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Assignment 10

5.2-1

In HIRE-ASSISTANT, assuming that the candidates are presented in a random or- der, what is the probability that you hire exactly one time? What is the probabilitythat you hire exactly n times?

If the best candidate is presented first, we will hire exactly once. In this scenario, there are (n-1)! possible orderings with the best candidate at the beginning. Therefore, the probability of hiring exactly once is (n-1)!/n!, which simplifies to 1/n.

On the other hand, when the candidates are arranged in increasing order, you will hire exactly n times. There is only one possible ordering for this situation, so the probability of hiring exactly n times is 1/n!.

5.2-2

In HIRE-ASSISTANT, assuming that the candidates are presented in a random or-der, what is the probability that you hire exactly twice?

- Candidate 1 is always hired
- The best candidate (candidate whose rank is n)is always hired
- If the best candidate is candidate 1, then that's the only candidate hired.

In order for HIRE-ASSISTANT to hire exactly twice, candidate 1 should have rank I, where $1 \le i \le n-1$ and all candidates whose ranks are $i+1, i+2, \ldots, n-1$ should be interviewed after the candidate whose rank is *n* (the best candidate).
Let Ei be the event in which candidate 1 have rank *i*, so we have P(Ei)=1/n for $1 \le i \le n$.
Our goal is to find for $1 \le i \le n-1$, given Ei occurs, i.e., candidate 1 has rank *i*, the candidate whose rank is *n* (the best candidate) is the first one interviewed out of the $n-i$ candidates whose ranks are $i+1, i+2, \ldots, n$.

$$\sum_{i=1}^{n-1} P(E_i) \cdot \frac{1}{n-i} = \sum_{i=1}^{n-1} \frac{1}{n} \cdot \frac{1}{n-i}.$$

5.2-4

Use indicator random variables to solve the following problem, which is known as the hat-check problem. Each of n customers gives a hat to a hat-check person at a restaurant. The hat-check person gives the hats back to the customers in a random order. What is the expected number of customers who get back their own hat?

Let *X* be the number of customers who get back their own hat and
Xi be the indicator random variable that customer *i* gets his hat back. The probability that an individual gets his hat back is 1/n .Thus we have

$$E[X] = E\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} E[X_i] = \sum_{i=1}^{n} \frac{1}{n} = 1.$$

5.2-5

Let A[1…n]be an array of n distinct numbers. If i<j and A[i] > A[j] , then the pair (i,j) is called an inversion of A. (See Problem 2-4 for more on inversions.) Suppose that the elements of A form a uniform random permutation of {1,2….n}. Use indicator random variables to compute the expected number of Inversions.

Let Xij for *i*<*j* be the indicator of *A*[*i*]>*A*[*j*]. We have that the expected number of inversions

$$E\left[\sum_{i<j} X_{i,j}\right]=n(n-1)/4$$