
IT 585 -Advanced Machine Learning

Quiz 1

March 10, 2023

Duration: 45 minutes

Maximum Marks: 10

Note: In case you have any doubt, make an appropriate assumption, state the assumption clearly, and proceed. Proofs should be complete

1. Kernel functions implicitly define some mapping function $\phi(\cdot)$ that transforms an input instance $\mathbf{x} \in \mathbb{R}^d$ to a high dimensional feature space Q by giving the form of dot product in Q : $K(\mathbf{x}_i; \mathbf{x}_j) = \phi(\mathbf{x}_i) \cdot \phi(\mathbf{x}_j)$. Assume we use radial basis kernel function $K(\mathbf{x}_i; \mathbf{x}_j) = \exp(-\frac{1}{2}\|\mathbf{x}_i - \mathbf{x}_j\|^2)$. Thus we assume that there's some implicit unknown function $\phi(\mathbf{x})$ such that

$$\phi(\mathbf{x}_i) \cdot \phi(\mathbf{x}_j) = K(\mathbf{x}_i; \mathbf{x}_j) = \exp(-\frac{1}{2}\|\mathbf{x}_i - \mathbf{x}_j\|^2)$$

Prove that for any two input instances \mathbf{x}_i and \mathbf{x}_j , the squared Euclidean distance of their corresponding points in the feature space Q is less than 2. That is, prove that

$$\|\phi(\mathbf{x}_i) - \phi(\mathbf{x}_j)\|^2 < 2$$

[3 Marks]

2. Prove that $VC(H) \leq \log_2 |H|$ where H is a hypothesis space. (A hypothesis on a set of n points, defines which of two classes each point belong to. A hypothesis space is a family of all possible hypotheses)

[2 Marks]

3. Let $X = R$. Let H be the set of all classifiers h that, for some set of non-intersecting intervals $R_1 \dots R_p$ classify a point x as $h(x) = 1$ if $x \in \bigcup_{i=1}^p R_i$, and $h(x) = 0$ otherwise (p is fixed and given). Find $VC(H)$ (Prove that your answer is correct).

[5 Marks]