Probability and Statistics

1]

1)  Expirement : (Rolled die)

2) $S = \{1, 2, 3, 4, 5, 6\}$
- All possible Outcome
- Sample space

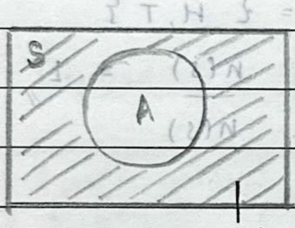
3) Event : Even no. on dice

$E : \{2, 4, 6\}$

4) $\text{Prob}[\text{Even no}] = \dfrac{n(E)}{n(S)} = \dfrac{3}{6} = \dfrac{1}{2} = \dfrac{\text{Req}}{\text{total}}$

2]

1) **Different types of Events.**

ⓐ 

$S = \{1, 2, 3, 4, 5, 6\}$

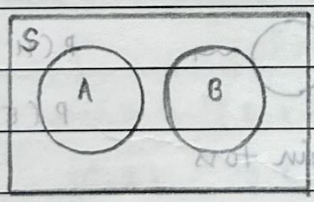
$A = \{1, 2\}$

$\therefore A' = S - A$

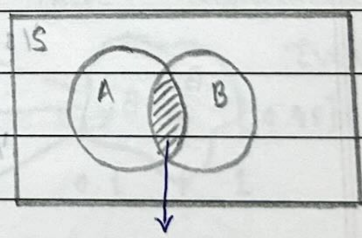
$\therefore \dfrac{n(A')}{n(S)} = \dfrac{n(S)}{n(S)} - \dfrac{n(A)}{n(S)}$

$\therefore P(A') = 1 - P(A)$

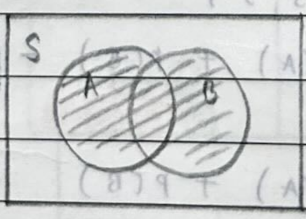
★. Complementary Events $(A' = \bar{A} = A^c)$

ⓑ 

Disjoint Events

ⓒ 

intersection

ⓓ 

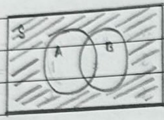
union ⇝ A∪B

Either A or B

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

★ $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

(e)



neither A nor B $= (\overline{A \cup B})$

$= S - (A\cup B)$

De-Morgan's

$\therefore P(\overline{A\cup B}) = P(\bar{A} \cap \bar{B}) = 1 - P(A\cup B)$

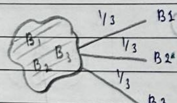
(f) Sure event : $\bigcirc \rightsquigarrow S = \{H, T\}$

$\therefore P(S) = \dfrac{n(S)}{n(S)} = 1$

(g) Null Event : (ϕ) $E = \{\ \}$

$\therefore P(E) = 0$

★ (h) Equally Likely Event :



$\bigcirc \rightsquigarrow$ $P(H) = 1/2$

$P(E) = 1/2$

Coin toss

★ (i) ⚃ $\rightsquigarrow \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 4, 6\}$ $B = \{1, 3, 5\}$

$P(A \text{ or } B) = P(A\cup B) = P(A) + P(B) - \boxed{P(A\cap B)}$

$\downarrow$

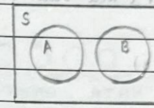
0

★ law of addition $\boxed{P(A\cup B) = P(A) + P(B)}$

Mutually Exclusive Event :-

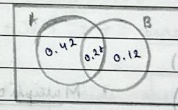
$\therefore A \cap B = \phi \rightarrow P(A\cap B) = 0 \rightarrow$ A & B are mutually Exclusive event

- $A \cap B \neq \phi$ then A and B are not mutually E.E.



Disjoint / Mutually Exclusive Events

(j) Dependent / Independent Events

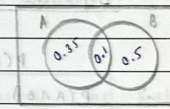


iff, $P(A\cap B) = P(A).P(B)$

then Independent Event

ex: $0.28 = [0.7][0.4]$

$0.28 = 0.28$



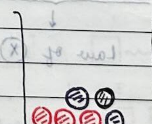
iff, $P(A\cap B) \neq P(A).P(B)$

then dependent Event

ex: $0.1 = [0.45][0.6]$

$0.1 \neq 1$

ex:



$\therefore$ Dependent Case

1) prob of getting 2 red balls without replacement

$\therefore \left(\dfrac{3}{6}\right) \times \left(\dfrac{2}{5}\right)$

$\therefore P(R \text{ in } 1^{st} \cap R \text{ in } 2^{nd})$

$$\therefore\ \boxed{P(R \text{ in } 1^{st} \cap R \text{ in } 2^{nd}) = P(R \text{ in } 1^{st}) \cdot P\left(\frac{R \text{ in } 2^{nd}}{R \text{ in } 1^{st}}\right)}$$

2) Prob of getting 2 red balls with replacement.

$\therefore$ P(Red balls in 1^{st} & Red ball in 2^{nd})

$A \cap B$

P (Red ball in 1^{st}). P (Red ball in 2^{nd})

$$= \underbrace{\left(\frac{3}{6}\right)}_{P(A)} \cdot \underbrace{\left(\frac{3}{6}\right)}_{P(B)}$$

⇝ Independent Case ⇝ $P(A \cap B) = P(A) . P(B)$

★ Dependent Case :

$$P(A \cap B) \neq P(A) . P(B)$$

$\therefore\ \boxed{P(A \cap B) = P(A) \cdot P(B/A)}$ • Multiplication rule

$\therefore\ \boxed{P(A \cap B) = P(B) \cdot P(A/B)}$

Mutually Exclusive	Independent Event
↓	↓
$P(A \cup B) = P(A) + P(B)$	$P(A \cap B) = P(A) . P(B)$
↓	↓
Law of (+)	Law of (×)

☼ Coin : $[n(s) = 2^n]$

n = 1 ⇒ $S = \{H, T\}$ ⇒ $n(s) = 2$

n = 2 ⇒ $S = \{HH, HT, TH, TT\}$ ⇒ $n(s) = 4 = 2^2$

n = 3 ⇒ S = { HHH HTH THH TTH
HHT HTT THT TTT } ⇒ $n(s) = 8 = 2^3$

n = 4

{ HHHH HTHH THHH TTHH
HHHT HTHT THHT TTHT
HHTH HTTH THTH TTTH
HHTT HTTT THTT TTTT } $n(s) = 16 = 2^4$

$n \geq 4$ ⇝ binomial distribution

ex: Two coins are simultaneously tossed. The probability of two heads simultaneously appearring is..

a) $1/8$
b) $1/6$
c) $1/2$
d) $1/4$

$$P = \frac{n(E)}{n(S)} = \frac{1}{4} \quad \{\because n(E) = 1,\ E = \{H,H\}\}$$

★ Die : $[n(s) = 6^n]$

n = 1 ⇝ $S = \{1, 2, 3, 4, 5, 6\}$ ⇒ $n(s) = 6^1$

n = 2 ⇝ $n(s) = 6^2 = 36$

$S = \{ (11)\ (12)\ (13)\ (14)\ (15)\ (16)$
$(21)\ (22)\ (23)\ (24)\ (25)\ (26)$
$(31)\ (32)\ (33)\ (34)\ (35)\ (36)$
$(41)\ (42)\ (43)\ (44)\ (45)\ (46)$
$(51)\ (52)\ (53)\ (54)\ (55)\ (56)$
$(61)\ (62)\ (63)\ (64)\ (65)\ (66) \}$

$n > 2$ ⇝ Die ⇝ B.D

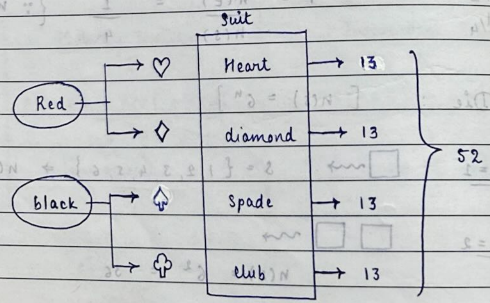
ex: A fair dice is rolled twice. The probability than an odd number will follow an even number is..

a) $1/2$
b) $1/6$
c) $1/3$
d) $1/4$ ✓

Even die 1	odd die 2	
2, 1	4, 1	6, 1
2, 3	4, 3	6, 3
2, 5	4, 5	6, 5

$\therefore P = \frac{9}{36} = \frac{1}{4}$

★ Cards [52]



number card = 9

A 2 3 4 5 6 7 8 9 10 | J Q K
non-face cards (10) | 3 face cards

ex: Two cards are drawn at random in succession with replacement from a deck of 52 well shuffled card probability of getting both 'aces' is..

a) $\frac{1}{169}$ ✓ b) $\frac{2}{169}$ c) $\frac{1}{13}$ d) $\frac{2}{13}$

$P(2\text{ Aces})$

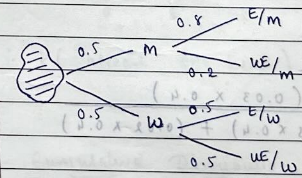
$= P(\text{Aces in } 1^{st}\ \&\ \text{Ace in } 2^{nd})$

$= P(\text{Ace in } 1^{st}) \cdot P(\text{Ace in } 2^{nd})$

$= \frac{4}{52} \cdot \frac{4}{52}$

$= \frac{1}{169}$

ex: A group consist of equal number of men and women. Of this group 20% of men and 50% of the women are unemployed. If a person is selected at random from this group, the probability of selected person being employed is...



