

ID NUMBER: _____	VALUE OF M:
NAME: _____	

NOTES:

1. The question paper has EIGHT (8) questions of FIVE (5) marks each.
2. **Time allowed: 2 HOURS.**
3. Write your answers in the boxes provided below.
4. ATTACH your rough-work sheet(s) with the filled-in question paper returned.

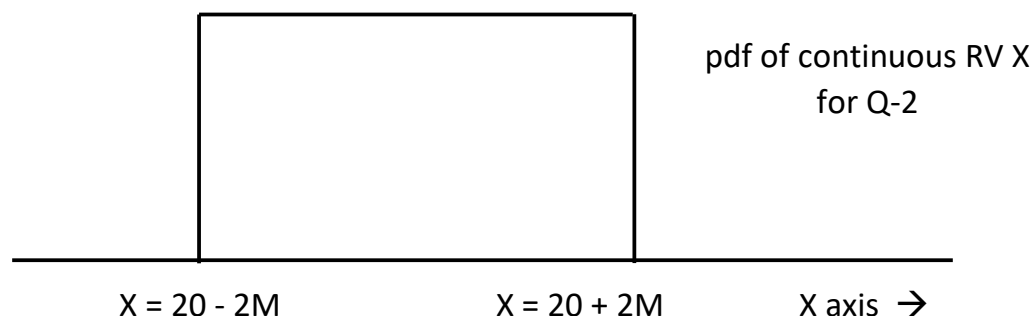
CALCULATION OF M

Let d1, d2 be the RIGHTMOST TWO DIGITS of your ID number, in that order. Example: If ID = 202211345, then d1 = 4 and d2 = 5.

First calculate $X = d1 + 2*d2$. Then calculate $M = \text{remainder}(X/3) + 2$
 $= \text{modulo}(X,3) + 2$. ENTER THE VALUE IN THE BOX ABOVE.

<u>ANSWERS</u>	
A-1	
A-2	
A-3	
A-4	
A-5	
A-6	
A-7	
A-8	

Q-1 A company purchases voltmeters from 3 suppliers A, B and C, in the ratio of 40%, 40% and 20% respectively. It has been found that the percentage of defective voltmeters from the three suppliers are $(5-M)\%$, $(5+M)\%$ and 5% , respectively. A randomly selected voltmeter from the company's stock is found to be defective. What is the probability that the selected voltmeter came from supplier B?



Q-2. Consider the probability density function shown in the diagram above. Find the variance of X , that is $\text{Var}(X)$.

Q-3 Door panels are produced on a machine in large numbers. The weight of a door panel, measured in KG, is a continuous random variable with probability distribution $N(20,1)$. What is the probability that the weight of a randomly selected door panel is within $\pm 0.6 \cdot M$ KG of the mean weight? Use the standard normal table given on page 3.

Q-4 A pair of fair dice is given. These dice are modified to create a pair of UNFAIR dice, using the following procedure:

- On each of the two fair dice, the face with SIX (6) dots on it is modified to have M dots. All other faces are left unchanged.

The pair of unfair dice is then thrown. What is the probability that this pair of unfair dice will throw up the number $2 \cdot M$?

5. Random variable X has distribution $N(250,10)$. For a two-tailed test of hypothesis, the level of significance (LOS) α is specified as M percent. Find the critical region of X , using the values provided on page 3.

6. The sample space S of discrete random variable X has $N = 5 \cdot M$ elements, labeled $X_1, X_2, X_3, \dots, X_N$. These N outcomes are equi-probable. A partition U is defined on the sample space S as follows: $U = \{ X_1, X_2 \}, \{ X_3, X_4 \}, \{ X_5 \}, \{ X_6 \}, \dots, \{ X_N \}$. Find the entropy $H(U)$.

7. Consider the Markov chain known as “random walk with reflecting barriers”. The transition probability matrix (TPM) below defines such a process with FIVE (5) states. The states are numbered 1, 2, 3, 4, 5. At time t_0 (initial time), all FIVE states are equiprobable. Find the probability that the process is NOT in state 3 at time t_2 .

0	1	0	0	0
0.5	0	0.5	0	0
0	$1-M/10$	0	$M/10$	0
0	0	0.5	0	0.5
0	0	0	1	0

8. The arrival rate of packets at a router is $3 \cdot M$ per millisecond, and the service rate is $3 \cdot (M+2)$ packets per millisecond. The process is a single server Poisson queue. Calculate (a) the average number of packets, and (b) the expected waiting time of packets in the router.

STD NORMAL CDF					
x	cdf		x	cdf	
0.0	0.5000		1.1	0.8643	
0.1	0.5398		1.2	0.8849	
0.2	0.5793		1.3	0.9032	
0.3	0.6179		1.4	0.9192	
0.4	0.6554		1.5	0.9332	
0.5	0.6915		1.6	0.9452	
0.6	0.7257		1.7	0.9554	
0.7	0.7580		1.8	0.9641	
0.8	0.7881		1.9	0.9713	
0.9	0.8159		2.0	0.9772	
1.0	0.8413				

Selected values of the standard normal variable, for Q-5:

x:	1.645	1.751	1.881	2.054	2.326
cdf $f(x)$:	0.95	0.96	0.97	0.98	0.99