MULTINOMIAL dis^N

2 Categories.

ASK 1 person

Bernoulli dis^n

[Head, Tails]

$P_S = P$ Person likes Salmon

2 Categories ask many people

Binomial dis

P_S and n people

$n=1$

$K=1$

(K) many categories and ask 1 person

Categorical dis^n

(P_S, P_T, P_C) such that $\sum P_i = 1$

$P(X_i = i) = P_i$

Special case

Multinomial

many categories (K) and ask many people (n)

$K=3$

(P_S, P_T, P_C) and n people ← no. of observations

$\sum P_i = 1$

$\sum x_i = n$ ← Conditions

x_i: no. of people which like i^{th} Category.

no. of people who like i

$$P[X_1 = x_1, X_2 = x_2, \dots, X_k = x_k]$$

$$= \binom{n}{x_1} P_1^{x_1} \binom{n-x_1}{x_2} P_2^{x_2} \cdots \binom{n-x_1-\cdots-x_{k-1}}{x_k} P_k^{x_k}$$

$$= \frac{n!}{x_1! \cdots x_k!} \prod P_i^{x_i}$$

↑ pmf for multinomial dis^n

$P_S = 0.5$

$P_T = 0.3$ $\quad n = 6$

$P_C = 0.2$

$P(X_S = 1, X_T = 2, X_C = 3)$

$= 0.0216$

low

Question

Beta dist^n

3 biased coins

Coin 1: $P(H) = 0.4$

Coin 2: $P(H) = 0.6$

Coin 3: $P(H) = 0.8$

Tossed 3 times and got: HHHTT

a) which coin was used for tossing ??

Ans

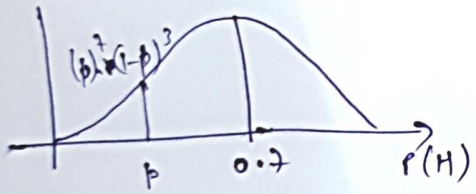
If coin 1 : $(0.4)^3 (0.6)^2 = 0.023$ /s → P: 0.295

Coin 2 : $(0.6)^3 (0.4)^2 = 0.0346$ /s → 0.443

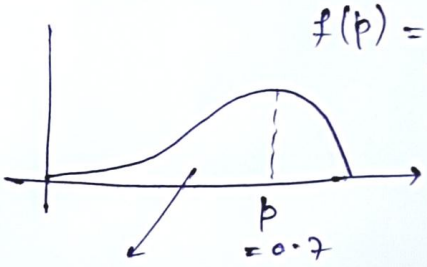
Coin 3 : $(0.8)^3 (0.2)^2 = 0.0205$ /s → 0.262

s

b) 7H 3T



$$f(p) = \frac{p^7 (1-p)^3}{B(8,4)}$$



Area = $B(8,4)$

Beta (8,4)

Flip coin many times : a Heads, b tails.

x : prob. of getting head.

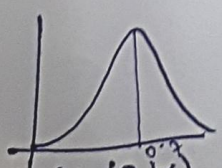
$$f(x) = \frac{x^a (1-x)^b}{B(a+1, b+1)}$$

$$E = \frac{a}{a+b}$$

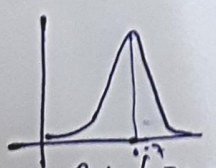
7H 3T

70H 30T

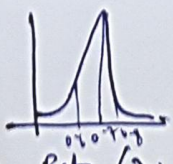
700H 300T



Beta(8,4)

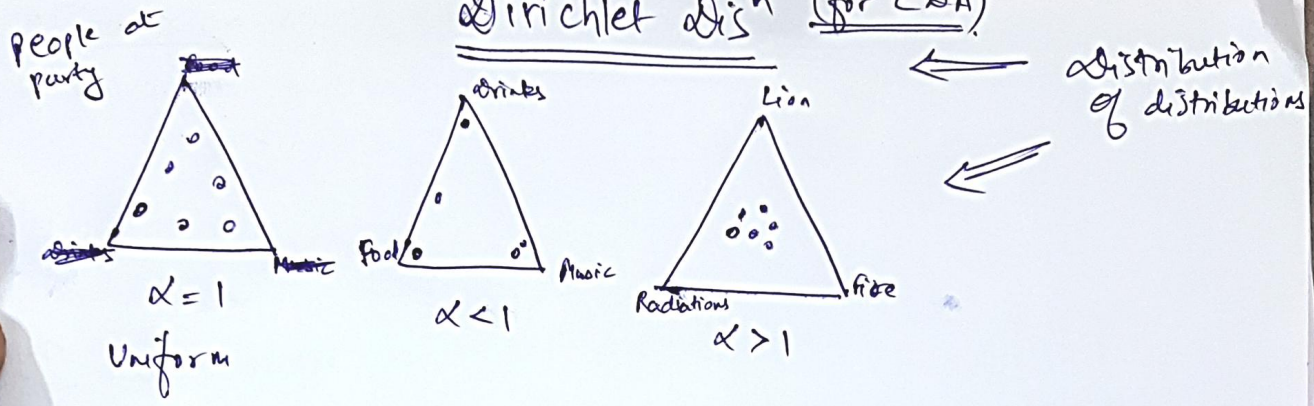


Beta (71, 31)



Beta (701, 301)

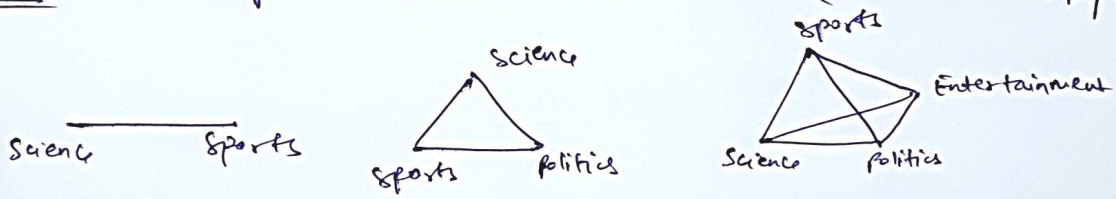
Dirichlet dist^n (for LDA)



Distribution of distributions

$$f(x_1, \dots x_K ; \alpha_1, \dots, \alpha_K) = \frac{1}{B(\alpha)} \prod_{i=1}^{K} x_i^{\alpha_i - 1}$$

Note: More topics $\Longrightarrow$ More dimensions (n-dimensional simplex)



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3 topics ← Science, Sports, Politics

4 words ← Referendum, Planet, Galaxy, Ball

and few documents.

2 dirichlet dist^n