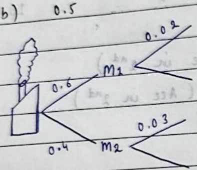
$\therefore P(E) = \left(\frac{1}{2} \times 0.8\right) + (0.5 \times 0.5)$

$\therefore P(E) = 0.65$

ex: In a factory, two machines M_1 and M_2 manufacture 60% and 40% of the auto components resp. out of total production, 2% of M_1 and 3% M_2 are found to be defective. If a randomly drawn auto component from combined lot is found defective, what is the probability that it was manufactured by M_2?

a) 0.35 b) 0.45

→ b) 0.5 d) 0.4



$\therefore P\left(\frac{M_2}{D}\right) = ?$

(M1) $P(M_2/D) = \frac{P(D/M_2) \cdot P(M_2)}{\Sigma P(D)}$

$= \frac{P(D/M_2) \cdot P(M_2)}{P(M_1) \cdot P(D/M_2) + P(M_2) \cdot P(D/M_2)}$

$= \frac{0.03 \times 0.4}{(0.6 \times 0.02) + (0.03 \times 0.4)}$

$\therefore P(M_2/D) = 0.5$

(M2) $\therefore P = \frac{\text{Req path}}{\text{Total path}}$

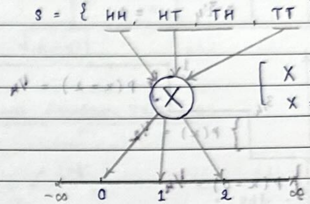
$\therefore P(M_2/D) = \frac{(0.03 \times 0.4)}{(0.03 \times 0.4) + (0.02 \times 0.4)}$

$\therefore P(M_2/D) = 0.5$

★ Random Variables

Expirement : Tossing 2 coin

$S = \{HH, HT, TH, TT\}$



[X = no. of heads
  X = x]

⇝ X = no. of heads

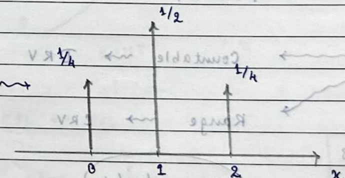
X = x	0	1	2
P(X = x)	1/4	1/2	1/4

Probability mass function / probability denity function ($f(x)$)

$\therefore \boxed{\Sigma P_x = 1}$

pdf plot /

Stem plot ⇝



⇝ $P(X = 0.5) = 0$

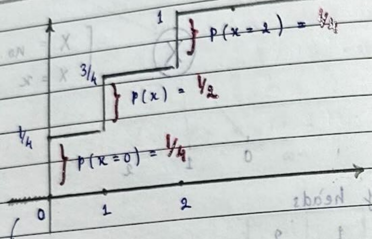
⇝ $P(X = 0.999) = 0$

$\therefore P(\text{atleast one head}) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

• Cumulative Distribution function :- ($F(X)$) cdf

X	0	1	2
P(X)	1/4	1/2	1/4
F(X)	1/4	1/2 + 1/4 = 3/4	1/4 + 1/2 + 1/4 = 1

Stair case plot



$F(x<0) = 0$

cdf
min value = 0
max value = 1

$p(x=2) = ?$

then $F(X)$: $F(X=2) - F(X=1)$

$(p(x=2) + p(x=1) + p(x=0))$

$- (p(X=1) + p(X=0))$

$= p(x=2)$

1) $X = x$ ⇝ Countable ⇝ DRV

⇝ Range ⇝ CRV

X	+1	-0.333
P(X)	1/4	3/4

$x = -/0/+$

ex :

X	0	1	2
P(X)	k/4	2k/4	k/4

K = ?

K = Normalization factor

Normalis. Constant

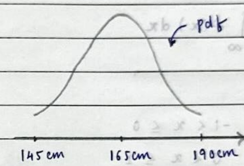
① X = DRV

② pdf : $\Sigma p = 1$

$\therefore \frac{k}{4} + \frac{2k}{4} + \frac{k}{4} = 1$

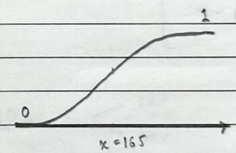
$\therefore \boxed{k = 1}$

★ X = Continuous Random Variable , $x \in (-\infty, \infty)$



• pdf - $\boxed{\int_{-\infty}^{\infty} f(x)\,dx = 1}$

X = height of boys



• cdf - $\boxed{\int_{-\infty}^{x} f(x)\,dx}$

★★ cdf ↔ pdf

$F(x) = \int_{-\infty}^{x} f(x)\,dx$

$\frac{d}{dx} F(x) = \frac{d}{dx} \int_{t=-\infty}^{x} f(t)\,dt$

$= 1\, f(x) - 0$

$\boxed{\frac{d}{dx} F(x) = f(x)}$

cdf pdf

✱ Leibenitz rule

$\frac{d}{dx} \int_{\phi_1(x)}^{\phi_2(x)} f(t)\,dt$

$= \phi_2'(x)\, f(\phi_2(x)) - \phi_1'(x)\, f(\phi_1(x))$

- **In Short :**

$$X \begin{cases} \to DRV \to \Sigma \\ \to CRV \to \int \end{cases}$$

★ **DRV :** 1) Pdf : $\sum_{-\infty}^{\infty} P(X) = 1$

2) cdf : $\sum_{-\infty}^{x} P(x)$

★ **CRV :** 1) Pdf : $\int_{-\infty}^{\infty} f(x)\, dx = 1$

2) cdf : $\int_{-\infty}^{x} f(x)\, dx$

** **ex :**

$$f(x) = \begin{cases} k(1+x), & -1 < x \le 0 \\ k(1-x), & 0 < x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find : i) k

ii) Plot f(x)

iii) cdf

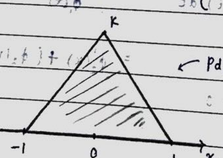
i) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$\therefore \int_{-1}^{0} k(1+x)\, dx + \int_{0}^{1} k(1-x)\, dx = 1$

$\boxed{k = 1}$

ii) now,

$$f(x) = \begin{cases} 1+x, & -1 < x \le 0 \\ 1-x, & 0 < x \le 1 \end{cases}$$



← Pdf

$A(\Delta) = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 2 \cdot k = 1$

$\boxed{k = 1}$

iii)

$\therefore F(-1 < X \le 0) = F(X \le 0) = \int_{-1}^{x} (1+x)\, dx$

$= \left[\frac{(1+x)^2}{2} \right]_{-1}^{x}$

$= x + \frac{x^2}{2} + \frac{1}{2}, \quad -1 \le x < 0$

$\therefore F(0 < x \le 1) = F(x \le 1) =$

$= \int_{-1}^{0} (1+x)\, dx + \int_{0}^{x} (1-x)\, dx$

$= x - \frac{x^2}{2} + \frac{1}{2}, \quad 0 \le x \le 1$

- cdf :

$$F(x) = \begin{cases} 0, & x < -1 \\ x + x^2/2 + 1/2, & -1 < x \le 0 \\ x - x^2/2 + 1/2, & 0 < x \le 1 \\ 1, & x > 1 \end{cases}$$

★ **1D RV :** $\begin{cases} \to CRV \\ \to DRV \end{cases}$ $\boxed{\text{2DRV} : \begin{cases} \to CRV \\ \to DRV \end{cases}}$

** **Q1)** Type 1

$P(X_1 = x_1, X_2 = x_2) = \frac{1}{27}(x_1 + 2x_2), \quad x_1 = 0, 1, 2$

$x_2 = 0, 1, 2$

$X_1 \downarrow$ \ $X_2 \to$	0	1	2
0	0	2/27	4/27
1	1/27	3/27	5/27
2	2/27	4/27	6/27

} $P(X_1, X_2)$

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- $p(x_1)$ only

X_1	0	1	2
$p(x_1)$	6/27	1/3	4/9

$\underline{\Sigma q^n}$ $p(x_1) = \sum_{x_1=0}^{2} p(x_1, x_2) = p(x_1, 0) + p(x_2, 1) + p(x_1, 2)$

$= \frac{x_1}{27} + \frac{x_1+2}{27} + \frac{x_1+4}{27}$

$= \frac{x_1+2}{9}$ //

- $p(x_2)$:

X_2	0	1	2
$p(x_2)$	1/9	3/9	5/9

Q2) $f(x,y) = \begin{cases} c(x^2+2y) & x = 0,1,2 \ / \ y = 1,2,3,4 \\ 0 & \text{elsewhere} \end{cases}$

Type 2

$X \downarrow$ \ $Y \rightarrow$	1	2	3	4
0	2c	4c	6c	8c
1	3c	5c	7c	9c
2	6c	8c	10c	12c

$\therefore \Sigma x \Sigma y = 1$

$\therefore 80c = 1$

$\therefore \boxed{c = \frac{1}{80}}$

i) $P(x=2, y=3) = \frac{10}{80} = \frac{1}{8}$ //

ii) $P(x \le 1, y > 2) = P[x = 0,1, y = 3,4]$

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$= P(X=0, Y=3) + P(X=0, Y=4) + P(X=1, Y=3) + P(X=1, Y=4)$

$= \frac{3}{8}$ //

Q-3) Type 3

$Y \downarrow$ \ $X \rightarrow$	1	2	3
1	1/12	1/6	0
2	0	1/9	1/5
3	1/18	1/4	2/15

i) $P(Y=3 / X=3) = \frac{P(Y=3, X=3)}{P(X=3)}$

$= \frac{2/15}{5/15}$

$= \frac{2}{5}$ //

ii) $P(X=1 / Y=2) = \frac{P(X=1, Y=2)}{P(Y=2)} = 0$ //

iii) $P(Y \le 2) = P(Y=1) + P(Y=2) = \frac{101}{180}$ //

*) $P(X+Y < 4) = P(X=1, Y=1) + P(X=1, Y=2) + P(X=2, Y=1) = \frac{1}{4}$ //

2D CRY $\rightsquigarrow \iint$

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Q1) $f(x,y) = \begin{cases} x^3y^3/16 , & 0 < x < 2 , \ 0 < y < 2 \\ 0 , & \text{elsewhere} \end{cases}$

Type 1

$\Rightarrow$ i) $f(x)$

$$f(x) = \int_{y=0}^{2} f(x,y)\, dy = \int_0^2 \frac{x^3y^3}{16}\, dy$$

$$= \frac{x^3}{16}\left(\frac{y^3}{4}\right)_0^2$$

$$= \boxed{\frac{x^3}{4}} , \quad \boxed{0 < x < 2}$$

ii) $f(y)$

$$f(y) = \int_{x=0}^{2} f(x,y)\, dx$$

$$= \int_0^2 \frac{x^3y^3}{16}\, dx$$

$$= \frac{y^3}{16}\int_0^2 x^3\, dx$$

$$= \boxed{\frac{y^3}{4}} , \quad \boxed{0 < y < 2}$$

iii) Is x and y are independent.

