Constructing a dual solution for set cover: Primal Dual method

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→ We donot want to actually solve a linear program

Insted want to quickly construct a feasible dual sol^{n} without actually solving dual LP

Algorithm :-

$y \leftarrow 0$

$I \leftarrow 0$

while there exist $e_i \notin \bigcup_{j \in I} S_j$ do

Increase the dual variable y_i until there is some l with $e_i \in S_l$ such that

$$\sum_{j : e_j \in S_l} y_j = w_l$$

$I \leftarrow I \cup \{l\}$

Intuition :-

we start with dual feasible sol^{n} $y = 0$

Actually till some e_i is not covered we increase y_i by

$$\epsilon = \min_{j : e_i \in S_j} \left(w_j - \sum_{k : k \in S_j} y_k \right)$$

Now for S_l inequality becomes tight

$$\sum_{k : e_k \in S_l} y_k = w_l$$

Thm : Primal-Dual Algo gives f-approximation

Proof :- We know for $j \in I$

$$w_j = \sum_{i : e_i \in S_j} y_i$$

$$\therefore \sum_{j \in I} w_j = \sum_{j \in I} \sum_{i : e_i \in S_j} y_i$$

$$= \sum_{i=1}^{n} y_i \cdot \left| \{ j \in I : e_i \in S_j \} \right|$$

$$\leq \sum_{i=1}^{n} f_i y_i$$

$$\leq f \sum_{i=1}^{n} y_i$$

$$\leq f \cdot OPT$$

□