# IT 584 - Approximation Algorithms

## End Semester Exam

May 10, 2023
Duration: 150 minutes
Maximum Marks: 40

***Note: This is an open notebook exam. You can have your notebook/handwritten notes/printed copies of handwritten notes with you. However printed scribe notes/textbooks are not allowed. In case you have any doubt, make an appropriate assumption, state the assumption clearly, and proceed. Proofs should be complete and general. Write to the point. Verbose answers with no actual relevant content will be penalized.***

1. We have a standard six-sided die. Let $X$ be the number of times that a 6 occurs over $n$ throws of the die. Let $p$ be the probability of the event $X \geq n/4$ . Compare the best upper bounds on $p$ that you can obtain using Markov's inequality, Chebyshev's inequality, and Chernoff bounds. **[2+2+3 Marks]**

2. Give a greedy algorithm for **unweighted** version of set cover problem (all weights are 1) by modifying the greedy algorithm for weighted set cover. Instead of just replacing each weight by one, explain what it means in terms of the set. Apply it on the following data. Compare the optimal solution(obtained by observation) and the greedy solution. $E = \{1, 2, 3, \ldots, 13\}, S1 = \{1, 2\}, S2 = \{2, 3, 4, 5\}, S4 = \{6, 7, 8, 9, 10, 11, 12, 13\}, S4 = \{1, 3, 5, 7, 9, 11, 13\}, S5 = \{2, 4, 6, 8, 10, 12, 13\}$. **[3+1+3 Marks]**

3. Apply the Christofides algorithm for TSP to the graph of cities given in Figure 1. Explain each step of the algorithm clearly with a corresponding figure. Finally give the solution obtained by the algorithm. **[8 Marks]**

4. Consider that for an instance of the Uncapacitated Facility Location problem you have 3 facilities say $1, 2$ and 3 and 3 clients $a, b, c$. The fixed cost is 1 for each facility. Consider the cost matrix $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ where the rows represent the facilities and columns the clients. Write down the integer program corresponding to this instance of the UFL. Give an LP

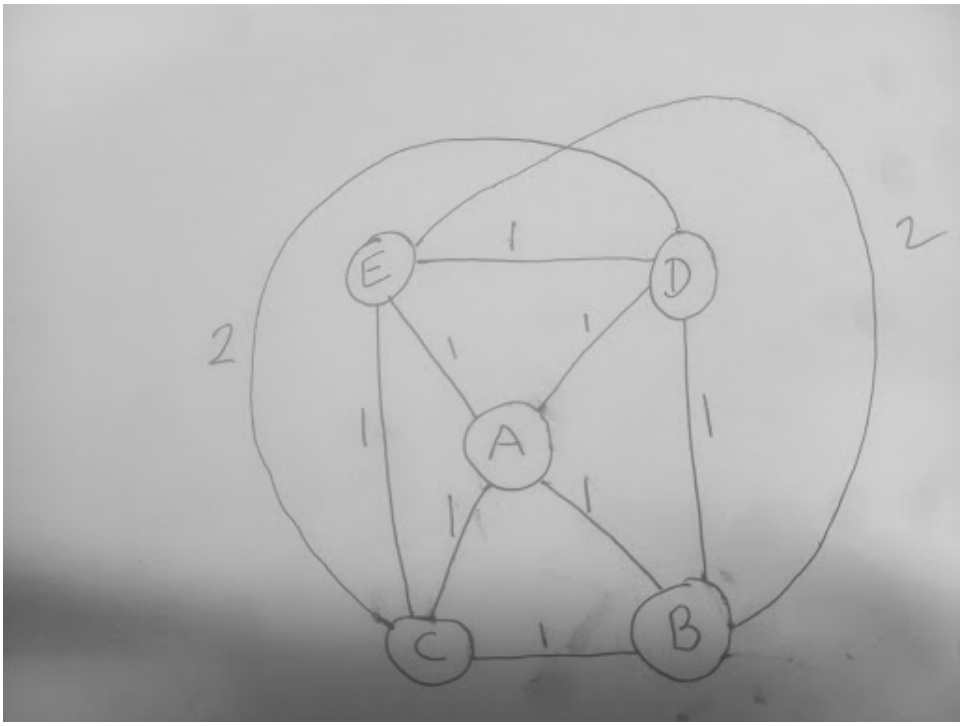


Figure 1: Graph of 5 cities with costs

relaxation for the same and give the dual of the LP. Give an optimal solution for the problem by observation. Please state and signify the meaning of the variables you use in your integer and LP formulations. **[2+1+2+1 Marks]**

5. Give a tight example for the greedy approximation of $k$-center problem. [***Hint****: You can choose $n$= 5 points and $k$=2*] **[4 Marks]**

6. Consider the modification of set cover problem known as *partial cover problem*, in which one finds a collection of subsets indexed by $I$ that minimizes $\sum_{j \in I} w_j$ such that

$$|\bigcup_{j \in I} S_j| \geq p|E|$$

where $0 < p < 1$ is some constant. Give a polynomial time algorithm to find a solution to the partial cover problem in which the value is no more than $c(p)OPT$ where $c(p)$ is a constant that depends on $p$ and $OPT$ is the value of the optimal solution to the **set cover problem**. You need to give the algorithm and also derive the approximation factor of the algorithm. **[2+6 Marks]**