$\therefore f(x,y) = f(x) \cdot f(y)$

$\therefore \frac{x^3y^3}{16} = \frac{x^3}{4} \cdot \frac{y^3}{4}$

$\therefore$ Independent.

iv) Check as it is pdf.

$\therefore \iint f(x,y)\, dx\, dy = 1$

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Q-2) $f(x,y) = \begin{cases} 1/8\,(6-x-y) , & 0 < x < 2 , \ 2 < y < 4 \\ 0 , & \text{elsewhere} \end{cases}$

Type 2

$\Rightarrow$ i) $P(x < 1 , y < 3)$

$$= \int_{-\infty}^{1}\int_{-\infty}^{3} f(x,y)\, dx\, dy$$

$$= \int_0^1\int_2^3 \frac{1}{8}(6-x-y)\, dy\, dx$$

$$= \frac{3}{8} //$$

ii) $P(x < 1 / y < 3)$

$$= \frac{P(x<1 , y<3)}{P(y<3)} = \frac{3/8}{5/8} = 3/5 //$$

• $P(y<3) = \int_{-\infty}^{3} f(y)\, dy = \int_2^3 f(y)\, dy = \int_2^3 \frac{5-y}{4}$

$$= 5/8 //$$

iii) $f(y) = \int_0^2 f(x,y)\, dx$

$$= \int_0^2 \frac{1}{8}(6-x-y)\, dx$$

$$= \boxed{\frac{5-y}{4}} , \quad \boxed{2 < y < 4}$$

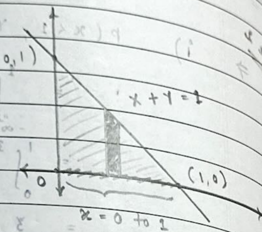
M2 : $P(y<3) = \int_{y=2}^{3}\int_{x=0}^{2} f(x,y)\, dx \cdot dy$

Q2) $f(x,y) = \begin{cases} e^{-(x+y)} & ; \quad x > 0 \quad y > 0 \\ 0 & ; \quad \text{elsewhere.} \end{cases}$

Type 3

$\Rightarrow \quad p(x+y<1)$

$$= \int_{x=0}^{1} \int_{y=0}^{1-x} f(x,y)\, dx\, dy$$



$$= \int_0^1 \left[\int_0^{1-x} e^{-y}\, dy \right] e^{-x}\, dx$$

$$= 1 - 2e^{-1}$$

$$\boxed{\begin{array}{l} x : 0 \text{ to } 1 \\ y : 0 \text{ to } 1-x \end{array}}$$

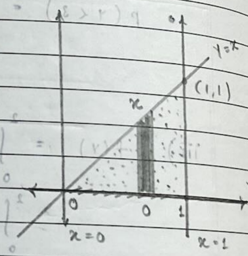
Q-3) $f(x,y) = 8xy \quad , \quad 0 < x < 1 \quad , \quad 0 < y < x$

$= 0 \quad , \quad$ elsewhere.

Type 4

$\Rightarrow$ i) $f(x)$

$$= \int_{y=0}^{x} f(x,y)\, dy$$



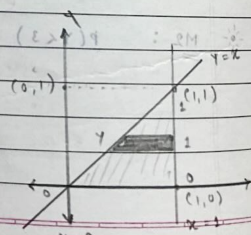
$$= \int_0^x 8xy\, dy$$

$$= 8x \left(\frac{y^2}{2} \right)_0^x$$

$$= \boxed{4x^3} \quad , \quad \boxed{0 < x < 1}$$

ii) $f(y)$

$$= \int_{x=y}^{1} f(x,y)\, dx$$



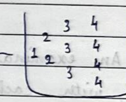
$$= \int_y^1 8xy\, dx$$

$$= y \left(\frac{8x^2}{2} \right)_y^1$$

$$= \boxed{4y(1-y^2)} \quad , \quad \boxed{0 < y < 1}$$

★. Average = Expectation

X	1	2	3	4
P(X)	1/10	2/10	3/10	4/10



DRV $\rightsquigarrow$ $E(X) = \sum_{-\infty}^{\infty} x \cdot p(x)$

$$= 1 \cdot \frac{1}{10} + 2 \cdot \frac{2}{10} + 3 \cdot \frac{3}{10} + 4 \cdot \frac{4}{10}$$

$\therefore \boxed{E(X) = 3}$

Type 1

Q: A fair coin is tossed till head appears, expectation of no of tosses?

① $\therefore E(X) = (\sum x\, p(x))$

$X =$ no. of tosses

②

Event	H	TH	TTH	.....
X	1	2	3	.....
P(X)	1/2	1/4	1/8	

$\therefore\ E(x) = \sum x\,p(x)$

$= 1 \times \left(\frac{1}{2}\right) + 2 \times \left(\frac{1}{2}\right)^2 + 3 \times \left(\frac{1}{2}\right)^3 + \dots$

$= \frac{1}{2}\left[1 + 2.\left(\frac{1}{2}\right) + 3.\left(\frac{1}{2}\right)^2 + \dots\right]$

$= \frac{1}{2}\left[1 - \left(\frac{1}{2}\right)\right]^{-2}$

$= 2$

$\therefore (1-x)^{-2} = 1 + 2x + 3x^2 + \dots$

Gate 2004

ex: An examination paper has 150 mcq of one mark each, with each Ques having four choices. Each incorrect answer fetches -0.25 mark. Suppose 1000 student choose all their answer randomly with uniform probability. The sum total of the expected marks obtained all these student is

a) 0
b) 2550
c) 7525
d) 9375

1) X = marks

X	1	-0.25
$p(X)$	$1/4$	$3/4$

$E(X) = \sum X\,p(X) = \frac{1}{16}$ mrk/Q

2) $E(X)$ of 1 student (150 Q)

$= \frac{1}{16}$ mrk $\times$ 150 Q / Q

$= \frac{150}{16}$ mrk / std

$\Rightarrow E(1000 \text{ std}) = 150 \times \frac{1000}{16}$ mrk

$= 9375$ mrks

CRV $\rightsquigarrow$ $E(x) = \int_{-\infty}^{\infty} x.\,f(x)\,dx$

Q) $f(x) = \begin{cases} kx(2-x), & 0 < x < 2,\ k > 0 \\ 0, & \text{otherwise} \end{cases}$

find $E(X)$.

$\Rightarrow$ 1) $\int_0^2 kx.(2-x)\,dx = 1$

$\therefore k.\int_0^2 x.(2-x)\,dx = 1$

$\therefore k.\int_0^2 2x - x^2\,dx = 1$

$\therefore \boxed{k = 3/4}$

2) $E(x) = \int_0^2 \frac{3}{4}x(2-x)\,dx$

$= 1$

***

$\therefore E(X) = \sum_{x=-\infty}^{\infty} X\,P(X)$ [DRV]

$\therefore E(X^2) = \sum_{x=-\infty}^{\infty} X^2\,P(X)$

$\therefore E(X^3) = \sum_{x=-\infty}^{\infty} X^3\,P(X)$

lly $\boxed{E[g(x)] = \sum_{-\infty}^{\infty} g(x).\,p(x)}$

{ Expectation of function of RV (x) }

Q)

X	0	1	2	3
P(X)	0.1	0.3	0.5	0.1

find $E(Y) = ?$

where $Y = x^2 + 2x$

$\Rightarrow$ 1) $E(Y) = E(x^2 + 2x) = \sum_{x=0}^{3} (x^2 + 2x) \cdot P(X)$

$= (1+2)\,P(1) + (2^2+4)\,P(2) + (3^2+6)\,P(3)$

$= 6.4$

CRV : $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$

$E(x^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx$

$\therefore \boxed{E(g(x)) = \int_{-\infty}^{\infty} g(x) \cdot f(x)\,dx}$

Q] $f(x) = \frac{2}{9} x \left(2 - \frac{x}{2}\right), \quad 0 < x < 3$

$E(Y) = ?$ Where $Y = (x+1)^2$

$\Rightarrow E((x+1)^2) = \int_{0}^{3} (x+1)^2 \cdot \frac{2}{9} x \left(2 - \frac{x}{2}\right) dx$

$= 8.1$

★ Property of Expectation

1) $E(C) =$ DRV $\longrightarrow \sum_{-\infty}^{\infty} c\,p(x) = c \cdot \underbrace{\sum_{-\infty}^{\infty} p(x)}_{=1}$

CRV $\longrightarrow \int_{-\infty}^{\infty} c \cdot f(x)\,dx = c \cdot \underbrace{\int_{-\infty}^{\infty} f(x)\,dx}_{=1}$

