Homework 3

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PHYS 273

60/60

1a.) $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{(17\ N/m)}{(0.22\ kg)}} = \boxed{8.790\ rad/s}$

1b.) $Q = \frac{\omega_0}{\gamma} \Rightarrow \gamma = \frac{\omega_0}{Q} \Rightarrow \frac{b}{m} = \frac{\omega_0}{Q}$

$$b = \frac{m\sqrt{\frac{k}{m}}}{Q} = \frac{(0.22kg)\sqrt{\frac{(17N/m)}{(0.22kg)}}}{(3.45)}$$

$$= \boxed{0.561\ kg/s}$$

1c.) $\gamma < 2\omega_0$, so this oscillator is underdamped.

$$\omega_u = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}} = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$$= \sqrt{\frac{(17N/m)}{(0.22kg)} - \frac{(0.5606\ kg/s)^2}{4(0.22\ kg)^2}}$$

$$= \boxed{8.698\ rad/s}$$

The angular frequency ω_u is less than ω_0.

1d.) $\omega_u t = 2\pi Q \Rightarrow t = \frac{2\pi Q}{\omega_u} = \frac{2\pi(1)}{(8.6977\ rad/s)} = 0.7224\ s$

$$A(t) = Ae^{-\gamma t/2}$$

$$A(0.7224) = (10\ cm)e^{-\frac{(0.5606\ kg/s)(0.7224\ s)}{2(0.22\ kg)}} = \boxed{3.984\ cm}$$

1e.) $t = \frac{2\pi(2)}{(8.6977\ rad/s)} = 1.4448\ s$

$$A(1.4448) = (10cm)e^{-\frac{(0.5606\ kg/s)(1.4448\ s)}{2(0.22\ kg)}} = \boxed{1.587\ cm}$$

1f.) $F = \frac{F_d}{m} = \frac{1.6\ \text{N}}{0.22\ \text{kg}} = \boxed{7.273\ \text{m/s}^2}$

1g.) $\omega = \sqrt{\omega_0^2 - \frac{\gamma^2}{2}} = \sqrt{(8.7905\ \text{rad/s})^2 - \frac{(2.5480\ \text{rad/s})^2}{2}}$

$= \boxed{8.604\ \text{rad/s}}$

1h.) $A = \frac{F}{\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}} = \frac{(7.2727\ \text{m/s}^2)}{\sqrt{((8.7905\ \text{rad/s})^2 - (0.25\ \text{rad/s})^2)^2 + (2.5480\ \text{rad/s})^2(0.25\ \text{rad/s})^2}}$

$= \boxed{0.0942\ \text{m}}$

1i.) $A = \frac{F}{\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}} = \frac{(7.2727\ \text{m/s}^2)}{\sqrt{((8.7905\ \text{rad/s})^2 - (0.50\ \text{rad/s})^2)^2 + (2.5480\ \text{rad/s})^2(0.50\ \text{rad/s})^2}}$

$= \boxed{0.0944\ \text{m}}$

This is almost exactly the same as the amplitude I calculated in part h. The two values differ by ≈ 0.0002 m.

1j.) $A = \frac{(7.2727\ \text{m/s}^2)}{\sqrt{((8.7905\ \text{rad/s})^2 - (8.70\ \text{rad/s})^2)^2 + (2.5480\ \text{rad/s})^2(8.70\ \text{rad/s})^2}}$

$= \boxed{0.327\ \text{m}}$

1k.) $\frac{A_{high}}{A_{low}} = \frac{0.3272\ \text{m}}{0.09419\ \text{m}} = \boxed{3.474}$

The Q of the system is 3.45, which differs from this ratio by ≈ 0.024, so the values are approximately the same.

1l.)

$$A = \frac{(7.2727\ m/s^2)}{\sqrt{((8.7905\ rad/s)^2-(20.0\ rad/s)^2)^2+(2.5480\ rad/s)^2(20.0\ rad/s)^2}}$$

$$= \boxed{0.0223\ m}$$

This amplitude is smaller than the low-frequency amplitude from part h.

2a) $A = F((\omega_0^2-\omega^2)^2+\gamma^2\omega^2)^{-1/2}$ ⇐ Want to maximize

$$\frac{dA}{d\omega} = -\frac{1}{\cancel{2}}F((\omega_0^2-\omega^2)^2+\gamma^2\omega^2)^{-3/2}(\cancel{2}(\omega_0^2-\omega^2)(-2\omega)+\cancel{2}\gamma^2\omega)$$

$$= \frac{F(2\omega(\omega_0^2-\omega^2)-\gamma^2\omega)}{((\omega_0^2-\omega^2)^2+\gamma^2\omega^2)^{3/2}} = 0$$

$$F(2\omega(\omega_0^2-\omega^2)-\gamma^2\omega) = 0$$

$$2\omega(\omega_0^2-\omega^2)-\gamma^2\omega = 0$$

$$\omega(2\omega_0^2-2\omega^2-\gamma^2) = 0$$

Interested in when $-2\omega^2+2\omega_0^2-\gamma^2 = 0$

$$-2\omega^2 = -2\omega_0^2+\gamma^2$$

$$\omega^2 = \omega_0^2 - \frac{\gamma^2}{2}$$

$$\boxed{\omega = \sqrt{\omega_0^2-\frac{\gamma^2}{2}}}$$

2b.)

