{2} = \frac{1}{20} \quad \frac{1}{v} = \frac{3}{2} + \frac{1}{20} = \frac{30 + 1}{20} = \frac{31}{20}$$

$$v = 40 \text{ cm}$$

Hence correct option is (a).

37.  $\frac{1}{f} = \frac{n_g}{n_L} - 1 = \frac{1}{R_1} - \frac{1}{R_2}$

Here  $R_1 = 30$  cm and  $R_2 = 50$  cm

$$\frac{1}{f} = \frac{1.5}{1.4} - 1 = \frac{1}{30} - \frac{1}{50}$$

On solving

$$f = 1050 \text{ cm}$$

Hence correct option is (d).

38. From the figure, it is clear that  $\angle 1 = \angle 3 = \angle 2$

Hence correct option is (b).

39. Here  $A = 60^\circ$ ,  $m = 60^\circ$

$$\frac{\sin \frac{A}{2}}{\sin \frac{A}{m}} = \frac{\sin 60}{\sin 30}$$

$$\frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

Hence correct option is (a).

## JEE Corner

### Assertion and Reason

1. Due to shifting of image on refraction Shayam appear nearer to Ram and light suffer two refraction. Hence, both (a) and (b) are correct but reason does not explain the assertion.

Correct option is (b).

2. Applying lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Let  $u$  = object distance from lens

$v = d$   $u$  distance of image from lens.

$$\therefore \frac{1}{d} - \frac{1}{u} = \frac{1}{f}$$

$$u^2 - du - df = 0$$

$$u = \frac{d \pm \sqrt{d^2 + 4f}}{2}$$

$\therefore u$  is real hence  $d > 4f$

Thus mean distance  $v = 4f$ , if

$u = 2f$ ,  $v = 2f$ . Hence both assertion and reason are true, and reason explain or may not explain assertion. Hence correct option is (a, b)

3. Correct option is (b).

4. Correct option is (c).

5. Since both assertion and reason are true built reason is not explain assertion Hence correct option is (b).

6. Using mirror formula.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Here  $u =$

$$f \text{ (concave lens)}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$v = f$$

Hence image is formed at principle focus thus assertion is false but reason is true.

Hence correct option is (d).

7. 
$$\frac{\sin \frac{A}{2}}{\sin A/2}$$

Here  $A = 60^\circ$ ,  $\sqrt{2}$

$$\sqrt{2} = \frac{\sin \frac{60}{2}}{\sin 30}$$

$$\frac{60}{2} = 450^\circ \quad m = 30$$

Hence both assertion and reason are true and reason explain assertion correctly.

Correct option is (a).

8. Focal length of combination

$$\frac{1}{F} = \frac{1}{F_{\text{convex}}} + \frac{1}{F_{\text{convex}}} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\text{if } \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f_1} + \frac{1}{f_2}$$

$F$  negative

Hence assertion is true. Since power is a measure of converging or divergence of a lens. Hence reason is not true. Correct option is (c).

9. Since glass slab produced a net shift. Hence  $v$  is increased. Thus magnified image is obtained but image may be real or virtual depending on the position of slab.

Correct option is (b)

10. In this case image distance of  $O_1$  and  $O_2$  are same from the lens.

$$\therefore \frac{1}{v} = \frac{1}{u} + \frac{1}{f} \text{ and reason is true.}$$

Hence correct option is (d).

11. Assertion is false since only ray emerge if refractive index of the colour less than the prism and angle of incidence is less than critical angle but reason is true. Correct option is (a).

12. If two object is placed between pole and focus image is real hence assertion is true. Also reason is correct.

Hence correct option is (b)

13. Since both assertion and reason are true and reason explanation is correct.

Hence correct option is (a).

## Objective Questions (Level 2)

### ■ Single option correct

1. We have  $\frac{\text{Real depth}}{\text{App. depth}}$

$$\frac{4}{3} = \frac{1}{\text{App. depth}}$$

$$\text{App. depth} = \frac{3}{4}$$

Hence the distance between bird and mirror

$$2 \times \frac{3}{4} = \frac{11}{4} \text{ m}$$

Since plane mirror form image behind the mirror (for real object) at same distance as object hence the distance between bird and its image

$$\frac{11}{4} + \frac{11}{4} = \frac{11}{2} \text{ m}$$

Correct option is (d).

$$2. \frac{2}{v} = \frac{1}{u} + \frac{2}{R}$$

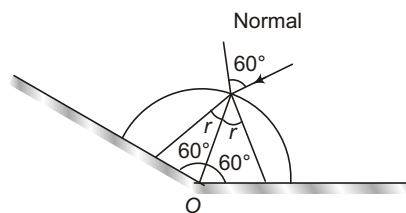
Here  $u_1 = 1$

and

$$\frac{1.5}{v} = \frac{1}{1} + \frac{0.5}{R} \quad v = 3R$$

Hence correct option is (b).

3. From figure,  $r = 30^\circ$



$$\therefore r = 90^\circ - 60^\circ = 30^\circ$$

Hence  $\frac{\sin 60}{\sin 30} = \sqrt{3}$

The correct option is (d).

4. The lens become diverging if

$$\begin{array}{cccc} & 1 & 2 & 2 & 3 \\ & 1 & 3 & 2 & 2 \\ \text{or} & 2 & 2 & 1 & 3 \end{array}$$

Hence correct option is (b).

5.  $\frac{1}{v_1} = \frac{1}{16} - \frac{1}{f}$   $\frac{1}{v_1} = \frac{16f}{16f - f}$

$$m_1 = \frac{v_1}{16} = \frac{16f}{16(16f - f)} = \frac{f}{16f - f} \quad \dots(i)$$

and  $\frac{1}{v_2} = \frac{1}{6} - \frac{1}{f}$   $\frac{1}{v_2} = \frac{6f}{6f - f}$

$$v_2 = \frac{6f}{5} \quad (\because \text{image is virtual})$$

$$m_2 = \frac{6f}{6(6f - f)} = \frac{f}{6f - f} \quad \dots(ii)$$

But  $m_1 = m_2$   $\frac{f}{16f - f} = \frac{f}{6f - f}$

$$6f - f = 16f - f \quad 2f = 22f$$

$$f = 11 \text{ cm}$$

Hence correct option is (d).

6. Let real depth at any instant  $t$  of the water is  $h$  then volume of water  $V = R^2 h$

$$\frac{dV}{dt} = R^2 \frac{dh}{dt} \quad \dots(i)$$

Let apparent depth at this instant is  $h$

$$\therefore \frac{\text{Real depth}}{\text{Apparent depth}}$$

$$\frac{n_2}{n_1} = \frac{h}{h_1} \quad h_1 = \frac{n_1}{n_2} h$$

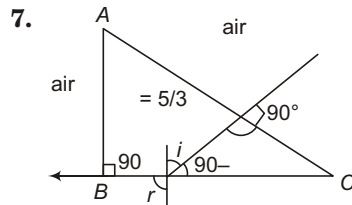
Now  $\frac{dh}{dt} = x \quad \frac{n_1}{n_2} \frac{dh}{dt}$

$$\frac{dh}{dt} = \frac{x n_2}{n_1} \quad \dots(ii)$$

From Eq. (i) and (ii), we get

$$\frac{dV}{dt} = \frac{R^2 x n_2}{n_1}$$

Hence correct option is (b).



$$\therefore \sin i = \sin [90 - (90 - r)] = \sin r$$

Applying Snell's law on face BC.

$$\frac{\sin 37}{\sin r} = \frac{5}{3} \quad \frac{3}{5} \sin 37 = \sin r$$

$$\sin r = \frac{4}{3} \sin 37 = \frac{5}{3} \cdot \frac{3}{5} = 1$$

$$r = 90$$

Hence deviation  $= 90 - 37 = 53$

Correct option is (b).

8. Let refractive index of liquid is

For position of fish w.r.t. bird is

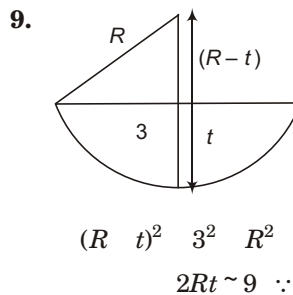
$$\frac{\text{Real depth}}{\text{App. depth}} = \frac{x}{h_1} \quad \dots(i)$$

For position of bird w.r.t. fish is

$$\frac{1}{\frac{y}{h_2}} = \frac{h_2}{h_1} \quad \dots(2)$$

From Eq. (i) and (ii) we get  $u = \frac{h_2}{h_1}$

Hence correct option is (a).



$$2R = \frac{900 \text{ mm}}{3 \text{ mm}} = 300 \text{ mm}$$

$$R = 150 \text{ mm} = 15 \text{ cm}$$

Hence correct option is (a).

$$10. \frac{1}{v_1} = \frac{1}{v_1} - \frac{1}{f} \quad v_1 = \frac{fv_1}{f} \quad \dots(i)$$

$$\frac{1}{v_2} = \frac{1}{v_2} - \frac{1}{f} \quad v_2 = \frac{fv_2}{f}$$

$$m_2 = \frac{v_2}{v_1} = \frac{f}{f} \quad \dots(ii)$$

$$\therefore m_1 = m_2 = \frac{f}{v_1} = \frac{f}{v_2} = \frac{v_1}{v_2}$$

Hence the correct option is (d).

$$11. \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{d}{f_1 f_2}$$

$$F = \frac{f_1 f_2}{f_1 + f_2} = d$$

as  $d$  increases  $f_1 + f_2$  decreases hence  $F$  increases. Hence image move to right.

Correct option is (b).

12. In this case, minimum deviation of ray 1 is same as ray 2.

Hence correct option is (c).

13. For critical angle at glass air surface

$$\sin c = \frac{1}{n_g} = \frac{2}{3} \quad \dots(i)$$

Now for glass water surface.

$$\frac{w}{g} = \frac{\sin c}{\sin r} = \frac{4/3}{3/2} = \frac{2}{3} \sin r$$

$$\sin r = \frac{4}{3}$$

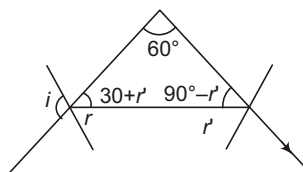
Now for water air surface

$$\frac{w}{a} = \frac{\sin r}{\sin r} = \frac{4}{3} = \frac{1}{\sin r}$$

$$\sin r = \frac{1}{4} \quad r = 90^\circ$$

Hence correct option is (d).

14. For limiting angle of incident emergent ray become parallel to the 2nd face



$$\therefore \frac{\sin r}{\sin 90} = \frac{\sqrt{3}}{7} \quad r = \sin^{-1} \frac{\sqrt{3}}{7}$$

$$\text{Now } r = 30^\circ \quad r = 90^\circ$$

$$r = 60^\circ \quad r = 90^\circ$$

$$r = (60^\circ \sin^{-1} \frac{\sqrt{3}}{7})$$

Now

$$\frac{\sin i}{\sin r}$$

$$\frac{\sqrt{7}}{3} = \frac{\sin i}{\sin [60^\circ \sin^{-1} \frac{\sqrt{3}}{7}]}$$

$$i = \sin^{-1} \{ \frac{\sqrt{7}}{3} \sin (60^\circ \sin^{-1} \frac{\sqrt{3}}{7}) \}$$

$$\sin^{-1} \frac{\sqrt{7}}{3} = \sin (60^\circ \sin^{-1} \frac{\sqrt{3}}{7})$$

$$\sin^{-1} \{ \frac{\sqrt{7}}{3} \sin 19^\circ \}$$

$$\sin^{-1} \{0.49\} \sim 30^\circ$$

Hence correct option is (a).

15. The image form the object itself if the rays incident parallel to optical axis on the mirror i.e., image of reflection is formed at  $O$ . It is possible when  $O$  is placed at focus i.e.,  $d = 10 \text{ cm}$

Hence correct option is (c).

16. The dot will appear at  $c$  for all values of  $\lambda$ . Since position does not in same medium.

Hence correct option is (b).





24. If the plane surface of plano-convex lens is silvered it behave the concave mirror of focal length  $f_m/2$

$$\therefore \begin{aligned} f_m &= 10 \text{ cm} \\ f_e &= 5 \text{ cm} \text{ hence } R = 10 \text{ cm} \end{aligned}$$

Correct option is (c).

25. Since lens made real and magnified image, hence it is a convex lens when lens dipped in water its focal length.

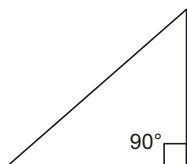
$$\frac{1}{f} = \frac{1}{f_g} - \frac{1}{f_w} \quad 1 = \frac{1}{R_1} - \frac{1}{R_2}$$

$$\frac{4/3}{3/2} = 1 = \frac{1}{R_1} - \frac{1}{R_2} \quad \frac{1}{9} = \frac{1}{R_1} - \frac{1}{R_2}$$

$\therefore f$  is ve lens behave as concave, hence the image is virtual and magnified.

Correct option is (c).

26. The prism transmit the light for which angle of incidence(c),  $2c = 90^\circ$   $c = 45^\circ$



$$\text{Hence } \frac{1}{\sin C} = \frac{1}{\sin 45^\circ} = \sqrt{2} = 1.414$$

Correct option is (b).

$$27. \frac{1}{v} = \frac{1}{u} - \frac{1}{f} \therefore |m| = \left| \frac{v}{u} \right| = 3$$

For convex lens  $v = 3u$

$$\frac{1}{48} = \frac{1}{16} - \frac{1}{f} \quad f = 12 \text{ cm (for real image)}$$

Similarly when distance is 6 cm, 3 times virtual image is formed hence mirror is convex with focal length 12 cm.

Correct option is (c).

$$28. v_g = n_g \frac{c}{g} = n_g \dots (i)$$

$$\text{and } \frac{c}{w} = n_w \dots (ii)$$

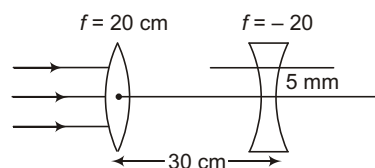
Dividing (i) and (ii)

$$\frac{w}{g} = \frac{g}{w} \quad \frac{4}{3} = \frac{4}{5}$$

$$g = \frac{5}{3}$$

Hence correct option is (a).

$$29. x = \frac{f_1 f_2}{f_1} \frac{d(f_1 - d)}{f_2} \text{ and } y = \frac{(f_1 - d)}{f_1} \frac{d}{f_2}$$



Here  $f_1 = f_2 = 20 \text{ cm}$ ,  $d = 30 \text{ cm}$

and  $55 \text{ mm} = 0.55 \text{ cm}$

Putting these values we get

$$x = 25 \text{ cm} \text{ and } y = 0.25 \text{ cm}$$

Correct option is (b).

30. Since for each angle of incidence at glass-air boundary remains  $0^\circ$  hence there will never be total internal reflection.

Correct option is (d).

31. Diameter Original diameter

$$\frac{4}{3} = 1 \text{ cm} \quad \frac{4}{3} \text{ cm}$$

Hence correct option is (a).

$$32. \frac{1}{f_1} = (1.5 - 1) \left( \frac{1}{20} - \frac{1}{20} \right)$$

$$f_1 = 20 \text{ cm}$$

Here  $u_1 = 30 \text{ cm}$

$$\frac{1}{v} = \frac{1}{u_1} - \frac{1}{f_1} \quad \frac{1}{v} = \frac{1}{30} - \frac{1}{20} \quad v_1 = 60 \text{ cm}$$

$$\text{Magnification } |m_1| = \left| \frac{v}{u} \right| = \frac{60}{30} = 2$$

(Inverted image)

For second lens,  $\frac{1}{v_2} = \frac{1}{60} - \frac{1}{20}$   $v_2 = 30$  cm

Magnification  $|m_2| = \left| \frac{30}{60} \right| = \frac{1}{2}$

Total magnification  $m_1 m_2 = 1$

Hence object size remains 3mm and it is formed at (120 - 30) cm = 90 cm from first lens.

Hence correct option is (b).

- 33.** From left hand side refraction occur from  $n_2 = 2$  to  $n_1 = 1$ .

$$n \frac{n_1}{n_2} = \frac{1}{2} \Rightarrow 0.5$$

$$\frac{n}{v} = \frac{1}{u} - \frac{n}{R}$$

$$\frac{0.5}{v} = \frac{1}{10} - \frac{0.5}{10} \Rightarrow \frac{0.5}{v} = \frac{1}{10} - \frac{0.5}{10}$$

$$v = 10 \text{ cm}$$

Hence correct option is (a).

- 34.** For lens  $v = 20$  cm,  $f = 10$  cm

Using  $\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$  we get  $v = 20$  cm

This image acts virtual object for convex mirror. For mirror  $v = (x - 20)$

and  $v = (20 - x)$  and  $f = 60$  cm

Using  $\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$

$$\frac{1}{(20 - x)} = \frac{1}{(x - 20)} + \frac{1}{60}$$

After solving we get  $x = 20$  cm.

Hence correct option is (c).

- 35.** In this case system behave as concave mirror or focal length  $f_e$

$$1.5 \times 20 = 30 \text{ cm}$$

Now using mirror formula

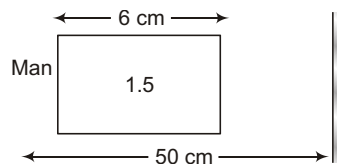
$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} \text{ we get}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{30}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{30} \Rightarrow v = 60 \text{ cm}$$

Hence correct option is (d).

- 36.** Shift in mirror  $= 6 \times \frac{1}{1.5} = 4$  cm



Hence man lie at 48 cm from mirror. The distance of image from observer  $= 2 \times 48 = 96$  cm

Hence correct option is (b).

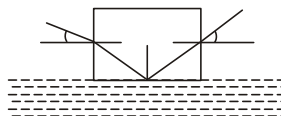
- 37.** Using lens formula  $\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$

$$\frac{1}{(f - 40)} = \frac{1}{(f - 10)} + \frac{1}{f}$$

On solving we get  $f = 20$  cm

Hence correct option is (c).

- 38.**



$$\frac{\sin i}{\sin(90^\circ - c)} = \frac{n_1}{n_2}$$

$$\frac{\sin i}{\cos c} = \frac{n_1}{n_2}$$

$$\text{Now } \cos c = \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

$$\sin i = \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}} \Rightarrow \sin i = \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

Hence correct option is (a).

39. For mirror  $u = 1 \text{ cm}$  (taking upward direction -ve)

$$\begin{aligned} f &= 2 \text{ cm} \\ \frac{1}{v} + \frac{1}{f} &= \frac{1}{u} \\ \frac{1}{v} + \frac{1}{2} &= \frac{1}{1} \\ v &= 2 \text{ cm} \end{aligned}$$

Hence mirror form virtual image behind mirror at 2 cm from pole. This image acts as virtual object for slab on see below the slab the shift is

$$t = 1 \times \frac{1}{3/2} = 2/3 \text{ cm}$$

Hence virtual image form on object thus correct option is (a).

40. We have  $\frac{2}{v} + \frac{1}{u} = \frac{2}{R} + \frac{1}{u}$

For real object  $u$  is -ve

$$\frac{2}{v} + \frac{1}{R} = \frac{1}{u}$$

$\therefore R$  is -ve and  $1/v > 1/R$

$$\frac{2}{v} > \frac{1}{u}$$

$$v < u$$

Hence, if  $1/v > 1/u$  then these cannot be real image of real object.

Hence, correct option is (a).

41. At oil-concave surface

$$\frac{1}{f_1} = (1.6 - 1) \left( \frac{1}{10} - \frac{1}{f_1} \right)$$

At other surface light goes from oil to glass

$$\begin{aligned} \frac{1}{f_2} &= \frac{1.5}{1.6} \left( \frac{1}{10} - \frac{1}{20} \right) \\ \frac{1}{f_2} &= \frac{0.3}{20 \times 1.6} \end{aligned}$$

Let focal length of combination is  $F$

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{0.6}{10} + \frac{0.3}{20 \times 1.6}$$

$$F = 28.57 \text{ cm}$$

Hence correct option is (d).

42. Using formula

$$\frac{2}{v} + \frac{1}{u} = \frac{2}{R}$$

For real image  $v$  is -ve

$$\frac{2}{v} + \frac{1}{u} = \frac{2}{R}$$

$$\frac{1.5}{v} + \frac{0.5}{R} = \frac{1}{u}$$

$$\frac{1.5}{v} + \frac{1}{2R} = \frac{1}{u} \quad v \text{ is -ve if } u > 2R$$

Hence correct option is (b).

43.  $v = \frac{fu}{f - u}$  when  $u < f$

$u$  lend to lens

$$\text{if } u < 0, v > 0$$

Hence correct option is (d).

44. Since image formed by diverging lens is always virtual.

Hence correct option is (a)

45. If the object place at first focus the image forms at  $\infty$ , rays incident the plane surface normally and retrace its path.

$$\text{Now } x = f_1 = \frac{1}{\frac{1}{2} - \frac{1}{1}} = \frac{60}{0.5} = 120 \text{ cm}$$

Hence correct option is (a).

46.  $\frac{2}{v_1} + \frac{1}{u_1} = \frac{2}{R} + \frac{1}{u_1}$  for  $u_1 = 1.6$

$$\frac{1.6}{v_1} + \frac{1}{(2)} = \frac{1.6}{1} + \frac{1}{10}$$

$$\frac{1.6}{v_1} + \frac{6}{10} = \frac{1}{2} + \frac{1}{10}$$

$$v_1 = 16 \text{ m}$$

For  $u_2 = 2$

$$\frac{2}{v_2} - \frac{1}{(2)} = \frac{2}{1} - \frac{1}{1}$$

$$\frac{2}{v_2} - 1 = \frac{1}{2} - \frac{1}{2} \quad v_2 = 4 \text{ m}$$

Hence separation between images  $v_1 - v_2$   
 $(16 - 4) \text{ cm}$   
 $12 \text{ cm}$

Hence correct option is (a).

47. System behave as a concave mirror

Here  $u = 10, v = 40 \text{ cm}$

Using mirror formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ we get}$$

$$\frac{1}{40} - \frac{1}{10} = \frac{1}{f}$$

## ■ More than one options correct

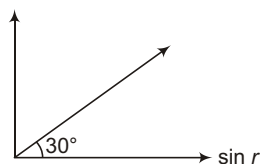
1. Since prisms are identical hence if a right prism produces deviation, inverted prism produces deviation.

if  $n = 2m$  deviation becomes zero.

if  $n = 2m - 1$  deviation produced is

Hence correct options are (a) and (b).

2.  $\sin i$



$$\therefore \sin i = \sin r$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

if speed of light in medium  $x$  is  $v$

then speed of light in medium  $y = \frac{v}{\sqrt{3}}$

Since  $y$  is denser w.r.t.  $x$  hence total internal reflection takes place when incidence is in  $y$ .

$$f = 8 \text{ cm}$$

Hence correct option is (a).