$\therefore E(C) = 1$

2) $E(aX) = a \cdot E(X)$

3) $E(aX + b) = E(ax) + E(b)$

$= a\,E(X) + b$

4) $E(X+Y) = E(X) + E(Y)$

5) $E(X-Y) = E(X) - E(Y)$

Q] $E(x) = 4$ Find $E(2X + Y - 3)$

$E(Y) = -2$

$\Rightarrow \therefore E(2X) + E(Y) - 3$

$\therefore 2E(X) + E(Y) - 3$

$\therefore 2(4) + (-2) - 3$

3

* If x and y are independent $\rightarrow E(XY) = E(X)\,E(Y)$ may/may not be

* $E(ax^n) = a \cdot E(x^n)$

★ Variance

$\therefore V(x) = (X - \bar{X})^2 \cdot P(X)$

$\therefore V(x) = E(X - \bar{X})^2$

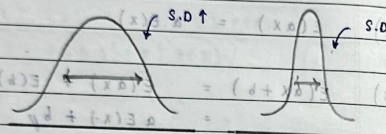
$$= E[x - E(x)]^2$$

$$= E[x^2 - 2x.E(x) + (E(x))^2]$$

$$\therefore V(x) = E(x^2) - 2E(x)E(x) + (E(x))^2$$

$$\therefore \boxed{V(x) = E(x^2) - (E(x))^2}$$

• $\boxed{S.D = \sqrt{V}}$



H.W ex: Gate

$$f(t) = \begin{cases} 1+t & , -1 \text{ to } 0 \\ 1-t & , 0 \text{ to } 1 \end{cases} \qquad S.D = ?$$

$$S.D = \sqrt{V}$$

$$\therefore V = E(t^2) - (E(t))^2$$

$$\therefore E(t) = \int_{-\infty}^{\infty} t\, f(t)\, dt$$

$$= \int_{-1}^{0} t(1+t)\, dt + \int_{0}^{1} t(1-t)\, dt$$

$$= \left(\frac{t^2}{2} + \frac{t^3}{3}\right)_{-1}^{0} + \left(\frac{t^2}{2} - \frac{t^3}{3}\right)_{0}^{1}$$

$$= -\left[\frac{1}{2} + \frac{(-1)}{3}\right] + \left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= 0$$

$$\therefore E(t^2) = \int_{-\infty}^{\infty} t^2 f(t)\, dt$$

$$= \int_{-1}^{0} t^2(1+t)\, dt + \int_{0}^{1} t^2(1-t)\, dt$$

$$= \left[\frac{t^3}{3} + \frac{t^4}{4}\right]_{-1}^{0} + \left[\frac{t^3}{3} + \frac{t^4}{4}\right]_{0}^{1}$$

$$= -\left[\frac{-1}{3} + \frac{1}{4}\right] + \left[\frac{1}{3} + \frac{1}{4}\right]$$

$$E(t^2) = \frac{1}{6}$$

$$V = E(t^2) - (E(t))^2$$

$$= \frac{1}{6} - (0)^2$$

$$= \frac{1}{6} - 0$$

$$V = 1/6 \rightsquigarrow \boxed{S.D = \sqrt{V} = \frac{1}{\sqrt{6}}}$$

Gate Q-1] pdf → $p(x) = k e^{-\alpha|x|}$, $x \in (-\infty, \infty)$ $k = ?$

M2

$$\int_{-\infty}^{\infty} k e^{-\alpha|x|}\, dx = 1$$

Above fun is an even fun'

$$\therefore 2\int_{0}^{\infty} k e^{-|x|}\, dx = 1$$

$$\therefore 2k\left[\frac{e^{-\alpha x}}{-\alpha}\right]_{0}^{\infty} = 1$$

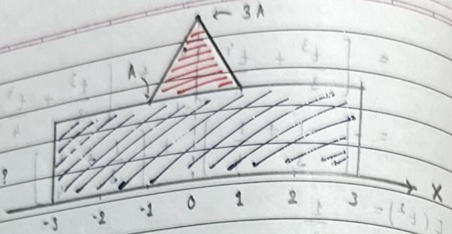
$$\therefore \frac{2k}{-\alpha}(e^{-\infty} - e^{0}) = 1$$

$$\therefore \boxed{k = \alpha/2}$$

Q2]

A = ?

It is pdf ?



⇒ 1) CRV

2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\therefore \int_{-3}^{3} f(x)\,dx = 1$

⇓

Area under funⁿ = 1

⇒ $\therefore A(\square) + A(\Delta) = 1$

$\therefore 6A + \frac{1}{\cancel{2}} \cdot \cancel{2} \cdot 2A = 1$

$\therefore 8A = 1$

$\therefore \boxed{A = 1/8}$

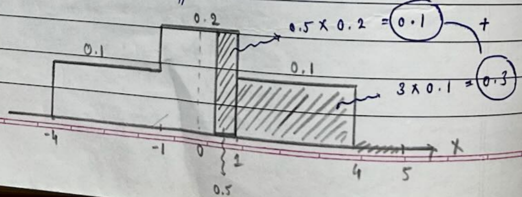
Q3] X = RV $f(x) = \begin{cases} 0.2, & |x| \le 1 \\ 0.1, & 1 < |x| \le 4 \\ 0, & \text{otherwise.} \end{cases}$

$P(0.5 < x < 5) = ?$

⇒ $\therefore \int_{0.5}^{5} f(x)\,dx = \int_{0.5}^{1} (0.2)\,dx + \int_{1}^{4} (0.1)\,dx + \int_{4}^{5} 0\,dx$

$= 0.4$ //

M2



Q4] $f(x) = \begin{cases} a + bx, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ $P(x < 0.5) = ?$

if $E(x) = 2/3$.

⇒ $\therefore E(x) = 2/3$

$\therefore \int_{-\infty}^{\infty} x f(x)\,dx = 2/3$

$\therefore \int_{0}^{1} x(a + bx)\,dx = \frac{2}{3}$

⇒ $\boxed{3a + 2b = 4}$ 1st eqⁿ

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\therefore \int_{0}^{1} (a + bx)\,dx = 1$

$\therefore \boxed{a + \frac{b}{2} = 1}$ 2nd eqⁿ

$a = 0$, $b = 2$

$\int_{0}^{0.5} (0 + 2x)\,dx = 0.25$ //

Q5] $f(x, y) = \begin{cases} x + y, & 0 \le x \le 1, \ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$

$P(x + y \le 1) = ?$

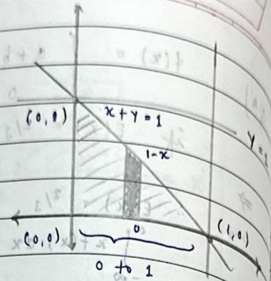
→ $P(x + y \le 1) = \iint f(x, y)\,dx\,dy$

$= \iint (x + y)\,dx\,dy$

$x = 0$ to 1

$y = 0$ to $1-x$

$$= \int_0^1 \int_0^{1-x} (x+y)\, dy\, dx$$



$$= \int_0^1 \left(xy + \frac{y^2}{2} \right)_0^{1-x} dx$$

$$= 0.33$$

Q6] $f(x) = x^2$, $-1 < x < 1$

$= 0$, otherwise

$\therefore P\left(-\frac{1}{3} < x < \frac{1}{3}\right) = ?$

$$\Rightarrow \therefore P\left(-\frac{1}{3} < x < \frac{1}{3}\right) = \int_{-1/3}^{1/3} f(x)\, dx$$

$$= \int_{-1/3}^{1/3} x^2\, dx \rightsquigarrow 2 \int_0^{1/3} x^2\, dx$$

$$= \frac{2}{3} \left[x^3 \right]_0^{1/3}$$

$$= 2/81$$

★ Variance property.

1) $V(c) = 0$
2) $V(aX) = a^2 V(X)$
3) $V(-X) = V(X)$
4) $V(X+Y) = V(X) + V(Y) + 2\,\text{cov}(X, Y)$

X & Y indep $\rightsquigarrow$ $V(X+Y) = V(X) + V(Y)$

5) $V(X-Y) = V(X) + V(Y) - 2\,\text{cov}(X, Y)$
6) $V(aX+b) = V(aX) + V(b)$

$= a^2 V(X)$

• Moment generating function

$E(x^1) \rightsquigarrow$ 1st moment

$E(x^2) \rightsquigarrow$ 2nd moment

$E(x^3) \rightsquigarrow$ 3rd moment

Now, $e^x = x^0 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

$$e^{tx} = 1 + \frac{tx}{1!} + \frac{t^2 x^2}{2!} + \ldots$$

$$\therefore E(e^{tx}) = 1 + \frac{t E(x)}{1!} + \frac{t^2 E(x^2)}{2!} + \ldots$$

$\rightsquigarrow$ for 1st moment, $\boxed{\left.\frac{d}{dt} E(e^{tx})\right|_{t=0} = E(x) + \frac{E(x^2)\, 2t}{2!} + \ldots}$

$$\therefore \boxed{E(x^n) = \left[\frac{d^n}{dt^n} E(e^{tx}) \right]_{t=0}}$$

OR

$$\therefore \boxed{E(x^n) = \left[\frac{d^n}{dt^n} M_0(t) \right]_{t=0}}$$

ex:

X	1	2	3	4	5	6
P(X)	1/6	1/6	1/6	1/6	1/6	1/6

1) Find MGF
2) $E(X)$
3) $E(X^2)$

1) $E(e^{tx}) = \sum_{x=1}^{6} e^{tx}\, p(x)$

$= e^{t}.p(1) + e^{2t}.p(2) + e^{3t}p(3) + e^{4t}p(4) + e^{5t}p(5) + e^{6t}p(6)$

$\therefore E(e^{tx}) = \frac{1}{6}\left[e^{t} + e^{2t} + e^{3t} + e^{4t} + e^{5t} + e^{6t}\right]$

2) $E(x) = \left[\frac{d\, E(e^{tx})}{dt}\right]_{t=0}$

$= \frac{1}{6}\left[e^{t} + 2e^{2t} + 3e^{3t} + 4e^{4t} + 5e^{5t} + 6e^{6t}\right]$

$= \frac{1}{6}\left[1+2+3+4+5+6\right]$

$= 7/2$

3) $E(x^2) = \left(\frac{d^2\, E(e^{tx})}{dt^2}\right)_{t=0}$

$= \frac{1}{6}\left(e^{t} + 4e^{2t} + 9e^{3t} + 16e^{4t} + 25e^{5t} + 36e^{6t}\right)$

$= \frac{1}{6}\left(1+4+9+16+25+36\right)$

$= \frac{91}{6}$

Gate ex: find the value of λ such that the function $f(x)$ is a valid probability density function ...