$$A = \frac{F}{\sqrt{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}}$$

$$A_{max} = \frac{F}{\sqrt{\left(\cancel{\omega_0^2}-\left(\cancel{\omega_0^2}-\frac{\gamma^2}{2}\right)\right)^2+\gamma^2\left(\omega_0^2-\frac{\gamma^2}{2}\right)}}$$

$$= \frac{F}{\sqrt{\frac{\gamma^4}{4}+\gamma^2\omega_0^2-\frac{\gamma^4}{2}}} \Longleftarrow \gamma^2\omega_0^2-\frac{\gamma^4}{4} = \gamma^2\left(\omega_0^2-\frac{\gamma^2}{4}\right)$$

$$= \boxed{\frac{F}{\gamma\sqrt{\omega_0^2-\frac{\gamma^2}{4}}}}$$

3a.) $m = 16\,g \Rightarrow 0.016\ kg$, $K = 0.24\ N/cm \Rightarrow 24\ N/m$

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{(24\ N/m)}{(0.016\ kg)}} = \boxed{38.730\ rad/s}$$

$$Q = \frac{\omega_0}{\gamma} = \frac{m\omega_0}{b} = \frac{(0.016\ kg)(38.7298\ rad/s)}{(0.060\ kg/s)}$$

$$= \boxed{10.328}$$

3b.) $F = \frac{F_d}{m} = \frac{0.75\ N}{0.016\ kg} = \boxed{46.875\ m/s^2}$

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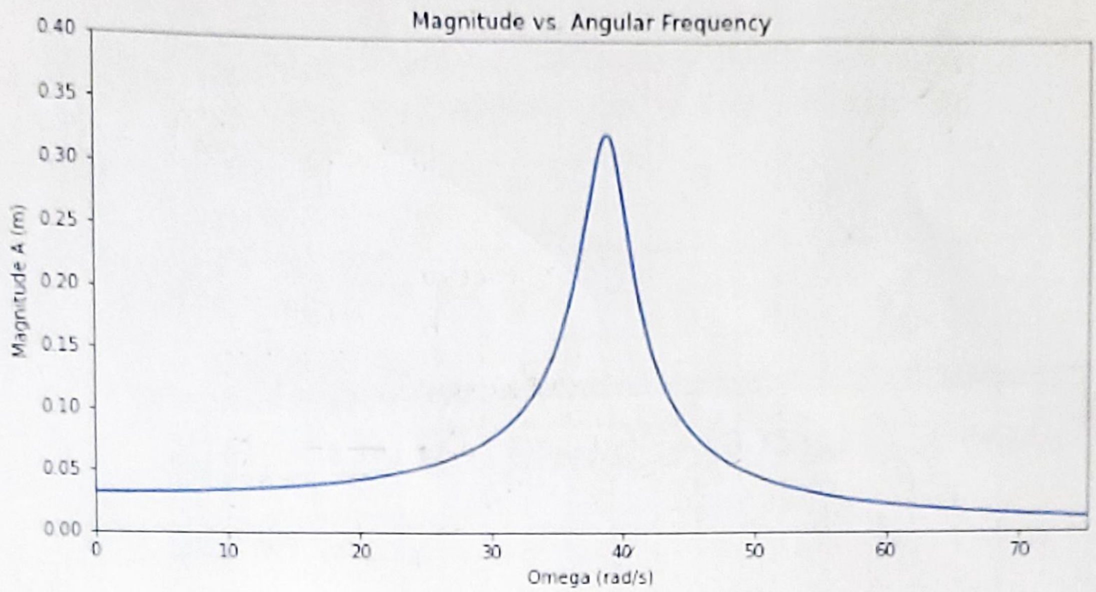
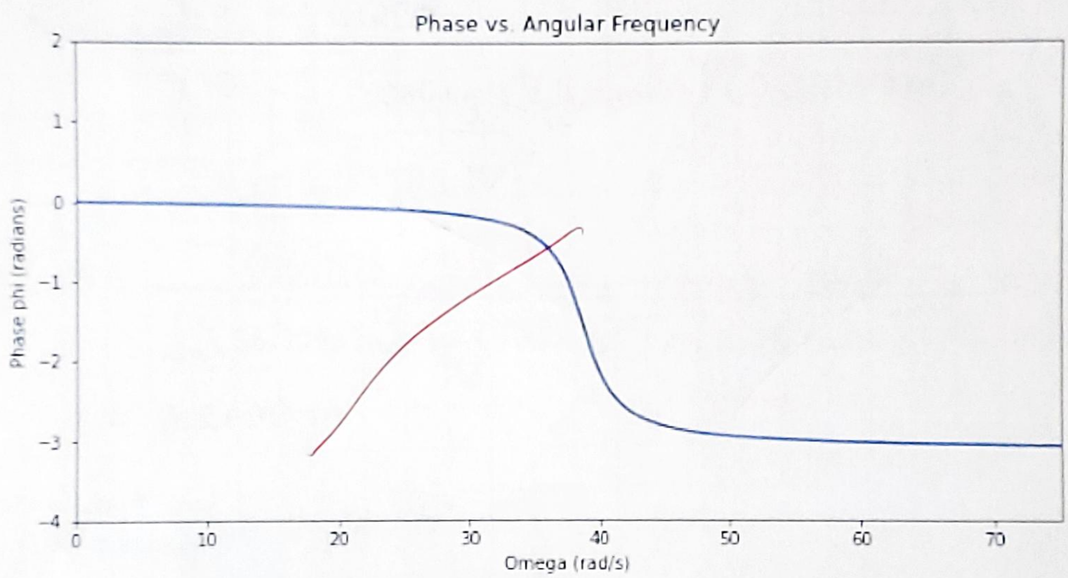
Run Code

```
plt.ylabel('Phase phi (radians)')
plt.title('Phase vs. Angular Frequency')
plt.plot(w2,phi2,color='blue')

plt.show()
```





