Q1) $V(x) = -V_0 \exp\left[-\left(\frac{x}{l}\right)^2 + 2\left(\frac{x}{l}\right)\right]$

a) $\frac{dV}{dx} = -V_0 \exp\left[-\left(\frac{x}{l}\right)^2 + 2\left(\frac{x}{l}\right)\right]\left(\frac{-2x}{l^2} + \frac{2}{l}\right) = 0$

$\therefore \frac{2x}{l^2} = \frac{2}{l}$

$\Rightarrow \underline{\underline{x = l}}$

$\frac{d^2V}{dx^2} = V_0 \exp\left[-\left(\frac{x}{l}\right)^2 + 2\left(\frac{x}{l}\right)\right] \cdot \frac{2}{l^2}$

$- V_0 \exp\left[-\left(\frac{x}{l}\right)^2 + 2\left(\frac{x}{l}\right)\right]\left(\frac{-2x}{l^2} + \frac{2}{l}\right)$

$\therefore \left.\frac{d^2V}{dx^2}\right|_{x=l} = V_0 \cdot \exp(1) \cdot \frac{2}{l^2} > 0$

$\therefore$ we have a minimum at $x = l = x_0$

b) $V(x_0) = -V_0 \exp(1)$

$V'(x_0) = 0$ → we have a minimum at $x = x_0$

$V''(x_0) = \frac{2V_0}{l^2} \exp(1)$.

$$\therefore V(x) = \sum_n \frac{V^{(n)}(x_0)}{n!}(x-x_0)^n$$

$$= V(x_0) + \frac{V'(x_0)}{1!}(x-x_0) + \frac{V''(x_0)}{2!}(x-x_0)^2$$

$$= -V_0\exp(1) + \frac{0}{1!}(x-x_0) + \frac{2V_0}{l^2 \cdot 2!}(x-x_0)^2 + \cdots$$

c) $$\omega = \sqrt{\frac{V''(x_0)}{m}} = \sqrt{\frac{2V_0\exp(1)}{ml^2}}$$

Q2)

a) I for a rod about its end $= \frac{1}{3}ml^2$

$$\omega = \sqrt{\frac{mgd}{I}} = \sqrt{\frac{\cancel{m}g\,l/2}{1/3\,\cancel{m}\,l^{\cancel{2}}}} = \sqrt{\frac{3g}{2l}}$$

distance of pivot from com.

b) we know: $$\ddot{\theta} + \frac{mgl}{I}\theta = 0$$

$$\Rightarrow \ddot{\theta} = -\omega^2\theta \quad ; \quad \omega \equiv \sqrt{\frac{mgd}{I}}$$

$$\therefore \theta(t) = A\cos(\omega t + \phi)$$

It is given that $\theta(0) = 0$

$$\Rightarrow \theta(0) = A\cos(\phi) = 0$$

$$\therefore \cos(\phi) = 0$$

$$\dot{\theta}(t) = -A\omega \sin(\omega t + \phi)$$

$$\dot{\theta}(0) = -A\omega\sin(\phi) = \Omega_0$$

$$\Rightarrow A = \frac{-\Omega_0}{\omega \sin(\phi)}$$

we know, when $\cos(\phi) = 0$, $\sin(\phi) = \pm 1$

$\therefore$ max A occurs when $\sin(\phi) = -1$

$$\therefore \quad \underline{\underline{A_{max} = \frac{\Omega_0}{\omega}}}$$