$f(x) = \lambda(x-1)(-x+2)$ for $1 \le x \le 2$

$= 0$, otherwise

$\therefore \int_1^2 f(x)\,dx = 1$

$\therefore \int_1^2 \lambda(x-1)(2-x)\,dx = 1$

$\therefore \boxed{\lambda = 6}$

ex: The probability density function $f(x) = a e^{-b|x|}$ where x is a random variable whose allowable value range is from $x = -\infty$ to $x = +\infty$. The cdf for this function for $x \geq 0$ is

→ a) $\frac{a}{b} e^{bx}$ c) $-\frac{a}{b}(e^{bx})$

b) $\frac{a}{b}(2 - e^{-bx})$ d) $-\frac{a}{b}(2 + e^{-bx})$

$\therefore F(x \geq 0) = \int_{-\infty}^{x} f(x)\,dx$

$= \int_{-\infty}^{0} a e^{-b(-x)}\,dx + \int_0^x a e^{-bx}\,dx$

$= a\left(\frac{e^{bx}}{b}\right)_{-\infty}^{0} + a\left(\frac{e^{-bx}}{-b}\right)_0^x$

$= \frac{a}{b}(2 - e^{-bx})$

Gate ex: The probability density function of a random variable X is $f(x) = \frac{x}{4}(4 - x^2)$; for $0 \le x \le 2$

0 otherwise

The mean of the random variable is ___

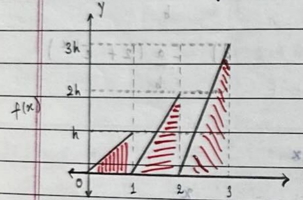
→ $\therefore E(x) = \int_0^2 x \cdot \frac{x}{4}(4 - x^2)\, dx$

$$= \frac{1}{4}\int_0^2 (4x^2 - x^4)\, dx$$

$$= \frac{16}{15}$$ //

Gate ex: The graph of a function $f(x)$ is shown in figure for $f(x)$ to be a valid probability function the value of h is _

~~a)~~ $1/3$
b) $2/3$
c) 1
d) 3



$$\therefore \int_{-\infty}^{\infty} f(x)\, dx = 1$$

$\therefore$ Area $= 1$

$$\therefore \frac{1}{2}\cdot 1 \cdot h + \frac{1}{2}\cdot 1 \cdot 2h + \frac{1}{2}\cdot 1\cdot 3h = 1$$

$$\therefore 3h = 1$$

$$\therefore \boxed{h = \frac{1}{3}}$$

Gate ex: Consider the joint probability mass function of random variable x and y as shown in table below for instance, $P\{X=1, Y=2\} = 0.3$

	X = 1	X = 2
Y = 1	0.2	0.3
Y = 2	0.3	0.1
Y = 3	0.1	

$P[X=2 / Y=2] = ?$

a) 0.1
~~b)~~ 0.25
c) 0.4
d) 0.75

$$P(X=2/Y=2) = \frac{P(X=2, Y=2)}{P(Y=2)}$$

$$= \frac{0.1}{0.3 + 0.1}$$

$$= 0.25$$ //

Gate ex: A watch uses two electronic circuits (ECs). Each ECs has a failure probability of 0.1 in one year of operation. both ECs are required for functioning of the watch. The probability of the watch functioning for one year without failure is...

a) 0.99
b) 0.90
~~c)~~ 0.81
d) 0.80

P(ECs success) = 0.9

$\therefore$ P(watch fun^n) = P(watch doesn't fail)

= P(both ECs are not fail)

= P(EC_1 success) · P(EC_2 success)

= (0.9) · (0.9)

= 0.81

Gate

ex: X and Y are two independent random variables with variances 1 and 2 respectively. Let $Z = X - Y$. The Variance of Z is ..

a) 0

b) 1

c) 2

d) 3

$\because Z = X - Y$

$\therefore V(Z) = V(X - Y)$

$\therefore V(Z) = V(X) + V(-Y)$

$\therefore V(Z) = V(X) + V(Y)$

$\therefore V(Z) = 1 + 2$

$\therefore V(X) = 3$

Gate

ex: You have gone to a cyber cafe with a friend. You found that the cyber cafe has only three terminals. All terminals are unoccupied. You & your friends have to make a random choice of selecting a terminal. What is the probability that both of you will NOT select the same terminal?

A) $\frac{1}{9}$ c) $\frac{2}{3}$

B) $\frac{1}{3}$ d) 1

T_1 T_2 T_3

$S = \{ \cancel{T_1 T_1}, \; T_2, T_1, \; T_3 T_1$

$T_1, T_2, \; \cancel{T_2 T_2}, \; T_3 T_2$

$T_1, T_3, \; T_2 T_3, \; \cancel{T_3 T_3} \}$

$\therefore P = \frac{6}{9} = \frac{2}{3}$

Gate

ex: In a housing society, half of the families have a single child per family while the remaining half have two children per family. The probability that a child picked at random has a sibling is ..

N

1^{st}: $\frac{N}{2}$ 2^{nd}: $\frac{N}{2}$

$1 \times \frac{N}{2}$ $2 \times \frac{N}{2}$

$\therefore$ Total No. of children $= \frac{N}{2} + N = \frac{3N}{2}$

$\therefore P = \frac{Req}{total} = \frac{N}{\frac{3N}{2}} = \frac{2}{3}$

Gate

ex: The probability of getting a "head" in a single toss of a biased coin is 0.3. The coin is tossed repeatedly till head is obtained. If the tosses are independent then the probability of getting "head" for the first time in the fifth toss is ..

$P(H) = 0.3$

$P(T) = 0.7$

T T T T H

$(0.7)(0.7)(0.7)(0.7)(0.3)$

$= (0.7)^4 \cdot (0.3)$

$= 0.072$

★ 3 coins are tossed

P [Exactly 2 heads in 3 tosses] given

$= P(HHT) + P(HTH) + P(THH)$

$= P(H)P(H)P(T) + P(H)P(T)P(H) + P(T)P(H)P(H)$

let $P(H) \rightsquigarrow p$ (success)

$P(T) \rightsquigarrow q$ (failure)

$= p.p.q + p.q.p + q.p.p$

$= \boxed{3.p^2.q}$

3	p^2	q
↓	↓	↓
No. of ways	$(\text{success})^x$ p^x	$(\text{failure})^{n-x}$ q^{n-x}

Total = n

1] n = no of trials

x = req for complete story. (success for Complete event)

p = Success for 1 trial

q = failure for 1 trial

$= 1 - p$

2] Binomial distribution

$\therefore \boxed{P(x) = {}^nC_x \, p^x \, q^{n-x}}$

3] Story → success → p ; → failure → q } $p + q = 1$

⇒ Independent events

4] $X = 0, 1, 2, \ldots$ [Discrete RV]

5] $E(x) = \sum x\,p(x) = np$

$V(x) = E(x^2) - (E(x))^2 = npq$

ex

X	0	1	2	3
P(x)	1/8	3/8	3/8	1/8

$\therefore E(x) = \sum x\,p(x) = 3/2$

or $E(x) = np = 3 \cdot \left(\frac{1}{2}\right) = 3/2$

$\boxed{n = 1}$

binomial Distⁿ $\rightsquigarrow$ bernoulli Distⁿ

${}^nC_x \, p^x q^{n-x}$ → $\boxed{p^x . q^{1-x}}$

- Independent event
- $p + q = 1$
- X = DRV $\boxed{X \leftrightarrow p}$

⇒ $E(x) = p$

⇒ $V(x) = pq$

Gate

ex: The probability of a defective piece being produced in a manufacturing process is 0.01. The probability that out of 5 successive pieces, only one is defective is

a) $(0.99)^2 (0.01)$

b) $(0.99)(0.01)^4$

c) $5 \times 0.99 \times (0.01)^4$

d) $5 \times (0.99)^4 \times 0.01$ ✓

$p = 0.01$	
$q = 0.99$	
$x = 1$	
$n = 5$	

B.D

$\therefore P(x) = {}^{n}C_x \, p^x \, q^{n-x}$

$= {}^{5}C_1 \cdot (0.01)^1 (0.99)^4$

$\therefore P(x) = 5 \times (0.99)^4 (0.01)$

Gate ex: A lot has 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly 2 of the chosen items are defective is

→

a) 0.0036
b) 0.1937
c) 0.2234
d) 0.3874

$x = 2$, $p = 0.1$, $q = 0.9$, $n = 10$

$P(x) = {}^{10}C_2 \, (0.1)^2 (0.9)^8$

$= 45 \times (0.1)^2 \times (0.9)^8$

$= 0.1937$

Gate ex: If 20 per cent managers are technocrats. the probability that a random commitee of 5 managers consists of exactly 2 technocrats is...

a) 0.2048
b) 0.4000
c) 0.4096
d) 0.9421

$p = 0.2$, $q = 0.8$, $x = 2$, $n = 5$

$P(x) = {}^{n}C_x \, p^x \, q^{n-x}$

$= {}^{5}C_2 \cdot (0.2)^2 \cdot (0.8)^3$

$= 0.2048$

Gate ex: The probability that a thermistor randomly picked up from a production unit is defective is 0.1. The probability that out of 10 thermistors randomly picked up 3 are defective is...

A) 0.001
B) 0.057
C) 0.107
D) 0.3

$p = 0.1$, $q = 0.9$, $x = 3$, $n = 10$

$P(x) = {}^{10}C_3 \cdot (0.1)^3 \cdot (0.9)^7$

$P(x) = 0.057$

Gate ex: A fair die is rolled four times find the probability that six shows up twice.

a) 1/2
b) 1/36
c) 16/325
d) 25/216

$p = 1/6$, $q = 5/6$, $x = 2$, $n = 4$

$\therefore P(x) = {}^{4}C_2 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^2$

$\therefore P(x) = \frac{25}{216}$

Gate ex: A coin is tossed 4 times. What is the probability of getting heads excactly 3 times?

a) 1/4
b) 1/2
c) 3/8
d) 3/4

x	3
n	4
p	1/2
q	1/2

$\therefore p(x) = {}^4C_3 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)$

$\therefore p(x) = 4 \times \frac{1}{8} \times \frac{1}{2}$

$\therefore p(x) = \frac{1}{4}$

Gate

ex In an expirement, (+)ve and (-)ve values are equally likely to occur. The probability of obtaning at most one negative in five trials is..

a) 1/32
b) 2/32
c) 3/32
d) 6/32

x	0, 1
n	5
p	1/2
q	1/2

$\therefore P(X \le 1) = P(X=0) + P(X=1)$

$= {}^5C_0 \times \left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^5 + {}^5C_1 \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^4$

$= \left(\frac{1}{2}\right)^5 + 5 \times \left(\frac{1}{2}\right)^5$

$= \frac{6}{32}$

Gate

ex The probability that a screw manufactured by a company is defective is 0.1. The company sells screw in packet contaning 5 screws and gives a guarrantee of replacement if one or more screws in the packet are found to be defective. The probability that a packet would be have to be replaced is..