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3d) $A_{max} = \dfrac{F}{\gamma\sqrt{\omega_0^2 - \frac{\gamma^2}{4}}}$

$= \dfrac{(46.875\ m/s^2)}{(3.75\ rad/s)\sqrt{(38.7298\ rad/s)^2 - \frac{(3.75\ rad/s)^2}{4}}}$

$= \boxed{0.323\ m}$

The peak amplitude on the graph appears to be a little less than halfway between 0.30 and 0.35, so ≈ 0.32 m. This agrees with my calculated peak amplitude.

3e) $A = \dfrac{F}{\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}}$

$= \dfrac{(46.875\ m/s^2)}{\sqrt{((38.7298\ rad/s)^2 - (8.0\ rad/s)^2)^2 + (3.75\ rad/s)^2(8.0\ rad/s)^2}}$

$= 0.03264\ m$

$\langle P_{damping}\rangle = -\frac{1}{2}b\omega^2A^2$

$= -\frac{1}{2}(0.060\ kg/s)(8.0\ rad/s)^2(0.03264\ m)^2$

$= \boxed{-0.00205\ W}$

3f) $A = \dfrac{(46.875\ m/s^2)}{\sqrt{((38.7298\ rad/s)^2 - (40\ rad/s)^2)^2 + (3.75\ rad/s)^2(40\ rad/s)^2}}$

$= 0.2600\ m$

$\langle P_{damping}\rangle = -\frac{1}{2}b\omega^2A^2$

$= -\frac{1}{2}(0.060\ kg/s)(40\ rad/s)^2(0.2600\ m)^2$

$= \boxed{-3.245\ W}$

3g.) $A = \dfrac{(46.875\ m/s^2)}{\sqrt{((38.7298\ rad/s)^2 - (200\ rad/s)^2)^2 + (3.75\ rad/s)^2(200\ rad/s)^2}}$

$= 0.001217\ m$

$\langle P_{damping} \rangle = -\frac{1}{2} b\omega^2 A^2$

$= -\frac{1}{2}(0.060\ kg/s)(200\ rad/s)^2(0.001217\ m)^2$

$= \boxed{-0.00178\ W}$

4a.) $F_{total} = ma$

$F_d \cos\omega t = ma$

$F_d \cos\omega t = m\ddot{x}$

$\boxed{\ddot{x} = \frac{F_d}{m}\cos\omega t}$

4b.) $x(t) = Ce^{i\omega t}$ $\quad \dot{x} = i\omega Ce^{i\omega t}$ $\quad \ddot{x} = i^2\omega^2 Ce^{i\omega t} = -\omega^2 Ce^{i\omega t}$

$\ddot{x} = \frac{F_d}{m}\cos\omega t$

$\Rightarrow -\omega^2 C\cancel{e^{i\omega t}} = \frac{F_d}{m}\cancel{e^{i\omega t}}$

$\boxed{C = \frac{-F_d}{m\omega^2}}$

4c.) $\frac{\text{output}}{\text{input}} = \frac{C}{F_d} = \boxed{\frac{-1}{m\omega^2}}$

1d.) for damped, driven oscillator:

$$\frac{C}{F_d} = \frac{1}{m}\frac{1}{\omega_0^2-\omega^2+i\gamma\omega}$$

When ω is very large, $\omega \gg \omega_0$, so

$$\frac{C}{F_d} = \frac{1}{m}\frac{1}{\cancel{\omega_0^2}-\omega^2+\cancel{i\gamma\omega}}$$

$$= \boxed{-\frac{1}{m\omega^2}}$$

This is the same result as what I got for the unconstrained particle.