48. The focal length of lens combination is  $2f$

$$\text{Hence } \frac{1}{2f} = \frac{(1.8 - 1.2)}{(1.2 - 1)} - \frac{1}{R}$$

$$\frac{1}{16} = \frac{0.6}{0.2R}$$

$$R = 48 \text{ cm}$$

Hence correct option is (a).

49. If plane surface is silvered the system acts as a concave mirror having focal length

$$\frac{R}{2} = 24 \text{ cm}$$

Hence correct option is (c).

The correct options are (b) and (d).

3. The correct options are (b) and (c).

4. The lens forms a real image if  $D < 4F$  (displacement method)

$$\text{and } f = \frac{D^2 - x^2}{4D}$$

and the magnification  $m_1 m_2 = 1$

Hence correct options are (b), (c) and (d).

5. Deviation produced by prism

$$(1) A = (1.5 - 1) 4 = 2$$

if the mirror is rotated  $2^\circ$  ray becomes horizontal after reflection from mirror.

Again if mirror is rotated by  $1^\circ$  reflection ray deviates by  $2^\circ$  from horizontal and after passing through prism again ray becomes horizontal.

Hence correct options are (a) and (b).

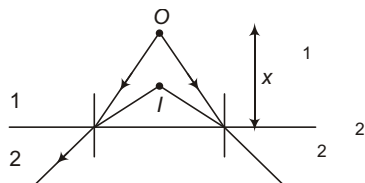
### ■ Match the Columns

1. Correct match is

- (a)  $q, r$  (b)  $p, s$   
(c)  $p, r$  (d)  $p, r$

2. (a)  $p, r$  (b)  $q, s$   
(c)  $q, r$  (d)  $q, r$

3.

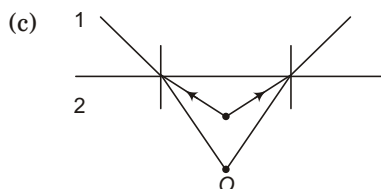


Hence image distance is less than  $x$  and virtual.

- (a)  $q, s$

Similarly as in (a) the correct match for

- (b)  $q, r$



- (c)  $p, s$

- (d)  $p, r$

4. (a)  $q,$

- (b)  $r$

- (c)  $r$

- (d)  $p$

5. Since  $\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$

here  $n = 1.5$  for (a)  $n = 1.4$

$$\frac{1}{f} = \frac{1.5}{1.4} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$f$  is ve

hence

- (a)  $p, s \because f$  increases, power decreases

- (b)  $q, s \because f$  is ve and increase in magnification

Similarly

- (c)  $q, s$  and

- (d)  $p, s$

6. For real object at  $2c$  convex lens form image at  $2c$  similarly for virtual object concave lens does.

- (a)  $q, s$

- (b)  $q, r$

- (c)  $q, r$

- (d)  $q, r$

# 28 Interference and Diffraction of Light

## Introductory Exercise 28.1

1. Because they are incoherent *ie*, does not remain constant.

2. Since laser is highly coherent and monochromatic source of light

3.  $I = I_0 \cos^2 \theta / 2$

$$\frac{3I_0}{4} = I_0 \cos^2 \theta / 2$$

$$\cos \theta = \sqrt{2} \frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1} \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6}$$

$$x = \frac{\lambda D}{d} \sin \theta = \frac{\lambda D}{d} \sin \frac{\pi}{6}$$

But

$$x = \frac{y d}{D}$$

$$y = \frac{D}{d} x = \frac{D}{d} \frac{\lambda D}{d} \sin \theta$$

$$y = \frac{1.2}{0.25} \frac{600}{10^9} \frac{10}{6} = 48 \text{ m}$$

4.  $2t = n \frac{\lambda}{2}$  for minimum thickness  $n = 1$

$$t = \frac{\lambda}{4} = \frac{3}{4 \times 1.5} = 0.5 \text{ cm}$$

5. Here  $a_1 = 3a$  and  $a_2 = a$

$$R^2 = (3a)^2 + (a)^2 = 2 \times 3a^2 = 6a^2 \Rightarrow R = \sqrt{6} a$$

$$I = 9I_0 + I_0 - 6I_0 \cos \theta$$

$$I = [10 - 6 \cos \theta] I_0$$

$$I = 10 I_0 - 6 I_0 \cos^2 \frac{\theta}{2} = 10 I_0 - 6 I_0 \cos^2 \frac{\theta}{2}$$

$$10 I_0 - 12 \cos^2 \frac{\theta}{2} = 6 I_0$$

$$4 I_0 = 12 \cos^2 \frac{\theta}{2} \Rightarrow \cos^2 \frac{\theta}{2} = \frac{1}{3}$$

$$4 I_0 = 12 \cos^2 \frac{\theta}{2} \Rightarrow \cos^2 \frac{\theta}{2} = \frac{1}{3}$$

$$\text{Now, } I_{\max} = \frac{I_0}{9}$$

$$I = \frac{4}{9} I_{\max} = 12 \cos^2 \frac{\theta}{2}$$

$$6. x = \frac{y d}{D} \sin \theta = \frac{y d}{D} \sin \theta$$

$$\text{if } \theta = \frac{\pi}{2} \Rightarrow x = \frac{y d}{D}$$

$$x = \frac{y d}{D} \sin \theta$$

For maxima  $x = n \frac{\lambda D}{d}$

$$\frac{y d}{D} \sin \theta = n \frac{\lambda D}{d}$$

This become minima.

For minima  $x = n \frac{\lambda D}{d}$

$$\frac{y d}{D} \sin \theta = n \frac{\lambda D}{d}$$

$$\frac{y d}{D} \sin \theta = n \frac{\lambda D}{d} \text{ this become maxima.}$$

Hence maxima and minima are interchanged.

7. For two slit experiment

$$d \sin \theta = n \lambda$$

$$\sin \theta = \frac{n \lambda}{d}$$

But  $\sin \theta = 1 \Rightarrow \frac{n \lambda}{d} = 1$

$$n = \frac{d}{\lambda} = \frac{4 \times 10^{-6}}{6 \times 10^{-7}} = 6.67$$

$$n = 6$$

8. Since amplitude of each wave is equal. The amplitude of resultant wave is zero if waves are equally displaced in phase

$$\text{i.e., } \frac{360}{8} = 45^\circ$$

Hence phase difference must be  $45^\circ$

## AIEEE Corner

### ■ Subjective Question (Level-1)

1.  $R^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \theta$

(i)  $R = 2a, a_1 = a_2 = a$   
 $4a^2 = a^2 + a^2 + 2a^2 \cos \theta$

$$\cos \theta = 1 \Rightarrow \theta = 0$$

(ii)  $2a^2 = 2a^2 + 2a^2 \cos \theta$

$$90^\circ$$

(iii)  $a^2 = 2a^2 + 2a^2 \cos \theta$

$$\cos \theta = \frac{1}{2} \Rightarrow \theta = 120^\circ$$

(iv)  $0 = 2a^2 + 2a^2 \cos \theta \Rightarrow \theta = 180^\circ$

2.  $\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{a_1^2 + a_2^2 + 2a_1a_2}{a_1^2 + a_2^2 - 2a_1a_2}$   

$$= \frac{5}{3} = \frac{8^2}{2^2} = 16$$

$$I_{\max} : I_{\min} = 16 : 1$$

3.  $I = I_{\max} \cos^2 \frac{\theta}{2}$

$$\frac{I_{\max}}{2} = I_{\max} \cos^2 \frac{\theta}{2}$$

$$\cos \frac{\theta}{2} = \cos \frac{\pi}{4}$$

Now,  $\frac{2}{2} = x$

$$\frac{2}{2} = x$$

$$x = \frac{\pi}{4}$$

But  $x = \frac{y d}{D}$

$$y = \frac{D x}{d}$$

$$y = \frac{1 \text{ m}}{4 \times 1 \text{ mm}} = \frac{1 \times 500 \times 10^{-9}}{4 \times 10^{-3}}$$

$$y = 1.25 \times 10^{-4} \text{ m}$$

4.  $I = I_{\max} \cos^2 \frac{\theta}{2}$

(a)  $\frac{I_{\max}}{2} = I_{\max} \cos^2 \frac{\theta}{2} \Rightarrow \frac{\theta}{2} = \frac{\pi}{2}$

Now  $\frac{2}{2} = x$

$$x = \frac{\pi}{4} \text{ and } x = \frac{y d}{D} = \frac{D}{4d}$$

(b)  $\frac{1}{4} I_{\max} = I_{\max} \cos^2 \frac{\theta}{2}$

$$\cos \frac{\theta}{2} = \cos \frac{\pi}{3} \Rightarrow \frac{\theta}{2} = \frac{\pi}{3}$$

$$\frac{2}{2} = x$$



$$\frac{2}{3} \frac{2}{3} x \quad x \frac{2}{3}$$

Now  $x \frac{yd}{D} \quad y \frac{D}{3d}$

5. (a)  $I = I_{\max} \cos^2 \frac{\pi}{2}$

$$I_{\max} = I_0 \text{ and } 60$$

$$I = I_0 \cos^2 30 = \frac{3}{4} I_0 = 0.75 I_0$$

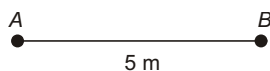
(b)  $\frac{2}{3} x$

$$\frac{2}{3} x$$

$$x = \frac{2}{6} \frac{480}{6} \text{ nm}$$

$$x = 80 \text{ nm}$$

6. A B 6m



For constructive interference difference wavelength  $0, \lambda, 2\lambda$

$$\therefore \frac{6}{\lambda} = d_{AB} = 5$$

Hence only constructive interference occur at  $0$

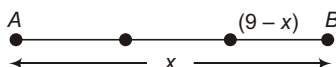
$$x = \frac{5}{2} \lambda = 2.5 \text{ m}$$

For destructive interference  $\frac{\lambda}{2}, \frac{3\lambda}{2}$

Only possibility at  $6$

Which occur at  $x = 1 \text{ m}$  and  $x = 4 \text{ m}$  from A.

7. The wavelength  $\frac{c}{n} = \frac{3 \times 10^8}{120 \times 10^6} = 2.5 \text{ m}$



For constructive interference

$$x = (9 - x) \lambda = n \lambda, \text{ where } n = 0, 1, 2, \dots$$

$$x = 4.5, 5.75, 7, 8.25$$

The other points are 3.25, 2.75

8. In Young's double slit experiment

$$w = \frac{D}{d}$$

$$2.82 \times 10^{-3} \text{ m} = \frac{2.2}{.460 \times 10^{-3}}$$

$$590 \text{ nm}$$

9.  $x_n$  for bright fringe is given by

$$x_n = \frac{nD}{d}$$

$$x_1 = \frac{D}{d} \text{ and } x_2 = \frac{2D}{d}$$

$$x = \frac{D}{d} \text{ the angular separation}$$

$$\sin(\theta) = \frac{x}{D} = \frac{5 \times 10^{-7}}{2 \times 10^{-3}} = 2.5 \times 10^{-4}$$

$$(\theta) = \sin^{-1}[0.00025]$$

$$0.014$$

10. When whole apparatus is immersed in water

$$n_w = \frac{4}{3} \quad 700 \times 10^{-9} \text{ m}$$

$$w = \frac{D}{d} = \frac{48 \times 10^{-2} \times 4 \times 10^{-7}}{3 \times 25 \times 10^{-5}} = 0.90 \text{ mm}$$

11.  $w_1 = \frac{D_1}{d}$  and  $w_2 = \frac{D_2}{d}$

$$w_1 - w_2 = \frac{(D_1 - D_2)}{d}$$

$$3 \times 10^{-5} = \frac{1.5 \times 10^{-2}}{10^{-3}}$$

$$\frac{3}{1.5} \times \frac{10^{-8}}{10^{-2}} = 2 \times 10^{-6} \text{ m} = 2 \text{ nm}$$

12. For bright fringe  $x_n = \frac{Dn}{d}$

For first light ( $480 \text{ nm}$ ) the third order  
Bright fringe is  $x_3 = \frac{1 \times 3 \times 480 \times 10^{-9}}{5 \times 10^{-3}}$

For second light ( $600 \text{ nm}$ )

$$x_3 = \frac{1 \times 3 \times 600 \times 10^{-9}}{5 \times 10^{-3}}$$

$$x_3 = \frac{3 \cdot 10^9 (600 \cdot 480)}{5 \cdot 10^3}$$

$$\frac{3 \cdot 120}{5} \cdot 10^6$$

$$x_3 = 72 \cdot 10^6, \quad x_2 = 72 \text{ m}$$

13. Fringe width :

$$\frac{D}{d} = \frac{(500 \cdot 10^9)(75 \cdot 10^2)}{(0.45 \cdot 10^3)} \text{ m}$$

$$0.83 \cdot 10^3 \text{ m} = 0.83 \text{ mm}$$

Distance between second and third dark line = one fringe width = 0.83 mm.

14. For first order bright fringe

$$x = \frac{D}{d}$$

$$4.94 \cdot 10^3 = \frac{3 \cdot 600 \cdot 10^9}{d}$$

$$d = \frac{18 \cdot 10^7}{4.94 \cdot 10^3} = \frac{18}{5.94} \cdot 10^4 \text{ m}$$

Let for wavelength first dark fringe is obtained at this point for first dark fringe

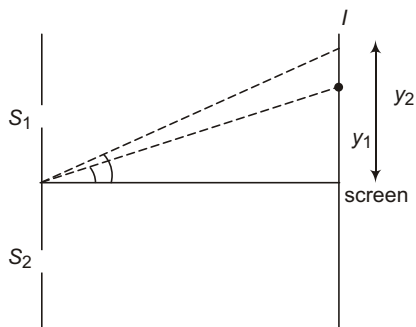
$$x = \frac{D}{2d}$$

$$4.94 \cdot 10^3 = \frac{3 \cdot 4.94}{2 \cdot 18 \cdot 10^4}$$

$$2 \cdot 18 \cdot 10^7 = 3$$

$$12 \cdot 10^7 \text{ m} = 1200 \text{ nm}$$

15.



For dark fringe

$$d \sin \theta = (2n - 1) \frac{\lambda}{2}$$

For first dark fringe  $n = 1$  and for 2nd,  $n = 2$

$$d \sin \theta = \frac{\lambda}{2}$$

$$d \sin \theta = \frac{3\lambda}{2} \quad \sin \theta = \frac{3\lambda}{2d}$$

$$\frac{y_1}{\sqrt{D^2 + y_1^2}} = \frac{\lambda}{2d}$$

$$\text{and } \frac{y_2}{\sqrt{D^2 + y_2^2}} = \frac{3\lambda}{2d}$$

$$\frac{y_1}{\sqrt{(35 \cdot 10^2)^2 + y_1^2}} = \frac{550 \cdot 10^9}{2 \cdot 1.8 \cdot 10^6} \quad \dots(i)$$

$$\text{and } \frac{y_2}{\sqrt{(35 \cdot 10^2)^2 + y_2^2}} = \frac{3 \cdot 550 \cdot 10^9}{2 \cdot 1.8 \cdot 10^6}$$

On solving  $y_2 - y_1 = 12.6 \text{ cm}$

16. Wavelength of source  $\lambda = \frac{h}{\sqrt{2meV}}$

$$\frac{6.62 \cdot 10^{-34}}{\sqrt{2 \cdot 9.1 \cdot 10^{-31} \cdot 100 \cdot 1.6 \cdot 10^{19}}}$$

$$1.24 \cdot 10^{-10} \text{ m}$$

$$w = \frac{D}{d} = \frac{3 \cdot 1.24 \cdot 10^{-10}}{10 \cdot 10^{-10}} \text{ m} = 36.6 \text{ cm}$$

17. Given  $\lambda = 546 \text{ nm} = 5.46 \cdot 10^{-7} \text{ m}, D = 1 \text{ m}$

$$d = 0.3 \text{ mm} = 0.3 \cdot 10^{-3} \text{ m}$$

(a) At distance  $y = 10 \text{ mm} = 10 \cdot 10^{-3} \text{ m}$  from the central fringe path difference will be

$$y = \frac{y \cdot d}{D} = \frac{10 \cdot 10^{-3} \cdot 0.3 \cdot 10^{-3}}{1} = 3 \cdot 10^{-6} \text{ m}$$

Corresponding phase difference

$$\frac{2\pi}{\lambda} \cdot y = \frac{2\pi}{5.46 \cdot 10^{-7}} \cdot 3 \cdot 10^{-6} \text{ rad}$$

$$I = I_0 \cos^2 \frac{\theta}{2}$$

$$I_0 \cos^2 \frac{1978}{2} = I_0 \cos^2 989$$

$$I = 3 \times 10^{-4} I_0$$

(b) Fringe width  $w = \frac{D}{d}$

$$w = \frac{5.46 \times 10^{-7} \times 1}{0.3 \times 10^{-3}} = 1.82 \text{ mm}$$

$$\text{Number of fringes} = \frac{10 \text{ mm}}{1.82 \text{ mm}} = 5.49$$

Hence the number of fringe is five.

18. Shift due to sheet of thickness  $10^{-3}$  m and refractive index 1.6 is

$$x_1 = \frac{(1.6 - 1)tD}{d}$$

$$x_1 = (1.6 - 1) \frac{10^{-3} \times 10^{-6} \times 1.5}{1.5 \times 10^{-3}}$$

$$x_1 = 0.6 \times 10^{-3} \times 6 \times 10^{-3} \text{ m}$$

Shift due to sheet of thickness  $15 \times 10^{-3}$  m and refractive index 1.2 is

$$x_2 = \frac{(1.2 - 1) \times 15 \times 10^{-3} \times 1.5}{1.5 \times 10^{-3}}$$

$$x_2 = 3 \times 10^{-3} \text{ m}$$

Since these shifts are in opposite direction of central maxima hence net shift

$$x = x_1 - x_2 = 6 \times 10^{-3} \text{ m} - 3 \times 10^{-3} \text{ m} = 3 \times 10^{-3} \text{ m}$$

19. Let  $\lambda$  is the wavelength of light  $D$  is screen distance from source and  $d$  is the separation between slits (all are in metres)

$$\text{Shift} = x = \frac{(1.6 - 1)tD}{d}$$

$$x = \frac{(1.6 - 1) \times 1.964 \times 10^{-6} \times D}{d}$$

$$x = \frac{1.1784 D}{d} \times 10^{-6} \text{ m}$$

Now when  $t$  is removed and  $D$  is doubled the distance between successive maximum (or minima) i.e., fringe width

$$w = \frac{2D}{d}$$

but according to question  $x = w$

$$\frac{1.1784 \times 10^{-6} D}{d} = \frac{2D}{d}$$

$$0.589 \times 10^{-6} \text{ m} = 589 \text{ nm}$$

20. Let  $n$  bright fringe ( $5500 \text{ \AA}$ ) coincide with 10th

bright fringe of  $6000 \text{ \AA}$

$$n \times 5500 \text{ \AA} = 6000 \text{ \AA} \times 10$$

$$n \sim 11$$

Similarly first bright fringe coincide with 1st fringe. Now fringe width

$$w = \frac{14.74 \times 12.5}{10} = 0.224 \text{ mm}$$

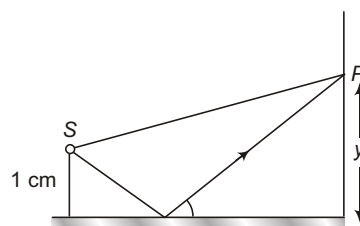
Hence position of 10th bright fringe

$$14.74 \times 0.224 \sim 14.55 \text{ mm}$$

Position of zero order bright fringe

$$12.75 \times 0.224 \sim 12.25 \text{ mm}$$

21. Here  $d \sim 1 \text{ cm}$



$$D = 100 \text{ m}$$

$$500 \text{ nm}$$

For first dark fringe

$$y = \frac{2t}{2d} \frac{100 \cdot 500 \cdot 10^9}{2 \cdot 10^2}$$

$$y = 2.5 \text{ mm}$$

22. The destructive interference will be  $2t = n\lambda$  for thinnest  $n = 1$

$$2t = \frac{650 \cdot 10^9}{2 \cdot 1.42}$$

$$t = 114 \text{ nm}$$

23. Here  $t = 0.485 \text{ m}$ ,  $t = 485 \text{ nm}$ ,  $n = 1.53$

- (a) Condition for constructive interference in refractive system

$$2t = n\lambda \quad \text{where, } n = 1, 2$$

$$\text{For } n = 1, \lambda = \frac{4t}{2 \cdot 1.53} = \frac{4 \cdot 485}{2 \cdot 1.53} = 2968.2 \text{ nm}$$

which does not lie in visible region put

$$n = 2, 3, \dots$$

we get  $424 \text{ nm}, 594 \text{ nm}, \dots$

- (b) For constructive interference in transmitted system

$$2t = n\lambda, \text{ putting } n = 1, 2,$$

only  $495 \text{ nm}$  is lie in visible region.

24. For constructive interference

$$2t = n\lambda \quad \text{for } t \text{ to be minimum } n = 1$$

$$t = \frac{6000}{4 \cdot 1.3} = 1154 \text{ \AA}$$

25. For destructive interference

$$2t = n\lambda \quad \text{for } t \text{ to be minimum}$$

$$n = 1$$

$$t = \frac{800}{2 \cdot 2} \quad \dots(i)$$

For constructive interference

$$2t = (2n - 1)\frac{\lambda}{2}$$

$$2 = \frac{800}{2} = (2n - 1)\frac{\lambda}{2}$$

$$1600 = (2n - 1)\lambda$$

For  $n = 1$ ,  $600 \text{ nm}$  which does not lie in visible region

For  $n = 2$

$$1600 = 3\lambda$$

$$\frac{1600}{3} = 533 \text{ nm}$$

for  $n = 3$ ,  $\frac{1600}{5} = 320 \text{ nm}$  which does lie

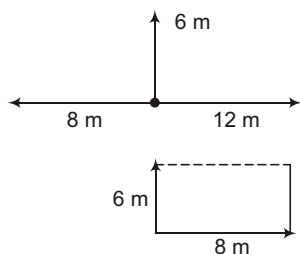
in visible

Hence  $533 \text{ nm}$

## Objective Questions (Level-1)

### ■ (Single option correct)

1. Using phasor method



$$R = \sqrt{8^2 + 6^2} = 10 \text{ m}$$

Hence correct option is (b).

$$2. \text{ Here } \frac{I_1}{I_2} = B^2$$

$$I_{\max} = [\sqrt{I_1} + \sqrt{I_2}]^2$$

$$I_{\min} = [\sqrt{I_1} - \sqrt{I_2}]^2$$

$$\frac{I_{\min}}{I_{\max}} = \frac{\sqrt{\frac{I_1}{I_2}} - 1}{\sqrt{\frac{I_1}{I_2}} + 1} = \frac{1 - 1}{1 + 1} = 0$$

$$\frac{(I_{\max} - I_{\min})}{(I_{\max} + I_{\min})} = \frac{1 - 1}{1 + 1} = 0$$

$$\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{1}{2}$$

Hence, correct option is (d).

