IE404
Digital Image Processing

Dr. Manish Khare

Image Enhancement in Frequency Domain

Background

- In spatial domain filtering, each output pixel is a function of an input pixel and its neighbors
- Any periodic function can be represented as the sum of sines and/or cosines of different frequencies, multiplied by a different coefficient
  - The sum is called Fourier series
- Functions that are not periodic but whose area under the curve is finite can be represented as the integral of sines and cosines multiplied by a weight function
  - Known as Fourier transform
- Function represented by Fourier transform can be completely reconstructed by an inverse transform with no loss of information
  - Allows working in Fourier domain and return to the original domain without any loss of information

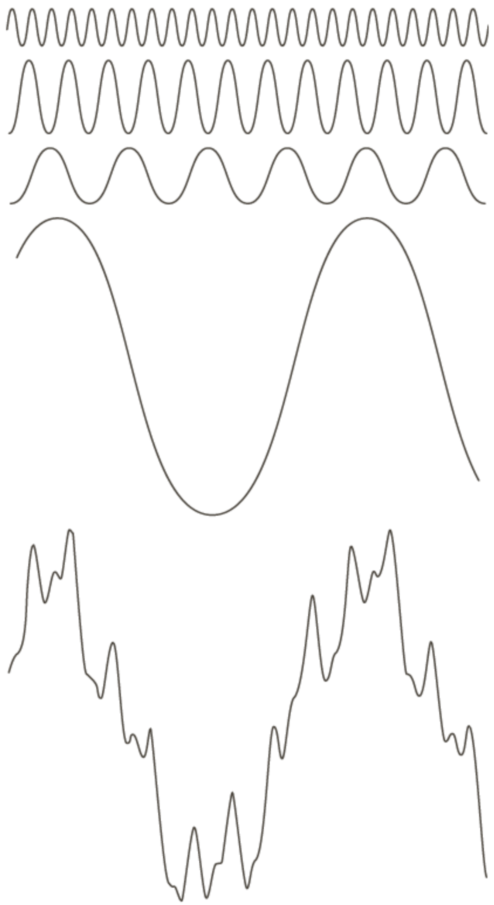


FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

- The frequency domain refers to the plane of the two dimensional discrete Fourier transform of an image.
- The purpose of the Fourier transform is to represent a signal as a linear combination of sinusoidal signals of various frequencies.

Introduction to the Fourier Transform and the Frequency Domain

- The one-dimensional Fourier transform and its inverse
  - Fourier transform (continuous case)

$$F(u)=\int_{-\infty}^{\infty} f(x)e^{-j2\pi ux}dx \quad \text{where } j=\sqrt{-1}$$

  - Inverse Fourier transform:

$$\boxed{e^{j\theta}=\cos\theta+j\sin\theta}$$

$$f(x)=\int_{-\infty}^{\infty} F(u)e^{j2\pi ux}du$$

- The two-dimensional Fourier transform and its inverse
  - Fourier transform (continuous case)

$$F(u,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}dxdy$$

  - Inverse Fourier transform:

$$f(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{j2\pi(ux+vy)}dudv$$

Introduction to the Fourier Transform and the Frequency Domain

- The one-dimensional Fourier transform and its inverse
  - Fourier transform (discrete case) DCT

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M} \quad \text{for } u = 0,1,2,\ldots,M-1$$

  - Inverse Fourier transform:

$$f(x) = \sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M} \quad \text{for } x = 0,1,2,\ldots,M-1$$

- Since $e^{j\theta} = \cos\theta + j\sin\theta$ and the fact $\cos(-\theta) = \cos\theta$ then discrete Fourier transform can be redefined

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)[\cos 2\pi ux/M - j\sin 2\pi ux/M]$$

$$\text{for } u = 0,1,2,\ldots,M-1$$

- Frequency (time) domain: the domain (values of u) over which the values of $F(u)$ range; because u determines the frequency of the components of the transform.
- Frequency (time) component: each of the M terms of $F(u)$.

Introduction to the Fourier Transform and the Frequency Domain

- $F(u)$ can be expressed in polar coordinates:

$$F(u) = |F(u)|e^{j\phi(u)}$$

$$\text{where } |F(u)| = \left[R^2(u) + I^2(u)\right]^{\frac{1}{2}} \quad \text{(magnitude or spectrum)}$$

$$\phi(u) = \tan^{-1}\left[\frac{I(u)}{R(u)}\right] \quad \text{(phase angle or phase spectrum)}$$

  - $R(u)$: the real part of $F(u)$
  - $I(u)$: the imaginary part of $F(u)$

- Power Spectrum

$$P(u) = |F(u)|^2 = R^2(u) + I^2(u)$$

The One-Dimensional Fourier Transform Example

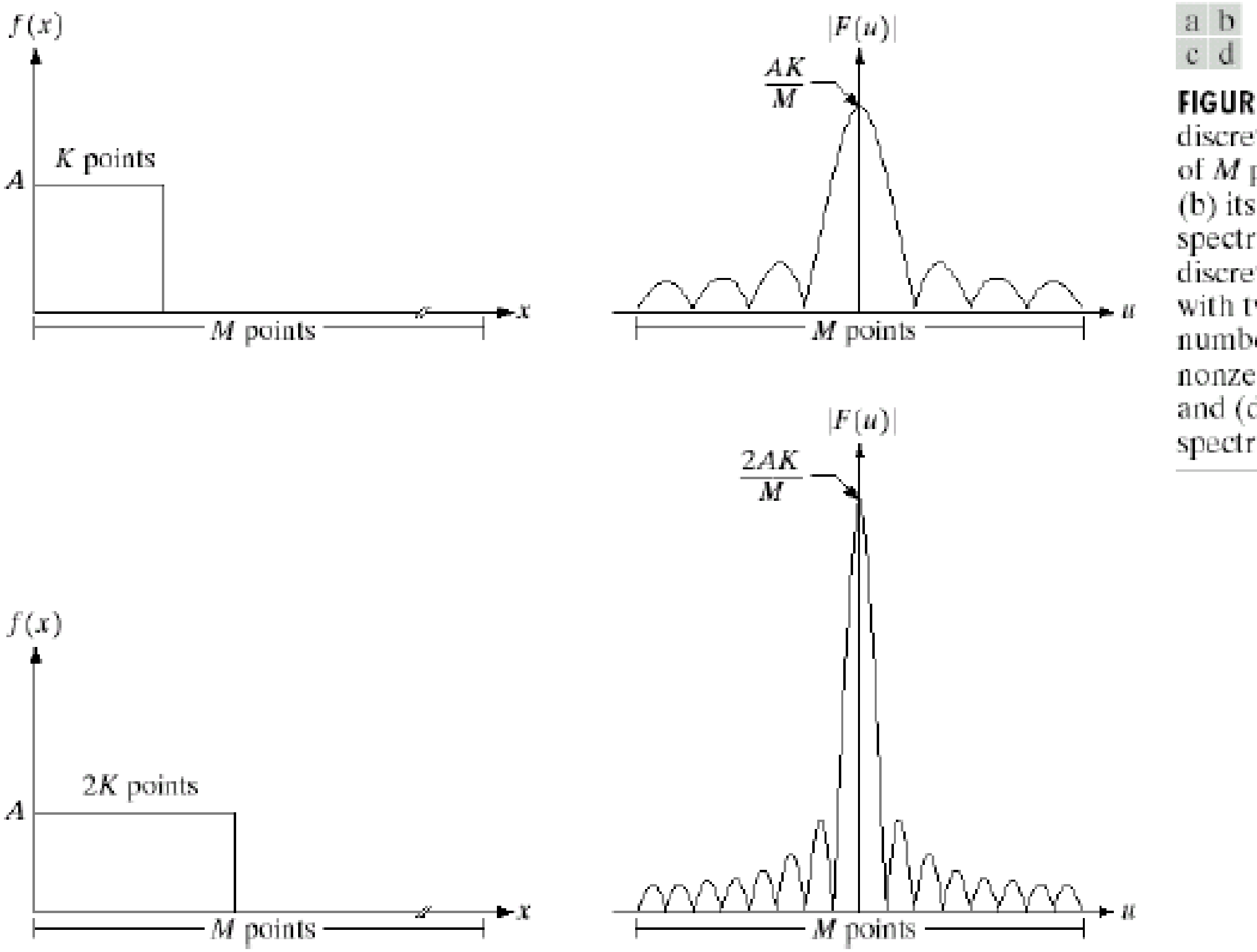


FIGURE 4.2 (a) A discrete function of M points, and (b) its Fourier spectrum. (c) A discrete function with twice the number of nonzero points, and (d) its Fourier spectrum.

Introduction to the Fourier Transform and the Frequency Domain

- The two-dimensional Fourier transform and its inverse
  - Fourier transform (discrete case) DTC

$$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

$$\text{for } u=0,1,2,\ldots,M-1,\ v=0,1,2,\ldots,N-1$$

  - Inverse Fourier transform:

$$f(x,y)=\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F(u,v)e^{j2\pi(ux/M+vy/N)}$$

$$\text{for } x=0,1,2,\ldots,M-1,\ y=0,1,2,\ldots,N-1$$

- u, v : the transform or frequency variables
- x, y : the spatial or image variables

- We define the Fourier spectrum, phase angle, and power spectrum as follows:

$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v)\right]^{\frac{1}{2}} \quad \text{(spectrum)}$$

$$\phi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right] \quad \text{(phase angle)}$$

$$P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v) \quad \text{(power spectrum)}$$

  – $R(u,v)$: the real part of $F(u,v)$
  – $I(u,v)$: the imaginary part of $F(u,v)$

- Some properties of Fourier transform

$$\Im\left[f(x,y)(-1)^{x+y}\right] = F\left(u-\frac{M}{2}, v-\frac{N}{2}\right) \quad \text{(shift)}$$

$$F(0,0) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y) \qquad \text{(average)}$$

$$F(u,v) = F^*(-u,-v) \qquad \text{(conujgate symmetric)}$$

$$|F(u,v)| = |F(-u,-v)| \qquad \text{(symmetric)}$$

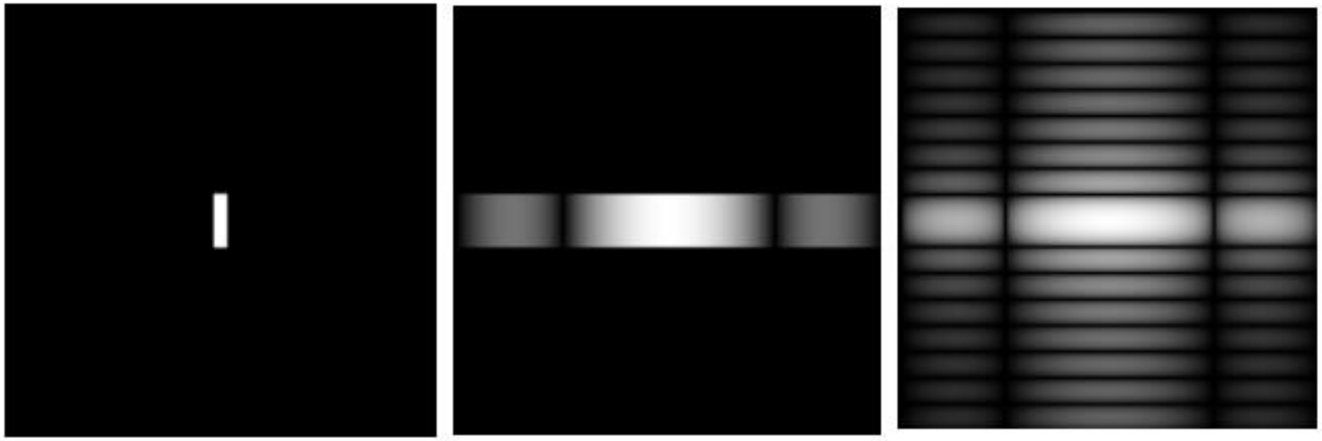
	Spatial Domain†		Frequency Domain†
1)	$f(x, y)$ real	$\Leftrightarrow$	$F^*(u, v) = F(-u, -v)$
2)	$f(x, y)$ imaginary	$\Leftrightarrow$	$F^*(-u, -v) = -F(u, v)$
3)	$f(x, y)$ real	$\Leftrightarrow$	$R(u, v)$ even; $I(u, v)$ odd
4)	$f(x, y)$ imaginary	$\Leftrightarrow$	$R(u, v)$ odd; $I(u, v)$ even
5)	$f(-x, -y)$ real	$\Leftrightarrow$	$F^*(u, v)$ complex
6)	$f(-x, -y)$ complex	$\Leftrightarrow$	$F(-u, -v)$ complex
7)	$f^*(x, y)$ complex	$\Leftrightarrow$	$F^*(-u - v)$ complex
8)	$f(x, y)$ real and even	$\Leftrightarrow$	$F(u, v)$ real and even
9)	$f(x, y)$ real and odd	$\Leftrightarrow$	$F(u, v)$ imaginary and odd
10)	$f(x, y)$ imaginary and even	$\Leftrightarrow$	$F(u, v)$ imaginary and even
11)	$f(x, y)$ imaginary and odd	$\Leftrightarrow$	$F(u, v)$ real and odd
12)	$f(x, y)$ complex and even	$\Leftrightarrow$	$F(u, v)$ complex and even
13)	$f(x, y)$ complex and odd	$\Leftrightarrow$	$F(u, v)$ complex and odd

†Recall that x, y, u, and v are *discrete* (integer) variables, with x and u in the range $[0, M - 1]$, and y, and v in the range $[0, N - 1]$. To say that a complex function is *even* means that its real *and* imaginary parts are even, and similarly for an odd complex function.

