
IT 584 -Approximation Algorithms

Quiz 1

February 28, 2023

Duration: 45 minutes

Maximum Marks: 10

Note: In case you have any doubt, make an appropriate assumption, state the assumption clearly, and proceed. Proofs should be complete

1. (5 marks) Given a graph $G = \{V, E\}$, a subset of edges E_M of E is said to be a matching if no two edges of E_M share a vertex. A matching of maximum cardinality in E is said to be a maximum matching. A matching that is maximal under inclusion is called a maximal matching. Both maximum matching and maximal matching can be solved in polynomial time. Design a 2-approximation algorithm for the problem of finding a minimum cardinality maximal matching in an undirected graph

Hint : Use the polynomial time algorithm to get maximum matching, prove and use a relation between size of any maximal matching and maximum matching and try to lower bound the OPT for the required problem.

2. (2 marks) Suppose that it is known that the number of items produced in a factory during a week is a random variable with mean 50.
 - (a) What can be said about the probability that this weeks production will exceed 75?
 - (b) If the variance of a weeks production is known to equal 25, then what can be said about the probability that this weeks production will be between 40 and 60?

3. (1 mark) (a) In an election with two candidates using paper ballots, each vote is independently misrecorded with probability $p = 0.02$. Use a Chernoff bound to bound the probability that more than 4% of the votes are misrecorded in an election of 1,000,000 ballots.

(2 marks) (b) Assume that a misrecorded ballot always counts as a vote for the other candidate. Suppose that candidate A received 510,000 votes and that candidate B received 490,000 votes. Use Chernoff bounds to bound the probability that candidate B wins the election owing to misrecorded ballots