3. For  $n$ th dark fringe

$$x_n = (2n - 1) \frac{D}{2d}$$

For 1st dark fringe  $n = 1$

$$x_1 = \frac{D}{2d}$$

Angular position  $\sin^{-1} \frac{x_1}{D}$

$$\sin^{-1} \frac{1}{2a}$$

$$\sin^{-1} \frac{5460 \times 10^{-10}}{2 \times 0.1 \times 10^{-3}}$$

$$\sin^{-1} [273 \times 10^{-5}]$$

$$\sin^{-1} [0.00273]$$

$$0.16$$

Hence correct option is (b)

4. For 10th bright fringe  $x_{10} = \frac{10D}{d}$

For 6th dark fringe  $x_6 = (2 \times 6 - 1) \frac{D}{2d}$

But

$$\frac{11}{2a} D = \frac{x_6 - x_{10}}{d} = \frac{10D}{d} - \frac{20}{11}$$

But  $\frac{20}{11} = 1.8$

Hence correct option is (a).

$$\text{5. Shift} = \frac{(1) \lambda D}{d}$$

$$\frac{(1.5 \times 10^{-6} \times 100 \times 10^{-2})}{2.5 \times 10^{-3}}$$

$$\frac{5 \times 10^{-6}}{2.5 \times 10^{-3}} = 2 \times 10^{-3} \text{ m} = 2 \text{ mm}$$

Hence correct option is (a).

6.  $x_n = \frac{Dn}{d}$  for  $n$ th bright fringe

$$x_n = \frac{D}{2d} (2n - 1) \text{ for } n\text{th dark fringe}$$

$$x - x_n = x_n = \frac{D}{2d} [2n - 2n - 1]$$

$$x = \frac{D}{2d}$$

Hence correct option is (c).

7. Since at centre path difference for all colour is always zero hence centre will be white.

Hence correct option is (a).

8.  $n_1 = 1, n_2 = 2$

$$60 \times 4000 \times n_2 = 6000 \times n_2 \times 40$$

Hence correct option is (a).

9. Initial fringe width  $w_1 = \frac{D}{d}$

$$\text{Final fringe width } w_2 = \frac{(D \times 5 \times 10^{-2})}{d}$$

$$|w| = |w_2 - w_1| = \frac{5 \times 10^{-2}}{d}$$

$$3 \times 10^{-5} = \frac{5 \times 10^{-2}}{10^{-3}}$$

$$\frac{3 \times 10^{-6}}{5} = 6000 \text{ Å}$$

Hence correct option is (a).

10.  $\frac{I_{\max}}{I_{\min}} = \frac{49}{9}$

Now  $\frac{I_{\max}}{I_{\min}} = \frac{\sqrt{I_1} \sqrt{I_2}^2}{\sqrt{I_1} \sqrt{I_2}} = \frac{\sqrt{I_1}^2}{\sqrt{I_2}^2} = \frac{1^2}{1^2}$

$\frac{I_{\max}}{I_{\min}} = \frac{\sqrt{\frac{49}{9}}}{\sqrt{\frac{49}{9}}} = \frac{7}{7} = 1$

$\frac{I_{\max}}{I_{\min}} = \frac{25}{4}$

Hence correct option is (a)

11.  $x_n = \frac{Dn}{d}$

$x_3 = \frac{3D}{d} = \frac{7.5 \times 10^3 \text{ m}}{7.5 \times 10^3 \times 0.2 \times 10^3} = \frac{3}{1}$

## ■ Assertion and Reason

1. We have  $I = 4I_0 \cos^2 \frac{\theta}{2}$  if  $\frac{\theta}{2} = \frac{\pi}{3}$

we get  $I = I_0$

Hence assertion is true.

Now path difference  $= \frac{\lambda}{2}$  phase difference

Path difference  $= \frac{\lambda}{2} = \frac{2}{3} \times \frac{\lambda}{3}$

Hence reason is true.

But reason is not the explanation of assertion. Hence correct option is (b).

2. Here assertion and reason are both true but reason is not correct explanation of assertion. Correct option is (b).

$0.5 \times 10^6 = 500 \text{ nm}$

Hence correct option is (b).

12. Let  $n$ th fringe of  $6500 \text{ Å}$  coincide with  $n$ th fringe of  $5200 \text{ Å}$ .

$x_n = \frac{Dn}{d} = \frac{6500 \text{ Å}}{d} = \frac{5200 \text{ Å}}{d}$

$\frac{n}{n} = \frac{5200}{6500} = \frac{4}{5}$

It means 4th fringe of  $6500 \text{ Å}$  coincide with 5th fringe of  $5200 \text{ Å}$  hence the distance

$x = \frac{4 \times 120 \times 10^{-2} \times 6500 \times 10^{-10}}{2 \times 10^{-3}}$

$x = 0.156 \text{ cm}$

Hence correct option is (a).

13. Since number of minima does not depends on orientation hence  $n_1 = n_2$

Hence correct option is (a).

3. Assertion is wrong since fringes are symmetrical *ie*, fringes obtained both above and below point  $O$ . Reason is true.

Correct option is (a).

4. Here both assertion and reason are true and reason correctly explain assertion. Hence correct option is (a).

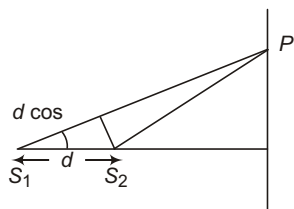
5. Both assertion and reason are true and reason correctly explain the assertion.

Hence correct option is (a).

6. Assertion is true since locus of all fringes is circle. But reason is wrong since fringes may have any shape.

Correct option is (c).

7.



Path difference at point  $P$  is  $x = d \cos \theta$

The path difference decreases as  $\theta$  increases.

$\therefore$  as  $\theta$  increases,  $\cos \theta$  decreases

Hence order of fringe  $n = \frac{d \cos \theta}{\lambda}$  decreases

as we go above  $P$ . Hence assertion is wrong (false).

For 11th order maxima path difference is more hence reason is true but assertion is false correct option is (d).

8. Here assertion is true and reason is false and reason does not correctly explain assertion. Correct option is (c).

$$9. \therefore d \sin \theta = n \lambda$$

$$\sin \theta = \frac{n \lambda}{d} \text{ but } \frac{d}{\lambda} = 4$$

$$\sin \theta = \frac{n}{4}$$

Now if  $\theta = 30^\circ$

$$n = 4 \sin 30^\circ = 2$$

Hence assertion is true.

Also reason is true and does not correctly explain assertion correct option is (b).

10. Here assertion is false. Since shift  $\left( \frac{(1)tD}{d} \right)$  is independent of  $\lambda$ .

Hence shift of red colour  $\neq$  shift of violet colour.

and reason is true

$$\therefore \frac{v}{\lambda} = \frac{R}{\lambda}$$

Hence correct option is (d)

## Objective Questions (Level 2)

$$1. I = 4I_0 \cos^2 \frac{x}{\lambda}$$

$$I_0 = 4I_0 \cos^2 \frac{x_1}{\lambda}$$

$$\frac{x_1}{\lambda} = \frac{\pi}{3} \quad \dots(i)$$

$$\text{and } 2I_0 = 4I_0 \cos^2 \frac{x_2}{\lambda}$$

$$\frac{x_2}{\lambda} = \frac{\pi}{4} \quad \dots(ii)$$

$$\frac{x}{\lambda} = \frac{x_1}{\lambda} + \frac{x_2}{\lambda} = \frac{\pi}{3} + \frac{\pi}{4} = \frac{7\pi}{12}$$

Hence correct option is (c).

$$2. I = 4I_0 \cos^2 \frac{x}{\lambda} = k \cos^2 \frac{x}{\lambda}$$

$$I = k \cos^2 \frac{x}{\lambda} = k \cos^2 \frac{x}{\lambda} \quad \dots(i)$$

$$\frac{I}{k} = \cos^2 \frac{x}{\lambda}$$

$$\frac{k}{4} = k \cos^2 \frac{x}{\lambda}$$

$$\cos \frac{x}{\lambda} = \frac{1}{2}$$

$$\frac{x}{\lambda} = \cos^{-1} \frac{1}{2}$$

$$\frac{x}{\lambda} = \frac{\pi}{3} \text{ or } \frac{x}{\lambda} = \frac{2\pi}{3}$$

$$x = \frac{\pi}{3} \lambda \text{ or } x = \frac{2\pi}{3} \lambda$$

Hence correct option is (b).

3. Light of wavelength  $\lambda$  is strongly reflected if

$$2ut = n \frac{\lambda}{2} \quad n = 0, 1, 2 \quad \dots(i)$$

$$2ut = 2 \times 1.5 \times 5 \times 10^7 \text{ m} = 1.5 \times 10^6 \text{ m}$$

Putting  $\lambda = 400 \text{ nm}$  in eq. (1) and using eq. (ii)

$$1.5 \times 10^6 \text{ m} \quad n = \frac{1}{2} \quad 4 \times 10^7 \text{ m}$$

$$n = 3.25$$

Putting  $n = 3.25$

$$1.5 \times 10^6 \text{ m} \quad n = \frac{1}{2} \quad 7 \times 10^7$$

$$n = 1.66$$

Hence  $n$  can take values 2 and 3.

$$\text{From (i) if } n = 2, \quad \frac{4ut}{2 \times 2 \times 1} = 600 \text{ nm}$$

$$\text{if } n = 3 \quad 429 \text{ nm}$$

only 600 is given in the options.

Hence correct option is (b).

$$4. \quad x_n = \frac{D}{d} n = \frac{100}{0.01} \frac{10^{-2}}{10^{-3}} n$$

$$x_n = \frac{n}{10^{-5}} \quad \text{For } 4000 \text{ Å}$$

$$x_n = \frac{n \times 4000 \times 10^{10} \text{ m}}{10^{-5}} = 4 \times 10^5 n$$

$$0.04 = n$$

$$x_n = 4n \text{ mm}$$

Similarly for  $7000 \text{ Å}$

$$x_n = 7n \text{ mm} \quad n = 5, 6$$

hence only  $x = 5$

Passes through hole  $5000 \text{ Å}$

Hence correct option is (b).

$$5. \quad I = 4I_0 \cos^2 \frac{x}{d} \quad \text{where } \frac{x}{d}$$

$$\frac{1}{1} \frac{6000 \times 10^{10}}{10^{-3}} = (6 \times 10^4)$$

$$75\% I_0 = I_0 \cos^2 \frac{x}{d}$$

$$\frac{3}{16} = \cos^2 \frac{x}{d}$$

$$\frac{\sqrt{3}}{4} = \cos \frac{x}{d}$$

$$x = \cos^{-1} \sqrt{\frac{3}{4}} \quad x = -\cos^{-1} \sqrt{\frac{3}{4}}$$

$$x = 0.20 \text{ mm}$$

Hence correct option is (d).

$$6. \quad \text{Number of fringes shifted} = \frac{(1.5 - 1)t}{\lambda}$$

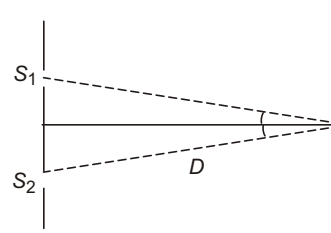
$$= 4 \frac{(1.5 - 1)t}{6000 \text{ Å}}$$

$$\frac{4}{0.5} = 6 \times 10^7 \text{ m} \quad t$$

$$t = 4.8 \text{ m}$$

Hence correct option is (a).

7. For  $n$ th order minima



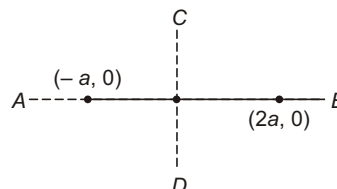
$$y_n = \frac{(2n - 1)D}{2d} \quad \text{for 3rd minima } n = 3$$

$$y_3 = \frac{5D}{2d} = \frac{5}{2}$$

$$\therefore \theta \sim \tan^{-1} \frac{d}{D}$$

Hence correct option is (b).

$$8. \quad AB = d = 3a = \frac{3a}{5}$$



$$AB = 15$$

$$\text{Hence total maxima} = 14 + 4 + 4$$

$$= 60$$

Hence correct option is (a).



$$9. I = 4I_0 \cos^2 \frac{x}{4}$$

$$I = 4I_0 \cos^2 \frac{x}{4}$$

$$I = 4I_0 \cos^2 \frac{x}{4}$$

$$\frac{4I_0}{2} = 2I_0$$

$$\frac{I_0}{I} = \frac{1}{2}$$

Hence correct option is (b).

$$10. I = I_0 \cos^2 \left( \frac{1}{2} \right) t$$

$$\text{at } I = I_0$$

Hence correct variation is (c)

$$11. I = I_0 \cos^2 \frac{x}{4}$$

$$\frac{3}{4} I_0 = I_0 \cos^2 \frac{x}{4}$$

$$\frac{x}{4} = \frac{\pi}{6}$$

$$x = \frac{\pi}{6} \text{ but } x = \left( \frac{1}{2} \right) t$$

$$(1.5 - 1) t = \frac{\pi}{6}$$

$$t = \frac{\pi}{3} = \frac{6000}{3} \text{ \AA}$$

$$t = 2000 \text{ \AA} = 0.2 \text{ \mu m}$$

Hence correct option is (a).

$$12. \text{Net shift} = \left( \frac{1}{2} - 1 \right) t = \left( \frac{1}{2} - 1 \right) t$$

$$\left( \frac{1}{2} - 1 \right) t = (1.52 - 1.40) t$$

$$\text{Net shift} = 0.12 \times 10.4 \text{ \mu m}$$

$$12 \times 104 \times 10^{-9} \text{ m}$$

$$1248 \times 10^{-9} \text{ m}$$

$$1248 \text{ nm}$$

Net shift =  $n \lambda$

where  $n$  is ve integer

$$n \lambda = 1248$$

Now, for 416 nm,  $n = 3$

For 624 nm,  $n = 2$

Hence correct option is (c).

$$13. w = \frac{D}{d} \frac{6300 \text{ \AA}}{1 \text{ mm}} \frac{1.33 \text{ m}}{1.33}$$

$$\frac{63 \times 10^{-8} \times 1.33 \text{ m}}{10^{-3} \times 1.33}$$

$$0.63 \text{ mm}$$

Hence correct option is (a).

$$14. x = \frac{7D}{nd} = \frac{3D}{nd} = 4 \frac{D}{nd}$$

$$x = 4 \times 0.63 \text{ mm} = 2.52 \text{ mm}$$

Correct option is (a).

$$15. x = 2 \text{ of fringe width}$$

$$\left( \frac{1}{2} - 1 \right) t = \frac{2 \times 0.63 \text{ mm}}{2}$$

$$t = \frac{2 \times 0.63}{0.53} = 1.57 \text{ mm}$$

Hence correct option is (b).

16. Since on introducing thin glass sheet fringe width does not change hence fringe width = 0.63 mm.

The correct option is (a).

### ■ More than one options correct

1. At centre path difference between all colours is zero hence centre is white since violet has least wavelength hence next to central will be violet and since intensity is different for different for colours hence there will be not a completely dark fringe

Hence correct options, are (b), (c) and (d).

2. Correct options are (a), (c) and (d).

$$3. \because I = 4I_0 \cos^2 \frac{\phi}{2} \text{ at centre } \phi = 0 \quad I = 4I_0$$

and at distance 4 mm above  $o$  is again maxima hence its intensity is also  $4I_0$ .

Hence correct options are (a) and (c).

4. The correct options, are (a), (c) and (d)

Since phase difference constant

Light should be monochromatic.

5. Since in this case fringe pattern shift upward hence the correct options are (a), (b) and (c).

6.  $\because n_1 - 1 > n_2 - 1$  for maxima

Using this option (a) is satisfied and  $(2n_1 - 1) \lambda_1 = (2n_2 - 1) \lambda_2$  for minima

Using this 3rd option is satisfied

Hence correct options are (a) and (c).

### ■ Match the Columns

$$1. \because I = 4I_0 \cos^2 \frac{\phi}{2}$$

$$\text{if } \phi = 60^\circ \quad I = 3I_0$$

$$\text{if } \phi = 90^\circ \quad I = 2I_0$$

$$\text{if } \phi = 0^\circ \quad I = 4I_0$$

$$\text{if } \phi = 120^\circ \quad I = I_0$$

Hence correct match is

$$(a) \rightarrow s; (b) \rightarrow q; (c) \rightarrow p; (d) \rightarrow r$$

$$2. \quad x = \frac{2}{3} \lambda$$

$$\frac{2}{3} \lambda \text{ if } x = \frac{\lambda}{3}$$

$$\frac{2}{3} \lambda = 120^\circ$$

$$\text{if } x = \frac{\lambda}{6}$$

$$60^\circ \text{ if } x = \frac{\lambda}{4}, \quad 90^\circ$$

$$\text{Using } I = 4I_0 \cos^2 \frac{\phi}{2}$$

The correct match are

$$(a) \rightarrow q; (b) \rightarrow p; (c) \rightarrow r; (d) \rightarrow s$$

3. Distance between third order maxima and central maxima  $\frac{3D}{d} = 3w$

Distance between 3rd order minima and central

$$\text{Maxima } = 3 \times \frac{1}{2} \frac{D}{d} = 2.5 \frac{D}{d} = 2.5w$$

Distance between first minima and forth order

$$\text{maxima } = \frac{4D}{d} - \frac{D}{2d} = 2.5w$$

Distance between first minima and forth order

$$\text{maxima } = \frac{4D}{d} - \frac{D}{2d} = 3.5w$$

Distance between 2nd order maxima and fifth order minima  $4.5w - 2w = 2.5w$

hence correct match is

$$(a) \rightarrow q; (b) \rightarrow p; (c) \rightarrow r; (d) \rightarrow p$$

4. Since fringe shift in the division of sheet placed source hence

(a)  $p$

Similarly for other the correct match

(b)  $r, s$ ; (c)  $p$ ; (d)  $p$

5. When  $y = \frac{D}{2d}$  there will a dark fringe at 0.

hence (a)  $q$

$$\text{when } y = \frac{D}{6d} \quad 3 \times \frac{D}{2d}$$

The intensity becomes  $3I$

(b)  $p$

$$\text{when } y = \frac{D}{4d} \quad \text{Intensity} = 2I$$

(c)  $s$

$$\text{when } y = \frac{D}{3d} \quad \text{Intensity} = I$$

(d)  $r$

6. When a thin plate (transparent) is placed in front of  $S_1$  zero order fringe shift above from  $O$  hence

(a)  $r$

When  $S_1$  is closed interference disappear and uniform illuminance is obtained on screen hence

(b)  $p, q$

Similarly (c)  $r, s$  and

When  $s$  is removed and two real sources  $s_1$  and  $s_2$  emitting light of same wavelength are placed interference disappear. Since sources become non-coherent hence (d)  $p, q$

# 29. Modern Physics I

## Introductory Exercise 29.1

1. The positron has same mass  $m$  as the electron. The reduced mass of electron positron atom is

$$\frac{m}{m} \frac{m}{m} \frac{1}{2} m$$

$$R_H \frac{me^4}{8 \frac{2}{0} ch^3}$$

$$R_P \frac{R_H}{2}$$

$$\frac{1}{H} R_H \frac{1}{2^2} \frac{1}{3^2}$$

$$\frac{1}{P} R_P \frac{1}{2^2} \frac{1}{3^2}$$

$$\frac{P}{H} \frac{R_H}{R_P} 2$$

$$P \quad 2 \quad H \quad 2 \quad 6563 \text{ \AA} \quad 13126 \text{ \AA} \\ 1.31 \text{ m}$$

$$\frac{1}{He} R_H \frac{1}{2^2} \frac{1}{3^2} z^2$$

$$\frac{1}{He} \frac{1}{H} z^2 \quad He \quad \frac{H}{2^2} \frac{6563 \text{ \AA}}{2^2}$$

$$He \quad 164 \text{ nm}$$

2.  $\frac{1}{R} \frac{1}{2^2} \frac{1}{n^2}$  for largest wavelength  $n = 3$

$$\frac{1}{R} \frac{1}{4} \frac{1}{9}$$

$$\frac{36}{5R} \frac{36}{1.097 \cdot 10^7}$$

$$656 \text{ nm}$$

3. For H-atom  $r_n = \frac{0 n^2 h^2}{me^2}$

$$u_n = \frac{e^2}{2 \frac{0}{n} h}$$

$$T_n = \frac{2 \frac{r_n}{u_n}}{2} = \frac{n^2 h^2}{me^2} \frac{2 \frac{0}{n} h}{e^2}$$

$$\frac{4 \frac{0}{n} n^3 h^3}{me^4}$$

$$r_n = \frac{1}{T_n} \frac{me^4}{4 \frac{2}{0} n^3 h^3}$$

$$r_1 = \frac{me^4}{4 \frac{2}{0} h^3}$$

$$1 = \frac{9.1 \cdot 10^{-31} (1.6 \cdot 10^{-19})^4}{4 (8.85 \cdot 10^{-12})^2 (6.6 \cdot 10^{-34})^3}$$

$$1 = 6.58 \cdot 10^{15} \text{ Hz}$$

$$2 = \frac{\frac{1}{2^3}}{8} \frac{1}{8} \frac{6.58 \cdot 10^{15}}{8}$$

$$0.823 \cdot 10^{15} \text{ Hz}$$

$$(b) \quad \frac{1}{R} \frac{1}{1^2} \frac{1}{2^2}$$

$$\frac{c}{4} \cdot 3 \cdot 10^8 \cdot R \frac{3}{4}$$

$$\frac{9 \cdot 10^8 \cdot 1.097 \cdot 10^7}{4}$$

$$2.46 \cdot 10^{15} \text{ Hz}$$

- (c) Number of revolutions

$$v_2 \quad T \quad 0.823 \cdot 10^{15} \quad 1 \cdot 10^8$$

$$8.23 \cdot 10^6 \text{ revolution}$$

4. Reduce  $\frac{m}{m} \frac{m_p}{m_p} \frac{207 \text{ m}}{(207 \text{ m} \ 1836 \text{ m})} \frac{1836 \text{ m}}{1836 \text{ m}} \text{ mass}$

$$r_1 = 4 \cdot 0 \cdot \frac{h^2}{4 \cdot 2 \cdot e^2} \cdot 4 \cdot 0 \cdot \frac{h^2}{4 \cdot 2 \cdot (186m) e^2}$$

Putting the value we get

$$r_1 = 2.55 \cdot 10^{13} \text{ m}$$

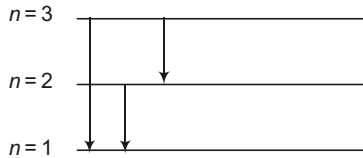
$$E_1 = \frac{e^4}{8 \cdot 2 \cdot h^2} = 2810 \text{ eV}$$