TABLE 4.1 Some symmetry properties of the 2-D DFT and its inverse. $R(u, v)$ and $I(u, v)$ are the real and imaginary parts of $F(u, v)$, respectively. The term *complex* indicates that a function has nonzero real and imaginary parts.

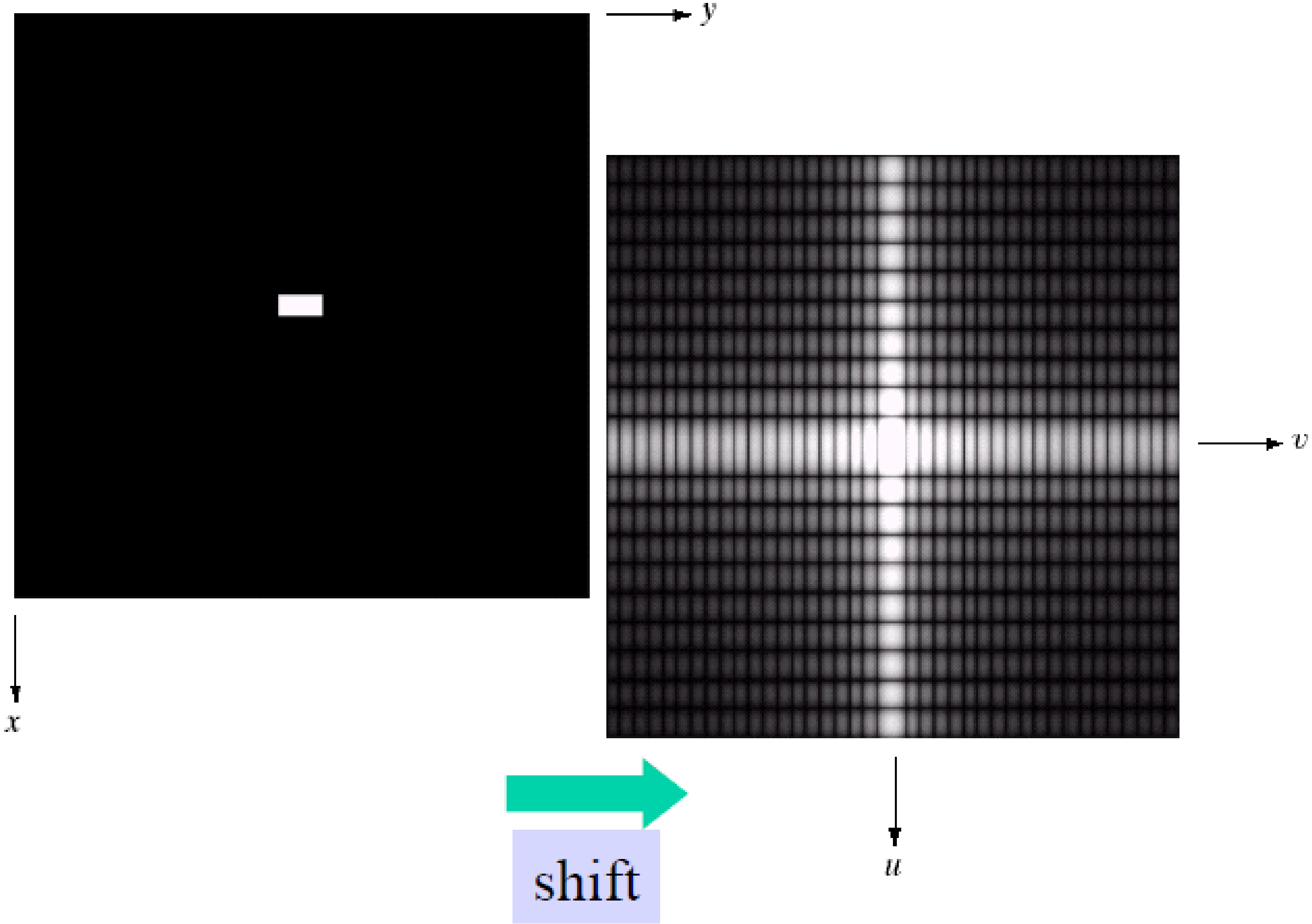
The 2-Dimensional DFT and Its Inverse

- The 2D DFT **F(u,v)** can be obtained by
- 1. taking the 1D DFT of every row of image **f(x,y). i.e. F(u,y)**
- 2. taking the 1D DFT of every column of **F(u,y), i.e. F(u,v)**



(a)f(x,y) (b)F(u,y) (c)F(u,v)

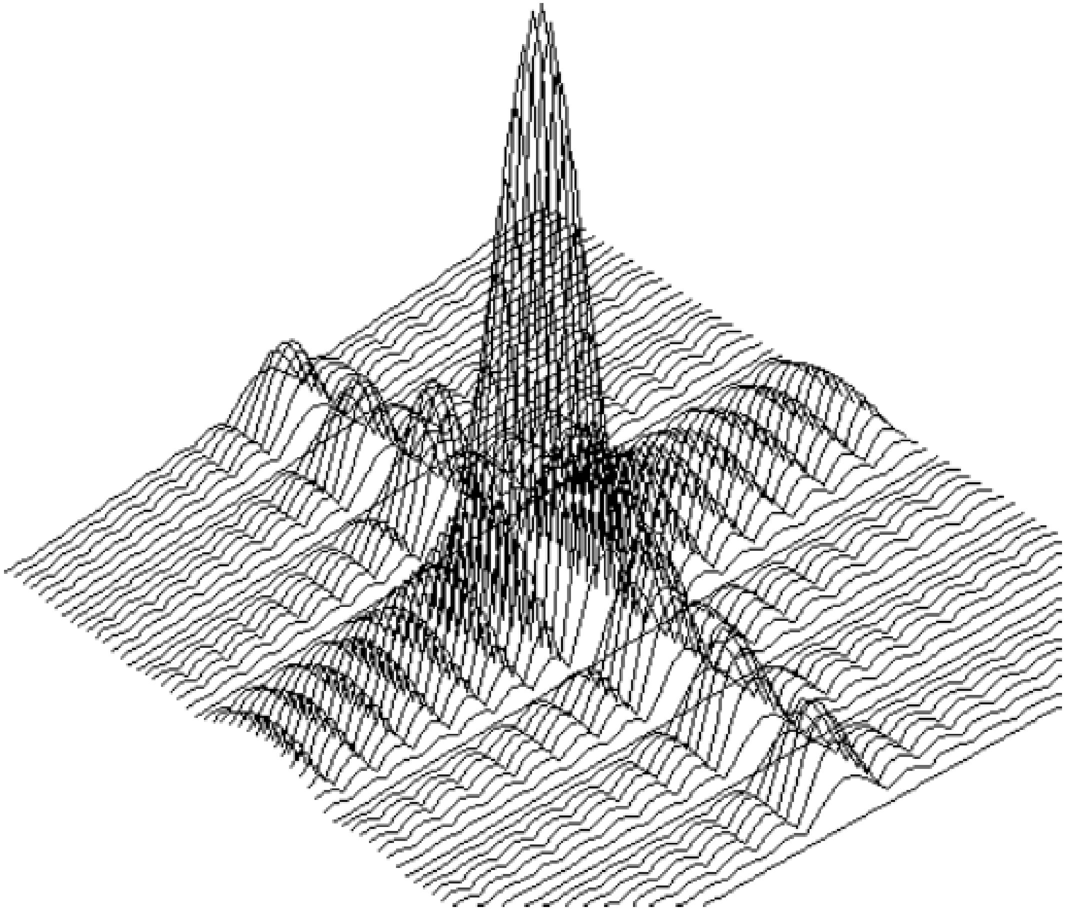
The 2-Dimensional DFT and Its Inverse



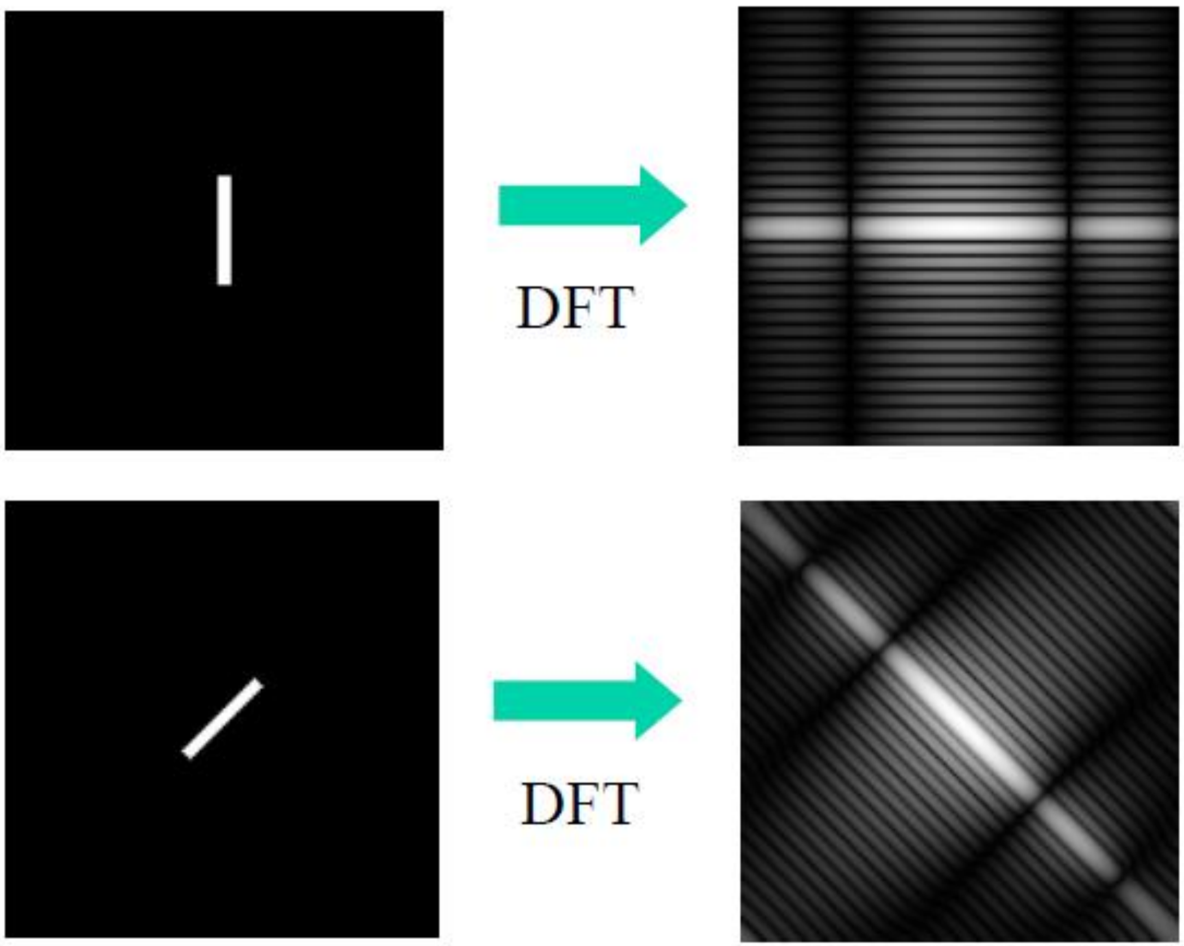
a b

FIGURE 4.3 (a) Image of a 20 × 40 white rectangle on a black background of size 512 × 512 pixels. (b) Centered Fourier spectrum shown after application of the log transformation given in Eq. (3.2-2). Compare with Fig. 4.2.

