IT584 Approximation Algorithms	February 15, 2023
Lecture 10	
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1 DNF Counting Problem

The DNF (Disjunctive Normal Form) Counting Problem is a computational problem where you're given a boolean formula in disjunctive normal form, and you need to count the number of satisfying assignments (i.e., combinations of true/false values for variables that make the formula true). This problem is known to be P-complete, which means it's one of the hardest problems in the complexity class P.

1.1 Disjunctive Normal Form

$C_1 \vee C_2 \vee C_3 \vee \ldots \vee C_t$

- We need to satisfy only one clause.
- Easier to find a satisfying assignment than CNF (Conjunctive Normal Form).
- Hard to count the number of satisfying assignments for DNF.

1.2 DNF Counting Algorithm

- Input: DNF formula with n variables
- Output: An approximation of $C(F)$ (i.e. the count of satisfying assignments)

Return $y = \left(\frac{x}{m}\right)^{2^n}$

.

Why is $C(F)$ greater than 0 and not 0?

$$X_k = \begin{cases} 1, & \text{if the } k\text{th iteration assignment is satisfying,} \\ 0, & \text{otherwise.} \end{cases}$$

$x = \sum_{k=1}^{m} X_k$

What is the probability of $X_k = 1$?

$E[Y] = C(F)$, where $m =$?

$m \geq \frac{3 \cdot 2^n \ln(2/\delta)}{\epsilon^2 C(F)}$

If $C(F) \geq \frac{2^n}{\alpha(n)}$, then it's good, but if $C(F) \ll 2^n$, then there's a problem.

FPRAS

Let $F = C_1 \vee C_2 \vee C_3 \vee \ldots \vee C_t$

- No clause includes a variable and its negation.
- Need to satisfy at least 1 clause in F.

- For each clause i, exactly 2^{n-l_i} satisfying assignments where $|l_i|$ is the number of literals in clause C_i.

$SC_i =$ Set of assignments satisfying clause i
$U = \{(i, a) \mid 1 \leq i \leq t \ , \ a \in SC_i\}$

$|U| = \sum_{i=1}^{t} |SC_i|$

$|SC_i|$ is computable.
$C(F) = \left| \bigcup_{i=1}^{t} SC_i \right|$

$C(F) \leq |U|$ why?

Use U to get $C(F)$.
Define $S = \{(i, a) \mid 1 \leq i \leq t, a \in SC_i, a \notin SC_j, \text{ for } j < i\}$
By uniform sampling can get $\frac{|S|}{|U|}$.

$\frac{|S|}{|U|} \geq \frac{1}{t}$ Why?

$$
\begin{aligned}
\Pr((i, a) \text{ is chosen}) &= \Pr(i \text{ is chosen}) \times \Pr(a \text{ is chosen} \mid i \text{ is chosen}) \\
&= \frac{|SC_i|}{|U|} \times \frac{1}{|SC_i|} \\
&= \frac{1}{|U|} \\
&= 1
\end{aligned}
$$