Ionization energy  $E_1 = 2.81 \text{ keV}$

5. (a)  $\frac{h}{mv} = \frac{6.6 \cdot 10^{34}}{46 \cdot 10^3 \cdot 30} = 4.8 \cdot 10^{34} \text{ m}$

(b)  $\frac{6.6 \cdot 10^{34}}{9.1 \cdot 10^{31} \cdot 10^7} = 7.3 \cdot 10^{11} \text{ m}$

6. (a) After absorbing 12.3 eV the atom excited to  $n = 3$  state



$$\frac{1}{L} = R \cdot 1 \cdot \frac{1}{n^2}$$

$$\frac{1}{L_1} = R \cdot 1 \cdot \frac{1}{9} = \frac{8R}{9}$$

$$L_1 = \frac{9}{8R} = \frac{9}{8 \cdot 1.097 \cdot 10^7} = 102 \text{ nm}$$

$$\frac{1}{L_2} = R \cdot 1 \cdot \frac{1}{4}$$

$$L_2 = \frac{4}{3R} = \frac{4}{3 \cdot 1.097 \cdot 10^7} = 122 \text{ nm}$$

$$\frac{1}{B} = R \cdot \frac{1}{2^2} \cdot \frac{1}{3^2} = \frac{5R}{36}$$

$$B = \frac{36}{5R}$$

$$B = \frac{36}{5 \cdot 1.097 \cdot 10^7} = 653 \text{ nm}$$

7.  $\frac{K}{K} \frac{K}{K} \frac{L}{L} \frac{L}{L} \frac{K}{K} \frac{K}{K}$

$$\frac{c}{K} \frac{c}{K} \frac{c}{L} \frac{L}{L} \frac{0.71}{0.71} \frac{0.63}{0.63}$$

$$\frac{1}{L} \frac{1}{K} \frac{1}{K} \frac{L}{L} = 5.59 \text{ nm}$$

8.  $\frac{hc}{E}$



$$K = \frac{6.6 \cdot 10^{34}}{(E_1 - 2870) \cdot 1.6 \cdot 10^{19}} = 0.71 \cdot 10^9$$

$$K = \frac{6.6 \cdot 10^{34}}{(E_1 - 2870) \cdot 1.6 \cdot 10^{19}} = 0.63$$

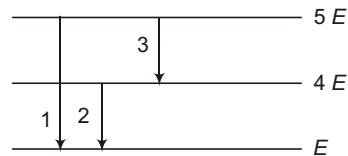
Solving this we get

$$E_1 = 4613 \text{ eV}$$

$$K_B = \frac{6.6 \cdot 10^{34}}{(4613 - E_3) \cdot 1.6 \cdot 10^{19}} = 0.63$$

Solving this we get  $E_3 = 2650 \text{ eV}$

9.  $\frac{4E}{h} = f \dots(i)$



$$\frac{3E}{h} = \frac{3}{4} \cdot \frac{4E}{h} = \frac{3f}{4}$$

$$\frac{E}{h} = \frac{f}{4}$$

## Introductory Exercise 29.2

1.  $eV_0 = \frac{hc}{\lambda} = W$

$$eV_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-7} \times 1.6 \times 10^{-19}} = 4.3 \text{ eV}$$

$$eV_0 = 6.2 \text{ eV} - 4.3 \text{ eV} = 1.9 \text{ eV}$$

$$V_0 = 1.9 \text{ volt}$$

2.  $P = 1.5 \text{ mW} = 1.5 \times 10^{-3} \text{ W}$

Energy of each photon

$$\frac{6.62 \times 10^{-34} \times 3 \times 10^8}{4 \times 10^{-7}}$$

$$4.96 \times 10^{-19} \text{ J}$$

Number of photons incident per second

$$\frac{P}{\text{Energy of each photon}} = \frac{1.5 \times 10^{-3}}{4.96 \times 10^{-19}}$$

$$\sim 3 \times 10^{15}$$

The number of photoelectrons produce

$$0.1\% \times 3 \times 10^{15} = 3 \times 10^{12}$$

$$\text{Current } i = 3 \times 10^{12} \times 1.6 \times 10^{-19} \text{ A}$$

$$4.8 \times 10^{-7} \text{ A} = 0.48 \text{ A}$$

3.  $K_{\max} = hf - hf_0$

$$K_{\max} = (f - f_0)$$

4.  $K_{\max} = \frac{hc}{\lambda} - W$

$$\frac{6.62 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-7} \times 1.6 \times 10^{-19}} - 3 \text{ eV}$$

$$[6.20 - 3] \text{ eV} = 3.20 \text{ eV}$$

The minimum kinetic energy = 0.

5.  $K_{\max} = h[f - f_0]$

$$1.2 \text{ eV} = h[f - f_0] \quad \dots(i)$$

$$4.2 \text{ eV} = h[1.5f - f_0] \quad \dots(ii)$$

Dividing Eq. (i) and Eq. (ii)

$$\frac{1.2}{4.2} = \frac{f - f_0}{1.5f - f_0}$$

$$3f - 2f_0 = 7f - 7f_0$$

$$5f_0 = 4f$$

$$f_0 = \frac{4}{5}f = 0.8f$$

$$1.2 = 1.6 \times 10^{-19} \times 6.62 \times 10^{-34} \times \frac{f_0}{0.8} \times f_0$$

$$\frac{1.2 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = \frac{2f_0}{8} \times \frac{f_0}{4}$$

$$f_0 = 1.16 \times 10^{15} \text{ Hz}$$

## Subjective Questions (Level I)

1. Here  $\lambda = 280 \text{ nm} = 28 \times 10^{-8} \text{ m}$

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{28 \times 10^{-8}} \text{ J}$$

$$E = \frac{19.8 \times 10^{-34} \times 10^{16}}{28} \text{ J} = \frac{198 \times 10^{-19}}{28} \text{ J}$$

$$E = \frac{198 \times 10^{-19}}{28 \times 1.6 \times 10^{-19}} \text{ eV} = \frac{198}{28 \times 1.6} \sim 4.6 \text{ eV}$$

We have  $E = mc^2 \Rightarrow m = \frac{E}{c^2} = \frac{198 \times 10^{-19}}{28 \times 9 \times 10^{16}}$

$$m = 8.2 \times 10^{-36} \text{ kg}$$

Momentum  $p = mc = 8.2 \times 10^{-36} \times 3 \times 10^8$   
 $2.46 \times 10^{-27} \text{ kg-m/s}$

2. Intensity of light at a distance 2 m

From the source  $\frac{1}{4(2)^2} \times \frac{1}{16} \text{ W/m}^2$

Let plate area is  $A$

Energy incident on unit time is

$$E_1 = \frac{1}{16} A$$

Energy of each photon

$$\frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4.8 \times 10^{-7}}$$

Number of photons striking per unit area

$$n = \frac{\frac{1}{16} A \times 4.8 \times 10^{-7}}{A \times 6.6 \times 10^{-34} \times 3 \times 10^8}$$

$$4.82 \times 10^{16} \text{ per m}^2 \text{ s}$$

3. Here  $p = 8.24 \times 10^{28} \text{ kg-m/s}$

(a) Energy of photon  $E = pc$

$$E = 8.24 \times 10^{28} \times 3 \times 10^8$$

$$E = 2.47 \times 10^{19} \text{ J}$$

$$\text{Energy in eV} = \frac{E \text{ in joule}}{1.6 \times 10^{-19}} = \frac{2.47 \times 10^{19}}{1.6 \times 10^{-19}}$$

$$\text{Energy (in eV)} = 1.54 \text{ eV}$$

(b) Wavelength

$$\frac{h}{p} = \frac{6.6 \times 10^{-34}}{8.24 \times 10^{28}} = 804 \text{ nm}$$

This wavelength is in Infrared region.

4. We have  $c = f \lambda \Rightarrow f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{6 \times 10^{-7} \text{ m}}$

$$f = 5 \times 10^{14} \text{ Hz}$$

We have  $p = \frac{E}{t}$   $E = pt$  power per sec

energy

$$P = \frac{E}{t}$$

$$75 = (h \nu) n$$

$$\frac{75}{6.6 \times 10^{-34} \times 5 \times 10^{14}}$$

$$n = 2.3 \times 10^{20} \text{ photons/sec}$$

5. (a)  $E = 2.45 \text{ MeV} = 2.45 \times 1.6 \times 10^{19} = 10^6 \text{ J}$

$$E = h \nu \Rightarrow \frac{E}{h} = \frac{3.92 \times 10^{13}}{6.6 \times 10^{-34}}$$

$$5.92 \times 10^{20} \text{ Hz}$$

(b) We have  $c = \frac{c}{\lambda}$

$$\frac{3 \times 10^8}{5.92 \times 10^{20}} = 5.06 \times 10^{-13} \text{ m}$$

6. We have  $p = \sqrt{2mK}$

$$p_1 = \sqrt{2mK_1} \text{ and } p_2 = \sqrt{2mK_2}$$

$$\frac{p_1}{p_2} = \sqrt{\frac{K_1}{K_2}}$$

$$\frac{1}{2} \sqrt{\frac{K_1}{K_2}} \quad [\because p_2 = 2p_1]$$

$$K_2 = 4K_1$$

(b)  $E_1 = p_1 c$  and  $E_2 = p_2 c \Rightarrow E_2 = 2p_1 c$

$$E_2 = 2E_1$$

7. (a) Since power = energy per unit line  
let  $n$  be the number of photons

$$P = nE = 10 \times n \times \frac{nc}{\lambda}$$

$$n = \frac{10}{hc} = \frac{10 \times 500 \times 10^9}{6.6 \times 10^{-34} \times 3 \times 10^8}$$

$$n = 2.52 \times 10^{19}$$

(b) Force exerted on that surface

$$F = \frac{P}{c} = \frac{10}{3 \times 10^8} = 3.33 \times 10^{-8} \text{ N}$$

8. Absorbing (power) light = 70% of incident light

$$P_a = 70\% \text{ of } 10 \text{ W} = 7 \text{ W}$$

Refractive power = 30% of 10 W

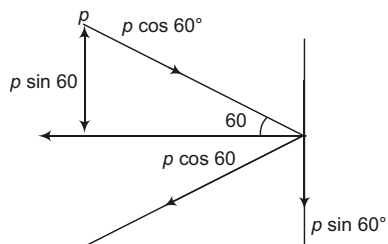
$$P_R = 3 \text{ W}$$

The force exerted  $\frac{P_a}{c} = \frac{2P_R}{c}$

$$\frac{7}{c} = \frac{2 \times 3}{c} \Rightarrow \frac{13}{3 \times 10^8}$$

$$4.3 \times 10^{-8} \text{ N}$$

9. Force rate of change of momentum



$$F = \frac{p}{t} \text{ here } t = 1 \text{ s}$$

$$F = 2p \cos 60$$

$$F = p$$

$$F = \frac{nh}{t}$$

where  $n$  is number of photons striking per second

$$F = \frac{1 \cdot 10^{19} \cdot 6.63 \cdot 10^{-34}}{663 \cdot 10^{-9}} = 10^{-8} \text{ N}$$

10. Here output energy = 60 W/s

$$\text{Pressure } p = \frac{2 \cdot 60}{3 \cdot 10^8} = 4 \cdot 10^{-7} \text{ N}$$

de-Broglie wavelength

11. Here  $m = 5 \text{ g} = 5 \cdot 10^{-3} \text{ kg}$ ,  $v = 340 \text{ m/s}$

by de-Broglie hypothesis wavelength

$$\frac{h}{mv} = \frac{6.62 \cdot 10^{-34}}{5 \cdot 10^{-3} \cdot 340}$$

$$3.9 \cdot 10^{-34} \text{ m}$$

Since  $\lambda$  is too small. No wave like property is exhibit.

$$12.(a) \quad \lambda_e = \frac{h}{m_e v} = \frac{6.6 \cdot 10^{-34}}{9.1 \cdot 10^{-31} \cdot 4.7 \cdot 10^6}$$

$$1.55 \cdot 10^{-10} \text{ m}$$

$$(b) \quad \lambda_p = \frac{6.6 \cdot 10^{-34}}{1836 \cdot 9.1 \cdot 10^{-31} \cdot 4.7 \cdot 10^6}$$

$$8.44 \cdot 10^{-14} \text{ m}$$

$$13. (a) \quad \frac{h}{p} = \lambda = \frac{6.6 \cdot 10^{-34}}{2.8 \cdot 10^{-10}}$$

$$p = 2.37 \cdot 10^{-24} \text{ kg-m/s}$$

$$(b) \because p^2 = 2m_e K \quad K = \frac{p^2}{2m_e}$$

$$K = \frac{(2.37 \cdot 10^{-24})^2}{2 \cdot 9.1 \cdot 10^{-31}}$$

$$K = 3.07 \cdot 10^{-18} \text{ J}$$

$$K (\text{in eV}) = \frac{K \text{ in J}}{1.6 \cdot 10^{-19}} = \frac{3.07 \cdot 10^{-18}}{1.6 \cdot 10^{-19}}$$

$$K = 19.2 \text{ eV}$$

14. Here  $T = 273 + 20 = 293 \text{ K}$

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \frac{h}{Mv_{\text{rms}}}$$

$$\frac{h}{\sqrt{3MRT}}$$

$$\frac{6.6 \cdot 10^{-34}}{\sqrt{3 \cdot 1836 \cdot 9.1 \cdot 10^{-31} \cdot 8.31 \cdot 293}}$$

$$1.04 \text{ \AA}$$

15. For hydrogen like atom

$$E = K \text{ Here } E = 3.4 \text{ eV}$$

$$K = 3.4 \text{ eV} = 3.4 \cdot 1.6 \cdot 10^{-19} \text{ J}$$

$$\frac{h}{p} = \frac{h}{\sqrt{2m_e K}}$$

$$\frac{6.6 \cdot 10^{-34}}{\sqrt{2 \cdot 9.1 \cdot 10^{-31} \cdot 3.4 \cdot 1.6 \cdot 10^{-19}}}$$

$$6.663 \text{ \AA}$$

16. In Bohr model the velocity of electron in  $n$ th orbit is given by

$$U_n = \frac{e^2}{2 \cdot 0 n h}$$

Putting the values of  $e$ ,  $h$  and  $n = 1$ , we get

$$U_1 = 2.19 \cdot 10^7 \text{ m/s and } U_4 = \frac{2.19 \cdot 10^6}{4} \text{ m/s}$$



$$\begin{aligned}
 & \frac{1}{m_e v_1} \text{ and } \frac{4}{m_e u_4} = \frac{h}{m_e \frac{u_1}{4}} \quad \frac{4}{1} \\
 & \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 2.19 \times 10^6} \\
 & 3.32 \times 10^{-10} \text{ m} \\
 & \frac{4}{4} \times \frac{4}{1} \times \frac{4}{4} \times 3.32 \times 10^{-10} \text{ m} \\
 & 13.28 \times 10^{-10} \text{ m}
 \end{aligned}$$

$$1.33 \times 10^{-9} \text{ m}$$

The radius of first Bohr orbit

$$r_1 = 0.529 \times 10^{-10}$$

The radius of fourth Bohr orbit

$$\begin{aligned}
 r_4 &= 16 \times 0.529 \times 10^{-10} \\
 2 r_1 &= 2 \times 3.14 \times 0.529 \times 10^{-10} \\
 &= 3.32 \times 10^{-10} \text{ m}
 \end{aligned}$$

## Bohr Atomic Model and Emission Spectrum

17. For hydrogen like atom we can write

$$e_n = \frac{z^2}{n^2} (13.6 \text{ eV})$$

For lithium atom  $z = 3$  we get

$$E_n = \frac{9}{n^2} (13.6 \text{ eV}) = \frac{122.4}{n^2} \text{ eV}$$

The ground state energy is for  $n = 1$

$$E_1 = \frac{122.4}{1^2} \text{ eV} = 122.4 \text{ eV}$$

Ionization potential  $E_1 = 122.4 \text{ eV}$

18. For hydrogen atom we can write

$$(a) E = K = 3.4 \text{ eV}$$

$$(b) PE = 2K = 2 \times 3.4 = 6.8 \text{ eV}$$

Since potential energy depends upon reference hence it will change.

19. Binding energy of an electron in He-atom is  $E_0 = 24.6 \text{ eV}$ . i.e., the energy required to remove one electron from He-atom  $= 24.6 \text{ eV}$

Now, He-atom becomes  $\text{He}^+$  and energy of  $\text{He}^+$  ion is given by

$$E_n = \frac{z^2 (13.6)}{n^2} \text{ for He } z = 2, \text{ we get}$$

$$E_1 = 4 \times 13.6 = 54.4 \text{ eV.}$$

Hence energy required to remove this electron  $= 54.4 \text{ eV}$ , thus total energy  $= 24.6 + 54.4 = 79 \text{ eV}$

20. For hydrogen atom  $E_n = \frac{13.6 \text{ eV}}{n^2}$

Putting  $n = 3$ , we get

$$E_3 = \frac{13.6 \text{ eV}}{9} = 1.51 \text{ eV}$$

Hence hydrogen atom is in third excited state the angular momentum

$$L = \frac{nh}{2} = \frac{3h}{2} = \frac{3 \times 6.62 \times 10^{-34}}{2 \times 3.14}$$

$$L = 3.16 \times 10^{-34} \text{ kg-m}^2/\text{s}$$

21. We have  $\frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{E}$

$$E = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1023 \times 10^{-10}} \text{ (in Joule)}$$

$$E = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1023 \times 10^{-10} \times 1.6 \times 10^{-19}} \text{ (in eV)}$$

$$\sim 12.1 \text{ eV}$$

$$E_n = E_1 = 12.1 \text{ eV} \text{ But } E_1 = 13.6 \text{ eV}$$

$$E_n \sim 1.51 \text{ eV}$$

$$\text{For H-atom } E_n = \frac{13.6}{n^2} = 1.51 = \frac{13.6}{n^2}$$

$$n = 3$$

Hence atom goes to 3rd excited state. The possible transitions are  $(3 \rightarrow 2, 3 \rightarrow 1, \text{ and } 2 \rightarrow 1)$  i.e., 3 transitions are possible and the largest wavelength  $= 1023 \text{ \AA}$

(From  $3 \rightarrow 1$ )

22. For hydrogen like atom

$$E_n = \frac{z^2}{n^2} (13.6) \text{ eV}$$

For Li  $z = 3$

$$E_n = \frac{122.4}{n^2} \text{ eV} \quad E_1 = 122.4 \text{ eV}$$

$$E_3 = \frac{122.4}{9} \text{ eV} = 13.6 \text{ eV}$$

$$\frac{E}{E_3} = \frac{E_1}{E_3} = \frac{122.4}{13.6} = 9 \quad \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{108.8 \times 1.6 \times 10^{-19}} = 113.74 \text{ Å}$$

23. The excited state energy He atom will be equal to the sum of energies of the photons having wavelength 108.5 nm and 30.4 nm.

$$E_n - E_1 = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\frac{663 \times 10^{-34} \times 3 \times 10^8}{10^{-10}} = \frac{1}{1085} - \frac{1}{304}$$

$$E_n - E_1 = \frac{83.7 \times 10^{-19} \text{ J}}{1.6 \times 10^{-19} \text{ J/eV}} = 52.3 \text{ eV} \quad \dots(i)$$

$$\text{For He atom } E_n = \frac{z^2(13.6)}{n^2} \text{ eV}$$

For He  $z = 2$

$$E_n = \frac{54.4}{n^2} \text{ eV}$$

$$E_1 = 54.4 \text{ eV}$$

$$E_n - E_1 = 54.4 \left( 1 - \frac{1}{n^2} \right) \text{ eV} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$52.3 = 54.4 \left( 1 - \frac{1}{n^2} \right)$$

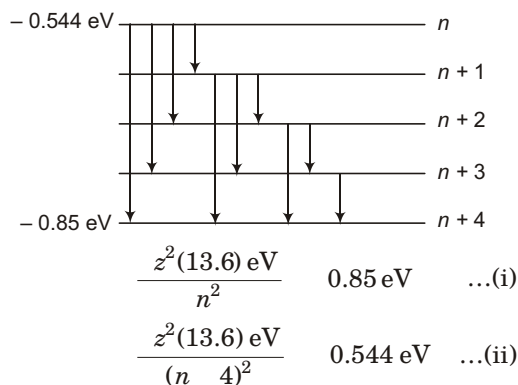
$$1 - \frac{1}{n^2} = 0.96$$

$$n = 5$$

24. For hydrogen like atom

$$E_n = \frac{z^2}{n^2} (13.6) \text{ eV}$$

Let for ten transitions quantum numbers of energy levels are  $n, n-1, n-2, n-3$  and  $n-4$



Dividing Eq. (i) and Eq. (ii)

$$\frac{(n-4)^2}{n^2} = \frac{0.85}{0.544} = 1.5625$$

$$\frac{n-4}{n} = 1.25$$

$$1 - \frac{4}{n} = 1.25$$

$$\frac{4}{n} = 0.25$$

$$n = \frac{4}{0.25} = 16$$

Putting this value of  $n$  in Eq. (i)

$$\frac{z^2(13.6)}{(16)^2} = 0.85$$

$$z^2 = \frac{256 \times 0.85}{13.6}$$

$$z^2 = 16 \quad z = 4$$

Hence atom no. of atom is 4

We know that  $E = \frac{hc}{\lambda}$

$\frac{hc}{E}$  for smallest wavelength  $E$  is maximum

$$\frac{6.6 \times 10^{-34} \times 3 \times 10^8}{[0.544 - (0.85)] \times 1.6 \times 10^{-19}} = 40441 \text{ Å}$$

25. Here  $E_1 = 15.6 \text{ eV}$

(a) Hence ionization potential

$$E_1 = 15.6 \text{ eV}$$

(b) We have  $\frac{hc}{E}$  for short wavelength

$E$  is maximum

$$\frac{6.6 \times 10^{-34} \times 3 \times 10^8}{0.53 \times 1.6 \times 10^{-19}} \sim 2335 \text{ \AA}$$