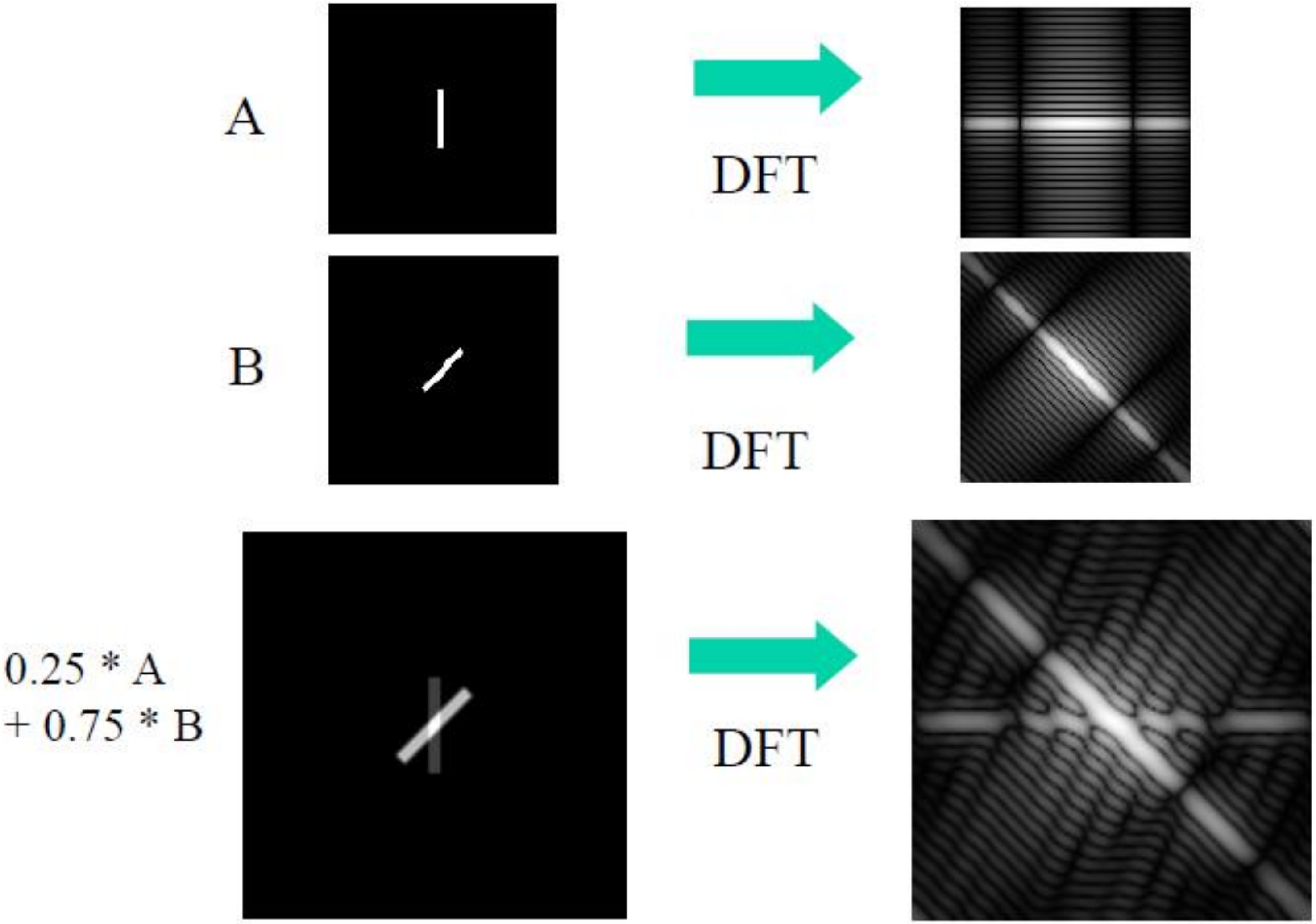
The 2-Dimensional DFT and Its Inverse



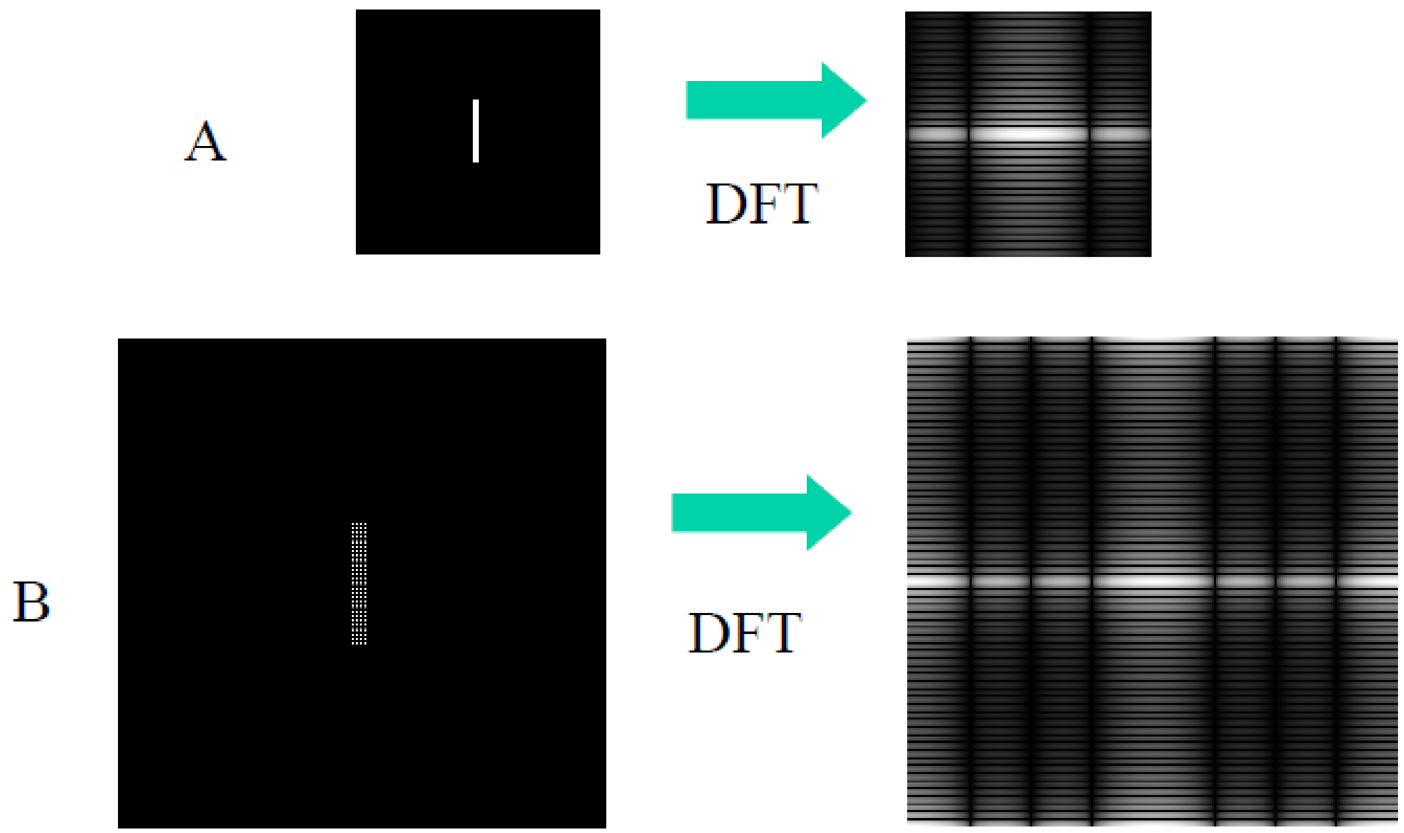
The Property of 2-Dimensional DFT - Rotation



The Property of 2-Dimensional DFT – Linear Combination



The Property of 2-Dimensional DFT – Expansion

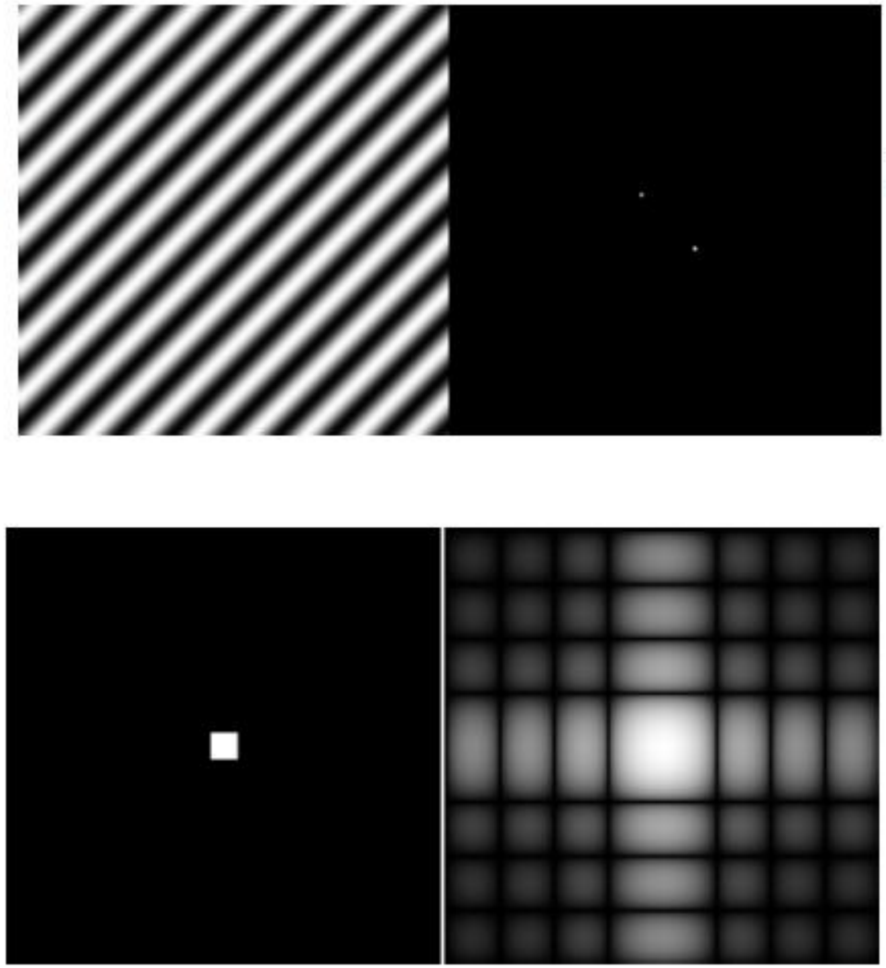


Expanding the original image by a factor of n (n=2), filling the empty new values with zeros, results in the same DFT.

2-Dimensional DFT with Different Functions

Sine wave

Its DFT

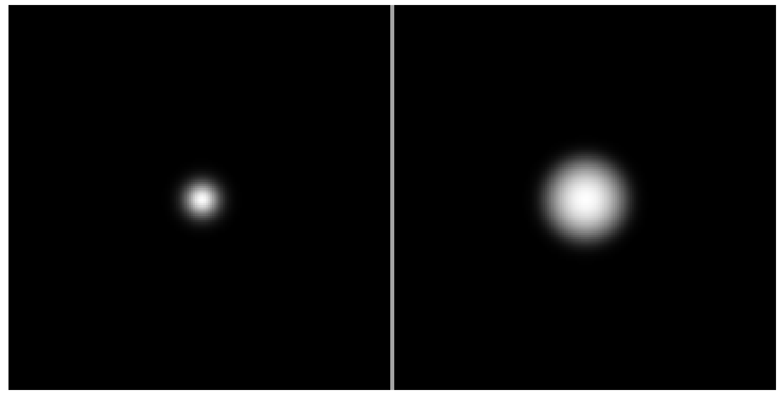


Rectangle

Its DFT

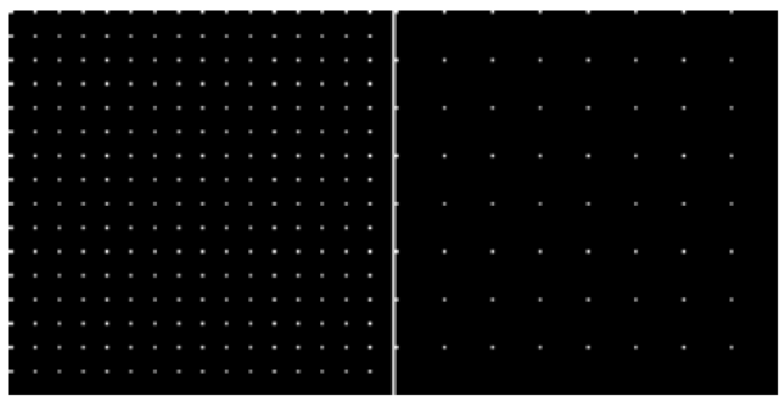
2-Dimensional DFT with Different Functions

