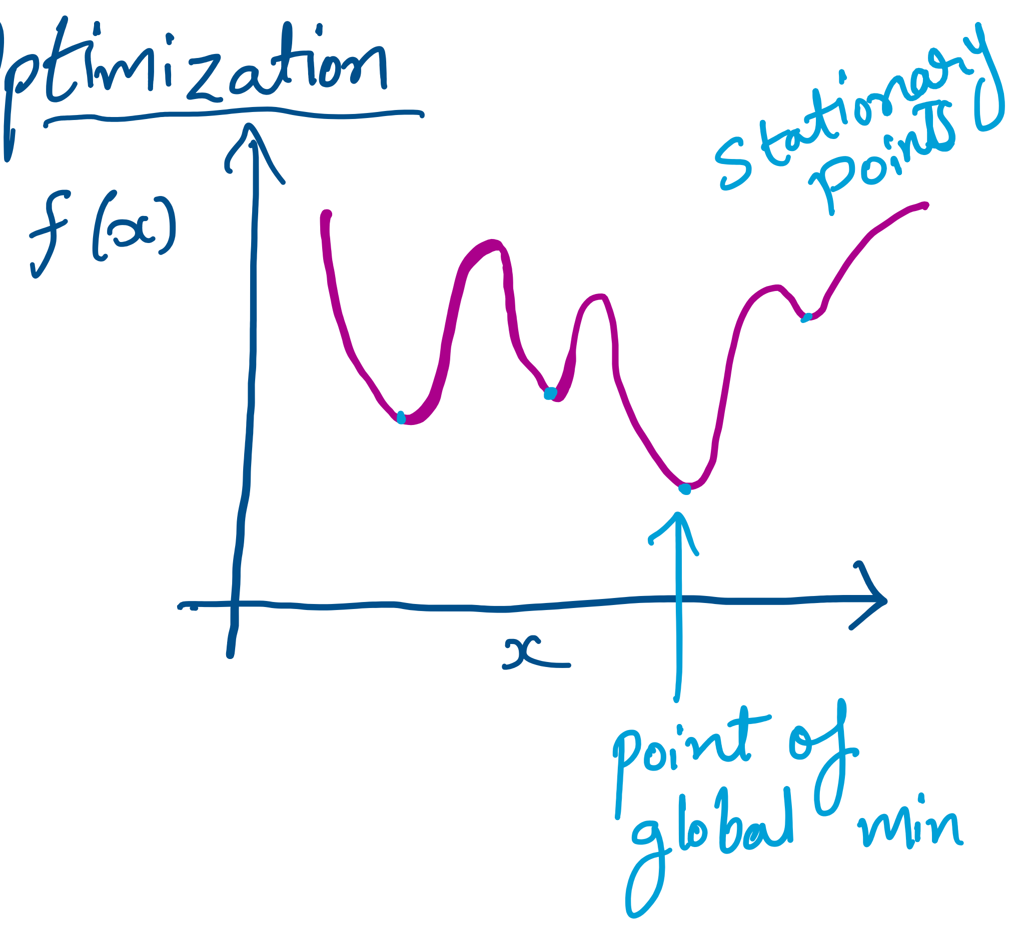
Convex Optimization

Wednesday, 29 November 2023 3:03 PM

Optimization



$f: \mathbb{R}^n \to \mathbb{R}$

Unconstrained optimization

$\min f(x)$
s.t. $x \in \mathbb{R}^n$

Constrained

$\min f(x)$
s.t. $x \in X \subseteq \mathbb{R}^n$

Some Important Results/Definitions

- Let $f: \mathbb{R}^n \to \mathbb{R}$ be continuously differentiable

x^* is pt. of local opt. $\Rightarrow$ $\nabla f(x^*) = \begin{bmatrix} \frac{\partial f(x)}{\partial x_1} \\ \vdots \\ \frac{\partial f(x)}{\partial x_n} \end{bmatrix} = 0$