(c) Excitation potential for  $n = 3$  state is

$$E_3 - E_1 = 3.08 - 15.6 = -12.52 \text{ V}$$

(d) From  $n = 3$  to  $n = 1$

$$E = E_3 - E_1 = 3.08 - 15.6 = -12.52 \text{ eV}$$

We know  $\frac{hc}{E}$

$$\frac{1}{\lambda} = \frac{E}{hc} = \frac{12.52}{6.6 \times 10^{-34} \times 3 \times 10^8}$$

$$\lambda = 1.01 \times 10^{-7} \text{ (m)} = 101 \text{ nm}$$

26. (a)  $E_1 = 6.52 \text{ eV}$

$$\lambda = 860 \text{ nm} = 8600 \text{ \AA}$$

Energy of this photon

$$\frac{12375 \text{ (eV)}}{8600} = 1.44 \text{ eV}$$

hence internal energy of atom after absorbing this photon is given by

$$E_i - E_1 = 1.44 \text{ eV} + 6.52 = 7.96 \text{ eV}$$

(b)  $\frac{12375 \text{ (eV)}}{4200} = 2.95 \text{ eV}$

hence internal energy of the atom after emission of this photon is given by

$$E_i - E_1 = 2.95 \text{ eV} + 6.52 = 9.47 \text{ eV}$$

27. Heve  $U = \frac{1}{2} m v^2 = F \left| \frac{dU}{dr} \right| = m^2 v^2 r$

$$\text{But } \frac{mv^2}{r} = m^2 v^2 r$$

$$v^2 = m^2 v^2 r^2$$

But by Bohr's postulate  $mvr = \frac{nh}{2\pi}$

$$m^2 v^2 r^2 = \frac{n^2 h^2}{4\pi^2}$$

$$m^3 v^2 r^4 = \frac{n^2 h^2}{4\pi^2}$$

$$r^4 = \frac{n^2 h^2}{4\pi^2 m^3 v^2}$$

$$r = \frac{\sqrt{n} h}{2\pi m v}$$

$$28. \frac{1}{K} R(z-1)^2 = \frac{1}{2^2} = \frac{1}{4}$$

$$\frac{1}{K} R(z-1)^2 = \frac{1}{3^2} = \frac{1}{9}$$

$$\frac{K}{K} = \frac{3}{4} \times \frac{9}{8} = \frac{27}{32}$$

$$K = \frac{27}{32} K = \frac{27}{32} \times 0$$

$$29. \frac{hc}{eV}$$

$$\frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 25 \times 10^3}$$

$$= 49.5 \text{ pm}$$

$$= 2 \times 49.5 \text{ pm} = 99 \text{ pm}$$

$$[1 \text{ pm} = 10^{-12} \text{ m}]$$

$$30. f_K = (2.48 \times 10^{15}) \text{ Hz} (z-1)^2$$

$$\frac{3 \times 10^8}{K} = 2.48 \times 10^{15} (2-1)^2$$

$$\frac{3 \times 10^8}{0.76 \times 10^{10} \times 2.58 \times 10^{15}} = (z-1)^2$$

$$z-1 \sim 40 \quad z = 41$$

$$31. \frac{hc}{eV} = 26 \text{ pm when } V_f = 1.5 \text{ V}$$

$$\frac{f}{e} = \frac{hc}{1.5 \text{ V}} = \frac{1}{1.5} \frac{hc}{eV} = \frac{12}{1.5} \frac{1}{1.5} \times 10^{-10} \text{ m}$$

$$\left( \frac{1}{i} - \frac{26}{3} \right) \frac{2}{3} \frac{1}{i}$$

$$\frac{3}{i} - \frac{26}{3} - \frac{3}{2} \frac{2}{i}$$

$$\frac{1}{i} - \frac{78}{10^{12}} \frac{6.6 \times 10^{34}}{1.6 \times 10^{19}} \frac{3}{V} \frac{10^8}{V}$$

$$V = \frac{6.6 \times 10^{26}}{1.6 \times 78 \times 10^{12} \times 10^{19}}$$

$$V = 15865 \text{ volt}$$

$$32. \sqrt{V} = a(z - b)$$

$$\frac{c}{a^2} (z - b)^2$$

$$\frac{1}{c} \frac{a^2}{c} (z - b)^2$$

$$\frac{1}{887 \text{ pm}} \frac{a^2}{c} (13 - b)^2 \quad \dots(i)$$

$$\text{and} \quad \frac{1}{146 \text{ pm}} \frac{a^2}{z} (30 - b)^2 \quad \dots(ii)$$

Dividing Eq. (i) and Eq. (ii)

$$\frac{146}{887} \frac{(13 - b)^2}{(30 - b)}$$

$$2.5 \frac{30 - b}{13 - b}$$

$$32.5 = 2.5b + 30 - b$$

$$2.5 = 1.5b$$

$$b = \frac{5}{3}$$

$$\frac{1}{26} \frac{a^2}{c} - \frac{26}{3} \frac{5}{3} \frac{2}{3}$$

$$\frac{1}{26} \frac{26 \frac{5}{3} \frac{2}{3}}{887 \text{ pm} \frac{5}{3} \frac{2}{3}}$$

$$\frac{887 \text{ pm} \frac{5}{3} \frac{2}{3}}{26 \frac{5}{3} \frac{2}{3}}$$

$$887 \text{ pm} \frac{34^2}{(73)^2}$$

$$\sim 198 \text{ pm}$$

$$33. \sqrt{f} = \sqrt{\frac{3RC}{4}} (z - 1)$$

$$\sqrt{4.2 \times 10^{18}} \sqrt{\frac{3 \times 1.1 \times 10^7 \times 3 \times 10^8}{4}} (z - 1)^2$$

$$\frac{4.2 \times 10^{18} \times 4}{9 \times 1.1 \times 10^{15}} (z - 1)^2$$

$$(z - 1) = \frac{41}{z} = \frac{42}{z}$$

$$34. P = V_i = 40 \text{ kW} = 10 \text{ mA} = 400 \text{ W}$$

$$\% \text{ of } P = \frac{400}{100} = 4 \text{ W}$$

$$(a) \text{ Total power of X-rays} = 4 \text{ W}$$

$$(b) \text{ Heat produced per second}$$

$$400 - 4 = 396 \text{ J/s}$$

## Photoelectric effect

35. Einstein photo electric equation is

$$K_{\max} = h\nu - W$$

$$eV_0 = \frac{hc}{\lambda} - W \therefore K_{\max} = eV_0$$

$$10.4 \text{ eV} = \frac{12375}{\lambda (\text{\AA})} - 1.7 \text{ eV}$$

$$\lambda (\text{\AA}) = \frac{12375}{12.1} = 1022 \text{\AA}$$

$$\text{For H-atom} \quad \frac{hc}{E} = \frac{E}{1022} = 12.1 \text{ eV}$$

This difference equal to  $n = 3 - n = 1$  transition.

$$36. K_{\max} = h\nu - W$$

$$K_{\max} = \frac{6.6 \times 10^{-34} \times 1.5 \times 10^{15}}{1.6 \times 10^{-19}} - 3.7$$

$$K_{\max} = 6.18 - 3.7 = 2.48 \text{ eV}$$

37. Here work function

$$W (\text{in eV}) = \frac{12375}{5000 \text{ \AA}} = 2.475 \text{ eV}$$

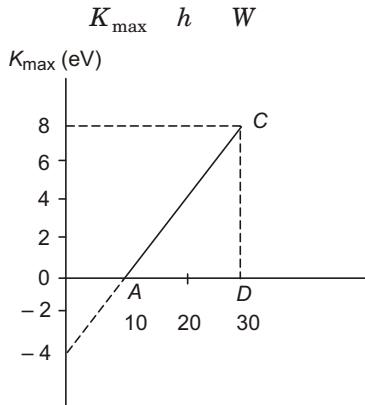
$$K_{\max} = eV_0 = 3 \text{ eV}$$

$$K_{\max} = \frac{hc}{\lambda} - W$$

$$3 = \frac{12375}{(\lambda \text{ in \AA})} - 2.475$$

$$\frac{12375}{5.475} = 2260 \text{ \AA}$$

38. Comparing the given graph with



$$\nu = 1 \times 10^{14} \text{ Hz}$$

(a)  $\nu_0$  threshold frequency  $= 10 \times 10^{14} \text{ Hz}$   
 $10^{15} \text{ Hz}$

(b)  $W = 4 \text{ eV}$

(c)  $h$  slope of the graph

$$\frac{CD}{AD} = \frac{8 \text{ eV}}{20 \times 10^{14} \text{ Hz}}$$

$$h = \frac{8 \times 1.6 \times 10^{-19}}{2 \times 10^{15}} = 6.4 \times 10^{-34} \text{ J-s}$$

$$39. \text{ Here } \frac{v_{1(\max)}}{u_{2(\max)}} = \frac{3}{1}$$

Using Einstein equation,  $K_{\max} = \frac{hc}{\lambda} - W$ ,

we get

$$\frac{1}{2}mv_{\max}^2 = \frac{hc}{\lambda} - W$$

where  $m$  is the mass of photoelectron

$$\frac{1}{2}m[v_{1(\max)}]^2 = \frac{hc}{\lambda_1} - W \quad \dots(i)$$

$$\text{and } \frac{1}{2}m[v_{2(\max)}]^2 = \frac{hc}{\lambda_2} - W \quad \dots(ii)$$

Dividing Eq. (i) and Eq. (ii), we get

$$\frac{v_{1\max}^2}{v_{2\max}^2} = \frac{\frac{hc}{\lambda_1} - W}{\frac{hc}{\lambda_2} - W} \quad (3)^2 = \frac{\frac{1}{\lambda_1} - \frac{W}{hc}}{\frac{1}{\lambda_2} - \frac{W}{hc}}$$

$$\frac{9hc}{2} = \frac{hc}{1} - 8W$$

$$hc = \frac{9}{6000} - \frac{1}{3000} = 8W$$

$$\frac{6.6 \times 10^{-34}}{6000 \times 10^{-10}} - \frac{3 \times 10^{-8}}{1.6 \times 10^{-19}} = 8W$$

$$W = 1.81 \text{ eV}$$

Putting the value of  $W$  in Eq. (i)

$$\frac{1}{2}m(u_{1\max})^2 = \frac{hc}{3000 \times 10^{-10}} - 1.81 \times 1.6 \times 10^{-19}$$

$$\frac{1}{2}m(u_{1\max})^2 = \frac{6.62 \times 10^{-34}}{3 \times 10^{-7}} - 2.896 \times 10^{-19}$$

$$\frac{1}{2}m(u_{1\max})^2 = 3.724 \times 10^{-19}$$

$$(u_{1\max})^2 = \frac{3.724 \times 10^{19} \times 2}{9.1 \times 10^{31}}$$

$$1.81 \times 1.6 \times 10^{19} \div m_e \times 9.1 \times 10^{31}$$

$$u_{1\max} = 9 \times 10^5 \text{ m/s}$$

$$\text{and } v_{2\max} = \frac{1}{3} v_{1\max} = 3 \times 10^5 \text{ m/s}$$

40. Here intensity  $I = 2 \text{ W/m}^2$  and

$$\text{Area } A = 1 \times 10^{-4} \text{ m}^2$$

Energy incident per unit time on the metal surface

$$E = IA = \frac{2 \times 10^{-4} \text{ W}}{1.6 \times 10^{19} \text{ s}} = 2 \times 10^{-4} \text{ J/s}$$

$$\frac{2 \times 10^{15}}{1.6} \text{ eV/s}$$

$$\text{Energy of each photon} = 10.6 \text{ eV}$$

Number of photons incident on surface

$$\frac{2 \times 10^{15}}{1.6 \times 10^6}$$

Number of photoelectrons emitted

$$\frac{0.53}{100} = \frac{2 \times 10^{15}}{1.6 \times 10^6}$$

$$6.25 \times 10^{11} \text{ per second}$$

$$\text{Minimum KE} = 0$$

$$\text{Maximum KE} = (10.6 - 5.6) \text{ eV} = 5 \text{ eV}$$

41.  $K_{\max} = \frac{hc}{\lambda} - W$

$$K_{\max} = \frac{6.6 \times 10^{34} \times 3 \times 10^8}{180 \times 10^9} - 2 \times 1.6 \times 10^{19}$$

$$\frac{1}{2} m_e v_{\max}^2 = \frac{6.6 \times 3 \times 10^{18}}{18} - 2 \times 1.6 \times 10^{19}$$

$$\frac{1}{2} m_e v_{\max}^2 = 11 \times 10^{19} - 3.2 \times 10^{19}$$

$$7.2 \times 10^{19} \text{ J}$$

$$v_{\max} = \sqrt{\frac{2 \times 7.2 \times 10^{19}}{9.1 \times 10^{31}}} = 1.25 \times 10^6 \text{ m/s}$$

$$r = \frac{mv_{\max}}{eB} = \frac{9.1 \times 10^{31} \times 1.25 \times 10^6}{1.6 \times 10^{19} \times 5 \times 10^5}$$

$$r = 0.148 \text{ m}$$

42. The given equation is

$$E = (100 \text{ V/m}) [\sin((5 \times 10^{15})t) \sin(8 \times 10^{15})t]$$

Light consist of two different frequencies.

Maximum frequency

$$\frac{8 \times 10^{15}}{2} = 1.27 \times 10^{15} \text{ Hz}$$

For maximum KE we will use Einstein's equation

$$(KE)_{\max} = h\nu - W = \frac{1.27 \times 10^{15} \times 6.62 \times 10^{-34}}{1.6 \times 10^{19}} - 2$$

$$(KE)_{\max} = 3.27 \text{ eV}$$

43. Here  $E = E_0 \sin 1.57 \times 10^7 (x - ct)$  frequency of the wave

$$\frac{1.57 \times 3 \times 10^{15}}{2 \times 3.14} = 0.75 \times 10^{15}$$

We have

$$eV_0 = \frac{6.6 \times 10^{-34} \times 0.75 \times 10^{15}}{1.8 \times 10^{19}} = 1.9 \text{ eV}$$

$$V_0 = 1.2 \text{ V}$$

## ■ Objective Questions (Level-1)

1. Einstein photo electric equation is

$$K_{\max} = h\nu - W$$

its slope  $h$  planck constant which is same for all metals and independent of intensity of radiation.

Hence correct option is (d).

2. Since current is directly proportional to intensity therefore as current is increased

intensity is increased since  $\frac{1}{V}$ , if  $V$  is

decreased  $\lambda_{\min}$  is increased.

Hence correct option is (c).

3. For hydrogen atom (Bohr's model)  $n$ th orbital speed  $v_n = \frac{e^2}{2_0 n h}$

For first orbit  $n = 1$

$$v_1 = \frac{e^2}{2_0 h} = \frac{(1.6 \times 10^{-19})^2}{2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}}$$

$$v \sim \frac{1}{137} \times 3 \times 10^8 = \frac{c}{137}$$

Hence correct option is (c).

4.  ${}_{86}A^{22} \quad {}_3 \quad {}_{80}X^{210} \quad {}_4 \quad {}_{84}B^{210}$

Hence correct option is (b)

5.  $\lambda_{\min} (\text{in } \text{\AA}) = \frac{12375}{V (\text{in volt})} = \frac{12375}{20 \times 1000} \sim 0.62 \text{\AA}$

Hence correct option is (c).

6. We have  $\frac{1}{2} m_e v_{\max}^2 = eV$

$$v_{\max} = \sqrt{\frac{2eV}{m}}$$

$$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 18 \times 1000}{9.1 \times 10^{-31}}}$$

$$v_{\max} \sim 8 \times 10^7 \text{ m/s}$$

Hence correct option is (a).

7. For hydrogen atom  $v_n = \frac{e^2}{2_0 n h}$

$$v_2 = \frac{e^2}{2_0 \times 2h}, v_3 = \frac{e^2}{2_0 \times 3h}$$

$$\frac{v_2}{v_3} = \frac{3}{2}$$

Let  $\lambda_2$  and  $\lambda_3$  are the de-Broglie wavelengths

$$\frac{\lambda_2}{\lambda_3} = \frac{\frac{h}{mv_2}}{\frac{h}{mv_3}} = \frac{v_3}{v_2}$$

$$\frac{\lambda_2}{\lambda_3} = \frac{2}{3}$$

Hence correct option is (a).

8. For hydrogen like atom

$$E_n = \frac{z^2}{n^2} (13.6 \text{ eV})$$

For ground state  $n = 1$

$$E_1 = \frac{z^2}{1^2} \times 13.6 \text{ eV}$$

But  $E_1 = 122.4 \text{ eV}$

$$122.4 \text{ eV} = \frac{z^2}{1^2} \times 13.6 \text{ eV}$$

$$\frac{z^2}{1^2} = 9$$

$$z = 3$$

Hence it is  $\text{Li}^{2+}$

The correct option is (c).

9.  $\lambda_{\min} = \frac{hc}{eV} = \frac{\lambda_{\min}}{100} \times \frac{V}{100}$

Percentage change in  $\lambda_{\min} = 2\%$

Hence  $\lambda_{\min}$  is decreased by 2%

correct option is (c)

10.  $E_n = \frac{z^2}{n^2} (13.6 \text{ eV})$  for first excited state  $n = 2$

$$E_2 = \frac{z^2}{4} (13.6) \text{ eV}$$

$$13.6 \text{ eV} = \frac{z^2}{4} \times 13.6 \text{ eV}$$

$$z = 2$$

Hence it is He

Correct option is (a).

11.  $\lambda_{\min} (\text{in } \text{\AA}) = \frac{12375}{V (\text{in volt})}$

$$V = \frac{12375}{1} = 12.375 \times 10^3 \text{ V}$$

$$V = 12.4 \text{ eV}$$

Hence correct option is (c).

12. We have  $\frac{h}{p}$

$$p = \frac{h}{0.5 \times 10^{-10}} = \frac{6.62 \times 10^{-34}}{0.5 \times 10^{-10}} = 13.26 \times 10^{-24} \text{ kg-m/s}$$

Hence correct option is (c).

13.  $W = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19}} = 1.2375 \times 10^{-16} \text{ J}$

Hence correct option is (a).

14. For H-like atom Balmer series is  $\frac{1}{n^2} - \frac{1}{m^2}$

For third Balmer line  $n = 5$

$$\frac{1}{1085 \times 10^{-10}} - \frac{1}{z^2} = 1.09 \times 10^7 \left( \frac{1}{4} - \frac{1}{5^2} \right)$$

$$z^2 = \frac{100 \times 1000}{1085 \times 1.097 \times 21} = 4$$

$$z = 2$$

Binding energy  $(13.6) z^2 \text{ eV} = 13.6 \times 4 \text{ eV} = 54.4 \text{ eV}$

Hence correct option is (a).

15. If  $V_1 = 0$  then total energy = KE

$$\text{KE} = 13.6 \text{ eV}$$

and the energy difference between two states  $10.2 \text{ eV}$

Hence total energy in this state

$$13.6 + 10.2 = 23.8 \text{ eV}$$

The correct option is (c).

16.  $eV_0 = \frac{hc}{3300} = W$  ... (i)

$$2eV_0 = \frac{hc}{2200}$$
 ... (ii)

Subtracted Eq. (i) from Eq. (ii), we get

$$eV_0 = \frac{hc}{3300} = \frac{3300}{2200} \times \frac{hc}{3300}$$

$$V_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{3 \times 2200 \times 1.6 \times 10^{-19} \times 10^{-10}} = \frac{30}{16} \times \frac{15}{8} = \frac{15}{8} \text{ V}$$

Hence the correct option is (c).

17.  $E = \frac{hc}{\lambda} = W$  ... (i)

$$4E = \frac{hc}{\lambda} = W$$

$$4E = \frac{3hc}{\lambda} = W$$
 ... (ii)

From Eqs. (i) and (ii)

$$4 \frac{hc}{\lambda} = W = \frac{3hc}{\lambda} = W$$

$$\frac{hc}{\lambda} = 3W$$

$$W = \frac{hc}{3}$$

Hence correct option is (b).

18. By Moseley's law

$$\sqrt{f} = a(z - b) \text{ for } K \text{ line } b = 1$$

$$\sqrt{f} = a(z - 1)$$

$$\sqrt{f} = a(31 - 1) = a \times 30$$
 ... (i)

and  $\sqrt{f} = a(51 - 1) = 9 \times 50$

$$\frac{\sqrt{f}}{\sqrt{f}} = \frac{3}{5}$$

$$f = \frac{25f}{9}$$

Hence correct option is (a).

19.  $\sqrt{f} = \sqrt{RC} (z - 1) \sqrt{\frac{1}{12} - \frac{1}{n^2}}$

for  $K, n = 2$



$$K, n = 3$$

$$\sqrt{f} = \sqrt{RC} (z-1) \sqrt{1 - \frac{1}{2^2}} = \sqrt{RC} (z-1) \sqrt{\frac{3}{4}}$$

$$\sqrt{f} = \sqrt{RC} (z-1) \sqrt{1 - \frac{1}{9}} = \sqrt{RC} (z-1) \sqrt{\frac{8}{9}}$$

$$\frac{\sqrt{f}}{\sqrt{f}} = \frac{\sqrt{\frac{8}{9} \cdot \frac{4}{3}}}{\sqrt{\frac{32}{27}}}$$

Correct option is (a).

$$20. \frac{1}{R} = \frac{1}{2^2} - \frac{1}{n^2} \text{ here } n = 3$$

$$\frac{1}{R} = \frac{1}{2^2} - \frac{1}{3^2} = \frac{9-4}{9 \cdot 4} = \frac{5R}{36}$$

Hence correct option is (c)

$$21. D = \frac{h}{\sqrt{2meV}} \text{ and } \min = \frac{hc}{eV}$$

$$\frac{D}{\min} = \frac{1}{c} \sqrt{\frac{eV}{2m}}$$

$$= \frac{1}{3 \cdot 10^8} \sqrt{\frac{1.8 \cdot 10^{11} \cdot 10000}{2}}$$

$$\frac{D}{\min} = \frac{3 \cdot 10^7}{3 \cdot 10^8} = \frac{1}{10}$$

Hence correct option is (c).

$$22. 5eV_0 = \frac{hc}{W} \quad \dots(i)$$

$$eV_0 = \frac{hc}{3} = W \quad \dots(ii)$$

From Eqs. (i) and (ii)

$$\frac{5hc}{3} = 5W = \frac{hc}{W}$$

$$\frac{5hc}{3} = \frac{hc}{4W}$$

$$\frac{hc(5-3)}{3} = 4W$$

$$\frac{2hc}{3} = 4W$$

$$W = \frac{hc}{6}$$

Hence correct option is (a).

$$23. eV_0 = h[2V_0 - V_0] = hV_0 \quad \dots(i)$$

$$eV = h[3V_0 - V_0] = h \cdot 2V_0 = 2hV_0 \quad \dots(ii)$$

From Eqs. (i) and (ii)

$$eV = 2 \cdot eV_0 = V = 2V_0$$

Hence correct option is (b).

24. For H-atom Lyman series is

$$\frac{1}{\lambda} = R \left[ \frac{1}{1^2} - \frac{1}{n^2} \right]$$

$$v = \frac{c}{\lambda} = RC \left[ \frac{1}{1^2} - \frac{1}{n^2} \right]$$

For H-like atom

$$RC = \frac{1}{12} \left[ \frac{1}{n^2} - z^2 \right]$$

$$\text{For Li}, n = 3 \quad 3^2 = 9v$$

Hence correct option is (c).