2D Gaussian function

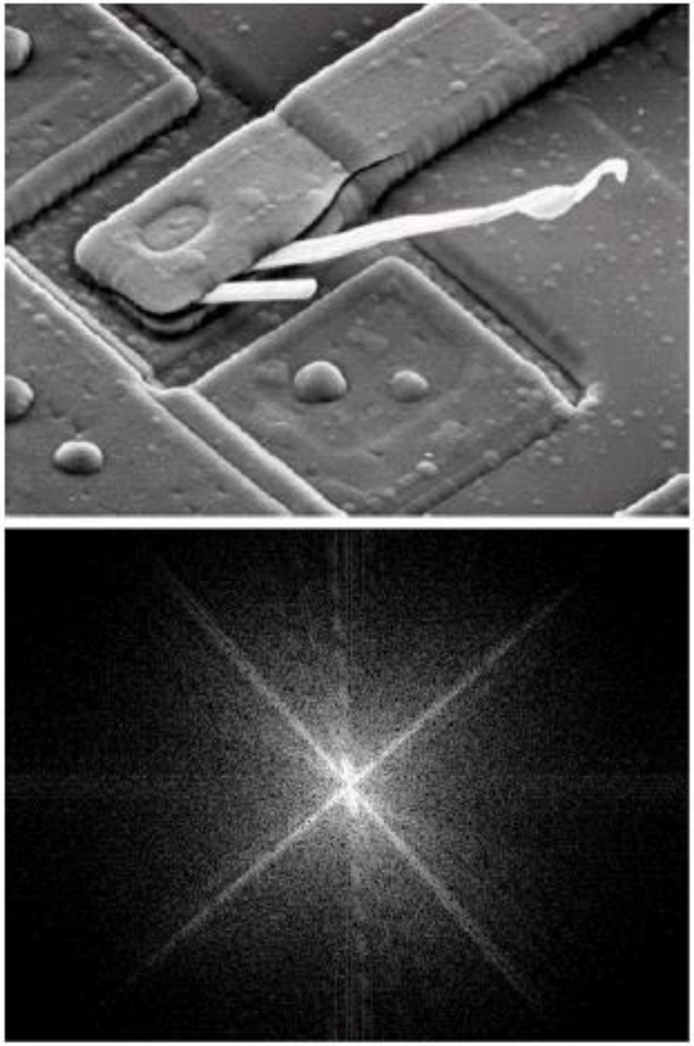


Its DFT

Impulses



Its DFT



a
b

FIGURE 4.4
(a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

Summary of DFT

Name	Expression(s)
1) Discrete Fourier transform (DFT) of $f(x, y)$	$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(ux/M + vy/N)}$
2) Inverse discrete Fourier transform (IDFT) of $F(u, v)$	$f(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi(ux/M + vy/N)}$
3) Polar representation	$F(u, v) = \lvert F(u, v) \rvert e^{j\phi(u,v)}$
4) Spectrum	$\lvert F(u, v) \rvert = \left[R^2(u, v) + I^2(u, v)\right]^{1/2}$ $R = \text{Real}(F); \quad I = \text{Imag}(F)$
5) Phase angle	$\phi(u, v) = \tan^{-1}\left[\frac{I(u, v)}{R(u, v)}\right]$
6) Power spectrum	$P(u, v) = \lvert F(u, v) \rvert^2$
7) Average value	$\bar{f}(x, y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) = \frac{1}{MN} F(0, 0)$

(Continued)

TABLE 4.2
Summary of DFT definitions and corresponding expressions.

Name	Expression(s)
8) Periodicity (k_1 and k_2 are integers)	$F(u, v) = F(u + k_1M, v) = F(u, v + k_2N) = F(u + k_1M, v + k_2N)$ $f(x, y) = f(x + k_1M, y) = f(x, y + k_2N) = f(x + k_1M, y + k_2N)$
9) Convolution	$f(x, y) \bigstar h(x, y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m, n)h(x - m, y - n)$
10) Correlation	$f(x, y) \star h(x, y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f^*(m, n)h(x + m, y + n)$
11) Separability	The 2-D DFT can be computed by computing 1-D DFT transforms along the rows (columns) of the image, followed by 1-D transforms along the columns (rows) of the result. See Section 4.11.1.
12) Obtaining the inverse Fourier transform using a forward transform algorithm.	$MNf^*(x, y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F^*(u, v)e^{-j2\pi(ux/M + vy/N)}$ This equation indicates that inputting $F^*(u, v)$ into an algorithm that computes the forward transform (right side of above equation) yields $MNf^*(x, y)$. Taking the complex conjugate and dividing by MN gives the desired inverse. See Section 4.11.2.

Name	DFT Pairs
1) Symmetry properties	See Table 4.1
2) Linearity	$af_1(x, y) + bf_2(x, y) \Leftrightarrow aF_1(u, v) + bF_2(u, v)$
3) Translation (general)	$f(x, y)e^{j2\pi(u_0x/M + v_0y/N)} \Leftrightarrow F(u - u_0, v - v_0)$ $f(x - x_0, y - y_0) \Leftrightarrow F(u, v)e^{-j2\pi(ux_0/M + vy_0/N)}$
4) Translation to center of the frequency rectangle, $(M/2, N/2)$	$f(x, y)(-1)^{x+y} \Leftrightarrow F(u - M/2, v - N/2)$ $f(x - M/2, y - N/2) \Leftrightarrow F(u, v)(-1)^{u+v}$
5) Rotation	$f(r, \theta + \theta_0) \Leftrightarrow F(\omega, \varphi + \theta_0)$ $x = r\cos\theta \quad y = r\sin\theta \quad u = \omega\cos\varphi \quad v = \omega\sin\varphi$
6) Convolution theorem[†]	$f(x, y) \star h(x, y) \Leftrightarrow F(u, v)H(u, v)$ $f(x, y)h(x, y) \Leftrightarrow F(u, v) \star H(u, v)$

(*Continued*)

TABLE 4.3
Summary of DFT pairs. The closed-form expressions in 12 and 13 are valid only for continuous variables. They can be used with discrete variables by sampling the closed-form, continuous expressions.

Name	DFT Pairs
7) Correlation theorem†	$f(x, y) \star h(x, y) \Leftrightarrow F^*(u, v)\, H(u, v)$ $f^*(x, y) h(x, y) \Leftrightarrow F(u, v) \star H(u, v)$
8) Discrete unit impulse	$\delta(x, y) \Leftrightarrow 1$
9) Rectangle	$\text{rect}[a, b] \Leftrightarrow ab \dfrac{\sin(\pi ua)}{(\pi ua)} \dfrac{\sin(\pi vb)}{(\pi vb)} e^{-j\pi(ua+vb)}$
10) Sine	$\sin(2\pi u_0 x + 2\pi v_0 y) \Leftrightarrow$ $j\dfrac{1}{2}\left[\delta(u + Mu_0, v + Nv_0) - \delta(u - Mu_0, v - Nv_0)\right]$
11) Cosine	$\cos(2\pi u_0 x + 2\pi v_0 y) \Leftrightarrow$ $\dfrac{1}{2}\left[\delta(u + Mu_0, v + Nv_0) + \delta(u - Mu_0, v - Nv_0)\right]$
The following Fourier transform pairs are derivable only for continuous variables, denoted as before by t and z for spatial variables and by μ and ν for frequency variables. These results can be used for DFT work by sampling the continuous forms.	
12) *Differentiation* (The expressions on the right assume that $f(\pm\infty, \pm\infty) = 0$.)	$\left(\dfrac{\partial}{\partial t}\right)^m \left(\dfrac{\partial}{\partial z}\right)^n f(t, z) \Leftrightarrow (j2\pi\mu)^m (j2\pi\nu)^n F(\mu, \nu)$ $\dfrac{\partial^m f(t, z)}{\partial t^m} \Leftrightarrow (j2\pi\mu)^m F(\mu, \nu); \dfrac{\partial^n f(t, z)}{\partial z^n} \Leftrightarrow (j2\pi\nu)^n F(\mu, \nu)$
13) *Gaussian*	$A2\pi\sigma^2 e^{-2\pi^2\sigma^2(t^2+z^2)} \Leftrightarrow Ae^{-(\mu^2+\nu^2)/2\sigma^2}$ (A is a constant)

†Assumes that the functions have been extended by zero padding. Convolution and correlation are associative, commutative, and distributive.

Basic steps for filtering in the frequency domain

- Frequency domain: space defined by values of the Fourier transform and its frequency variables (u; v).
- Relation between Fourier Domain and image:
  - u = v = 0 corresponds to the gray-level average
  - Low frequencies: image's component with smooth gray-level variation (*e.g.* areas with low variance)

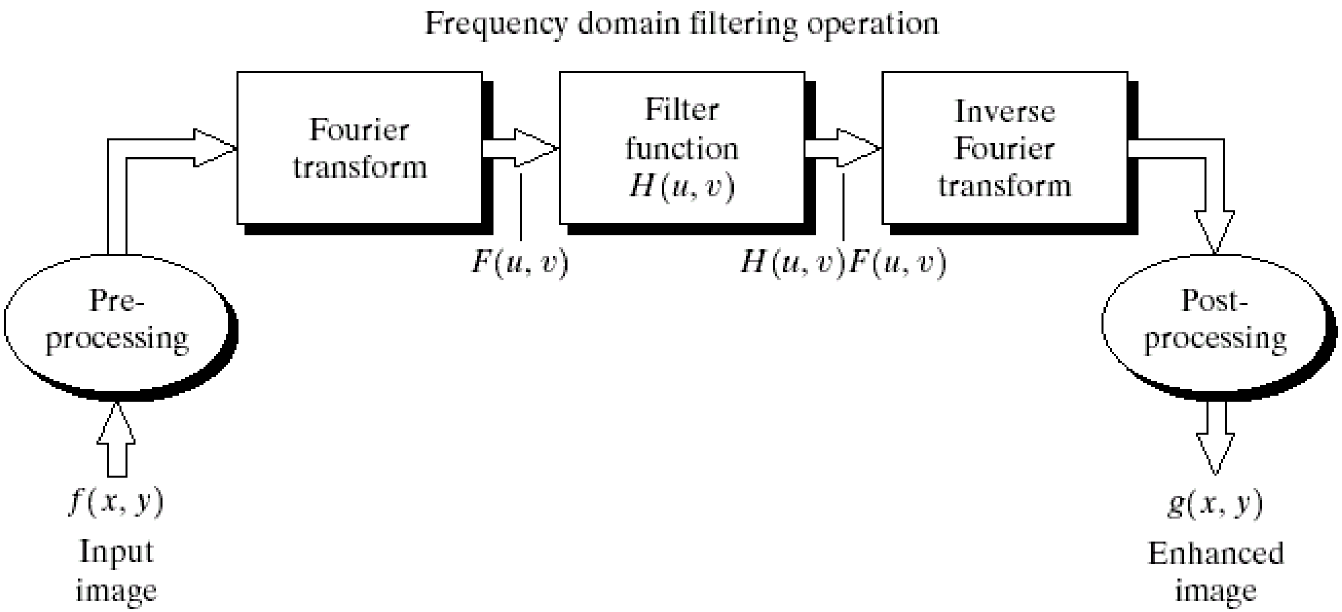


FIGURE 4.5 Basic steps for filtering in the frequency domain.

Basics of filtering in the frequency domain

- Let H(u, v) a *filter*, also called *filter transfer function*.
  - Filter: suppress certain frequencies while leaving others unchanged

$$G(u; v) = H(u; v)\ F(u; v)$$

- H(u, v) in image processing:
  - In general H(u, v) is real: zero-phase-shift filter
  - H multiply real and imaginary parts of F
  - $\Phi(u,v) = \tan^{-1}\frac{I(u,v)}{R(u,v)}$ does not change if H is real

Basics of filtering in the frequency domain

1. multiply the input image by $(-1)^{x+y}$ to center the transform to u = M/2 and v = N/2 (if M and N are even numbers, then the shifted coordinates will be integers)
2. computer F(u,v), the DFT of the image from (1)
3. multiply F(u,v) by a filter function H(u,v)
4. compute the inverse DFT of the result in (3)
5. obtain the real part of the result in (4)
6. multiply the result in (5) by $(-1)^{x+y}$ to cancel the multiplication of the input image.

From Spatial to Frequency Domain

$$f(x,y) * h(x,y) \Leftrightarrow F(u,v)\, H(u,v)$$

$$\delta(x,y) * h(x,y) \Leftrightarrow \mathcal{F}[\delta(u,v)]\, H(u,v)$$

$$h(x,y) \Leftrightarrow H(u,v)$$

- Multiplication in the frequency domain is a convolution in the spatial domain.
- Given h(x, y), we can obtain H(u, v) by taking the inverse Fourier transform.

