IT584 Approximation Algorithms	January 16, 2024

Lecture 4

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1 Scheduling Jobs with Deadline on a Single Machine

1.1 Definition

There are n jobs to be scheduled on a single machine. Each job j must be processed for P units of time and may begin no earlier than release date r_j.

The machine can process at most one job at a time and must process a job until its completion once it has begun processing. We assume that the schedule starts at time 0. Furthermore, assume that each job j has a specified due date d_j, and if we complete its processing at time C_j, then its lateness L_j is equal to C_j - d_j.

Objective:- Schedule jobs to minimize the maximum latencies. L_{max} = Max(L_j) where, j=1,...,n

1.2 Greedy Strategy

Suppose a job j is available at time t and if $r_j \leq$ t.
At each moment when the machine is idle, it should start processing the next job available with the earliest due date.
We first provide a good lower bound in the optimal value for this problem.Let L^*_{max} denote the optimal value.
Let S denote a subset of jobs, and let r(S) = min(r_j), p(S) = Sum(P_j), and d(S) = max(d_j).

Lemma 1 *For each subset S of jobs,*

$$L^*_{max} \leq r(S) + p(S) - d(S)$$

Proof Consider an optimal schedule and view it as a schedule for the jobs in subset S.
Let, j be the last job to be processed. Since none of the jobs follow j, job j cannot complete its job any earlier than,

$$C_j \geq r_j + P_j$$

Then lateness of job j is at least,

$$L_j = C_j - d_j \geq r_j + P_j - d_j$$

Hence, for each subset S of jobs,

$$L^*_{max} \geq r(S) + p(S) - d(S)$$

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Lemma 2 *EDD(Earliest Due Date) rule is a 2-approximation for our objective.(Release dates with negative due dates)*

Proof Consider the schedule produced by the EDD rule, and let job j be a job of maximum lateness in this schedule.
Let, C_j be a time when machine is idle, and t = Earliest time the machine was idle, and S = Subset of jobs that are completed before instant t.
From this, we can say that the machine was processing without any idle time for the entire period [t,C_j).
Hence, r(S) = t and, P(S) = C_j - t.
We know that,

$$C_j \leq r(S) + p(S) - d(S) \leq r(S) + p(S)$$

$L^*_{max} \geq r(S) + p(S) - d(S)$
$L^*_{max} \geq r(S) + p(S)$
$L^*_{max} \geq C_j$

When S = j,

$$L^*_{max} \geq r_j + P_j - d_j$$

Adding Equation 4 and 5
$2L^*_{max} \geq C_j + r_j + P_j - d_j$
$2L^*_{max} \geq r_j + P_j + L_{max} \qquad (Because C_j - d_j = L_{max})$

Hence, this completes the proof of our problem. ■