25. Ground state energy of H-atom = 13.6 eV

$$\text{For Li atom } E_n = \frac{(13.6)eV}{n^2} z^2$$

$$13.6eV = \frac{13.6eV}{n^2} \cdot 9$$

$$n = 3$$

Hence correct option is (c).

$$26. \frac{1}{2}mv_1^2 = h \cdot \frac{1}{\lambda} = W \quad \dots(i)$$

$$\frac{1}{2}mv_2^2 = h \cdot \frac{1}{\lambda} = W \quad \dots(ii)$$

$$\frac{1}{2}m[v_1^2 - v_2^2] = h \left[ \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right]$$

$$v_1^2 - v_2^2 = \frac{2h}{m} \left[ \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right]$$

Hence correct option is (b).

27. For Lyman series

$$\frac{1}{\lambda} = R \left[ \frac{1}{1^2} - \frac{1}{n^2} \right]$$

For largest wavelength  $n = 2$

$$\frac{1}{R} = R \left( \frac{1}{1^2} - \frac{1}{2^2} \right)$$

$$\frac{1}{R} = R \left( 1 - \frac{1}{4} \right)$$

$$\frac{4}{3R}$$

For He atom

$$\frac{1}{R_{\text{He}}} = R(z)^2 \left( \frac{1}{2^2} - \frac{1}{n^2} \right) = 4R \left( \frac{1}{4} - \frac{1}{n^2} \right)$$

$$\frac{3R}{4} = 4R \left( \frac{1}{4} - \frac{1}{n^2} \right)$$

$$\frac{3}{16} = \frac{1}{4} - \frac{1}{n^2}$$

$$\frac{1}{n^2} = \frac{1}{4} - \frac{3}{16} = \frac{4}{16} - \frac{3}{16} = \frac{1}{16}$$

$$n = 4$$

Hence correct option is (b).

28. We have  $\sqrt{\frac{1}{R}} = \sqrt{\frac{3R}{4}} (z - 1)$

$$\sqrt{\frac{1}{R}} = \sqrt{\frac{3 \cdot 1.0973 \cdot 10^7}{4}} (92 - 1)$$

$$\frac{4}{3 \cdot 91 \cdot 91 \cdot 1.0973 \cdot 10^7} = 0.15 \text{ \AA}$$

Hence correct option is (c).

29. For  $K$ ,  $K$  and  $L$  of X-rays

$$\frac{K}{Y_2} = \frac{K}{Y_1} = \frac{L}{Y_3}$$

Hence correct option is (b).

30. We have  $\frac{h}{\sqrt{2mqV}}$

$$p = \frac{h}{\sqrt{2m_p eV}} \text{ and } \frac{h}{\sqrt{2m_e V}}$$

$$\frac{p}{m_p} = \frac{\sqrt{2m}}{m_p} \sqrt{\frac{2 \cdot 4m_p}{m_p}} = \sqrt{8} = 2\sqrt{2}$$

Hence correct option is (c).

31. We have,  $\frac{h}{\sqrt{2m_e E_1}} = \frac{h}{\sqrt{2m_e E_2}}$

$$p = \frac{h}{\sqrt{2m_p E_3}} \therefore m_p = m_e$$

and  $\frac{E_1}{p} = \frac{E_3}{p} = \frac{E_2}{p}$

Hence correct option is (a).

32. KE in ground state = 13.6 eV

Total energy in  $n$  KE Energy  
difference between  $n$  to  $n = 1$

Total energy

$$13.6 \text{ eV} - 13.6 \text{ eV}$$

$$27.2 \text{ eV}$$

Hence correct option is (b).

33. Here  $P = 1000 \text{ W}$ ,  $f = 880 \text{ kHz}$ ,  $\lambda = 880 \cdot 10^3 \text{ Hz}$

Let  $n$  is the number of photon  $p$  emitted per second

$$n = \frac{P}{h} = \frac{1000}{6.62 \cdot 10^{-34} \cdot 880 \cdot 10^3}$$

$$1.7 \cdot 10^{30}$$

Correct option is (b).

34. Here  $\lambda = 3000 \text{ \AA} = 3 \cdot 10^{-7} \text{ m}$

Energy of incident radiation  $E = \frac{hc}{\lambda}$  joule

$$E = \frac{hc}{1.6 \cdot 10^{-19}} \text{ (in eV)}$$

$$E = \frac{6.62 \cdot 3 \cdot 10^{26}}{3 \cdot 1.6 \cdot 10^{26}} = 4.125 \text{ eV}$$

$\therefore E$  work function hence no emission of electrons it means sphere remain natured.

Hence correct option is (c).

35.  $\therefore E_n = \frac{13.6 \text{ eV}}{n^2}$  for  $n = 5$

$$E_5 = \frac{13.6 \text{ eV}}{5^2} = 0.54 \text{ eV}$$

Hence correct option is (a).

36.  $K_{\max} = E - W = (6.2 - 4.2) \text{ eV} = 2 \text{ eV}$

$$K_{\max} = 2 \times 1.6 \times 10^{19} \text{ J}$$

$$= 3.2 \times 10^{19} \text{ J}$$

Hence correct option is (b).

37.  $\frac{6.62 \times 10^{-34}}{5200 \times 10^{-10}} = 9.1 \times 10^{-31} \text{ v}$

$$v = \frac{6.625 \times 10^{-27}}{5.2 \times 9.1 \times 10^{-31}} \sim 1400 \text{ m/s}$$

Hence correct option is (c).

38.  $K_{\max} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{3000 \times 10^{-10} \times 1.6 \times 10^{-19}} = 1 \text{ eV}$

$$K_{\max} = 3.14 \text{ eV} = 3.14 \times 1.6 \times 10^{-19} \text{ J}$$

$$\frac{1}{2} m_e v_{\max}^2 = 3.14 \times 1.6 \times 10^{-19}$$

$$v_{\max} = \sqrt{\frac{3.14 \times 3.2 \times 10^{-19}}{9.1 \times 10^{-31}}} \sim 10^6 \text{ m/s}$$

Hence correct option is (d).

## JEE Corner

1. Here both assertion and reason are true and reason explain correctly assertion. Correct option is (a).

2. For photon  $E = \frac{hc}{\lambda}$  and  $p = \frac{E}{c}$

If  $\lambda$  is doubled,  $E$  and  $p$  are reduced to half. Hence assertion is true. Since speed of photon is always  $c$ . Hence reason is false. Hence correct option is (c).

3. If frequency is increased keeping intensity constant photoelectron emitted the plate reach other plate in less time hence saturation current can be increased. Reason can be true or not hence correct option is (a, b).

4. Here both assertion and reason is true and reason correctly explain assertion. Hence correct option is (a).

5. Here assertion is true since possible transition are 6 → 3, 6 → 4, 6 → 5, 5 → 3, 5 → 4, and 4 → 3. According to reason total  $\frac{n(n-1)}{2}$  transition has  $n = 3$   $\frac{6(3-1)}{2} = 6$

it may explain or may not explain assertion

Hence correct option is (a, b)

6. We have

$$eV_0 = h[\nu_0]$$

$$V_0 = \frac{h}{e} \nu_0 = \frac{h}{e} \nu_0 \dots (i)$$

if  $\nu_0$  does not become double hence assertion is false but reason is true. Hence correct option is (d).

7. Here both assertion and reason are true and reason may or may not explain assertion correct option is (a, b).

8. Here assertion and reason are both true.

$$\therefore \text{min } \frac{hc}{eV} \text{ if } V \text{ increases}$$

$$\text{min } \frac{hc}{eV} \text{ decreases}$$

but reason is not correct explanation of assertion hence correct option is (b).

9.  $\therefore E_n = \frac{13.6}{n^2}$

$$E_2 = E_1$$

Hence assertion is true and  $E = K \frac{v}{2}$

$$v \text{ is more in } n = 2$$

Here reason is also true but it is not correct explanation of assertion hence correct option is (b).

10. Here assertion is false but reason is true. Hence correct option is (a).

### ■ Objective Questions (Level 2)

$$1. F_a = F_c = \frac{GmM}{r^2} = \frac{mv^2}{r} \quad \dots(i)$$

$$\text{and} \quad mvr = \frac{nh}{2} \quad \dots(ii)$$

From Eq. (ii)

$$v = \frac{nh}{2mr}$$

Putting this value in Eq. (i)

$$\frac{GM}{r} = \frac{h^2 h^2}{4 m^2 r^2}$$

$$r = \frac{n^2 h^2}{4 m^2 GM}$$

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m \frac{GM}{r} = \frac{GMm}{2r}$$

$$\text{PE} = \frac{GMm}{r}$$

$$E = \text{KE} + \text{PE}$$

$$E = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

$$E = -\frac{2^2 G^2 M^2 m^3}{n^2 h^2} \text{ for ground state } n = 1$$

$$E = -\frac{2^2 G^2 M^2 m^3}{n^2}$$

Hence correct option is (b).

$$2. \text{ We have } i_n A_n = \frac{e}{T_n} r_n^2$$

$$i_n = \frac{e}{2\pi r_n} \frac{r_n^2}{u_n} = \frac{e u_n r_n}{2}$$

$$i_n = e \frac{v_1}{n} \frac{r_1}{2} \frac{n^2}{2} = \frac{e v_1 r_1}{2} n$$

$$\text{and} \quad i_1 = \frac{e v_1 r_1}{2}$$

$$i_1 = \frac{2}{n^2}$$

Hence magnetic moment decreases two times correct option is (b).

$$3. \text{ For H-like atom } E_n = \frac{(13.6)Z^2 \text{ eV}}{n^2}$$

$$\text{Here } E_2 = \frac{(13.6)Z^2}{4} \text{ and } E_1 = 13.6 Z^2$$

$$E_2 - E_1 = 40.8 \text{ eV}$$

$$13.6 Z^2 - 13.6 \frac{Z^2}{4} = 40.8$$

$$Z^2 - \frac{Z^2}{4} = \frac{40.8}{13.6} \Rightarrow \frac{3Z^2}{4} = 3 \Rightarrow Z^2 = 4$$

$$Z = 2$$

Energy needed to remove the electron from ground state is

$$E_1 = (13.6) Z^2 = 13.6 \times 4 = 54.4 \text{ eV}$$

Hence correct option is (a).

$$i_n = \frac{e}{T_n} = \frac{e}{2\pi r_n} \frac{e v_n}{u_n}$$

$$\therefore u_n = \frac{1}{n} \text{ and } r_n = \frac{1}{n^2}$$

$$i_n = \frac{1}{n^3} = \frac{i_1}{2^3}$$

$$i_1 = 8 i_2$$

Hence current increases 8 times correct option is (c).

5. Since five dark lines are possible hence atom is excited to  $n = 6$  state.

$$\text{The number of transition in emission line} = \frac{n(n-1)}{2}$$

$$\text{Number of emission transition} = \frac{6 \times 5}{2} = 15$$

Hence correct option is (c).

$$6. A_n = \frac{r_n^2}{k n^2} \text{ for hydrogen atom } r_n = k n^2$$

where  $k$  is constant.

$$A_n = \frac{k^2 n^4}{k n^2}$$

$$A_1 = \frac{k^2}{A_1} \Rightarrow A_1^2 = k^2 \Rightarrow A_1 = k$$

Taking log both sides  $\log \frac{A_n}{A_1} = 4 \log n$

Hence it is a straight line with slope 4

Correct option is (b)

7. For hydrogen atom  $i_n = \frac{1}{n^3}$  and

$$B_n = \frac{i_n}{r_n} = B_1 \cdot k \frac{1}{n^5} \quad [\because r_n \propto n^2]$$

$$B_2 = \frac{k}{2^5} \text{ and } B_1 = k$$

$$\frac{B_2}{B_1} = \frac{1}{2^5} = \frac{1}{32}$$

$$B_1 = 32 B_2$$

Hence magnetic field increases 32 times.

The correct option is (d).

8. For H-atom Lyman series is given by

$$\frac{1}{\lambda} = R \left( \frac{1}{1^2} - \frac{1}{n^2} \right) \text{ for first line } n = 2$$

$$\frac{1}{\lambda} = R \left( 1 - \frac{1}{2^2} \right)$$

$$\frac{1}{\lambda} = \frac{3R}{4}$$

Momentum of photon  $P_p = \frac{h}{\lambda}$

Let momentum of atom  $p_A$

$\therefore$  Initial momentum was zero. Hence using momentum conservation law, we get

$$p_A = p_B = Mv = \frac{h}{\lambda} = \frac{3hR}{4} \\ v = \frac{3hR}{4M}$$

Hence the correct option is (a).

9. Light wave equation is

$$200 \text{ V/m} \sin(1.5 \times 10^{15} \text{ sec}^{-1}) t \\ \cos(0.5 \times 10^{15} \text{ sec}^{-1}) t$$

$$\text{Here maximum frequency} = \frac{1.5 \times 10^{15}}{2}$$

Maximum incident energy

$$\frac{1.5 \times 10^{15}}{2} = \frac{6.6 \times 10^{-34}}{1.6 \times 10^{-19}}$$

$$0.98 \text{ eV}$$

Since work function = 2 eV maximum energy hence no emission of electrons.

Thus correct option is (d).

10. Since in Balmer series of H-like atom wavelengths (in visible region) are found same or smaller hence the gas was initially in second excited state.

Correct option is (c).

11. For H-atom  $T_H = 2 n^3$

and for H-like atom

$$T_x = \frac{2 n^3}{z^2}$$

For H-atom in ground state  $T_H = 2$

For H-like atom in first excited state

$$T_x = \frac{(2)^3}{z^2} = \frac{2 \times 8}{z^2}$$

$$\text{But } T_H = 2T_x = 2 \times \frac{2 \times 8}{z^2}$$

$$\frac{z^2}{2} = 16 \\ z = 4$$

Hence correct option is (c).

12. For K line of X-ray

$$\frac{1}{\lambda} = \frac{a^2}{c} (z - 1)^2$$

$\therefore z$  (atomic No.) for  $\text{Pb}^{204}$ ,  $\text{Pb}^{206}$ ,  $\text{Pb}^{208}$

are same hence  $\lambda_1 = \lambda_2 = \lambda_3$ .

Hence correct option is (c).

13. The correct option is (d).

$$14. \text{ Since } E_n = \frac{13.6 \text{ eV}}{n^2}$$

$$E_1 = 13.6 \text{ eV}$$

$$\text{and first excited state } E_2 = \frac{13.6}{4} \text{ eV}$$

$$E_2 - E_1 = 3.4 \text{ eV}$$

$$E_2 - E_1 = 10.2 \text{ eV} \text{ if } K = 10.2 \text{ eV}$$

The electron collide elastically with H-atom in ground state.

The correct option is (c).

15. For Lyman series  $\frac{1}{\lambda} = R \left( \frac{1}{1^2} - \frac{1}{n^2} \right)$  here  $n = 3$

$$\frac{1}{\lambda} = R \left( 1 - \frac{1}{9} \right)$$

$$\frac{1}{\lambda} = \frac{8R}{9}$$

$$P_{\text{photon}} = \frac{h}{\lambda} = \frac{8Rh}{9}$$

But

$$P_{\text{photon}} = P_{\text{H-atom}}$$

$$\frac{8Rh}{9} = M_p v$$

$$v = \frac{8}{9} \times \frac{1.097}{1837} \times \frac{10^7}{9.1} \times \frac{6.6}{10} \times \frac{10^{34}}{31}$$

$$v = 4 \text{ m/s}$$

Hence correct option is (a).

16. Power  $P = VI = 150 \times 10^3 = 15000 \text{ W}$

The 99% power heated the target hence

$$\text{Heating power} = \frac{99}{100} \times 15000 = 14850 \text{ W}$$

The rate at which target is heated per sec. (in cal)

$$\frac{14850}{4.2} \sim 3535 \text{ cal/sec} \quad \therefore 1 \text{ J} = \frac{1}{4.2} \text{ cal}$$

Hence correct option is (c).

17.  $E_n = \frac{13.6 \text{ eV}}{n^2}$

$$E_3 = \frac{13.6 \text{ eV}}{9} \text{ and } E_4 = \frac{13.6 \text{ eV}}{16}$$

$$E_4 - E_3 = (13.6 \text{ eV}) \left( \frac{1}{9} - \frac{1}{16} \right)$$

$$E = \frac{13.6 \text{ eV}}{16} \times \frac{7}{9} = 32.4 \text{ eV}$$

$$z^2 = \frac{16}{13.6} \times \frac{32.4}{7} = 49$$

$$z = 7$$

Hence correct option is (d).

18.  $\frac{h}{p} = \frac{h}{\sqrt{2m_p K}} = \frac{h}{\sqrt{2m_p eV}}$

$$10^{13} = \frac{6.6 \times 10^{34}}{\sqrt{2 \times 1836 \times 9.1 \times 10^{31} \times 1.6 \times 10^{19}}} \text{ V}$$

$$V = 8.15 \times 10^4 \text{ volt}$$

Hence correct option is (b).

19. Since  $E_n = \frac{1}{n^2}$  and  $L_n = n$

$$\text{Hence } E_n = \frac{1}{L_n^2}$$

The correct option is (d).

20. Since  $r_n = \frac{e u_n r_n}{2} \therefore u_n = \frac{1}{n}$  and  $r_n = n^2$

$$r_n = k n$$

Where  $k$  is constant for H-atom

For ground state  $r_1 = k$  ... (i)

For third excited state  $n = 4$

$$r_4 = 4k$$
 ... (ii)

From Eqs. (i) and (ii) we get  $r_4 = 4r_1$

Hence correct option is (d).

21. By conservation of momentum

$$M_H v = (M_H + M_H) v \quad v = \frac{v}{2}$$

Let initial KE of H-atom  $K$

Final KE of each-H-atom  $\frac{K}{2}$

$$\text{For excitation } \frac{K}{2} = E_2 - E_1 = \frac{13.6}{4} = 13.6$$

$$\frac{K}{2} = 10.2 \text{ eV}$$

$$K = 20.4 \text{ eV}$$

Hence correct option is (a).

22. We know that for H-like atom

$$E_n = K_n \frac{h^2}{2m_e K} \quad 3.4 \text{ eV}$$

$$\frac{h^2}{2m_e K} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}}}$$

$$6.6 \text{ \AA}$$

Hence both options (a) and (b) are correct.

Hence answer is (c) both are correct.

23-25.  $0.6e \frac{hc}{4950 \times 10^{10}} \text{ W} \dots(i)$

$1.1e \frac{hc}{2} \text{ W} \dots(ii)$

Subtracting Eq. (i) from Eq. (ii) we get

$$0.5e \frac{hc}{2} \frac{1}{4950 \times 10^{10}} = \frac{0.5 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34} \times 3 \times 10^8} - \frac{1}{4950 \times 10^{10}} \times \frac{1}{2}$$

$$\lambda_2 \sim 4111 \text{ \AA}$$

From Eq. (i)

$$W = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4.95 \times 10^{-7} \times 1.6 \times 10^{-19}} \sim 1.9 \text{ eV}$$

23.  $W = 1.9 \text{ eV}$

Hence correct option is (c).

24.  $4111 \text{ \AA}$

Hence correct option is (c)

25. Since magnetic field does not change the KE of electrons hence retarding potential remain same.

Hence correct option is (c).

26.  $(KE)_{\max} = 5 \text{ eV} - 3 \text{ eV} = 2 \text{ eV}$

$$\frac{h^2}{2m_e K} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 19 \times 1.6 \times 10^{-19}}}$$

$$\frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 2 \times 1.6 \times 10^{-19}}} = 8.69 \text{ \AA}$$

Hence correct option is (b).

27. Photo emission will stop when potential of sphere becomes stopping potential

$$\frac{1}{4} \frac{q}{r} = 2V$$

Since  $(KE)_{\max} = eV_0$

hence  $V_0 = 2 \times \frac{q}{8 \times 0r} \text{ coulomb}$

Hence correct option is (b)

28. Let  $t$  be the time for photo emission

$$\frac{1}{4} \frac{q}{r} = \frac{8}{t} \frac{0r}{q}$$

Intensity of light at 0.8 from source

$$I = \frac{3.2 \times 10^{-3}}{4 \times (0.8)^2} \sim 4 \times 10^{-4} \text{ W/m}^2$$

Energy incident on the sphere in unit time

$$E_1 = (8 \times 10^{-3})^2 \times 4 \times 10^{-4} = 8.04 \times 10^{-8} \text{ W}$$

Energy of each photon

$$E_2 = 5 \times 1.6 \times 10^{-19} = 8 \times 10^{-19} \text{ J}$$

Total number of photons incident on the sphere per second

$$\frac{E_1}{E_2} = \frac{8.04 \times 10^{-8}}{8 \times 10^{-19}} = 10^{11}$$

Since  $10^6$  photons emit one electron.

Hence the total number of photoelectron

per sec is  $n_2 = \frac{n_1}{10^1} = \frac{10^{11}}{10^6} = 10^5$

Therefore,

$$q = n_2 \times e \times t = \frac{10^5 \times 1.6 \times 10^{-19} \times t}{9 \times 10^9 \times 1.6 \times 10^{-14} \times t} = 2$$

$$t = \frac{2.8 \times 10^{-3}}{9 \times 1.6 \times 10^{-5}} = 111 \text{ s}$$

Hence correct option is (c).

### ■ More than one options are correct

1. Since  $\lambda_0 = \frac{hc}{eV}$  if  $v$  increases  $\lambda_0$  decreases hence the interval between  $\lambda_K$  and  $\lambda_0$  as well as  $\lambda_K$  and  $\lambda_0$  increases.

The correct options are (b) and (c).

2.  $R = n^2, V = \frac{1}{n}$  and  $E = \frac{1}{n^2}$  for Bohr model of H-atom

$$VR = n \text{ and } \frac{V}{E} = n$$

Hence, the correct options are (a) and (c).

3. For Bohr model of H-atom

$$L = n, r = n^2 \text{ and } T = n^3$$

Hence  $\frac{rL}{T}$  is independent of  $n$

$$\frac{L}{T} = \frac{1}{n^2} \text{ and } \frac{T}{r} = n, L = n^3$$

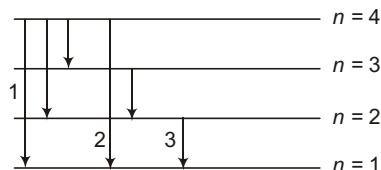
Hence correct options are (a), (b) and (c).

4.  $\therefore \frac{h}{mv} \text{ and } \frac{h}{\sqrt{2mK}}$

Hence heavy particle has smallest wavelength when speed and KE both particle are same.

The correct options are (a) and (c).

5. Since there are six different wavelength



Hence, final state will be  $n = 4$ .

Since two wavelengths are longer than  $\lambda_0$  [(From  $n = 4 \rightarrow 3$  and  $n = 3 \rightarrow 2$ )]

Hence initial state was  $n = 2$

and there are three transitions shown as (1), (2) and (3) belonging to Lyman series.

Hence correct options are (a), (b) and (d).