A Simple Filter: Notch Filter

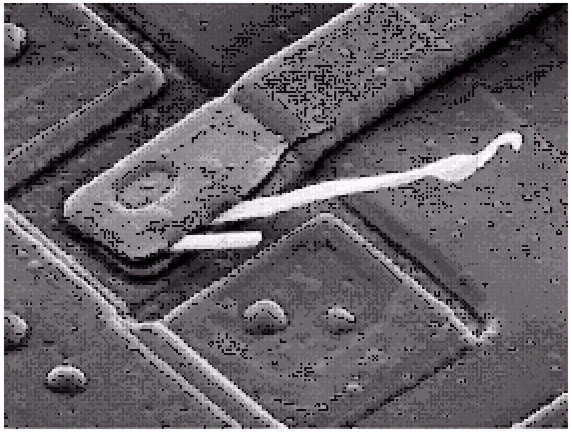
We wish to force the average value of an image to zero:

- $F(0,0)$ is the average value of the image
- if size of the image is $M \times N$ then the centered value of the Fourier transform is the average value $(\frac{M}{2}, \frac{N}{2})$

$$H(u,v) = \begin{cases} 0 \text{ if } (u,v) = (\frac{M}{2}, \frac{N}{2}) \\ 1 \text{ otherwise.} \end{cases}$$

notch filter (constant function with a hole at the origin)

$$H(u,v) = \begin{cases} 0 \text{ if } (u,v) = (\frac{M}{2}, \frac{N}{2}) \\ 1 \text{ otherwise.} \end{cases}$$



Some Basic Filters and Their Functions

Lowpass Filter

Highpass Filter

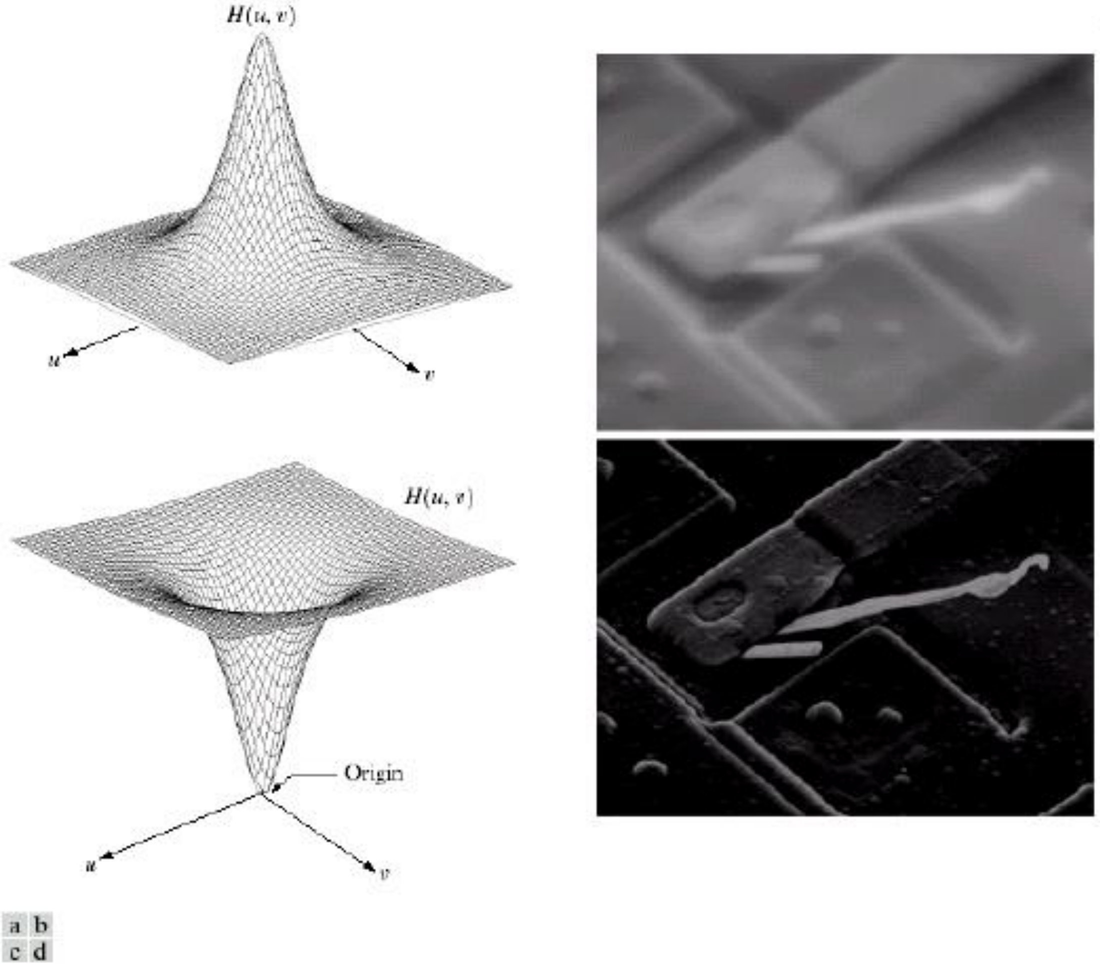
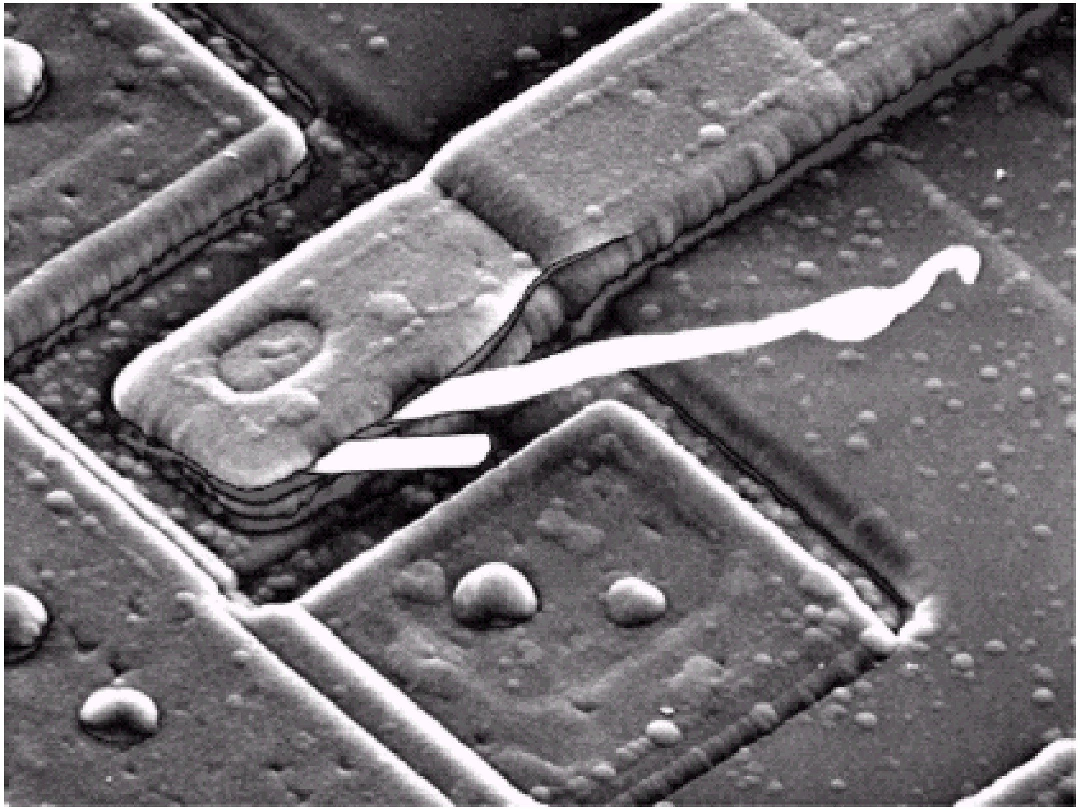


FIGURE 4.7 (a) A two-dimensional lowpass filter function. (b) Result of lowpass filtering the image in Fig. 4.4(a). (c) A two-dimensional highpass filter function. (d) Result of highpass filtering the image in Fig. 4.4(a).

FIGURE 4.8 Result of highpass filtering the image in Fig. 4.4(a) with the filter in Fig. 4.7(c), modified by adding a constant of one-half the filter height to the filter function. Compare with Fig. 4.4(a).



Correspondence between filter in spatial and frequency domains

➢ Convolution Theorem

– The discrete convolution of two functions $f(x,y)$ and $h(x,y)$ of size $M \times N$ is defined as

$$f(x,y) * h(x,y) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) h(x-m, y-n)$$

– Let $F(u,v)$ and $H(u,v)$ denote the Fourier transforms of $f(x,y)$ and $h(x,y)$, then

$$f(x,y) * h(x,y) \Leftrightarrow F(u,v) H(u,v) \qquad \text{Eq. (4.2-31)}$$

$$f(x,y) h(x,y) \Leftrightarrow F(u,v) * H(u,v) \qquad \text{Eq. (4.2-32)}$$

Correspondence between filter in spatial and frequency domains

- $A\delta(x-x_0, y-y_0)$:an impulse function of strength A, located at coordinates (x_0, y_0)

$$\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} s(x,y)A\delta(x-x_0, y-y_0) = As(x_0, y_0)$$

$$\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} s(x,y)\delta(x,y) = s(0,0)$$

where $\delta(x,y)$: a unit impulse located at the origin

- The Fourier transform of a unit impulse at the origin (Eq. (4.2-35)) :

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} \delta(x,y)e^{-j2\pi(ux/M+vy/N)} = \frac{1}{MN}$$

Correspondence between filter in spatial and frequency domains

- Let $f(x,y)=\delta(x,y)$, then the convolution (Eq. (4.2-36))

$$f(x,y)*h(x,y)=\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\delta(m,n)h(x-m,y-n)$$

$$=\frac{1}{MN}h(x,y)$$

- Combine Eqs. (4.2-35) (4.2-36) with Eq. (4.2-31), we obtain

$$f(x,y)*h(x,y)\Leftrightarrow F(u,v)H(u,v)$$

$$\underbrace{\delta(x,y)*h(x,y)}_{\boxed{\frac{1}{MN}h(x,y)}}\Leftrightarrow \underbrace{\Im[\delta(x,y)]H(u,v)}_{\boxed{\frac{1}{MN}H(u,v)}}$$

$$\Rightarrow \quad h(x,y)\Leftrightarrow H(u,v)$$

Correspondence between filter in spatial and frequency domains

- Let $H(u)$ denote a frequency domain, Gaussian filter function given the equation

$$H(u) = Ae^{-u^2/2\sigma^2}$$

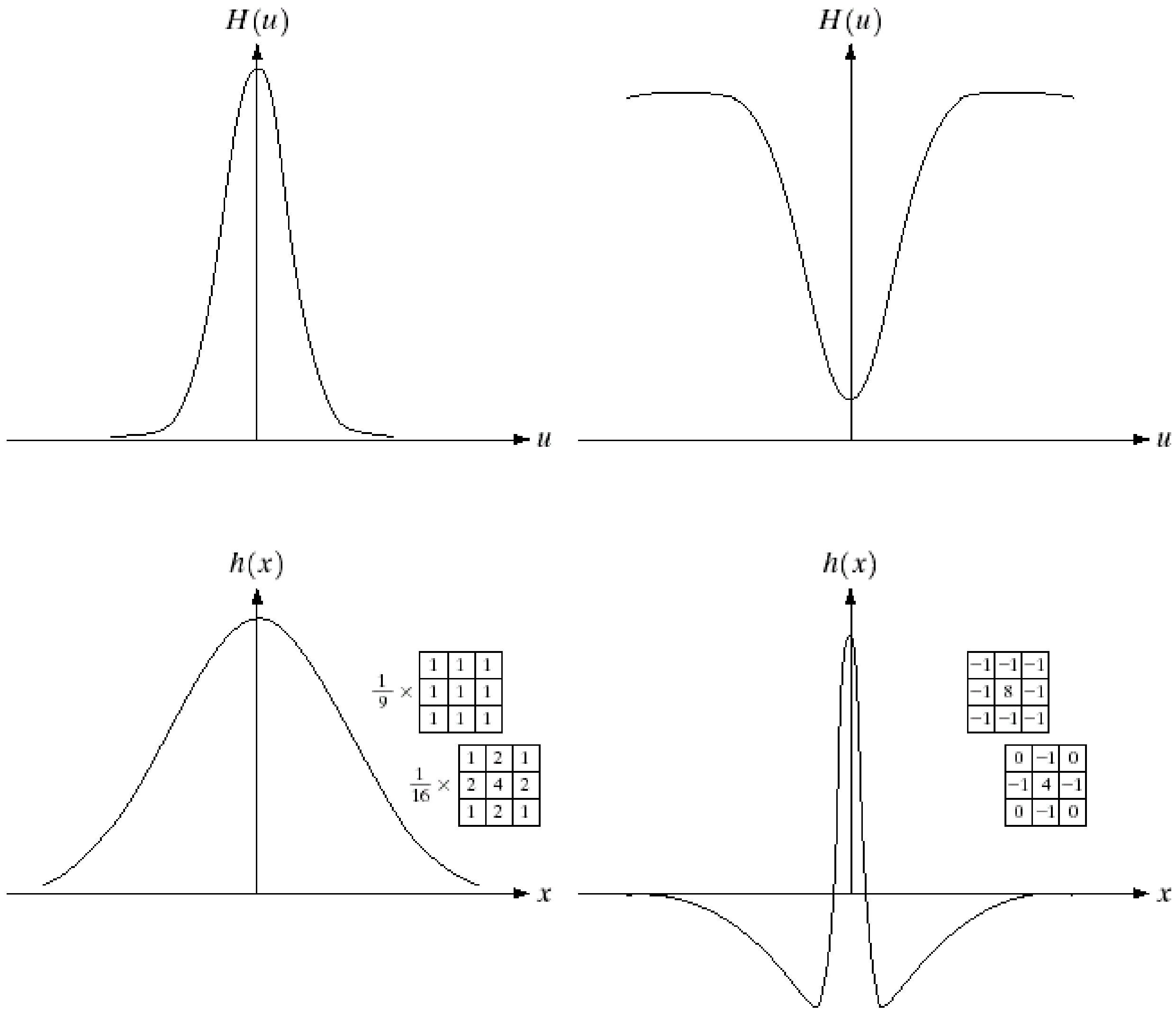
where σ : the standard deviation of the Gaussian curve.

- The corresponding filter in the spatial domain is

$$h(x) = \sqrt{2\pi}\sigma A e^{-2\pi^2\sigma^2x^2}$$

- Note: Both the forward and inverse Fourier transforms of a Gaussian function are real Gaussian functions.

Correspondence between filter in spatial and frequency domains



a b
c d

FIGURE 4.9 (a) Gaussian frequency domain lowpass filter. (b) Gaussian frequency domain highpass filter. (c) Corresponding lowpass spatial filter. (d) Corresponding highpass spatial filter. The masks shown are used in Chapter 3 for lowpass and highpass filtering.