X	0
n	5
p	0.1
q	0.9

(def)

$\therefore P(x \ge 5) = 1 - P(x=0)$

$\therefore P(x \ge 5) = 1 - [{}^5C_0 \times (0.1)^0 \times (0.9)^5]$

$= 1 - 0.59$

$= 0.4095$

Gate

ex A six faced fair dice is rolled five times. Then probability (in %) of obtaning 'ONE' at least four times is..

a) 33.3
b) 0.33
c) 3.33
d) 0.0033

n	5
X	≥ 4
P	1/6
q	5/6

$\therefore P(X \ge 4) = P(X=4) + P(X=5)$

$\therefore P(X \ge 4) = {}^5C_4 \times \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right) + {}^5C_5 \times \left(\frac{1}{6}\right)^5 \cdot \left(\frac{5}{6}\right)^0$

$= \frac{26}{7776}$

$= 0.0033$

In (%) $\rightsquigarrow 0.0033 \times \frac{100}{100} \rightsquigarrow 0.33\%$

Gate

ex An unbiased coin is tossed an infinite no of times. The probability that the fourth head appears at the tenth toss is

a) 0.067
b) 0.082
c) 0.073
d) 0.091

$\underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{H}$

$\underbrace{1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9}_{\text{9 tosses, Exactly 3 heads}}$ $\uparrow$ 4th head (10th tosses)

$\uparrow$ $\frac{1}{2}$

$n - 9$

$x - 3$

$p - \frac{1}{2}$

$q - \frac{1}{2}$

$= {}^9C_3 \cdot \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^6$

$= 0.1638$

$\therefore 0.1638 \times 0.5$

$\therefore 0.082$ //

Imp

ex Consider a sequence of tossing of a fair coin where the outcomes of tosses are independent. The probability of getting the head for the third time in the fifth toss is __

a) 5/16

b) 3/16

c) 3/5

d) 9/16

$\underbrace{\underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }}_{\text{4toss / 2 head}}$ $\underline{H}$ ~ 3rd time

1 2 3 4 5th toss

0.5

$= {}^4C_2 \times (0.5)^2 \cdot (0.5)^2$

$= 3 \times (0.5)^4 \times 0.5$

$= \frac{3}{16}$ //

★ Q] 10 Markers on table, 6 are defective & 4 are not defective if 3 are randomly taken from table. P(Exactly 1 marker is defective).

it is not binomial distribution because "dependent Events"

"Hypergeometric Distribution"

i) DRV

ii) 2 scenario

iii) Dependent Event (without replacement)

10

6D 4ND

3

1D 2ND

$\therefore P = \frac{\text{Req}}{\text{total}}$

$\therefore P = \frac{{}^6C_1 \times {}^4C_2}{{}^{10}C_3}$

$\therefore P = \frac{3}{10}$ //

★ $n \rightsquigarrow$ very large

$n \to \infty$

$BD \Rightarrow P(x) = {}^nC_x\ p^x q^{n-x}$

$= {}^nC_x \left(\frac{p}{q}\right)^x q^n$

$= {}^nC_x \left(\frac{p}{1-p}\right)^x (1-p)^n$

$\therefore E(X) = np$

$\therefore$ Mean $= np$

$\therefore p = \frac{\text{Mean}}{n}$

$\therefore \boxed{\begin{matrix} n \to \infty \\ p \to 0 \end{matrix}}$ (Poission Distrib^n)

BD $\Rightarrow$ PD

i) DRV ($x = 0, 1, 2 \ldots$)

ii) Independent event

iii) $p + q = 1$

iv) $n \to \infty$

$p \to 0$

$$P(x) = \frac{e^{-m} \cdot m^x}{x!} = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$\boxed{m = \lambda = np}$; $m = \lambda$ = mean = Exp = Avg

P.D →
$$\boxed{\begin{aligned} E(x) &= np \\ V(x) &= np \\ \text{mean} &= \text{var} = \lambda \end{aligned}}$$

Gate ex: Consider a poision distribution for the tossing of a biased coin. The mean for this distribution is μ. The standard deviation for this distribution is given by

a) $\sqrt{\mu}$ b) μ^2 c) μ d) $1/\mu$

$\therefore S.D = \sqrt{V}$

but, for PD → variance = mean = μ

$\therefore S.D = \sqrt{\mu}$

Gate ex: The poision distribution is given by $P(r) = \frac{m^r \exp(-m)}{r!}$

The first moment about the origin for the distrib^n is __

a) 0
b) m
c) $1/m$
d) m^2

$\therefore E(x) = \text{Mean} = m$

Gate ex: The number of accident occurring in a plant in a month follows poission distribution with mean as 5.2. The probability of occurrence of less than 2 accidents in the plant during a randomly selected month.

$x < 2$ accident in month

$\lambda = 5.2$

$$P(x<2) = P(x=0) + P(x=1)$$
$$= \frac{e^{-5.2}(5.2)^0}{0!} + \frac{e^{-5.2}(5.2)^1}{1!}$$
$$\therefore P(x<2) = 0.034$$

Gate ex: A traffic officer imposes on an average 5 number of penalties daily on traffic violators. Assume that the number of penalties on different days is independent and follows a poission distribution. The probability that there will be less than 4 penalties in a day is __

$\lambda = 5$

$$P(x<4) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$
$$= e^{-5}\left[\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!}\right]$$
$$= 0.265$$

Gate ex: The average number of accident occuring monthly on an assembly shop floor is 2. The probability that there will be at least one accident in this month is estimated to be __

$\lambda = 2$

a) 0.055

b) 0.456

c) 0.865

d) 0.950

$X \geq 1$

$\therefore P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - \frac{e^{-2} \cdot 2^0}{0!}$

$= 0.865$

Gate

ex: If a random variable X satisfies the posion distribⁿ with a mean value of 2 then prob that $X \geq 2$ is ..

a) $2e^{-2}$

b) $1 - 2e^{-2}$

c) $3e^{-2}$

d) $1 - 3e^{-2}$

$\lambda = 2$

$X \geq 2$

$\therefore P(X \geq 2) = 1 - P(X < 2)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - \left(\frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2^1}{1!} \right)$

$\therefore P(X \geq 2) = 1 - 3e^{-2}$

Gate

ex: An observer counts 240 veh/hr at a specific highway location. Assume that the vechicle arrival at the location is posion distributed, the probability of having one vehicle arriving over a 30 second time interval is ..

$\therefore \lambda = 240$ veh/hr

$= 240 \times \frac{30}{3600}$ veh/30^{sec}

$= \frac{240}{120}$ veh/30sec

$\therefore \lambda = 2$ veh/30sec

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$\therefore P(X = 1 \text{ veh}/\text{sec}_{30}) = \frac{e^{-\lambda} \lambda^x}{x!}$

$= \frac{e^{-2} \cdot 2^1}{1!}$

$= 0.27$

Gate

ex: It is estimated that the average no. of events during a year is three. What is the propability of occurrence of not more than two events over a two year duration? Assume that the number of events follows a posion distribution.

$\therefore \lambda = 3$ for 1 yrs

$\therefore \lambda = 6$ for 2 yrs $= \boxed{(3 * 2)}$

$\therefore P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$\therefore P(X \leq 2) = e^{-6} \left(1 + 6 + \frac{6^2}{2!} \right)$

$= 0.0619$

ex: passenger try repeatedly to get a seat reservation in any train running betⁿ two station untill they are successful. If there is 40% chances of getting reservation in any attempt by a passenger then the average number of attempts that passenger need to make to get a seat reserved is..

M1

1) $E(x) = \sum x \cdot p(x)$

$E(x) \downarrow$ no. of attempts to get seat reserved.

2)

Event	R	NR R	NR NR R	....
X	1	2	3	....
p(x)	0.4	(0.6)(0.4)	(0.6)(0.6)(0.4)	...

$$E(x) = 1 \times 0.4 + 2 \times (0.6)(0.4) + 3 \times (0.6)^2 (0.4) + \dots$$
$$= 0.4\,(1 + 2(0.6) + 3(0.6)^2 + \dots)$$
$$= 0.4\,(1 - 0.6)^{-2}$$
$$\therefore E(x) = 2.5$$

M2: **Geometric Distrib^n** [Position Fixed]

$X = 1 \rightarrow p$

$= 2 \rightarrow qp$

$= 3 \rightarrow qqp$

$\vdots \qquad \vdots$

$$\boxed{p(x) = p^1 q^{x-1}} \qquad x = 1, 2, 3 \dots$$

$p + q = 1$

$$\therefore E(x) = 1 \times p + 2qp + 3qqp + \dots$$
$$= p\,(1 + 2q + 3q^2 + \dots)$$
$$= p\,(1-q)^{-2}$$
$$= \frac{p}{(1-q)^2}$$

$$\therefore E(x) = \frac{p}{p^2} = \frac{1}{p} \qquad \therefore \boxed{E(x) = \frac{1}{p}}$$

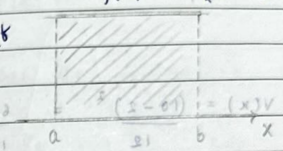
$$\therefore \boxed{V(x) = \frac{q}{p^2}}$$

$$\therefore E(x) = \frac{1}{0.4} = 2.5$$

Uniform Distrib^n:

- CRV $\rightarrow \int$
- pdf

$f(x) \leftarrow 1/b-a$



$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

- cdf

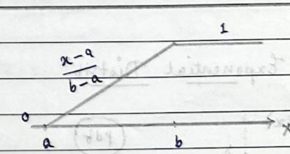
$F(x < a) = 0$

$F(x > b) = 1$

$$F(a < x < b) = \int_{-\infty}^{x} f(x)\,dx = \int_{a}^{x} \frac{1}{b-a}\,dx$$
$$= \frac{1}{b-a}\,(x)_a^x$$

$$\therefore \boxed{F(a < x < b) = \frac{x-a}{b-a}, \quad a < x < b}$$

- cdf $\rightsquigarrow$



- $\boxed{E(x) = \frac{b+a}{2}}$
- $\boxed{E(x^2) = \frac{a^2 + ab + b^2}{3}}$
- $\boxed{V(x) = \frac{(b-a)^2}{12}}$

ex A random variable is uniformly distributed over the interval 2 to 10. its variance will be ..