6.  $\therefore \sqrt{f} = a(z - b)$

$\sqrt{f}$  versus  $z$  is a straight line

$$\therefore f = \frac{c}{\sqrt{\frac{1}{a^2} - \frac{a}{\sqrt{c}}}}(z - b)$$

hence  $\sqrt{\frac{1}{a^2} - \frac{a}{\sqrt{c}}}$  versus  $z$  is a straight line

$$f = a^2(z - b)$$

$\log f = \log a^2 + \log(z - b)$  which is a straight line

Hence correct options are (a), (b) and (c).

### ■ Match the Columns :

1. Lyman series lies in UV region, Balmer series lies in visible region and Paschen and Brackett series lie in infrared region. Hence

(a)	r
(b)	q
(c)	p
(d)	p

2. For H-atom  $E_n = \frac{13.6}{n^2} \text{ eV}$

$$E_2 = \frac{13.6}{2^2} \text{ eV} = \frac{13.6}{4} \text{ eV}$$

Ionization energy from first excited state of H-atom

$$E = E_2 = \frac{13.6}{4} \text{ eV} \quad \dots(i)$$

For He ion

$$E_{H(\text{He})} = \frac{(13.6) \text{ eV}}{n^2} Z^2 \text{ for He } z = 2$$

$$E_n(\text{He}) = \frac{13.6 \text{ eV}}{n^2} \times 4$$

Ionization energy of He atom from ground state  $(13.6) \text{ eV} \times 4 = 4E$  from Eq. (i)

$$16E$$



$$E_2(\text{He}) = \frac{(13.6)\text{eV}}{4} \quad (13.6)\text{eV}$$

But  $E_2 = K_2 = K_2 (13.6)$

and  $\bar{U} = 2K = 2 (13.6) = 2 \cdot 4E = 8E$

From Eq. (i)

KE in ground state of He ion

$$(13.6)\text{eV} \cdot 4$$

$$4E = 4 \cdot 16E$$

Ionisation energy from 1st excited state

$$\frac{13.6\text{eV}}{4} \cdot 4 = 13.6\text{eV} \cdot 4E$$

Hence correct match are

(a)	s
(b)	r
(c)	s
(d)	p

3.  $K_{\max} = h \cdot W$  and  $V_0 = \frac{h}{e} \cdot \frac{W}{e}$

Slope of line 1 is  $h, Y_1 = W$

Slope of line 2 is  $\frac{h}{e}, Y_2 = \frac{W}{e}$

Hence correct match are

(a)	q
(b)	p
(c)	r
(d)	s

4.  $T = \frac{2}{v} \cdot r \therefore r = \frac{n^2}{z} \cdot U = \frac{z}{n}$

$$T = \frac{n^3}{z^2}$$

(a)  $r$

$\therefore L = n$  (b)  $s$

$\therefore V = \frac{z}{n}$  (c)  $s$

$\therefore R = \frac{n^2}{z}$  (d)  $q$

5. Balmer series is

$$\frac{1}{B} = R \cdot \frac{1}{2^2} - \frac{1}{n^2} \quad n = 3, 4, 5 \dots$$

For 2nd line  $n = 4$  and  $B$

$$\frac{1}{B} = \frac{1}{4} - \frac{1}{16} = R$$

$$\frac{16}{3R}$$

$$\frac{1}{B_1} = R \cdot \frac{1}{2^2} - \frac{1}{3^2}$$

$$B_1 = \frac{36}{5R} = \frac{36}{5} \cdot \frac{16}{3} = \frac{36}{16} \cdot \frac{3}{5}$$

$$B_1 = \frac{27}{20}$$

(a)  $p$

$$\frac{1}{B_3} = R \cdot \frac{1}{2^2} - \frac{1}{5^2} = R \cdot \frac{(25 - 4)}{25 \cdot 4} = \frac{21R}{100}$$

$$B_3 = \frac{100}{21R} = \frac{100}{21} \cdot \frac{16}{3}$$

$$\frac{100}{21} \cdot \frac{3}{16} = \frac{25}{28}$$

(b)  $(s)$

For Lyman series

$$\frac{1}{L} = R \cdot \frac{1}{1^2} - \frac{1}{n^2} \quad n = 2, 3, 4$$

$$\frac{1}{L_1} = R \cdot \frac{1}{1^2} - \frac{1}{2^2}$$

$$L_1 = \frac{4}{3R}$$

$$L_1 = \frac{5}{16} \cdot \frac{4}{3} \quad (c) \quad q$$

$$\frac{1}{L_2} = R \cdot 1 - \frac{1}{9} = \frac{8R}{9}$$

$$L_2 = \frac{9}{8R} = \frac{9}{8} \cdot \frac{16}{3} = \frac{9}{16} \cdot \frac{3}{8} = \frac{27}{128}$$

(d)  $s$

6. The proper match are

(a)  $s$

$\therefore$  X-ray is inverse process of photoelectric effect [high energy electrons convert in electromagnetic radiation]

(b)  $p$

$\therefore \quad c \quad \frac{1}{V} \quad (c) \quad q$

$\therefore$  Wavelength of continuous X-ray depends on voltage

(d)  $q$

7.  $\therefore (KE)_{\max} \propto f$  and stopping potential  $\propto f$

(a)  $p, r$

$\therefore$  stopping potential  $\propto f$  hence it remains same

(b)  $s$

$\therefore$  Current is directly proportional to Intensity. Hence sat current increases but stopping potential does not change

(c)  $q, s$

$\therefore (KE)_{\max} = \frac{hf}{e} - \frac{W}{e}$  and stopping potential

$$\frac{hf}{e} - \frac{W}{e}$$

If  $W$  is decreased  $(KE)_{\max}$  and stopping potential increased

(d)  $p, r$

# Modern Physics II

## Introductory Exercise 30.1

- $R_0 = N_0 \cdot 8000 \text{ Bq}$   

$$\frac{R_1}{R_0} = \frac{1}{8} = \frac{N}{N_0} e^{-\lambda t} \quad \log 8 = \lambda t$$
  
 $3 \log 2 = \frac{\log 2}{T_{1/2}} \cdot 9$   
 $T_{1/2} = \frac{9}{3} = 3 \text{ days}$   
 Average, life days  $1.44 T_{1/2} = 1.44 \cdot 3 = 4.33$
  - $R_0 = N_0$   
 $40 \cdot 3.7 \cdot 10^{10} = 10^6 \cdot \frac{0.693}{64.8} N_0$   

$$N_0 = \frac{40 \cdot 64.8 \cdot 3.7 \cdot 10^4}{0.693}$$
  
 $13.83 \cdot 10^7$   
 Now,  $N = N_0 e^{-\lambda t}$   
 $N_{10} = N_0 e^{-\frac{0.693}{64.8} \cdot 10}$   
 $13.83 \cdot 10^7 = e^{-\frac{0.693}{64.8} \cdot 10}$   
 $N_{12} = 13.83 \cdot 10^7 = e^{-\frac{0.693}{64.8} \cdot 12}$   
 $N_{10} = N_{12} = 13.83 \cdot 10^7$   

$$e^{-\frac{0.693}{64.8} \cdot 10} = e^{-\frac{0.693}{64.8} \cdot 12}$$
  
 $9.47 \cdot 10^9 \text{ nuclei}$
  - (a)  $R_0 = 10 \text{ mCi}, R = 8 \text{ mCi}$   

$$\log \frac{10}{8} = \lambda \cdot 4 = \frac{\log 2}{T_{1/2}} \cdot 3600$$
- $$\frac{0.223}{4 \cdot 3600}$$
- $$T_{1/2} = \frac{0.693}{1.55 \cdot 10^{-5} \text{ s}} = 12.4 \text{ h}$$
- $R_0 = 10 \text{ mCi} = 10 \cdot 3.7 \cdot 10^{10} = 3.7 \cdot 10^8 \text{ Bq}$   

$$N_0 = \frac{R_0}{\lambda} = \frac{3.7 \cdot 10^8}{1.55 \cdot 10^{-5}} = 2.39 \cdot 10^{13} \text{ (atoms)}$$
  - $R = R_0 e^{-\lambda t}$   
 $10 \text{ mCi} = e^{-\frac{0.693}{12.4} \cdot 30} \cdot 1.87 \text{ mCi}$
- $R_0 = N_0$   

$$\frac{R_0}{N_0} = \frac{6 \cdot 10^{11} \text{ Bq}}{10^{15}}$$
  
 $6 \cdot 10^4 \text{ s}$   
 $T_{1/2} = \frac{0.693}{6 \cdot 10^{-4}} = 1.16 \cdot 10^3 \text{ s}$
  - $N_x = N_y = N_0$   
 $T_{1/2x} = 50 \text{ min and } T_{1/2y} = 100 \text{ min}$   

$$N_x = (N_0)^{\frac{1}{2}} = N_0^{\frac{1}{2}} = \frac{N_0}{2}$$
  

$$N_y = N_0^{\frac{1}{2}} = N_0^{\frac{1}{2}} = \frac{N_0}{2}$$
  

$$\frac{N_x}{N_y} = \frac{\frac{N_0}{2}}{\frac{N_0}{2}} = \frac{1}{4}$$

## Introductory Exercise 30.2

1.  $E = Mc^2$

4.002602) u

Here,  $P = 10^9 \text{ J/s} = 10^9 \times 24 \times 60 \text{ J/day}$

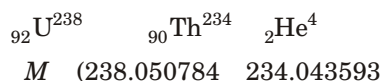
$$M = \frac{10^9 \times 24 \times 60}{(3 \times 10^8)^2} = 9.6 \times 10^{-4} \text{ kg}$$

$M = 0.0004589 \text{ u}$

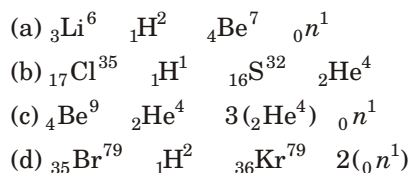
$E = Mc^2 = 931.5 \text{ MeV}$   
 $0.0004589 \times 931.5$   
 $4.27 \text{ MeV}$

2. Number of fission  $\frac{10^9 \text{ J/s}}{200 \times 10^6 \times 1.6 \times 10^{-19}}$   
 $3.125 \times 10^{19}$

3. Given reaction is



4. Complete reactions are



## AIEEE Corner

### ■ Subjective Questions (Level 1)

#### Radioactivity

1. (a) Initially the rate of disintegration is

$$\frac{dN}{dt} = \lambda N_0$$

After 5 min  $\frac{dN}{dt} = \lambda N$

$$\frac{N_0}{N} = \frac{\frac{dN}{dt} \big|_0}{\frac{dN}{dt}} = \frac{4750}{2700} = 1.76$$

Now  $N = N_0 e^{-\lambda t}$  or  $\log \frac{N}{N_0} = -\lambda t$

$$\frac{2.3026}{t} \log_{10} \frac{N_0}{N} = \lambda$$

$$\frac{2.3026}{5} \log_{10} (1.76) = \lambda$$

$0.113 \text{ /min}$

(b) Half-life  $\frac{0.693}{0.113} = 6.132 \text{ min}$

2. We have  $A = \lambda N$

$$6 \times 10^{11} = \frac{1 \times 10^{15}}{6 \times 10^4 \text{ s}^{-1}}$$

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{6 \times 10^4} = 1155 \text{ s}$$

$19.25 \text{ min}$

3.  $A = \lambda N$

$$A = 8 \text{ Ci} = 8 \times 3.7 \times 10^{10} \text{ decay/s}$$

$$\frac{0.693}{T_{1/2}} = \frac{0.693}{5.3 \text{ yr}}$$

$$\frac{0.693}{5.3 \times 365 \times 24 \times 60 \times 60 \text{ s}}$$

$N = \frac{A}{\lambda}$

$$\frac{8 \times 3.7 \times 10^{10} \times 5.3 \times 365 \times 24 \times 3600}{0.693}$$

$N = 7.2 \times 10^{19}$

$$6.023 \times 10^{23} \text{ nuclei } 60 \text{ g}$$

$$1 \frac{60 \text{ g}}{6.023 \times 10^{23}}$$

Hence  $7.2 \times 10^{19}$  nuclei

$$\frac{60 \times 7.2 \times 10^{19}}{6.023 \times 10^{23}} = 7.11 \times 10^3 \text{ g}$$

#### 4. Number of decay per second

$$\frac{m}{M} N_A = \frac{1}{238} \times 6 \times 10^{23} \times \frac{0.693}{4.5 \times 10^9 \text{ yr}}$$

$$\frac{6 \times 10^{23}}{238} \times \frac{0.693}{4.5 \times 10^9 \times 365 \times 24 \times 60 \times 60 \text{ s}}$$

$$1.23 \times 10^4 \text{ decay/s}$$

#### 5. Probability of decay

$$P = (1 - e^{-t}) (1 - e^{-t/T_{\text{mean}}})$$

$$P = (1 - e^{-5/10}) (1 - e^{-2}) = 0.39$$

#### 6. Since initially no $^{206}\text{Pb}$ nuclei is present and after time $t$ the ratio of $\frac{N_v}{N_{\text{Pb}}} = 3$

It means  $\frac{1}{4}$  of original  $^{238}\text{U}$  nuclei decays.

Hence  $N = N_0 e^{-\lambda t}$

$$N_0 = \frac{N_0}{4} = N_0 e^{-\lambda t}$$

$$\frac{3}{4} = e^{-\lambda t}$$

$$t = \frac{\log 4 - \log 3}{\lambda}$$

$$t = \frac{(\log 4 - \log 3)}{0.693} \times 4.5 \times 10^9 \text{ yr}$$

$$t = 1.88 \times 10^9 \text{ yr}$$

#### 7. $(R_1)_{0\text{P}_{32}} = \frac{1}{4} N_0$

$$(R_2)_{0\text{P}_{33}} = \frac{1}{2} N_0$$

$$R_0 = (R_1)_{0\text{P}_{32}} + (R_2)_{0\text{P}_{33}}$$

$$N_0 \left( \frac{1}{4} + \frac{1}{2} \right) = 3 \text{ mCi}$$

$$(R_1)_{t\text{P}_{32}} = \frac{1}{4} N_0 e^{-\lambda t}$$

$$(R_1)_{t\text{P}_{32}} = \frac{1}{4} N_0 e^{-\frac{\log 2}{14} \times 60 \times 365}$$

$$(R_2)_{t\text{P}_{33}} = \frac{1}{2} N_0 e^{-\frac{\log 2}{25} \times 60 \times 365}$$

$$R = (R_1)_{t\text{P}_{32}} + (R_2)_{t\text{P}_{33}}$$

$$N_0 \left( \frac{1}{4} e^{-\frac{\log 2}{14} \times 60 \times 365} + \frac{1}{2} e^{-\frac{\log 2}{25} \times 60 \times 365} \right)$$

$$\frac{3 \text{ mCi}}{\left( \frac{1}{4} + \frac{1}{2} \right)}$$

$$\frac{4 \times 1}{1} e^{-\frac{\log 2}{14} \times 60 \times 365} + \frac{2 \times 1}{2} e^{-\frac{\log 2}{25} \times 60 \times 365}$$

$$\frac{3 \text{ mCi}}{\frac{4}{14} \frac{\log 2}{25} + \frac{\log 2}{25}}$$

$$\frac{4 \log 2}{14} e^{-\frac{\log 2}{14} \times 60 \times 365} + \frac{\log 2}{25} e^{-\frac{\log 2}{25} \times 60 \times 365}$$

$$0.205 \text{ mCi}$$

#### 8. Complete reactions are



#### 9. Only reaction (b) is possible.

#### 10. $E = (7 \times 1.000783 + 7 \times 1.00867 - 14.00307) \times 931.5 \text{ MeV}$

$$E = 104.72 \text{ MeV}$$

#### 11. $E = [8m_p + 8m_n - m({}_8\text{O}^{16})] \times 931.5$

$$(8 \times 1.007825 + 8 \times 1.008665 - 15.994915) \times 931.5 = 127.6 \text{ MeV}$$

#### 12. (a) Number of nuclei in kg

$$\frac{6.023 \times 10^{23}}{235} \times 1$$

Energy

$$\frac{6.023 \times 10^{23}}{235} \times 200 \times 10^6 \times 1.6 \times 10^{-19}$$

$$8.09 \times 10^{13} \text{ J}$$

$$\begin{aligned}
 \text{(b) Mass} & \frac{8.09 \times 10^{13} \text{ J}}{30 \times 10^3 \text{ J/g}} \\
 & \frac{8.09 \times 10^{13}}{30 \times 10^3} \text{ g} \\
 & \frac{8.09}{3} \times 10^9 \times \frac{1}{10^3} \text{ kg} \\
 & 2.7 \times 10^6 \text{ kg}
 \end{aligned}$$

**13. Applying conservation of momentum**

$$\begin{aligned}
 M v &= M_{\text{Ti}} v_{\text{Ti}} \\
 v_{\text{Ti}} &= \frac{M v}{M_{\text{Ti}}} \\
 K &= \frac{1}{2} M v^2 = 6.802 \text{ MeV} \\
 K_{\text{Ti}} &= \frac{1}{2} M_{\text{Ti}} v_{\text{Ti}}^2 = \frac{1}{2} M_{\text{Ti}} \frac{M^2 v^2}{M_{\text{Ti}}^2} \\
 K_{\text{Ti}} &= \frac{M}{M_{\text{Ti}}} \times \frac{1}{2} M v^2 \\
 &= \frac{4}{208} \times 6.802 \text{ MeV} \\
 &= \frac{1}{52} \times 6.802 = 0.1308 \text{ MeV}
 \end{aligned}$$

**14. Power**  $100 \text{ MW} = 10^8 \text{ W} = 10^8 \text{ J/s}$

$$\frac{10^8}{1.6 \times 10^{13} \text{ J/MeV}} = \frac{10^{21} \text{ MeV}}{1.6 \text{ s}}$$

Energy per fission  $185 \text{ MeV}$

Hence number of fissions  $\frac{10^{21}}{1.6 \times 185} / \text{s}.$

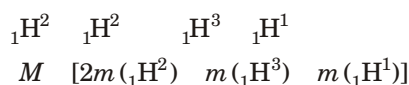
Number of nuclei in 1 kg

$${}_{\text{U}}^{235} \frac{6.023 \times 10^{26}}{235}$$

Hence  $t = \frac{6.023 \times 10^{26} \times 1.6 \times 185}{235 \times 10^{21}}$

$8.78 \text{ days}$

**15. (a) The given reaction is**



$$\begin{aligned}
 & [2(2.014102) - 3.016049 - 1.007825]u \\
 M &= 0.000433 \text{ u}
 \end{aligned}$$

$$\begin{aligned}
 Q &= M \times 931.5 = 0.000433 \times 931.5 \\
 &= 4.05 \text{ MeV}
 \end{aligned}$$

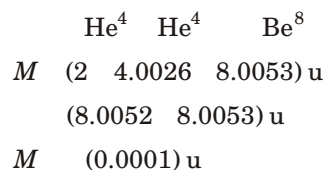
(b)  $M = (2 \times 2.014102 - 3.016049 - 1.008665) \times 931.5$

$M = 3.25 \text{ MeV}$

(c)  $M = (2.014102 - 3.016049 - 4.002603 - 1.008665) \times 931.5$

$M = 17.57 \text{ MeV}$

**16. The given reaction is**



$\therefore M$  is negative this reaction is not energetically favourable

$$\begin{aligned}
 E &= M \times 931.5 \\
 &= 1 \times 10^{-4} \times 931.5 \text{ MeV} \\
 &= 93.15 \text{ keV}
 \end{aligned}$$

**17. Number of nuclei in 1 kg water**

$$\begin{aligned}
 & \frac{6.023 \times 10^{26}}{18} \\
 \text{Heavy water} &= \frac{6.023 \times 10^{26}}{18} \times \frac{1.5 \times 10^2}{100} \\
 &= \frac{6.023 \times 1.5}{18} \times 10^{22}
 \end{aligned}$$

Energy released per fission

$$(2 \times 2.014102 - 3.016049 - 1.007825) \times 931.5$$

$$4.33 \times 10^{-3} \times 931.5 \times 10^6 = 1.6 \times 10^{19} \text{ J}$$

Hence total energy  $\frac{6.023 \times 1.5 \times 10^{22}}{18}$

$$4.33 \times 931.5 \times 1.6 \times 10^{16}$$

$3200 \text{ MJ}$

## ■ Objective Questions (Level-1)

1. Since during  $\beta^-$  decay a neutron in the nucleus is transformed into a proton, an electron and an antineutrino as  
 $n \rightarrow p + e^- + \bar{\nu}$ .

Hence Correct option is (c).

2. Since nuclear force is same for all nucleons.  
 Hence  $F_1 = F_2 = F_3$

Correct option is (a)..

3. Given reaction is  ${}_{90}\text{X}^{200} \rightarrow {}_{80}\text{Y}^{168}$

Difference in mass number  $= 200 - 168 = 32$

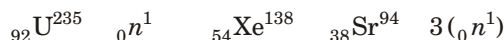
Hence Number of  $\alpha$ -particles  $= \frac{32}{4} = 8$

Difference in atomic number  $= 10$

hence number of  $\beta^-$ -particles  $= 6$

Hence correct option is (d).

4. The reaction is



The correct option is (b) three neutrons.

5. The reactions are  $A \rightarrow B + \alpha$  and  $C \rightarrow D + \alpha$   
 After one  $\alpha$  atomic number reduced by 2 and  
 after 2  $\alpha$  atomic number increased by.

Hence A and C are isotopes Correct option is (d).

6. Here  $m_p = 1.00785 \text{ u}$ ,  $m_n = 1.00866 \text{ u}$

and  $m = 4.00274 \text{ u}$

$$m = 2(m_p + m_n) - m$$

$$m = [2(1.00785 + 1.00866) - 4.00274] \text{ u}$$

$$m = 0.03028 \text{ u}$$

$$E = mc^2 = 931.5 \text{ MeV}$$

$$0.03028 \times 931.5 = 28.21 \text{ MeV}$$

Hence correct option is (c).

7.  $N = N_0 \left( \frac{7}{8} \right)^n = N_0 \left( \frac{1}{8} \right)^n$

$$\text{But } N = N_0 \left( \frac{1}{2} \right)^n = N_0 \left( \frac{1}{8} \right)^n$$

$$n = 3$$

$$3 = \frac{T}{T_{1/2}} \Rightarrow T = 3 T_{1/2}$$

$$T_{1/2} = \frac{8}{3} \text{ s}$$

Hence correct option is (d).

8.  $N = N_0 e^{-\lambda t}$  for mean life  $t = \frac{1}{\lambda}$

$$N = N_0 e^{-\frac{1}{\lambda} \lambda} = \frac{N_0}{e}$$

Hence the fraction disintegrated

$$\frac{N_0 - N}{N_0} = 1 - \frac{1}{e}$$

Correct option is (b).

9.  $N = N_0 \left( \frac{7}{8} \right)^n = N_0 \left( \frac{1}{8} \right)^n$

$$\text{But } N = N_0 \left( \frac{1}{2} \right)^n$$

$$\frac{1}{8} = \left( \frac{1}{2} \right)^n \Rightarrow n = 3$$

$$\text{Hence half life } T_{1/2} = \frac{15 \text{ min}}{3} = 5 \text{ min}$$

Correct option is (a).

