# Homework 9

## **Problem 1**

Two sections of string are spliced together and stretched so that they have equal tension. In the section on the left, wave pulses travel at 10.0 m/s, while in the section on the right they travel at 20.0 m/s.

(a) Which section of string has a higher mass per unit length? Justify your answer.

(b) If the impedance of the string in the section on the right is 12 kg/s, what are the linear mass densities $\mu$ for each section of string? (Hint: the impedance depends on the tension as well as the string's linear mass density.)

(c) Suppose a transverse wave pulse coming from the left end has a maximum (transverse) displacement of 6 mm. When it reaches the splice point, it splits into a transmitted pulse and a reflected pulse. What are the speed and the maximum displacement of the transmitted pulse? Also, what are the speed and the maximum displacement of the reflected pulse, and is it inverted or same-side ("upright") relative to the incident pulse? (Hint: check your work using the fact that the transmitted max displacement should equal the sum of the incident and reflected max displacements.)

## **Problem 2**

On a standard full-size violin, the length of the string that is free to vibrate (between the bridge and the nut) is $L$=328 mm, according to something I found on the web (http://www.makingtheviolin.com/Measurements). Both ends of the string are effectively fixed by resting in slight grooves at those two points. The four strings are normally tuned to E ($f$=659 Hz), A ($f$=440 Hz), D ($f$=294 Hz) and G ($f$=196 Hz). Note that these are not angular frequencies ($\omega$). Be careful with units in all of your calculations and answers; for instance, watch out for grams versus kilograms, and millimeters versus centimeters versus meters.

(a) In the string's fundamental (lowest-frequency) vibrational mode, that length is *half* of a wavelength (see Figure 24 in Morin Chapter 4, for instance). So what is the wavenumber $k$ for this mode? (Include appropriate units, of course.)

(b) What are the wavelengths ($\lambda$), wavenumbers ($k$) and frequencies ($f$) for the next three modes (harmonics) of the string? If you need to pick a specific string, use the A string.

(c) We normally think of violin strings forming standing waves, but one could also produce a traveling wave. What would be the speed $v$ of a traveling wave on the A string? (Don't confuse $v$ with the Greek letter nu, $\nu$, which Morin uses to represent frequency in Hz.)

(d) Musical instrument strings can be made in various ways, some with multiple layers, but here let's consider a simple, solid steel string. Another web search told me that the diameter of a violin A string is about 0.8 mm (so the radius is half of that). What, then, is the string's mass per unit length, $\mu$? (You will have to look up the density of steel. Your answer for $\mu$ should turn out to be somewhere between 0.001 and 0.01 kg/m (SI units), or between 0.01 and 0.1 g/cm.)

(e) What tension (in newtons) must the A string be tightened to in order to play at $f = 440$ Hz?

## Problem 3

(a) Wikipedia tells me that "dry air contains 78.09% nitrogen, 20.95% oxygen, 0.93% argon, 0.04% carbon dioxide, and small amounts of other gases." (Those four percentages add up to a little over 100%, which I assume is due to rounding and means that there is a negligibly small amount of other gases.) Use those numbers to calculate the average mass of an "air molecule" in kilograms. You should find that it is somewhere around $5\times10^{-26}$ kg.

(b) Estimate the temperature of the room you are in right now; try to be accurate within a couple of degrees. Convert that temperature to Kelvin. (Hint: your answer surely should be somewhere between 273 K and 373 K, the freezing and boiling points of water rounded to the nearest integer degree.)

(c) In lecture, we found an expression for the speed of sound involving the temperature (must be in Kelvin) and the mass of a gas molecule (must be in kg if you are using the Boltzmann constant with its usual SI units). Use that plus your answers to parts a and b to calculate the speed of sound in the room you're in right now in m/s. (Since air is mostly diatomic nitrogen and oxygen, you may use the gamma value for diatomic molecules. The argon and $CO_2$ aren't diatomic but they make up only about 1% of the air.) It should be close to the typical speed mentioned in Morin and elsewhere, but probably not exactly the same.

(d) How different is the speed of sound outdoors on a winter day? To answer that, look up an average January temperature for somewhere in Maryland and repeat the part c calculation using that temperature. Do you get a faster or slower speed? By how many percent does that differ from the room-temperature speed you found in part c?

## Problem 4

Calculate the speed of sound in pure helium gas at the same temperature you used in part 1b. Note that helium is a monatomic gas.