Smoothing Frequency-Domain Filters

- The basic model for filtering in Frequency domain

$$G(u,v) = H(u,v)F(u,v)$$

where $F(u,v)$: the Fourier transform of the image to be smoothed

$H(u,v)$: a filter transfer function

- Smoothing is fundamentally a lowpass operation in the frequency domain.
- There are several standard forms of lowpass filters (LPF).
  - Ideal lowpass filter
  - Butterworth lowpass filter
  - Gaussian lowpass filter

Ideal Lowpass Filters (ILPFs)

- The simplest lowpass filter is a filter that "cuts off" all high frequency components of the Fourier transform that are at a distance greater than a specified distance *D*0 from the origin of the transform.

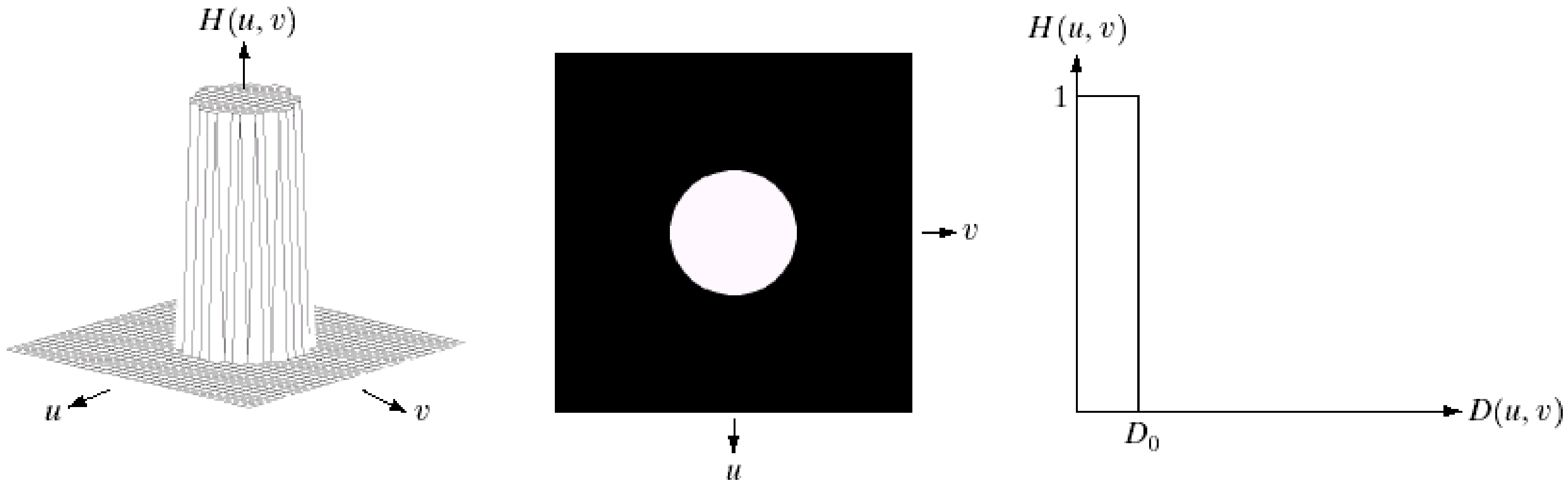
The transfer function of an ideal lowpass filter

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

where $D(u,v)$: the distance from point (u,v) to the center of ther frequency rectangle

$$D(u,v) = \left[(u - M/2)^2 + (v - N/2)^2\right]^{\frac{1}{2}}$$

Ideal Lowpass Filters (ILPFs)



a b c

FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

Cutoff Frequency

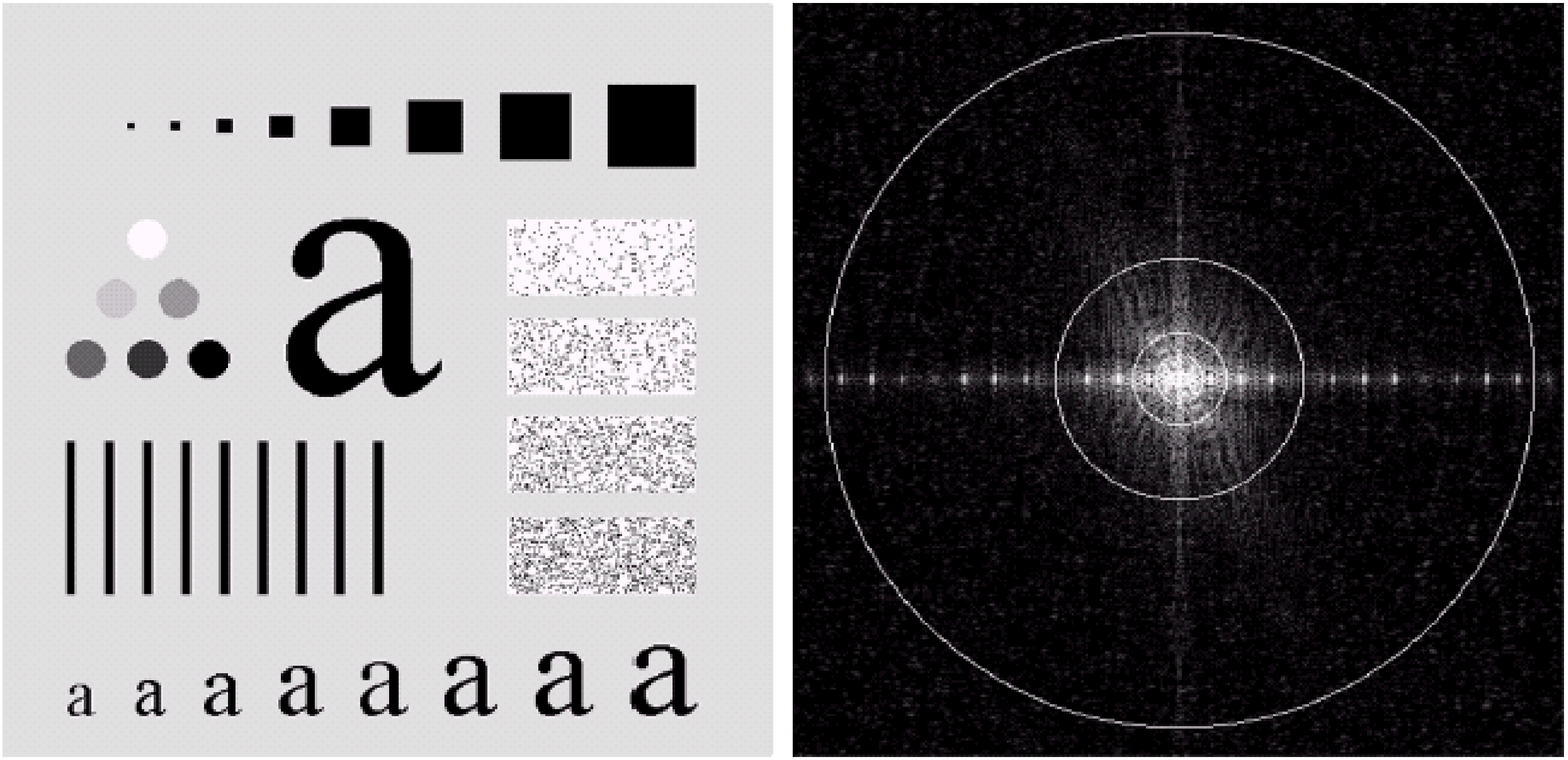
Compute circles that enclose specific amount of total image power P_T

$$P_T = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} P(u,v)$$

$$P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v)$$

$$\alpha = \frac{100}{P_T} \sum_u \sum_v P(u,v)$$

image power circles



a b

FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

Result of ILPF

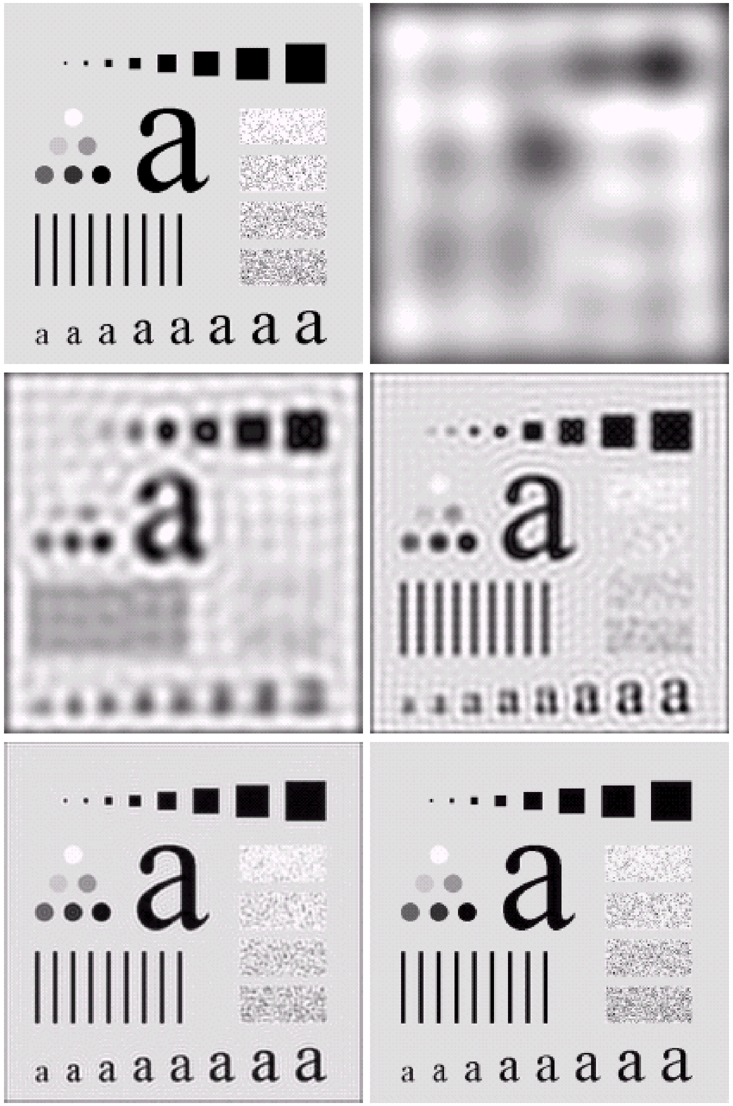


FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

Butterworth Lowpass Filter (BLFs)

BLPF of order n, with a cutoff frequency distance D_0 is defined as

$$H(u,v) = \frac{1}{1 + [\frac{D(u,v)}{D_0}]^{2n}}$$

$$D(u,v) = \sqrt{(u - \frac{M}{2})^2 + (v - \frac{N}{2})^2}$$

no clear cutoff between passed and filtered frequencies

Butterworth Lowpass Filter (BLFs)