a) $\frac{16}{3}$ b) 6 c) $\frac{256}{9}$ d) 36

$$V(x) = \frac{(b-a)^2}{12}$$

$$b = 10,\ a = 2, \Rightarrow V(x) = \frac{(10-2)^2}{12} = \frac{64}{12} = \frac{16}{3}$$

ex X is uniformly distributed random variable that takes values betⁿ 0 and 1. The value of $E(x^3)$ will be,

a) 0
b) 1/8
c) 1/4
d) 1/2

$$E(x^3) = \int_{-\infty}^{\infty} x^3 \cdot \frac{1}{b-a}\, dx$$

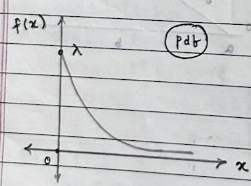
$$= \int_0^1 x^3 \cdot 1\, dx$$

$$\therefore E(x^3) = \frac{1}{4}$$

Exponential Distribⁿ

• CDF

pdf



$$\therefore \boxed{f(x) = \lambda e^{-\lambda x},\quad x \geq 0}$$

• cdf ~ $\boxed{F(x) = 1 - e^{-\lambda x}}$, $x \geq 0$

$\boxed{E(x) = \frac{1}{\lambda}}$ $\boxed{V(x) = \frac{1}{\lambda^2}}$ $\boxed{E(x^n) = \frac{n!}{\lambda^n}}$

ex: Assume that the duration in minutes of telephone conversation follows the exponential distribution

$f(x) = \frac{1}{5} e^{-x/5}$, $x \geq 0$. The probability that the conversation will exceed five minutes is ..

CRV	DRV
• UD	• BD
• expD	• PD
• NorD	• GD
	• HGD

a) 1/e
b) 1 − 1/e
c) 1/e²
d) 1 − 1/e²

• cdf

$$P(x \geq 5) = \int_5^{\infty} f(x)\, dx$$

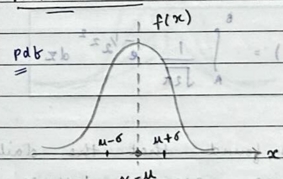
$$= \int_5^{\infty} \frac{1}{5} e^{-x/5}\, dx$$

$$= \frac{1}{5} \cdot \left[\frac{e^{-x/5}}{-1/5}\right]_5^{\infty}$$

$$\therefore P(x \geq 5) = \frac{1}{e}$$

Normal Distribⁿ : (Gaussian distribⁿ / bell shaped curve)

pdf



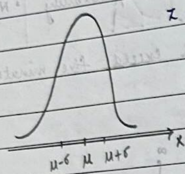
$$\therefore \boxed{f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}$$

- X = CRV
- μ, σ
- Symmetric about x = μ

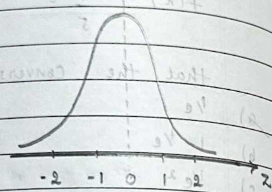
∴ Symmetry ⇒ $\int_{\mu}^{\infty} f(x)\,dx = 1/2$

$\int_{-\infty}^{\mu} f(x)\,dx = 1/2$

$z = \frac{x-\mu}{\sigma}$



⇒



$\therefore P(Z > 0) = 1/2$

$\therefore P(Z < 0) = 1/2$

- $P(x_1 < x < x_2) = \int_{x_1}^{x_2} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$

$\therefore Z = \frac{x-\mu}{\sigma}$

$\therefore \sigma Z = x - \mu$

$\therefore x = \mu + \sigma Z$

$\therefore dx = \sigma\, dz$

$$\boxed{P(Z) = \int_{A}^{B} \frac{1}{\sqrt{2\pi}} e^{-1/2 z^2} dz}$$

ex: A nationalized bank has found that the daily balance available in its saving accounts follows a normal distribution with a mean of Rs 500 and standard deviation of Rs 50. The percentage of saving account holders, who maintain an average daily balance more than Rs 500 is...

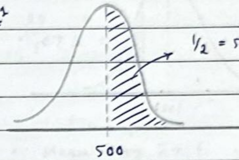
(1) $\mu = 500$

$\sigma = 50$

(2) ND

(3) $P(X > 500)$

M1



M2 (Z distrib^n)

$Z = \frac{X-\mu}{\sigma}$

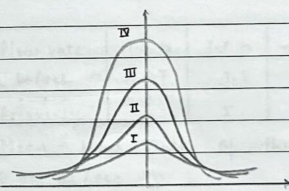
$\therefore P(X > 500)$

$\therefore P\left(\frac{X-\mu}{\sigma} > \frac{500-\mu}{\sigma}\right)$

$\therefore P\left(Z > \frac{500-500}{50}\right)$

$\therefore P(Z > 0) = 0.5 \rightarrow 50\%$

ex below, which one has the lowest variance?

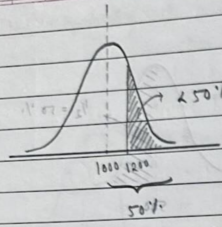


a) I
b) II
c) III
d) IV

ex The annual precipitation data of a city is normally distributed with mean and standard deviation as 1000 mm and 200 mm respectively. The probability that the annual precipitation will be more than 1200 mm is...

a) < 50 %
b) 50 %
c) 75 %
d) 100 %

→



ex : $f_x(x) = \frac{1}{\sqrt{8\pi}} e^{-\left\{\frac{(x-1)^2}{8}\right\}}$, $-\infty < x < \infty$

then $\int_{1}^{\infty} f_x(x)\,dx = ?$

a) 0 b) $1 - \frac{1}{e}$ c) $\frac{1}{2}$ d) 1

Statistics

1) **Mean**

$\{1, 8, 3, 2, 6\}$ → ungrouped data

- Mean $= \frac{\Sigma x_i}{n} = \frac{20}{5} = 4$

grouped data

x_i	f_i
8	1
6	1
5	3
7	2
4	2
9	1

- Mean $= \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{60}{10}$
- Mean $= 6$

ex The following data about the flow of liquid was observed in a continuous chemical process plant

flow rate lit/sec	7.5 to 7.7	7.7 to 7.9	7.9 to 8.1	8.1 to 8.3	8.3 to 8.5	8.5 to 8.7
frequency	1	5	35	17	12	10

Mean flow rate of the liquid is..

a) 8.00 lit/sec
b) 8.16 lit/sec
c) 8.06 lit/sec
d) 8.26 lit/sec

x_i	7.6	7.8	8.0	8.2	8.4	8.6
f_i	1	5	35	17	12	10

$\therefore$ Mean $= \frac{\Sigma x_i f_i}{\Sigma f_i} = 8.16$ lit/sec

2) Median

Ex 1 {2, 1, 3} ⇝ odd set

S1 : (Ascend / Desc)

1, 2, 3

S2 : 2 → Median

★ • odd data set ⇝ 1) Asc/ Desc

2) $\left(\frac{N+1}{2}\right)^{th}$ position = Median

Ex 2 {1, 4, 2, 3} ⇝ Even set

S1 : 1, 2 ⋮ 3, 4

S2 : $\frac{2+3}{2}$ = 2.5

★ • Even data set ⇝ 1) Asc/ Desc

2) $\dfrac{\left(\frac{N}{2}\right)^{th} + \left(\frac{N}{2}+1\right)^{th}}{2}$ = Median

ex The spot speeds (expressed in km/hr) observed at a road section are 66, 62, 45, 79, 32, 51, 56, 60, 53, 49. The median speed (expressed in km/hr) is ___

32 45 49 51 $\boxed{53}$ ⋮ $\boxed{56}$ 60 62 66 79 (Even)

$= \frac{53+56}{2}$

$= 54.5$

ex The marks obtained by a set of student are: 38, 84, 45, 70, 75, 60, 48

The mean, and median marks respectively are

a) 45 and 75

b) 55 and 48

c) 60 and 60

d) 60 and 70

38 45 48 60 70 75 84

∴ Median = $\left(\frac{7+1}{2}\right)^{th}$ position

= 4^{th} position

= 60

∴ Mean = $\frac{\Sigma x_i}{n}$ = 60

ex : Median (Grouped data)

Marks	No. of Stud	cf
20	6	6
25	20	26
28	24	50
29	28	78
33	15	93
38	4	97
42	2	99
43	1	100

$\Sigma f_i = 100$ ⇝ Even

∴ Median = $\dfrac{\left(\frac{N}{2}\right)^{th} + \left(\frac{N}{2}+1\right)^{th}}{2} = \frac{50^{th} + 51^{th}}{2} = \frac{28+29}{2}$

$= 28.5$

Q)

x_i	f_i	cf
65 - 85	4	4
85 - 105	5	9
105 - 125	13	22 - pcf
lm 125 - 145	20 fm	42
145 - 165	14	56
165 - 185	8	64
185 - 205	4	68

$\Sigma f_i = 68$

$\therefore$ Median =

$$l_m + \frac{(N/2 - pcf)}{f_m} \cdot h$$

$$= 125 + \frac{(34 - 22)}{20} \cdot 20$$

$$= 137$$

3) **Mode** { highest freq ⇒ x_i }

Ex1 { 1, 2, 2, 3, 3, 3 }

mode = 3 [unimode]

Ex2 { 1, 2, 2, 3, 3 }

mode = 2 & 3 [bi model]

Ex3 { 1, 2, 2, 3, 3, 4, 4 }

mode = 2 & 3 & 4 [Trimodal]

Ex4 { 1, 2, 3, 4, 5 }

mode : (No mode)

ex Marks obtained by 100 students in an examination are given in table.

