IT584 Approximation Algorithms	January 16, 2024
Lecture 4	
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1 Scheduling Jobs with deadlines on single machines

Suppose that there n jobs to be scheduled on a single machine, where the machine can process at most one job at a time and must process a job until its completion once it has begun processing. We assume that the schedule starts at time 0. Furthermore, assume that each job j has a specified due date d_j, and if we complete its processing at time c_j, then its lateness L_j is equal to c_j - d_j.

Definition:- There are n jobs to be scheduled on a single machine. Each job j must be processed for pj units of time. Job j may begin no earlier than release date rj

Objective:- Schedule jobs to **minimize** the maximum latencies. $L_{max} = \max_{j=1,\ldots,n} L_j$

Assumption 1: Let all $d_j > 0$, Then as $c_j < 0$, Hence $L_j \geq 0$ (Strictly)

1.1 Greedy Strategy

• A job j is available at time t if $C_j \leq t$.
• At each moment that the machine is idle, start processing the next job available with the earliest due date.
• We first provide a good lower bound in the optimal value for this problem.
• Let S denote a subset of jobs, and let $r(S) = \min_{j \in S} r_j$, $r(S) = \sum_{j \in S} p_j$ and $d(S) = \max_{j \in S} d_j$.

Table 1: An instance with four jobs with deadlines. Each job J_j (with the index j) has the processing time p_j and the deadline d_j .

j	p_j	dj
1	2	3
2	2	5
3	7	10
4	1	12

Scheduling Jobs with Deadlines: Earliest Due Date

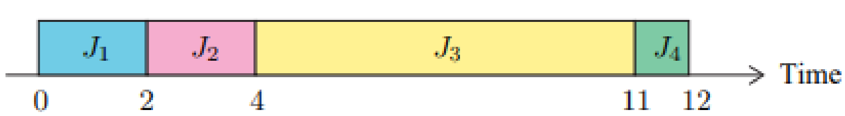

Figure 1: EDD schedule for jobs with deadlines in Table 1.

Lemma 1 *For each subset S of jobs,*

$$L^*{}_{\max} \leq r(S) + p(S) - d(S) \tag{1}$$

Proof:

Consider an optimal schedule and view it as a schedule for the jobs in subset S. Let, j be the last job to be processed. Since none of the jobs follow j, job j cannot complete its job any earlier than,

$$c_{\mathrm{j}} \geq r_{\mathrm{j}} + p_{\mathrm{j}} \tag{2}$$

Then lateness of job j is at least,

$$L_{\mathrm{j}} = c_{\mathrm{j}} - d_{\mathrm{j}} \tag{3}$$

$$L_{\mathrm{j}} \geq r_{\mathrm{j}} + P_{\mathrm{j}} - d_{\mathrm{j}} \tag{4}$$

Hence, for each subset S of jobs,

$$L^*{}_{\max} \geq r(S) + p(S) - d(S) \tag{5}$$

The above strategy gives me a SOL.

Lemma 2 *The EDD(Earliest Due Date) rule is a 2-approximation algorithm for the problem of minimizing the maximum lateness on a single machine subject to release dates with negative due dates.*

Proof:

• Consider the schedule produced by the EDD rule, and let job j be a job of maximum lateness in this schedule.
• Let, C_j be a time when machine is idle, and t = Earliest time the machine was idle, and S = Subset of jobs that are completed before instant t.
• From this, we can say that the machine was processing without any idle time for the entire period $[t,C_j)$
• Hence, $r(S) = t$ and $P(S) = c_j - t$

$$c_j \leq r(S) + p(S) - d(S) \leq r(S) + p(S) \tag{6}$$

In EDD schedule let j be the Job with max lateness C_j

$$L^*{}_{\max} \geq r(S) + p(S) - d(S) \tag{7}$$

$$L^*{}_{\max} \geq r(S) + p(S) \tag{8}$$

$$L^*{}_{\max} \geq c_j \tag{9}$$

Ontheotherhand, by applying Lemma1 with $S = j,$.

$$L^*{}_{\max} \geq r_j + p_j - d_j \tag{10}$$

$$L^*{}_{\max} \geq -d_j \tag{11}$$

Let sum of eq. 9 and eq.11

$$L^*{}_{\max} + L^*{}_{\max} \geq c_j - d_j \tag{12}$$

$$\mathbf{2}L^*{}_{\max} \geq c_j - d_j \tag{13}$$