IT 584 Approximation Algorithms

Tutorial on Markov, Chebysev and Chernoff bounds

[1] A random variable X is always strictly larger than -100. You know that $E[X] = -60$. Give the best upper bound you can on $P(X \geq -20)$.

[1] I have taken the liberty to use many of the questions from the course CS 328 2020 at IIT Gandhinagar offered by Prof. Anirban Dasgupta

Question 3

A post office handles, on average, 10000 letters a day.

- a) Using Markov's inequality, what can be said about the probability that it will handle at least 15000 letters tomorrow ?
- b) Suppose now that the variance σ^2 in the number of letters per day is 2000. Using Chebyshev's inequality what can be said about the probability that this post office handles between 8000 and 12000 letters tomorrow?
- c) Using Chebyshev's inequality how can we bound the probability that it will handle at least 15000 letters tomorrow? How does it compare with the bound in (a).

Question

A casino is testing a new class of simple slot machines. Each game, the player puts in \$1, and the slot machine is supposed to return either \$3 to the player with probability $4/25$, \$100 with probability $1/200$, or nothing with all remaining probability. Each game is supposed to be independent of other games. The casino has been surprised to find in testing that the machines have lost \$10,000 over the first million games. Derive a Chernoff bound for the probability of this event.

Question

We plan to conduct an opinion poll to find out the percentage of people in a community who want its president impeached. Assume that every person answers either yes or no. If the actual fraction of people who want the president impeached is p, we want to find an estimate X of p such that $Pr(|X - p| \leq \epsilon p) > 1 - \delta$ for a given ϵ and δ, with $0 < \epsilon, \delta < 1$.
We query N people chosen independently and uniformly at random from the community and output the fraction of them who want the president impeached. How large should N be for our result to be a suitable estimator of p? Use Chernoff bounds, and express N in terms of p, ϵ, and δ. Calculate the value of N from your bound if $\epsilon = 0.1$ and $\delta = 0.05$ and if you know that p is between 0.2 and 0.8.

Question

We have n i.i.d. Bernoulli (p) random variables. Let A be the average of these random variables. The following plots are on log-scale (in y-axis) and show various bounds and approximations for the $Pr[A > 1.1E[A]]$ as a function of n (depicted in linear scale in x-axis). The scatterpoints represent the actual probability of the deviation. In this set of plots, $p = 0.4$. Label which one corresponds to Markov's Inequality, to Chebyshev's Inequality, and to a Chernoff Bound.