IT584 Approximation Algorithms	January 18, 2024

Lecture 5

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1 Greedy algorithms and local search

1.1 Scheduling on parallel machines

We considered the problem of scheduling jobs on a single machine to minimize lateness. Here we consider a variation on that problem,in which we now have multiple machines and no release dates, but our goal is to minimize the time at which all jobs are finished.

There are n jobs, m identical machines, and the j^{th} job needs p_j units of time. Each job is available at time 0. Each machine can process at most, one job at a time. Our objective is to minimize the makespan or the length of the schedule

$$C_{\max} = \max_{j=1\ldots n} C_j \tag{1}$$

C_j- largest completion time over all jobs

There are m machines and n jobs. If $m \geq n$ or m = n, then each job will be assigned one machine and it will take p_j units of time to complete.

If $m \leq n$ then there is a problem and hence there is a need for Local Search Algorithm.

1.2 Local Search Algorithm

Set of local changes/moves that change on **FEASIBLE** solution to the other.

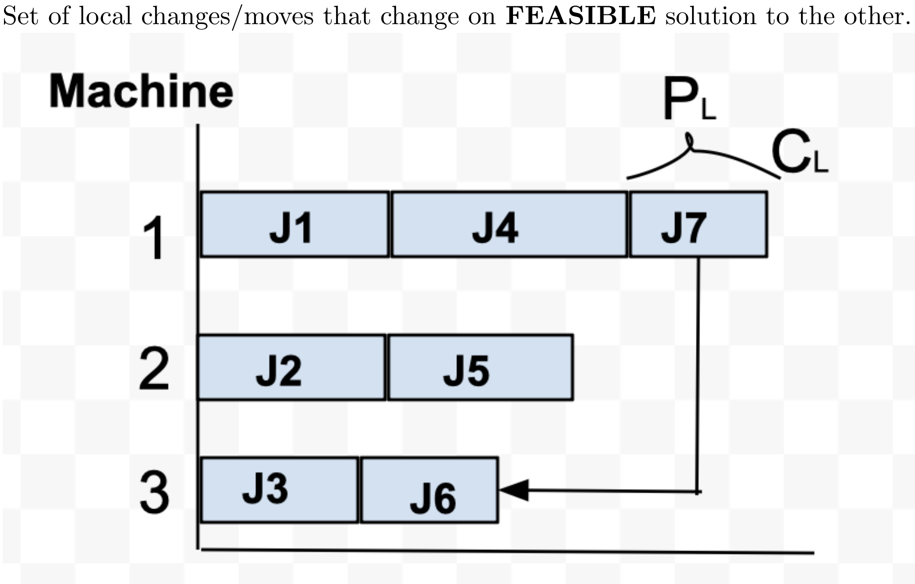

Figure 1: An example of local move in the local search algorithm for scheduling jobs on parallel machines. Job 7 on Machine 1 finishes last in the schedule, but the schedule can be improved by moving the Job 7 to Machine 3.

Algorithm:

• Start with any schedule.

• Consider job l that finishes last.

• Is there a free machine that finishes l earlier?
- If **YES** transfer l to the other machine, equivalent to checking if every machine that finishes its currently assigned job earlier than C_l - P_l.

• Repeat till last job to complete and further any job cannot be transferred.

If there are n jobs and m machines, then the possible solution is n^m.

$$C^*{}_{\max} \geq \max_j P_{\mathrm{j}} \tag{2}$$

[Minimize the maximum completion time.]

Analysis: Lower bound on optimal $C^*{}_{\max}$

$$C^*{}_{\max} \geq \max_{j=1,2\ldots n} P_{\mathrm{j}} \tag{3}$$

There is at least one machine which process more than or equal to the average.

$$P = \sum_{j=1}^{n} P_{\mathrm{j}} \tag{4}$$

There are total m machines so on an average P/m units of work on one machine.

$$C^*{}_{\max} \geq \sum_{j=1}^{n} \frac{P_{\mathrm{j}}}{m} \tag{5}$$

Let l be a job that completes last in the final schedule. $S_l = C_l - P_l$ is start time of 1.

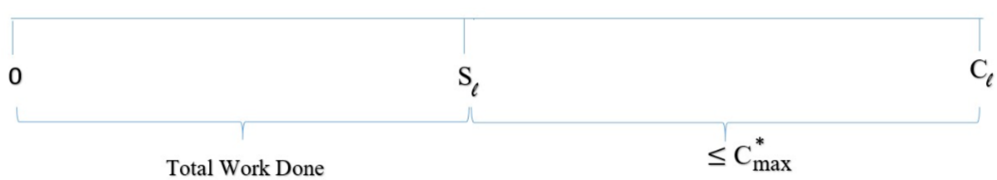

Figure 2: Every machine (other than on which l was processed) is busy from time 0 to S_l.

$$mS_l \leq \sum_{j=1}^{n} P_{\mathrm{j}} \tag{6}$$

$$S_l \leq \sum_{j=1}^{n} \frac{P_{\mathrm{j}}}{m} \tag{7}$$

$$C^*{}_{\max} \geq \sum_{j=1}^{n} \frac{P_{\mathrm{j}}}{m} \geq S_l \tag{8}$$

Therefore from previous lower bound and $S_l \leq C^*{}_{\max}$

Also, S_l to $C_l \leq C^*{}_{\max}$
Makespan $\leq 2C^*{}_{\max}$