Constrained Convex Optimization

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Let $f_0 : \mathbb{R}^n \to \mathbb{R}$ be some fn

$$\min f_0(x)$$

$$\text{s.t.} \quad f_i(x) \le 0 \quad \forall i = 1, \dots, m$$

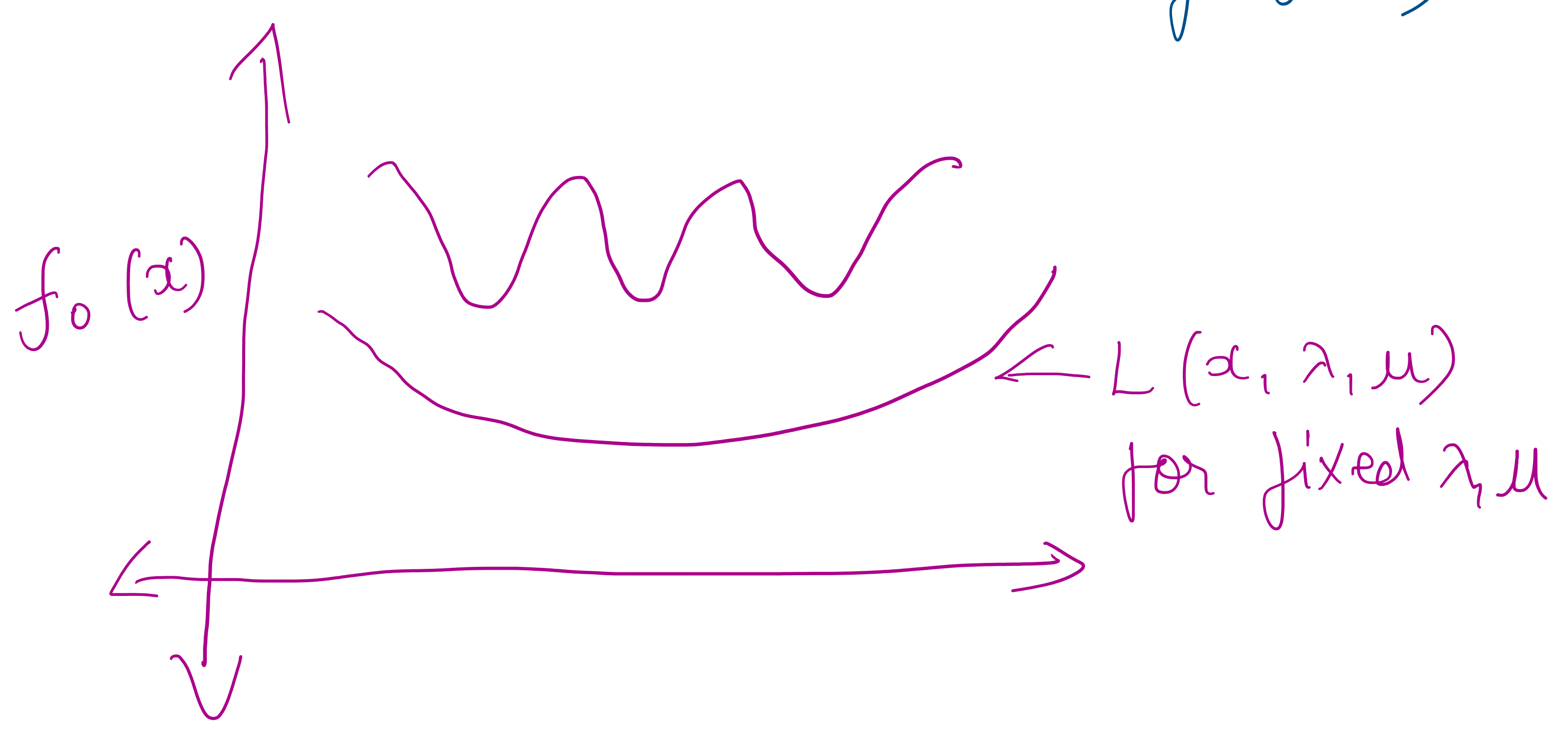
$$f_j(x) = 0 \quad \forall j = 1, \dots, l$$

Lets have $\lambda_1, \lambda_2, \dots \lambda_m$ & $\mu_1, \mu_2, \dots \mu_l$

Define (for $\lambda_i \ge 0$)

$$L(x, \lambda, \mu) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{j=1}^{l} \mu_j f_j(x)$$

$$\le f_0(x) \quad \text{(if } x \text{ is feasible)}$$



$$\min_x L(x, \lambda, \mu) \le f_0(x^*)$$

s.t. $\lambda \ge 0$

$$\boxed{\therefore \max_{(\lambda, \mu)} \left[\min_{\substack{x \\ \lambda \ge 0}} L(x, \lambda, \mu) \right] \le f_0(x^*)}$$

LAGRANGIAN DUAL

$$g(\lambda, \mu) = \min_x L(x, \lambda, \mu) \quad \text{s.t. } \lambda \ge 0$$

Dual function

Want to max $g(\lambda, \mu)$

If λ^*, μ^* are p.t. of maxima

$$\boxed{g(\lambda, \mu) \le g(\lambda^*, \mu^*) \le f_0(x^*)}$$

WEAK DUALITY THM

$|f_0(x^*) - g(\lambda^*, \mu^*)| \leftarrow$ Duality Gap

Why convex function imp ??

If P is convex problem $\xrightarrow{\text{Usually}}$ duality gap Zero (Strong Duality)

KKT Conditions :-

Let x^* & (λ^*, μ^*) be any primal & dual optimal points with zero duality gap. Then we must have

1) $f_i(x^*) \le 0 \;\; \forall i$ & $f_j(x^*) = 0 \;\; \forall j$ (PF)

2) $\lambda^* \ge 0$ (DF)

3) $\lambda_i^* f_i(x^*) = 0 \quad \forall i$ (CS)

4) $\nabla_x L(x, \lambda^*, \mu^*) = 0$

USE THIS FOR SVM