S.No	Marks obt (x_i)	No of stud (f_i)
1	25	20
2	30	20
3	35	40
4	40	20

What would be the mean, median and mode of the marks obtained by the students ?

a) Mean 33 ; Median 35 ; Mode 40
b) Mean 35 ; Median 32.5 ; Mode 40
c) Mean 33 ; Median 35 ; Mode 35 ✓
d) Mean 35 ; Median 32.5 ; Mode 35

1) Mean = $\frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{3300}{100} = 33$

2) Median = $\frac{35 + 35}{2} = 35$

3) Mode = 35

Q)

x_i	f_i
0 - 20	10
20 - 40	35
40 - 60	5 · f_0
$\boxed{60 - 80}$	$\boxed{61}$ · f_1
80 - 100	38 · f_2
100 - 120	29

$\therefore$ Mode

$$= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \cdot h$$

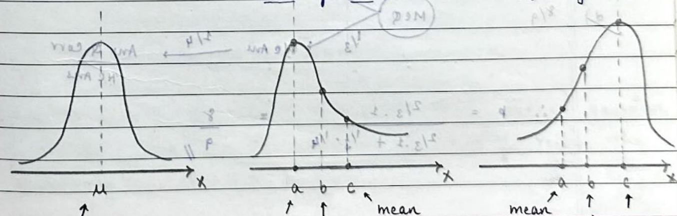
$$= 60 + \left[\frac{61 - 52}{122 - 52 - 38}\right] \cdot 20$$

$$= 65.625$$

★. **Skewness** :-

Positively Skewed

Negatively Skewed



mean = med = mode

mode ≤ med ≤ mean

mode ≥ med ≥ mean

$$\therefore \boxed{1\,\text{mode} + 2\,\text{mean} = 3\,\text{median}}$$

$$\therefore \sigma = \sqrt{\frac{\Sigma x_i^2 f_i}{\Sigma f_i} - \left(\frac{\Sigma x_i f_i}{\Sigma f_i}\right)^2}$$

$$\therefore \boxed{\text{coefficient of variation} = \frac{\sigma}{\mu}}$$

Q) $\sigma = 8.8$ kmp/h $\qquad \mu = 33$ kmph $\qquad$ C.V = ?

$$C.V = \frac{\sigma}{\mu} = 0.266$$

ex The probability that a student knows the correct answer to a multiple choice question is 2/3. If the student does not know the answer, then the student guesses the answer. The probability of the guessed answer being correct is 1/4. Given that the student has answered the question correctly, the conditional probability that the student knows the correct answer is..

a) 2/3
b) 3/4
c) 5/6
d) 8/9

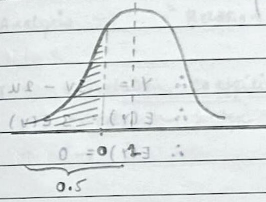
MCQ $\xrightarrow{2/3}$ K.C.Ans $\xrightarrow{1}$ Ans Q Corr / K C Ans

MCQ $\xrightarrow{1/3}$ G C Ans $\xrightarrow{1/4}$ Ans Q Corr / G C Ans

$$\therefore p = \frac{2/3 \cdot 1}{2/3 \cdot 1 + 1/3 \cdot 1/4} = \frac{8}{9}$$

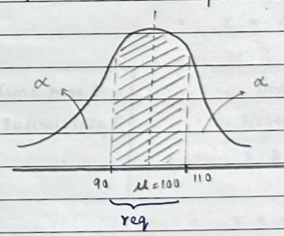
ex let X be a normal random variable with mean 1 and variance 4. The probability $P\{X<0\}$ is

a) 0.5
b) greater than zero and less than 0.5
c) greater than 0.5 and less than 1.0
d) 1.0



ex for a random variable X ($-\infty < x < \infty$) following normal distribution, the mean is $\mu = 100$. If the probability is $P = \alpha$ for $x \geq 110$, then the probability of X lying betⁿ 90 and 110, i.e. $P(90 \leq x \leq 110)$ will be equal to..

a) $1-2\alpha$
b) $1-\alpha$
c) $1-\alpha/2$
d) 2α



$$\therefore \alpha + \text{Req} + \alpha = 1$$

$$\therefore \text{Req} = 1-2\alpha$$

ex let U and V be two independent zero mean gaussian random variable of variance $\frac{1}{4}$ and $\frac{1}{9}$ resp. The probability $P(3V \geq 2U)$ is..

a) $4/9$ b) $1/2$ c) $2/3$ d) $5/9$

$\therefore P(3V \geq 2U)$

$= P(3V - 2U \geq 0)$

$= P(Y \geq 0)$

$= P\left(\frac{Y-\mu}{\sigma} \geq \frac{0-\mu}{\sigma}\right)$

$= P\left(Z \geq \frac{0-0}{\sigma}\right)$

$= P(Z \geq 0)$

$= 0.5$

$\therefore Y = 3V - 2U$

$\therefore E(Y) = 3E(V) - 2E(U)$

$\therefore E(Y) = 0$

Correlation & Regression

★ **Correlation :**

1)

x	1	2	3	4
Y	10	20	30	40

- data is given

2) Analysis → " Relationship " → Correlation
- Nature
- Strength

3) r = correlation coefficient

= correlation index

= $[-1, 1]$

★ **Types of correlation :**

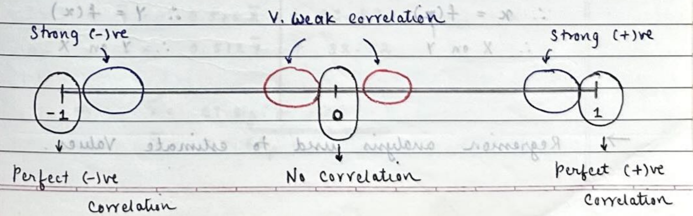
Study hr ↑	Sale ↓
mrks ↑	profit ↓

- direct correlation
- positive correlation
- x & y are in same direction
- $r = 0$ to 1

Exercise ↑	Time pass ↓
body fat ↓	Success rate ↑

- Inverse correlation
- Negative correlation
- x & y are in opposite direction
- $r = -1$ to 0

$r = -1$ to 1



★ Karl pearson's product moment method :

$$\therefore \boxed{r = \frac{Cov(x,y)}{\sigma_x \sigma_y}}$$

$$\therefore \boxed{Cov(x,y) = \frac{\sum (x-\bar{x})(y-\bar{y})}{n}}$$

$$\therefore \boxed{r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}}$$

ex

$Cov(x,y) = 11.12$

$\sigma_x = 2.33$

$\sigma_y = 6.69$

$$\therefore r = \frac{11.12}{(2.33)(6.69)} = 6.713$$

★ Regression :

Type 1

x	y
1	10
2	11
3	13
?	17

$\therefore x = f(y)$

$\therefore$ X on Y

Type 2

x	y
10	0
10.5	2
11	3
12	?

$\therefore Y = f(x)$

$\therefore$ Y on X

→ Regression analysis used to estimate Values.

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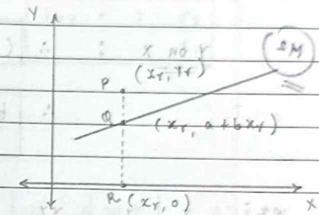
★ Y on X



$\therefore \boxed{Y = a + bX}$

$\therefore \boxed{\sum y = na + b\sum x}$

$\therefore \boxed{\sum xy = a\sum x + b\sum x^2}$

ex

3 values in x & y

$y = a + bx$ using least square method

$\sum x = 6$ $\quad \sum x^2 = 14$ $\quad \sum y = 21$ $\quad \sum xy = 46$

Find a & b.

1) $\sum y = na + b\sum x \Rightarrow 21 = 3a + b(6)$

$\sum xy = a\sum x + b\sum x^2 \Rightarrow 46 = a(6) + b(14)$

$\boxed{a = 3, b = 2}$

→ now, $\frac{\sum y}{n} = \frac{na}{n} + \frac{b\sum x}{n}$

$\therefore \boxed{\bar{Y} = a + b\bar{x}}$

ex :

$Y = 0.516x + 33.33$, mean $\bar{x}$ & $\bar{y}$.

$X = 0.512y + 32.52$

$\bar{Y} = 0.516\bar{X} + 33.33$

$\bar{X} = 0.512\bar{Y} + 32.52$

$\Rightarrow \bar{x} = 67.6$

$\bar{Y} = 68.61$

M2 Y on X : $\therefore \boxed{(Y-\bar{Y}) = b_{yx}(X-\bar{X})}$

$\therefore \boxed{b_{yx} = \dfrac{n\Sigma xy - \Sigma x \Sigma y}{n(\Sigma x^2) - (\Sigma x)^2}}$

ex:

X	Y	XY	X^2	Y^2
1	-3	-3	1	9
2	-1	-2	4	1
3	0	0	9	0
4	1	4	16	1
5	2	10	25	4
$\Sigma x = 15$	$\Sigma y = -1$	$\Sigma xy = 9$	$\Sigma x^2 = 55$	$\Sigma y^2 = 15$

Y on X = ?

m1 : $Y = a + bX$

$\Sigma y = na + b\Sigma x \rightarrow -1 = 5a + b(15)$

$\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow 9 = a(15) + b(55)$

$\boxed{a = -3.8}$

$\boxed{b = 1.2}$

Y on X :- $Y = -3.8 + (1.2)X$

m2 : $(Y-\bar{y}) = b_{yx}(x-\bar{x})$

$\bar{Y} = \dfrac{\Sigma Y}{n} = \dfrac{-1}{5}$

$\bar{X} = \dfrac{\Sigma x}{n} = 3$ $\quad b_{yx} = \dfrac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = 1.2$

$\therefore Y = 1.2x - 3.8$

★ $\boxed{b_{xy} = r \cdot \dfrac{\sigma_x}{\sigma_y}}$ $\quad \boxed{b_{yx} = r \cdot \dfrac{\sigma_y}{\sigma_x}}$

$\boxed{r^2 = b_{xy} \cdot b_{yx}}$

ex

$x = 19.13 - 0.87y \rightarrow$ X on Y, find r.

$y = 11.64 - 0.5x \rightarrow$ Y on X

$r^2 = b_{xy} \cdot b_{yx}$

$= (-0.87) \cdot (-0.5)$

$= \pm 0.66$

$r = +0.66$ ✗ $\quad r = -0.66$ ✓

$b_{xy} = r \dfrac{\sigma_x}{\sigma_y}$

b_{xy}: (-)ve

r: ? (-)ve

$\dfrac{\sigma_x}{\sigma_y}$: always (+)ve