10. Since radioactive substance loses half of its activity in 4 days it means its half life

$$T_{1/2} = 4 \text{ days}$$

Now  $A = 5\% \text{ of } A_0$

$$A = \frac{1}{20} A_0$$

$$\frac{A}{A_0} = \left( \frac{1}{2} \right)^n$$

$$\text{But } A = A_0 e^{-\lambda t} \Rightarrow \frac{1}{20} = e^{-\lambda t}$$

$$\log 20 = \lambda t \Rightarrow t = \frac{\log 20}{\lambda}$$

But  $\frac{\log_e 2}{T_{1/2}}$

$$t = T_{1/2} \frac{\log_e 20}{\log_e 2} = T_{1/2} \frac{\log_{10} 20}{\log_{10} 2} = 4.32 \times 4$$

$$t = 17.3 \text{ days}$$

Hence correct option is (c).

11. Total energy released per sec

$$1.6 \text{ MW} = 1.6 \times 10^6 \text{ J/s}$$

Energy released per fission = 200 MeV

$$200 \times 10^6 = 1.6 \times 10^{19} \text{ J}$$

$$2 \times 1.6 \times 10^{11} \text{ J}$$

Number of fission per second

$$\frac{1.6 \times 10^6}{2 \times 1.6 \times 10^{11}} = 5 \times 10^{16} / \text{s}$$

Hence correct option is (a).

12.  $\therefore R \propto A^{1/3}$

$$\text{Volume} \propto R^3 \propto \frac{4}{3} R_0^3 A$$

$$\text{Mass of nucleus} \propto A \propto 1.67 \times 10^{-27} \text{ kg}$$

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{A \times 1.67 \times 10^{-27} \text{ kg}}{\frac{4}{3} R_0^3 A}$$

$$= \frac{1.67 \times 10^{-27} \text{ kg}}{\frac{4}{3} R_0^3}$$

$\therefore$  is independent of A, hence ratio of densities  $= \frac{1}{2}$ .

Correct option is (d).

## ■ Assertion and Reason

- Here both assertion and reason are true but reason does not explain assertion. Hence correct option is (b).
- Here assertion is false but reason is true since for heavier nucleus binding energy per nucleon is least.

Correct option is (d).

13.  $N = N_0 e^{-\lambda t} = \frac{N}{N_0} e^{-\lambda t}$

$$\frac{N}{N_0} = e^{-\frac{0.693}{6.93} \times 10} = e^{-1} = \frac{1}{e}$$

Fractional change

$$\frac{N_0 - N}{N_0} = 1 - \frac{1}{e} \sim 0.63$$

Hence correct option is (b).

14. Since radioactive substance reduce to about 6% it means  $N = \frac{N_0}{16}$

We have  $N = (N_0) \left(\frac{1}{2}\right)^n$

$$\frac{1}{16} = \left(\frac{1}{2}\right)^n \Rightarrow n = 4$$

$$4 \times T_{1/2} = 2h \Rightarrow T_{1/2} = 30 \text{ min}$$

Hence correct option is (a).

15. Probability of a nucleus for survival of time  $t$ .

$$P_{(\text{survival})} = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$$

For one mean life  $t = \frac{1}{\lambda}$

$$P_{\text{survival}} = e^{-1} = e^{-1} = \frac{1}{e}$$

Hence Correct option is (a).

- Here both assertion and reason are true but reason is not correct explanation of assertion. Hence correct option is (b).
- Here a ssertion is true but reason is false since electromagnetic waves are produced by accelerating charge particles.

Correct option is (c).



5. Here assertion is wrong since  $\beta^-$ -decay process is  $n \rightarrow p + e^- + \bar{\nu}$  but reason is true hence correct option is (d).
6. Here assertion is true but reason is false. Correct option is (c).
7. Here both assertion and reason are true and reason may or may not be true. Correct option is (a, b)
8. Both assertion and reason are true but reason is not correct explanation of assertion. Hence correct option is (b).
9. Here reason is true but assertion is false  $\therefore 1 \text{ amu} = 931.5 \text{ MeV}$   
Correct option is (d)
10. Both assertion and reason are true but reason does not correctly explain assertion. Hence correct option is (b).
11. Here both assertion and reason are true and reason may or may not be correct explanation of assertion.  
Hence correct options are (a, b).

## Objective Questions (Level 2)

### ■ Single option correct

1. Let initially substance have  $N_i$  nuclei then

$$N = N_i e^{-\lambda t}$$

$$\frac{dN}{dt} = -\lambda N_i e^{-\lambda t}$$

At  $t = 4t$

we get

$$\frac{dN}{dt} \bigg|_{t=4t} = -\lambda N_i e^{-4\lambda t} = -\lambda N_0 \quad \dots(i)$$

At  $t = 4t$

$$\frac{dN}{dt} \bigg|_{t=4t} = -\lambda N_i e^{-4\lambda t} = -\lambda \frac{N_0}{16} \quad \dots(ii)$$

Dividing Eq. (i) and Eq. (ii) we get

$$e^{3\lambda t} = 16 \quad \dots(iii)$$

Now at  $t = \frac{11}{2} t$

$$\frac{dN}{dt} \bigg|_{t=\frac{11}{2}t} = -\lambda N_i e^{-\frac{11}{2}\lambda t}$$

$$= -\lambda N_i e^{-\frac{8}{2}\lambda t} e^{-\frac{3}{2}\lambda t}$$

$$= \frac{N_i e^{-4\lambda t}}{\sqrt{e^{3\lambda t}}} = \frac{N_0}{16 \sqrt{16}}$$

From Eqs. (ii) and (iii)

$$\frac{N_0}{64}$$

Hence, correct option is (b).

2. We have  $\frac{\log 2}{30} = \frac{\log 2}{60}$

$$\frac{\log 2}{20}$$

Now

$$N = N_0 e^{-\lambda t}$$

$$\frac{N_0}{4} = N_0 e^{-\frac{t \log 2}{20}}$$

$$\log 4 = \frac{t}{20} \log 2$$

$$2 \log 2 = \frac{t}{20} \log 2$$

$$t = 40 \text{ yr}$$

Hence correct option is (c).

3. From graph it is clear that number of nucleons in X is  $N_3$  and binding energy per nucleon is  $E_3$  for Y nucleon is  $N_2$  and BE per nucleon is  $E_2$ .

$$\text{Hence } X \rightarrow Y + E_3 N_3 - E_2 N_2$$

$$\text{Similarly } W \rightarrow E_1 N_1$$

The reaction is  $W \rightarrow X + Y$

The energy released is

$$(E_3 N_3 - E_2 N_2 - E_1 N_1)$$

Hence Correct option is (b).

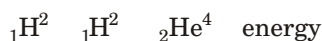
4. Energy (110 8.2 90 8.2 200 7.4)

$$200 (8.2 7.4)$$

$$200 0.8 160 \text{ MeV}$$

Hence correct option is (d).

5. The reaction is



$$\text{Energy} (4 7 2 1.1) \text{ MeV}$$

$$(28 4.4) 23.6 \text{ MeV}$$

Hence Correct option is (b).

6. Total energy released per second

$$16 \cdot 10^6 \text{ W}$$

$$16 \cdot 10^6 \text{ J/s}$$

Energy per fission 200 MeV

$$200 \cdot 10^6 \cdot 1.6 \cdot 10^{19}$$

$$2 \cdot 1.6 \cdot 10^{11} \text{ J}$$

$\therefore$  Efficiency 50%

Hence power (energy converted per second)

$$2 \cdot 1.6 \cdot 10^{11} \cdot \frac{50}{100} \cdot 1.6 \cdot 10^{11} \text{ J}$$

$$\text{Number of fission} = \frac{16 \cdot 10^6}{1.6 \cdot 10^{11}} \cdot 10^{18} / \text{s}$$

Hence correct option is (d).

7.  $\frac{dN}{dt} = A - N$

$\therefore$  After time  $N$  become

$$\text{conservation} \quad \frac{dN}{dt} = 0$$

$$N = \frac{A}{\log 2} \cdot \frac{AT}{\log 2}$$

Hence correct option is (d).

8. By conservation of momentum

$$M_H v + (M_H - M_H) v = \frac{v}{2}$$

Let initial KE of H-atom  $K$

Final KE of each H-atom  $\frac{K}{2}$

For excitation

$$\frac{K}{2} - E_2 - E_1 = \frac{13.6}{4} = 13.6 \text{ eV}$$

$$\frac{K}{2} = 10.2 \text{ eV}$$

$$K = 2 \cdot 10.2 = 1.6 \cdot 10^{19}$$

$$\frac{1}{2} M_H u^2 = 2 \cdot 10.2 = 1.2 \cdot 1.6 \cdot 10^{19}$$

$$u_H = \sqrt{\frac{2 \cdot 1.2 \cdot 10.2 \cdot 2 \cdot 1.6 \cdot 10^{19}}{1.673 \cdot 10^{27}}}$$

$$6.25 \cdot 10^4 \text{ m/s}$$

Hence correct option is (c).

9. Let us suppose just before the death no radioactive atoms were present hence original activity  $A_0$  is given as

$$A_0 = N_0 \lambda \quad \dots(i)$$

After death the radioactivity decreases exponentially i.e.,

$$A = \frac{dN}{dt} = N \lambda = N_0 e^{-\lambda t} \quad \dots(ii)$$

Dividing eq. (ii) by eq. (i) we get

$$\frac{A}{A_0} = e^{-\lambda t}$$

$$\text{or } t = \log \frac{A_0}{A} \text{ or } t = \frac{1}{\lambda} \log \frac{A_0}{A}$$

Now  $A_0 = 15 \text{ decay/min/gram}$

$$A = \frac{375}{200} \text{ decay/min/g}$$

but

$$\frac{0.693}{5730 \text{ yr}}$$

$$t \frac{5730}{0.693} \log \frac{15}{375} \frac{200}{5730} \frac{5730}{0.693} \log \frac{200}{25}$$

$$t \frac{5730}{0.693} \log 8$$

$$\frac{5730}{0.693} 3 \log 2 = 5730 \times 3$$

$$t = 17190 \text{ yr}$$

Hence correct option is (c).

10.  $N_P = N_0 e^{-(t_1 - t)}$  ... (i)

and  $N_Q = N_0 e^{-t}$

Now  $\frac{A_P}{A_Q} = \frac{N_P}{N_Q}$  and  $\frac{A_Q}{A_P} = \frac{N_Q}{N_P}$

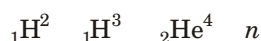
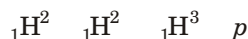
$$\frac{A_P}{A_Q} = \frac{N_P}{N_Q} e^{-(t_1 - t)}$$

$$t_1 \log \frac{A_Q}{A_P}$$

$$t_1 - \log \frac{A_Q}{A_P} = T \log \frac{A_Q}{A_P}$$

Hence correct option is (b).

11. The given reactions are



Mass defect

$$m = (3 \times 2.014 - 4.001 - 1.007 - 1.008)$$

$$m = 0.026 \text{ amu}$$

Energy Released

$$0.026 \times 931 \text{ MeV} = 3.87 \times 10^{12} \text{ J}$$

This energy produced by the three deuterons. Total energy released by

$10^{40}$  deuterons

$$\frac{10^{40}}{3} \times 3.87 \times 10^{12} \text{ J} = 1.29 \times 10^{28} \text{ J}$$

The average power  $P = 10^{16} \text{ W} = 10^{16} \text{ J/s}$

Therefore total time to exhaust all deuterons of the star will be

$$t = \frac{1.29 \times 10^{28}}{10^{16}} \text{ s} = 10^{12} \text{ s}.$$

Hence correct option is (c).

12.  $N_1 = N_0 e^{-\frac{\log 2}{t_1} t}$  ... (i)

$$N_2 = N_0 e^{-\frac{\log 2}{t_2} t}$$
 ... (ii)

$$R_1 = \frac{1}{2} N_1$$
 ... (iii)

and  $R_2 = \frac{1}{2} N_2$  ... (iv)

Let after time  $t$ ,  $R_1 = R_2$  then

$$\frac{R_1}{R_1} = 1$$

$$\frac{1}{2} \frac{N_1}{N_2} = 1$$

$$\frac{1}{2} \frac{N_2}{N_1} = e^{\log 2 \frac{t_2 - t_1}{t_1 t_2}} = e^{\log 2 \frac{t_2 - t_1}{t_1 t_2}}$$

$$\log \frac{t_2}{t_1} = \log 2 \frac{t_2 - t_1}{t_1 t_2}$$

$$t_1 = \frac{t_1 t_2}{0.693 (t_2 - t_1)} \log \frac{t_2}{t_1}$$

Hence Correct option is (a).

13. The given reaction is  ${}_Z X^{232} \rightarrow {}_{90} Y^A$



$$Z = 92 \text{ and } A = 228$$

$\therefore$  Initially  $X$  is in rest hence momentum of  $\alpha$ -particle after decay will be equal and opposite of  $Y$ .

$$M_Y v_Y = M \alpha v$$

$$v_Y = \frac{M}{M_Y} v$$

Total kinetic energy

$$K_T = \frac{1}{2} (M v^2 + M_Y v_Y^2)$$

$$K_T = \frac{1}{2} M v^2 + M_Y \frac{M^2 v^2}{M_Y^2}$$

$$K_T = \frac{1}{2} M v^2 = \frac{1}{2} \frac{M}{M_Y}$$

$$K_T = K = \frac{4}{228}$$

$$K = \frac{232}{228} K_T$$

Hence Correct option is (b).

14. Energy of emitted photon = 7 MeV

$$7 \times 10^6 \times 1.6 \times 10^{19} \text{ J}$$

$$11.2 \times 10^{13} \text{ J}$$

$$\text{Momentum of photon} = \frac{11.2 \times 10^{13} \text{ J}}{3 \times 10^8 \text{ m/s}}$$

$$\frac{11.2}{3} \times 10^{21} \text{ kg-m/s}$$

∴ Initial nucleus is stationary

Applying conservation of momentum principle

$$0 = \mathbf{P}_{\text{nuc}} + \mathbf{P}_{\text{photon}} \quad \mathbf{P}_{\text{nuc}} = -\mathbf{P}_{\text{photon}}$$

$$|\mathbf{P}_{\text{nuc}}| = |\mathbf{P}_{\text{photon}}|$$

$$P_{\text{nuc}} = \frac{11.2}{3} \times 10^{21} \text{ kg-m/s}$$

Mass of nucleus = 24 amu

$$24 \times 1.66 \times 10^{-27} \text{ kg}$$

$$\text{But } P_{\text{nuc}}^2 = 2mK_{\text{nuc}}$$

$$K_{\text{nuc}} = \frac{P_{\text{nuc}}^2}{2m} = \frac{11.2 \times 11.2 \times 10^{42}}{9 \times 2 \times 24 \times 1.66 \times 10^{-27}} \text{ Joule}$$

$$K_{\text{nuc}} = \frac{11.2 \times 11.2 \times 10^{42}}{18 \times 24 \times 1.66 \times 10^{-27} \times 1.6 \times 10^{-19}} \text{ eV}$$

$$1.1 \text{ keV}$$

Hence correct option is (b).

15. Let time interval between two instants is  $t_1$  then

$$N_1 = N_0 e^{-(t - t_1)}$$

$$\begin{aligned} \text{and } N_2 &= 2N_0 e^{-t} \\ A_1 &= N_1 = N_0 e^{-(t - t_1)} \\ A_2 &= N_2 = (2N_0) e^{-t} \\ \frac{A_1}{A_2} &= \frac{1}{2} e^{t - t_1} \\ \frac{2A_1}{A_2} &= e^{t - t_1} \end{aligned}$$

$$\log \frac{2A_1}{2A_1} = t - t_1$$

$$t_1 = \frac{1}{\log 2} \log \frac{A_2}{2A_1}$$

$$t_1 = \frac{T}{\log 2} \log \frac{A_2}{2A_1}$$

Hence Correct option is (c).

16. The given reaction is



$$M = [2m({}_1\text{H}^2) - m({}_1\text{H}^3) - m({}_1\text{H}^1)]$$

$$(2 \times 2.014102 - 3.016049 - 1.007825) \text{ amu}$$

$$4.33 \times 10^{-3} \text{ amu}$$

$$E = 4.33 \times 10^{-3} \times 931.5 \text{ MeV}$$

$$4 \text{ MeV}$$

Hence correct option is (c).

17. Number of fusion required to generate 1 kWh

$$\frac{1 \times 10^3 \times 3600}{4 \times 10^6 \times 1.6 \times 10^{19}}$$

$$\frac{36 \times 10^{18}}{6.4} = 5.6 \times 10^{18} \times 10^{18}$$

Hence correct option is (b).

18. The energy released = 4 MeV

This energy produced by two atoms.

Hence energy produced per atom

$$2 \text{ MeV} = 2 \times 1.6 \times 10^{13} \text{ J}$$

Hence number of atom fused to produced 1 kWJ

$$\frac{36}{2} \frac{10^5}{1.6} \frac{10^{13}}{10} \frac{18}{1.6} 10^{18}$$

Mass of deuterium which contain

$$\frac{18}{1.6} 10^{18} \text{ atom}$$

$$\frac{18}{1.6} \frac{10^{18}}{6.02 \times 10^{23}} 3.7 \times 10^{-5} \text{ kg.}$$

Hence correct option is (c).

## ■ More Than one Option is Correct

1.  $x = N_0, y = N_0$

$$\frac{x}{y} = \frac{N_0}{N_0} = 1$$

where  $\lambda$  is decay constant

Hence  $\frac{x}{y}$  is constant throughout.

$$\therefore \frac{x}{y} = \frac{1}{T} = \frac{1}{\frac{0.693}{T}} = \frac{T}{0.693}$$

$$\frac{x}{y} = T, xy = (N_0)^2$$

For one half life  $N = \frac{N_0}{2}$

$$(xy)_T = \frac{N_0}{2} \cdot \frac{N_0}{2} = \frac{N_0^2}{4} = \frac{xy}{4}$$

Hence correct options are (a), (b) and (d).

2. The correct options are (a), (b), (c) and (d).

3. A nucleus in excited state emits a high energy photon called as  $\gamma$ -ray. The reaction is



Hence by gamma radiation atomic number and mass number are not changed. Since after emission of one  $\gamma$  atomic number reduced by  $2(2^{-4})$  and after 2 atomic number is increased by (2). Hence correct options are (a), (b) and (c).

4. Here half lives are  $T$  and  $2T$  and  $N_x = N_0$ ,  $N_y = N_0$  after  $4T$  for first substance 4 half lives and after  $4T$  the second substance 2 Half lines.

$$N_x = N_0 \left(\frac{1}{2}\right)^4 = \frac{N_0}{16}$$

$$N_y = N_0 \left(\frac{1}{2}\right)^2 = \frac{N_0}{4}$$

$$x = \frac{N_x}{N_y} = \frac{N_0/16}{N_0/4} = \frac{1}{4}$$

Let their activity are  $R_x$  and  $R_y$ .

$$R_x = \lambda_x N_x \text{ and } R_y = \lambda_y N_y$$

$$y = \frac{R_x}{R_y} = \frac{\lambda_x}{\lambda_y} \frac{N_x}{N_y} = \frac{0.693}{0.693} \frac{T}{2T} = \frac{1}{4}$$

$$\frac{R_x}{R_y} = y = \frac{1}{2}$$

Hence correct options are (b) and (c).

5. Since nuclear forces are vary short range charge independent, no electromagnetic and they exchange ( $n \rightarrow p$  or  $p \rightarrow n$ ). Hence the correct options are (a), (b), (c) and (d).

6.  $\therefore R = R_0 A^{1/3}$

$$\frac{M}{4/3 \pi R^3} = \frac{A}{\frac{4}{3} \pi R_0^3 A}$$

is independent of  $A$ .

But  $\frac{1.67 \times 10^{-27} \text{ kg}}{\frac{4}{3} \pi (3.14 \times (1.3 \times 10^{-15})^3)}$

$$1.8 \times 10^{17} \text{ kg/m}^3$$

Hence correct options are (b) and (c).

## ■ Match the Columns

1. Here  $N_0$  is, let  $\lambda$  is the decay constant.

$$N = N_0 e^{-\lambda t} = x e^{-\lambda t}$$

$$\left| \frac{dN}{dt} \right| = \lambda x e^{-\lambda t} \text{ but } \left| \frac{dN}{dt} \right|_{t=0} = y$$

$$y = \lambda x e^0 = \lambda x$$

$$\frac{y}{x}$$

Hence

- (a) s

$$\text{Half life } T_{1/2} = \frac{\log 2}{\lambda} = \frac{\log 2}{\frac{y}{x}} = \frac{x}{y} \log 2$$

Hence

- (b) p

We have activity  $R = \lambda N = \lambda x e^{-\lambda t}$   
at  $t = \frac{1}{\lambda}$

$$R = \lambda x e^{-1} = x e^{-1} = \frac{x}{e}$$

but  $x = y$

$$R = \frac{y}{e}$$

Hence

- (c) r

Number of nuclei after time  $t = \frac{1}{\lambda}$

$$N = x e^{-1} = \frac{x}{e}$$

Hence

- (d) s

Thus correct match is

- (a) s  
(b) r  
(c) r  
(d) s

2. In reaction  $P + P \rightarrow Q$  energy is released

$\therefore$  Binding energy increases when two or more lighter nucleus combine to form heavier nucleus.

Hence correct match is

- (a) p

Similarly for reaction



Correct match is

- (b) p

for reaction

$P + R \rightarrow 2Q$  from graph BE per nucleon increases.

Hence energy is released.

Correct match is

- (c) p

For the reaction



We will check energy process if BE per nucleon is given. Hence data is not sufficient correct match is

- (d) s

3. Since  $A$  and  $B$  are radioactive nuclei of  $(A \rightarrow B)$  decreases with time. Hence correct match is

- (a) q

$\therefore A$  is converted into  $B$  and  $B$  is converted into  $C$  and decay rate of  $A \rightarrow B$  and  $B \rightarrow C$  are not known. Hence correct match is

- (b) s

Since at time passes  $A$  is converted to  $B$  and  $B$  is converted to  $C$ . Hence nuclei of  $(B \rightarrow C)$  increases. Correct match is

- (c) p

Similarly the correct match for (d) is

(d) s

4. After emission of 1 particle mass no decreased by 4 but after emission of 1 particle atomic number will increase or decrease by 1. Hence for (a)

(a) p, s

(b) p, r

(c) s

(d) q, r

5. For

(a) p

Since BE per nucleon of heavy nuclei is about 7.2 MeV. Hence

(b) s

X-ray photon have wavelength about  $1 \text{ \AA}$  the energy of this wavelength is of order of 10 keV.

(c) r

$\therefore$  Visible light energy of order of 2 eV.

Hence

(d) q