IT584 Approximation Algorithms January 19, 2023

Lecture 6

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1 Scheduling on Parallel Machine

1.1 Does it converge fast enough? Polynomial time?

- $C_{\max}$ for a sequence of schedules produced never increases.(if it remains the same, then the number of machines achieving the value decreases.)
- Transfer a job to the machine currently finishing earliest.
- Let $C_{\min}$ be the completion time of the machine that completes all its processing earliest.
- $C_{\min}$ never decreases, and $C_{\max}$ never increases.

1.2 Claim

- We never transfer a job twice; We will prove this by contradiction.
- Suppose, for the sake of contradiction, that we do.
- Say job j is transferred $i \to i' \to i^*$.
- When j is transferred from $i \to i'$, it starts at $C_{\min}$ for the current schedule.
- Similarly, for $i' \to i^*$, it starts at $C'_{\min}$.
- It is a linear-time algorithm.
- The algorithm terminates in n.

2 Theorem

The local search procedure for scheduling jobs on identical parallel machines is a 2-approximation algorithm

2.1 Refinement of Approximation Ratio

It is not hard to see that the analysis of the approximation ratio can be refined slightly. In deriving the inequality, we included job ℓ among the work to be done prior to the start of job ℓ

$$S_\ell \leq \sum_{j \neq \ell} \frac{p_j}{m}$$

We proceed with a step-by-step refinement:

Elimination of Denominator:

$$m \cdot S_\ell \leq \sum_{j \neq \ell} p_j$$

Adding Job Time:

$$m \cdot S_\ell + p_\ell \leq p_\ell + \sum_{j \neq \ell} p_j$$

Factoring and Total Schedule Length:

$$m \cdot S_\ell + p_\ell \leq (1 - \frac{1}{m})p_\ell + \sum_{j \neq \ell} p_j$$

Simplified Expression:

$$m \cdot S_\ell + p_\ell \leq (1 - \frac{1}{m})p_\ell + \frac{1}{m}\sum_{j=1}^{n} p_j$$

Relation to Optimal Make-span

$$m \cdot S_\ell + p_\ell \leq (2 - \frac{1}{m})C^*_{\max}$$

3 Conclusion

Applying the lower bounds to these terms reveals that the schedule produced by the local search has a length at most $(2 - \frac{1}{m})C^*_{\max}$.
It is not constant approximation. It depends on no. of machine.