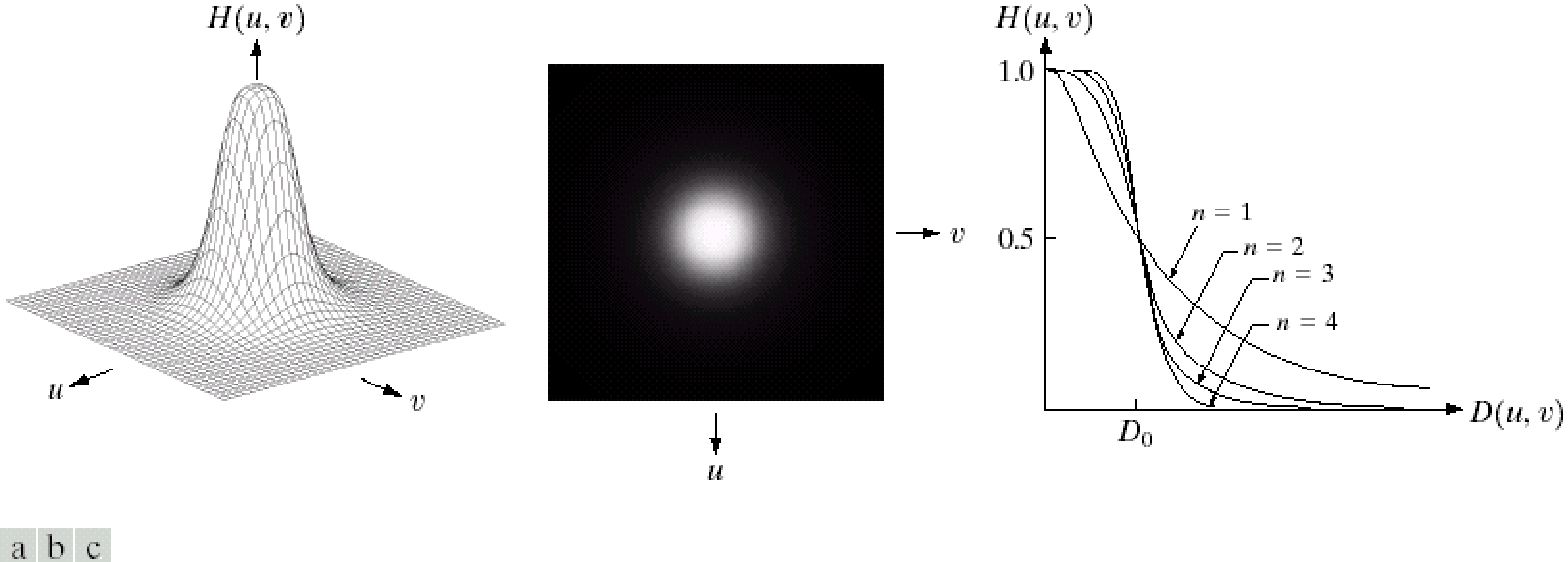
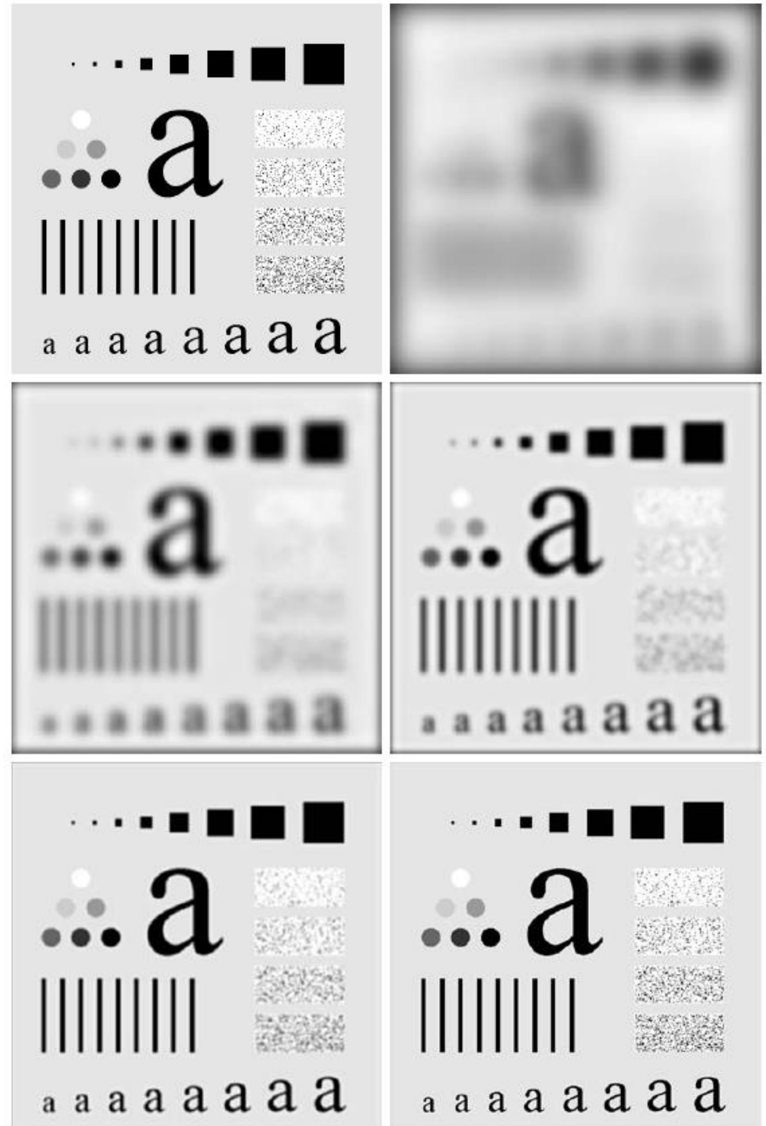


FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

Butterworth Lowpass Filters (BLPFs)

n=2

D_0=5,15,30,80,and 230



a b
c d
e f

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

Gaussian Lowpass Filter: GLPF

The form of a gaussian lowpass filter GLPF in 2D is:

$$H(u,v) = e^{\frac{-D^2(u,v)}{2\sigma^2}}$$

$$D(u,v) = \sqrt{(u - \frac{M}{2})^2 + (v - \frac{N}{2})^2}$$

The inverse Fourier transform of a GLPF is also a Gaussian

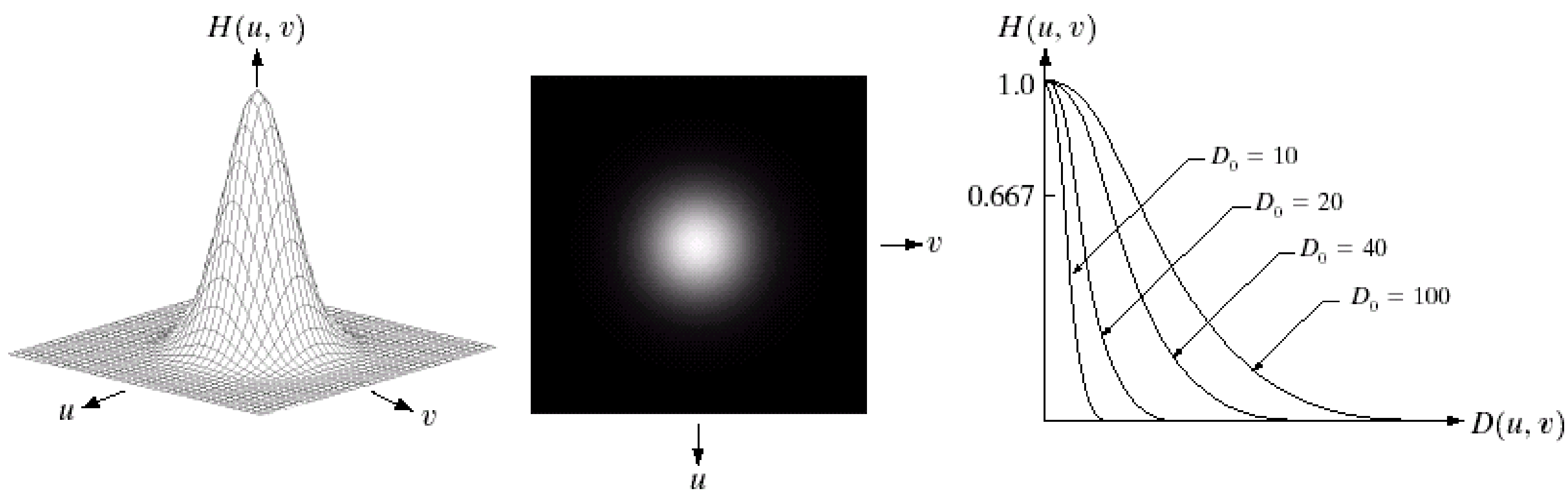
A spatial Gaussian filter will have no ringing

Gaussian Lowpass Filter: GLPF

σ: mesure of the spread of the Gaussian curve

Let $\sigma = D_0$, then:

$$H(u,v) = e^{\frac{-D^2(u,v)}{2D_0^2}}$$



a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

Example

Gaussian Lowpass Filters (FLPFs)

D_0=5,15,30,80,and 230

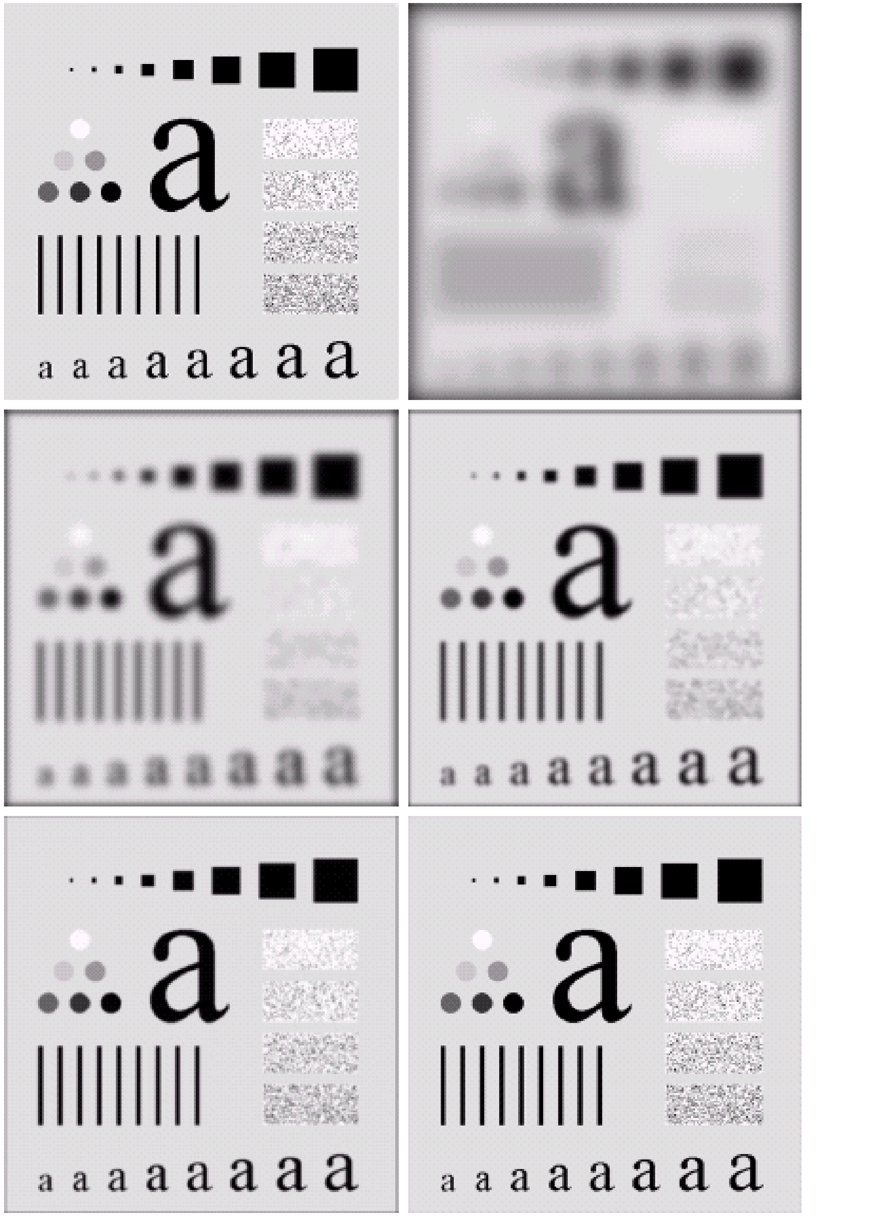
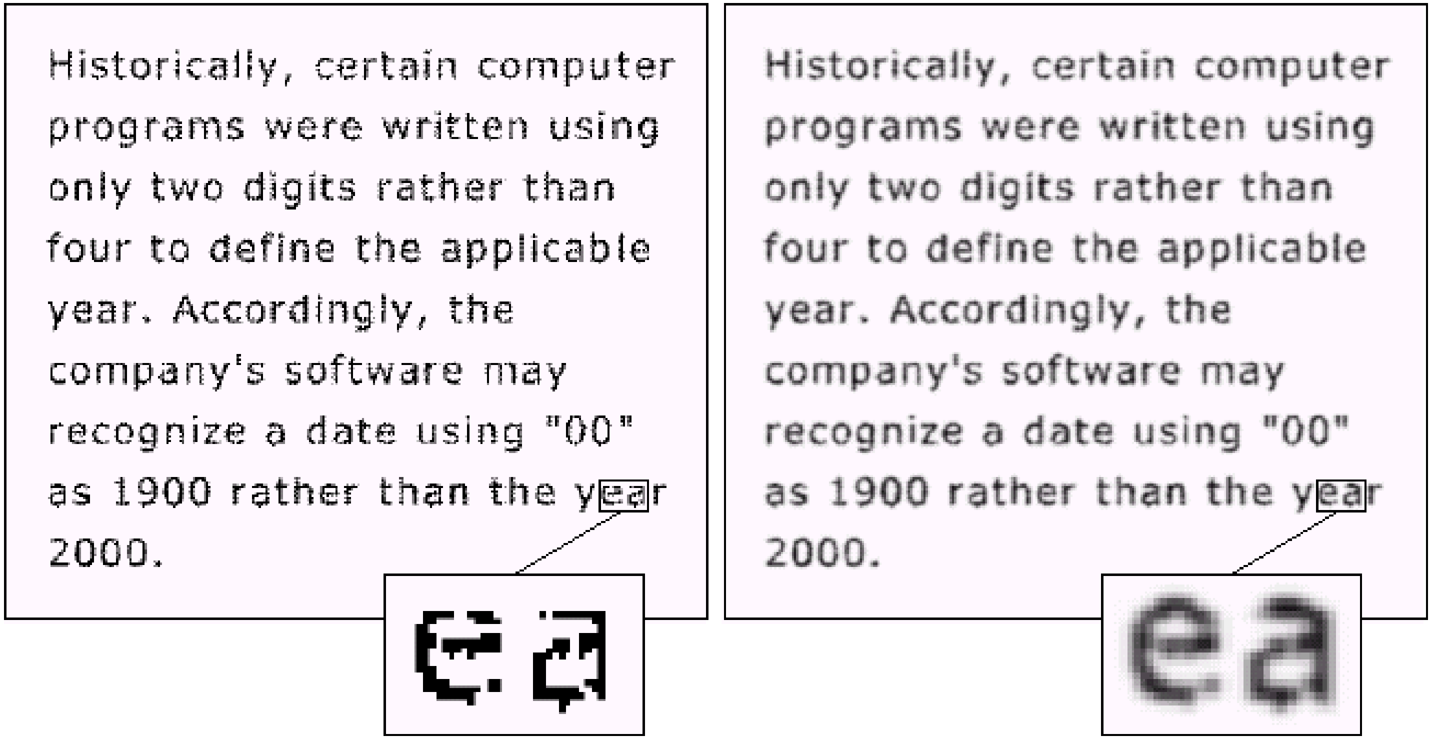


FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

Additional Examples of Lowpass Filtering

a b

FIGURE 4.19
(a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).



