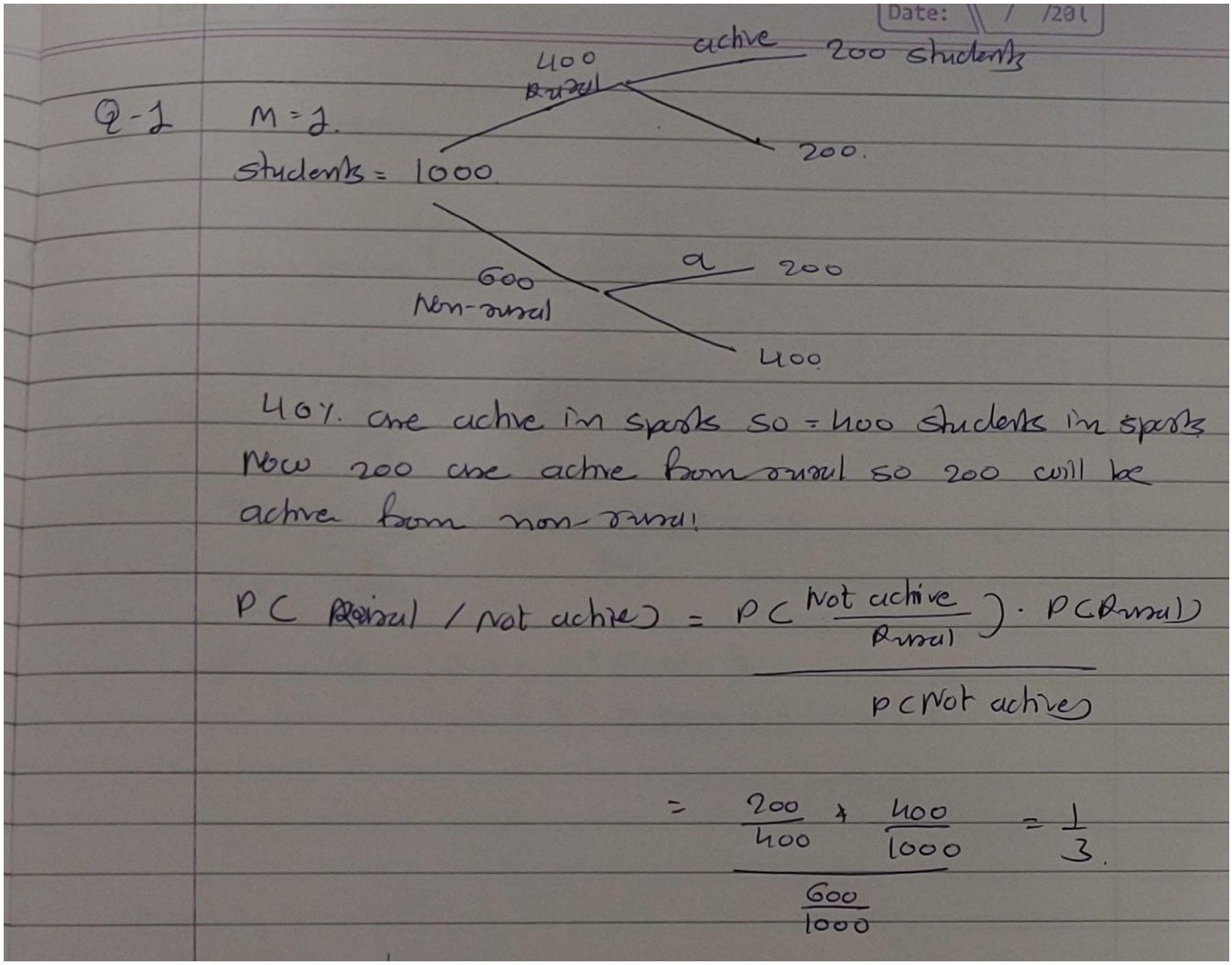
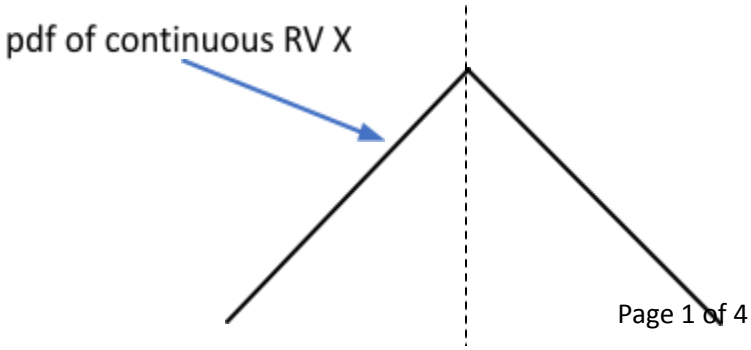
SC531 PROBABILITY & RANDOM VARIABLES

RETEST (12 marks, 40 minutes)

