Textbook Examples of Recursion

by Samarth Motka
Student ID: 202311023

August 7, 2023

Donald E. Knuth's paper *"Textbook Examples of Recursion"* discusses two well-known examples of recursion: McCarthy's "91 function" and Ikuo Takeuchi's "triple recursion". These functions are used to demonstrate the properties and challenges of recursion as a powerful tool in computer programming. McCarthy's "91 function" is defined as follows:

$$f(x) = if \;\; x > 100 \;\; then \;\; x\text{-}10 \;\; else \;\;\; f(f(x+11)) \tag{1}$$

It is known as the "91 function" because for all values of x less than or equal to 101, it evaluates to 91. McCarthy's function is extended in this study by presenting a new variation in which

$$f(x) = if \;\; x > 100 \;\; then \;\; x\text{-}10 \;\; else \;\; f^{91}(x+901) \tag{2}$$

where the inner function is applied 91 times. This modified function similarly converges to 91, and its behavior is observed for different given values of x.

The paper also discusses Takeuchi's triple recursion function. Ikuo Takeuchi deduced a recursive function for comparing the speeds of LISP systems. The Takeuchi's function is defined as follows:

$$t(x,y,z) = \begin{cases} y & \text{if } x \leq y \\ t(t(x-1,y,z), t(y-1,z,x), t(z-1,x,y)) & \text{otherwise} \end{cases} \tag{3}$$

The Takeuchi's function can run for a long time without using a larger number of stacks, making it useful for performance evaluations.The work attempts to provide fresh insights into the properties of these and similar functions. It also poses exciting open questions, demonstrating how seemingly simple recursive functions can lead to complicated problems.

At Last, the "91 function" or "triple recursion" is not intended for practical applications or calculations. Instead, it serves as a curiosity and a fun example to explore the behavior of recursion in programming languages.