Additional Examples of Lowpass Filtering

a b c

FIGURE 4.20 (a) Original image (1028 × 732 pixels). (b) Result of filtering with a GLPF with $D_0 = 100$. (c) Result of filtering with a GLPF with $D_0 = 80$. Note reduction in skin fine lines in the magnified sections of (b) and (c).

Additional Examples of Lowpass Filtering

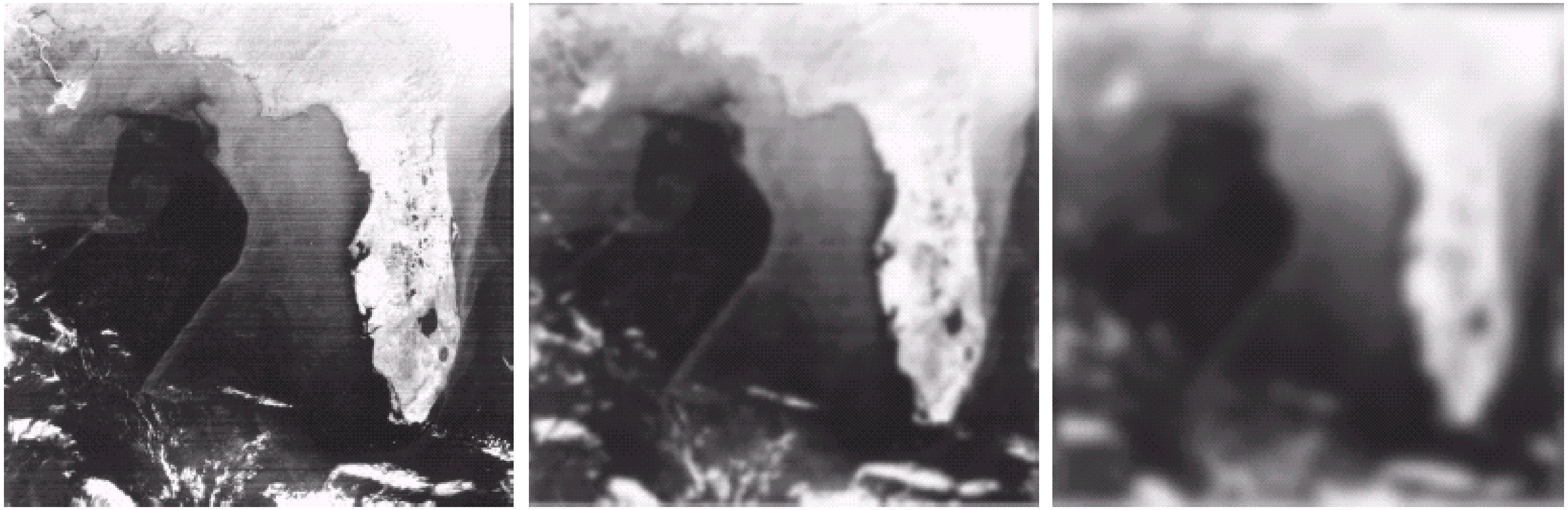


FIGURE 4.21 (a) Image showing prominent scan lines. (b) Result of using a GLPF with $D_0 = 30$. (c) Result of using a GLPF with $D_0 = 10$. (Original image courtesy of NOAA.)

Sharpening Frequency Domain Filter

Sharpening Frequency Domain Filter

- Highpass filter: image sharpening (low-frequency attenuation)
- In our case:
  - zero-phase-shift filter
  - radially symmetric

$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

$H_{lp}(u,v)$: transfer function of the corresponding lowpass filter

$H_{hp}(u,v)$: transfer function of the corresponding highpass filter

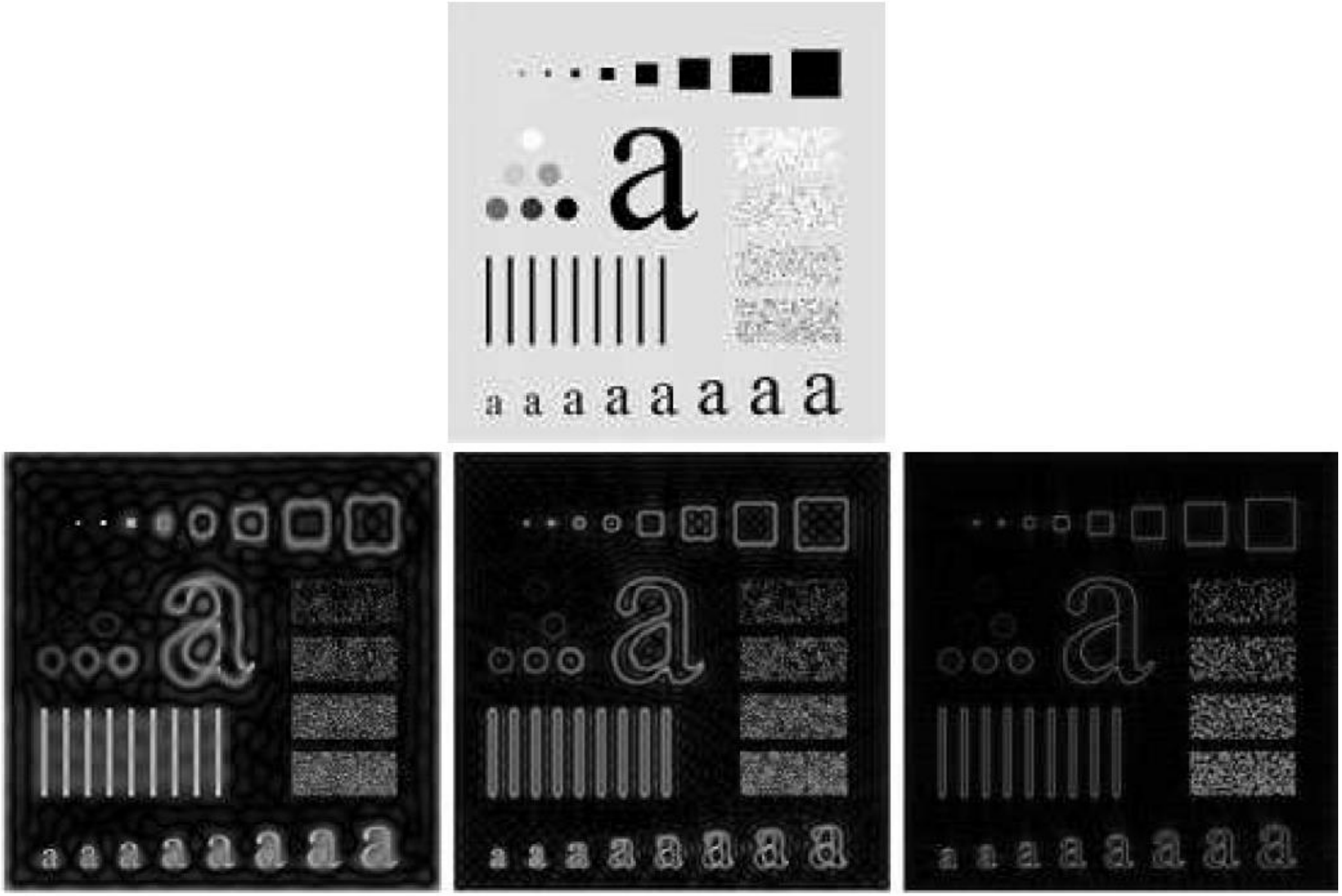
Ideal Highpass Filters

$$H(u,v) = \begin{cases} 0 \text{ if } D(u,v) \leq D_0 \\ 1 \text{ if } D(u,v) > D_0 \end{cases}$$

- D_0: nonnegative quantity
- $D(u,v)$: distance from a point (u,v) to the origin
- origin: $(\frac{M}{2}, \frac{N}{2})$ (centered)

Then,

$$D(u,v) = \sqrt{(u - \frac{M}{2})^2 + (v - \frac{N}{2})^2}$$



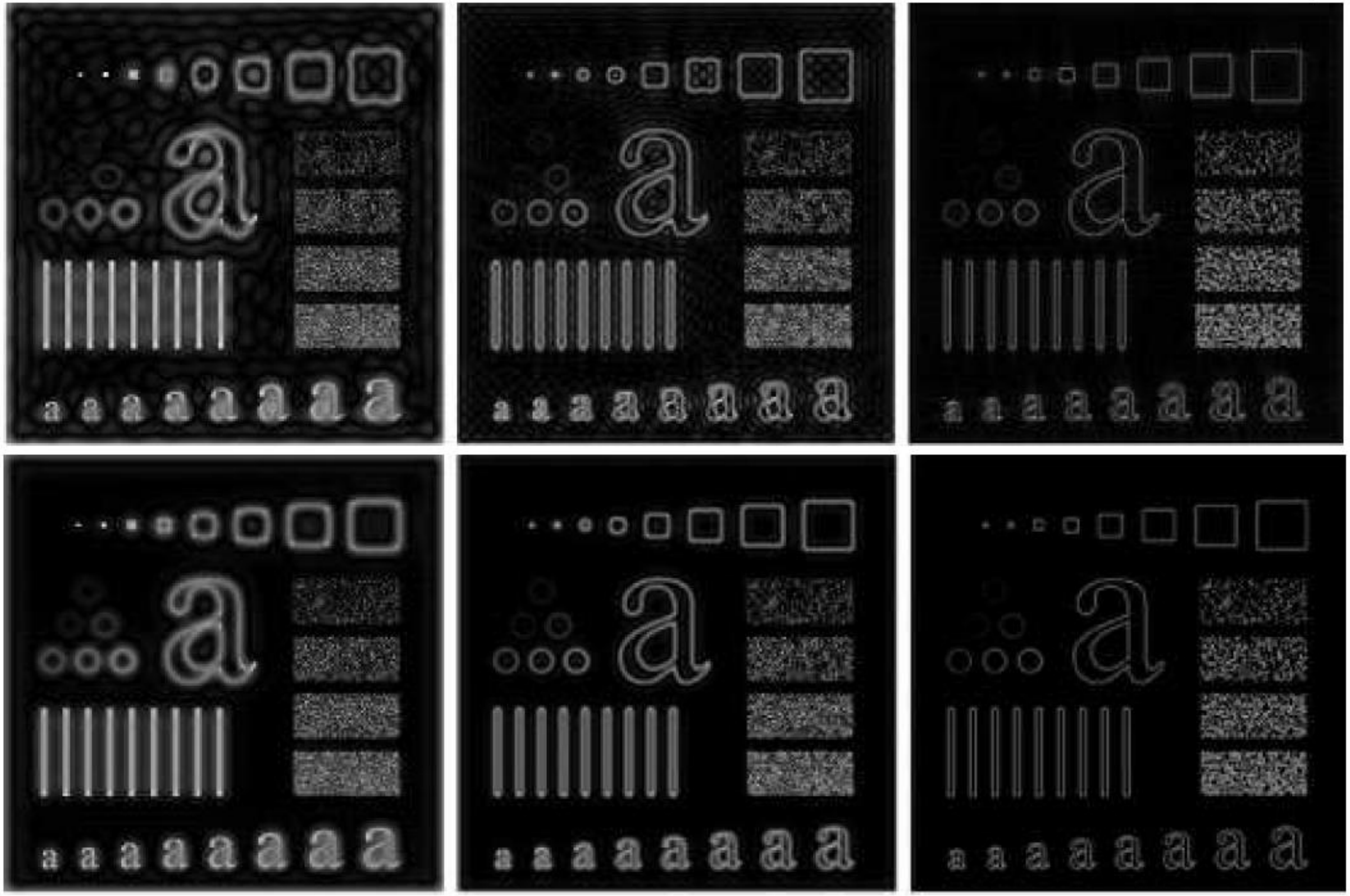
Butterworth Highpass Filters

- BLPF of order n, with a cutoff frequency distance D_0 is defined as

$$H(u,v) = \frac{1}{1 + [\frac{D_0}{D(u,v)}]^{2n}}$$

$$D(u,v) = \sqrt{(u - \frac{M}{2})^2 + (v - \frac{N}{2})^2}$$

no clear cutoff between passed and filtered frequencies



Gaussian Highpass Filters

- The form of a gaussian Highpass filter in 2D is:

$$H(u,v) = 1 - e^{\frac{-D^2(u,v)}{2D_0^2}}$$

$$D(u,v) = \sqrt{(u - \frac{M}{2})^2 + (v - \frac{N}{2})^2}$$

The inverse Fourier transform of a GLPF is also a Gaussian

A spatial Gaussian filter will have no ringing