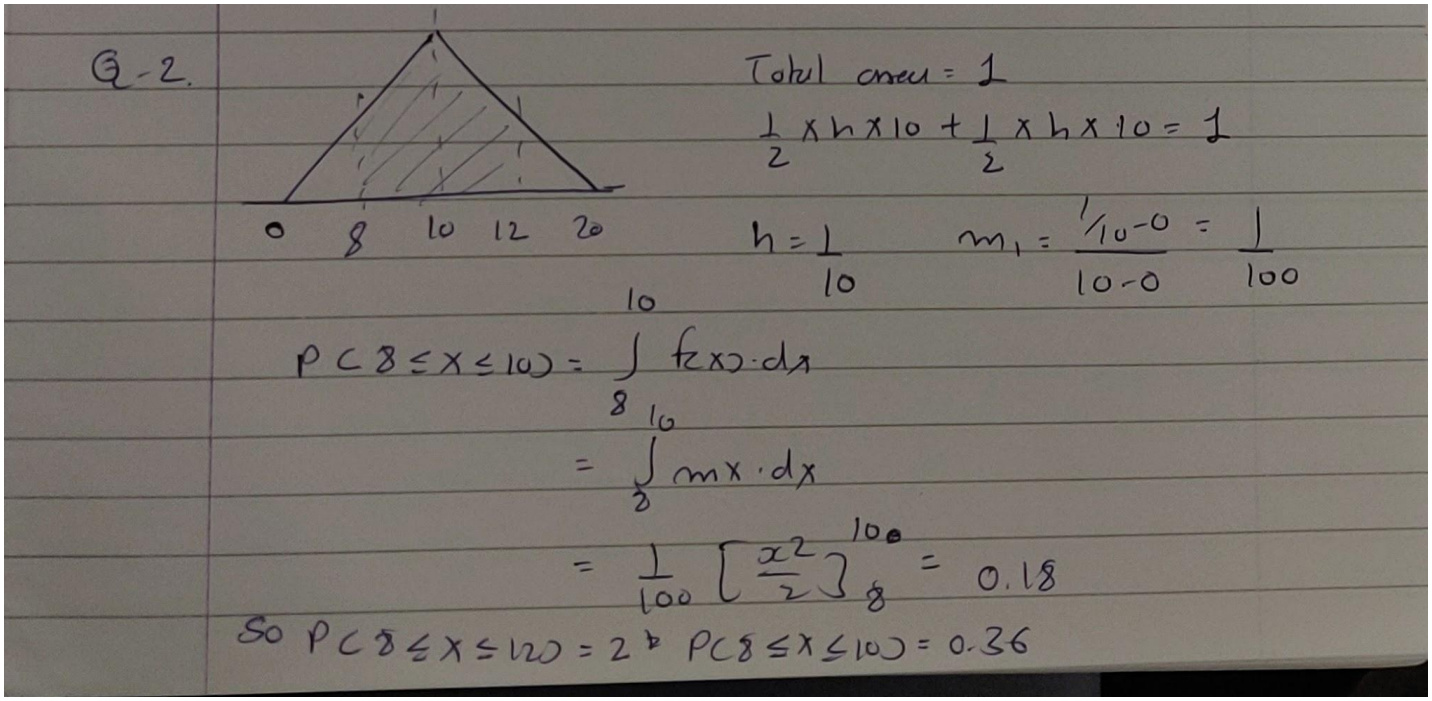
Q-1. A university has 1000*M students. 40% of the students are from rural background, and 40% of the students are active in sports. Half of the students from rural background are also active in sports. If a randomly selected student is NOT active in sports, what is the probability that he or she is from a rural background?





X = 0 X = 10 X = 20 X axis

Q-2. Given the probability density function shown above, find Prob(10-M <= X <= 10+M).



Q-3. The average working life of a certain power supply is claimed to be 10000 hours, with standard deviation of 400 hours. We test a sample of size 25 of the power supplies, and calculate the sample mean. Find the probability that the sample mean is in the range 10000 $\pm$ 40*M hours. The required table is given below.

Q-3

$\mu = 10000$

$\sigma = 400$

$X_1 = 10000 \pm 40 * M = 10080, 9920 \quad (M = 2)$

$$Z_1 = \frac{X_1 - M}{\frac{\sigma}{\sqrt{n}}} = \frac{10000 - 10080}{80} = 1$$

$$Z_2 = \frac{X_2 - M}{\frac{\sigma}{\sqrt{n}}} = \frac{9920 - 10080}{80} = -1$$

$P(-1 \leq Z \leq 1) = 2 * F(1) - 1$

$= 2 * 0.8413 - 1 = 0.68$

Q-4. Recall the Markov process defined as "random walk with reflecting barriers". The four states of the process are 1, 2, 3 and 4. The transition probability matrix is as given below, with $\alpha = M/10$. The initial probability distribution over states is (0, 1/2, 1/2, 0). What is the probability that the process is in state 2 after two time steps?

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ \alpha & 0 & \beta & 0 \\ 0 & \alpha & 0 & \beta \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Table of standard normal cumulative distribution

Index	z	F(z)	Index	z	F(z)
1	0.00	0.5000	11	1.00	0.8413
2	0.10	0.5398	12	1.10	0.8643
3	0.20	0.5793	13	1.20	0.8849
4	0.30	0.6179	14	1.30	0.9032
5	0.40	0.6554	15	1.40	0.9192
6	0.50	0.6915	16	1.50	0.9332
7	0.60	0.7257	17	1.60	0.9452
8	0.70	0.7580	18	1.70	0.9554
9	0.80	0.7881	19	1.80	0.9641
10	0.90	0.8159	20	1.90	0.9713
			21	2.00	0.9772