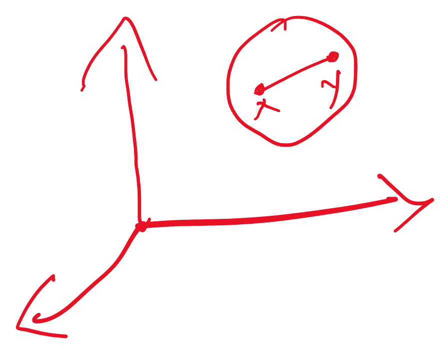
- Let f be twice differentiable

$$H(x) = \begin{bmatrix} \frac{\partial^2 f(x)}{\partial x_1} & \frac{\partial^2 f(x)}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f(x)}{\partial x_1 \partial x_n} \\ & & & \\ & & & \frac{\partial^2 f(x)}{\partial x_n^2} \end{bmatrix}$$

then $\nabla f(x^*) = 0$ & $H(x^*)$ is P.D. $\Rightarrow$ x^* is a p.t. of local optimum (min)

Convex set:- $X \subseteq \mathbb{R}^n$ convex is said to be if $\forall x, y \in X$

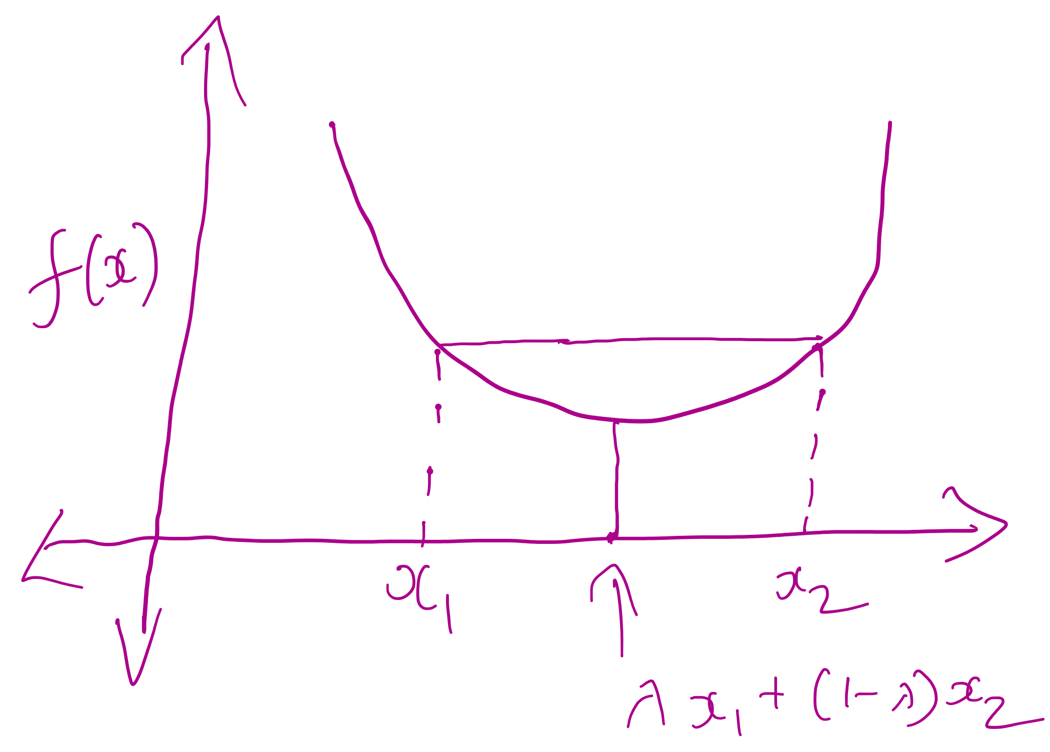
$\lambda x + (1-\lambda) y \in X \quad \forall\ 0 \leq \lambda \leq 1$



Convex fn :- $f: \mathbb{R}^n \to \mathbb{R}$ is defined over a convex set in following way

$\forall x_1, x_2$

$f(\lambda (x_1) + (1-\lambda) x_2) \leq \lambda f(x_1) + (1-\lambda) f(x_2)$



$f(\cdot)$ is convex
$\Longleftrightarrow H(x)$ is P.S.D $\forall x \in X$

$\boxed{\text{Why Convex fn are important?}}$

- Many useful ML loss fn convex

- For convex local min $\Rightarrow$ Global min