DAIICT **SC531 PROBABILITY & RANDOM VARIABLES**

SECOND IN-SEMESTER EXAMINATION

ID NUMBER: _____ NAME: _____

NOTES:

1. The question paper has FIVE questions of 4 marks each. **Time allowed: 60 minutes**.
2. Write your answers in the boxes provided below.
3. ATTACH your rough-work sheet(s) with the filled-in question paper returned.

CALCULATION OF M

Let d1, d2 be the RIGHTMOST TWO DIGITS of your ID number, in that order.
For example, if ID number = 2022113445, then d1 = 4 and d2 = 5.
First calculate integer X = d1 + 4*d2.
Then calculate
M = remainder(X/3) + 2 = modulo(X,3) + 2.
Enter the value in the box on the right.

M =

ANSWERS	Part (a)	Part (b)
A-1		
A-2		
A-3		
A-4		
A-5		

IN THE FOLLOWING, USE THE VALUE OF M WHICH YOU HAVE CALCULATED.

Q-1. Consider the following observed values of random variables X and Y, consisting of FIVE values each. For this pair of RVs, (a) Find the covariance Cov(X,Y) and (b) Find the regression coefficient r_{XY}.

X values	-2	-1	0	1	2
Y values	-3M	-2M	1	2M	3M

2. (a) For random variable X, it is known that the mean is 50 and the standard deviation is M. Using only Tchebycheff inequality, find an upper bound on the probability that the value of X will be outside the interval [48 ... 52].

(b) Further, it is given that the RV X of part (a) is normally distributed. Find the value ΔX such that Prob($|X-50| \leq \Delta X$) = 0.98. Use the standard normal values provided.

3. Let X represent hourly traffic at a railway station, which is assumed to be Poisson distributed with parameter $\lambda = M^2$. Samples of 100 such traffic readings are taken, and their average X_{mean} calculated.
(a) Using CLT, find the standard deviation of X_{mean}.
(b) Find the range [X_{min} ... X_{max}], centred on $\lambda = M^2$, such that the value of X is within that range with a probability of 90%. Use the standard normal values provided.

4. (a) Random variable X has mean value 100 and standard deviation 5. For a left-tailed test of hypothesis, the level of significance (LOS) α is taken as M (in percent). Find the critical region of X, using the standard normal values provided.

(b) The average fatality rate for a specific category of hospitalized patients is 20%. In a given hospital, out of 100 patients of this category, 16 have died. At LOS = M (in percent), can this hospital be considered significantly better than average, for this category of patients? Use the standard normal values provided.

5. (a) Discrete random variable X has N = 5*M equi-probable outcomes. A partition U is defined on the sample space S such that each event in U corresponds to exactly one outcome. Find the entropy H(U).

(b) The partition U of part (a) is modified by merging (that is, combining) ANY THREE of the events of U. Let the resulting partition be V. Find the entropy H(V).

Values of the standard normal variable, for Q-2, Q-3 and Q-4:

z:	1.645	1.751	1.881	2.054	2.326
CDF F(z):	0.95	0.96	0.97	0.98	0.99