IT 584 -Approximation Algorithms

MidSem Exam

April 3, 2023
Duration: 90 minutes
Maximum Marks: 30

Note: In case you have any doubt, make an appropriate assumption, state the assumption clearly, and proceed. Proofs should be complete and general. If you are asked to give an example, please explain the example.

1. (5 marks) The Greedy Algorithm for Vertex Cover Problem is as follows: Find a maximal matching in G and output the set of matched vertices. This is a 2-approximation algorithm. Given an example where the approximation is tight i.e. The algorithm outputs a solution which is exactly twice the size of the optimal solution.

2. (5 marks) Let $X_1, \ldots, X_m$ be independent and identically distributed indicator random variables, with $\mu = \mathbf{E}[X_i]$. For what value of m can we get an (ϵ, δ)- approximation for μ? Find and prove a bound on the value of m as a function of ϵ, δ, μ.

3. (3 marks) There may be several different min-cut sets in a graph. Using the analysis of the randomized min-cut algorithm, argue that there can be at most $n(n-1)/2$ distinct min-cut sets.

4. Let E be a set of items and for some $S \subseteq E$, let $f(S)$ give the value of subset S (f is called a set function).
   $f(\Phi) = 0$.
   A function f is submodular if for any $S, T \subseteq E$, we have $f(S)+f(T) \geq f(S \cup T)+f(S \cap T)$.
   A function f monotone if for any $S \subseteq T \subseteq E$, we have $f(S) \leq f(T)$.

   Now we have m subsets of E each with a non-negative weight $w_j >= 0$. We want to solve the *maximum coverage* problem i.e we want to choose k elements such that we maximize the weight of subsets that are covered. We say that a subset is covered if we have chosen some element from it. Mathematically, find $S \subseteq E$ such that $|S| = k$ and we maximize the total wight of subsets j such that $S \cap S_j \neq \Phi$. Give $(1 - 1/e)$- approximation algorithm for this problem.

- Step 1 : Show that the greedy algorithm for Float maximization in accounts extends to any submodular monotone function and accordingly give the general greedy algorithm and prove it gives $(1 - 1/e)$ for maximizing submodular monotone function. (3 marks)
- Step 2: Show that the objective for maximum coverage is actually a submodular and monotone function and its value is 0 for an empty set. Hence by the argument in Step 1 we get required approximation. (4 marks)

5. Let $E = \{1, 2, 3, 4, 5, 6\}, S_1 = \{1, 2\}, S_2 = \{1, 3\}, S_3 = \{2, 3, 4, 5, 6\}, S_4 = \{1, 3, 5\}$ and $S_5 = \{2, 4, 6\}$. The weights associated with the subsets are $w1 = 51, w2 = 52, w3 = 60, w4 = 56, w5 = 57$. Do the following:

   - Write the integer program corresponding to the minimum weight set cover problem for the above sets: (2 marks)
   - Write down the LP relaxation and the dual of the LP relaxation for the above integer program : (1+2= 3 marks)
   - Run the primal-dual algorithm on the data. Give step by step tracing of the algorithm and the final solution. Also get the value of optimal solution (just by looking at data you get the optimal value and solution). Compare it with solution given by the primal dual algorithm. (1+4 =5 marks)