Homework 2

Problem 1

Verify that x = A $e^{-\alpha t}$ cos (ωt) is a possible solution to the equation

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x = 0$$

and find α and ω in terms of γ and ω_0

Problem 2

Consider an oscillator with a mass of 3 kg, a spring constant of 12 N/m, and damping of 1.5 kg/s. (You can tell from the units whether I am giving you a b value or a γ value describing damping.)

(a) Determine whether this oscillator is under-damped, critically damped, or overdamped.

(b) If set into motion, how long (in seconds) will it take the amplitude of oscillations to be reduced by a factor of $1/e$? Anything which grows or decays following an exponential trend can be said to have a "time constant" defined by how long it takes to change by a factor of e, so in this case we can call this the *damping time constant* of the system.

(c) Generalize that: for an oscillator with mass m, spring constant k and damping b, what is the damping time constant in terms of one or more of those quantities. (Hint: when you think you have that expression, check that it has units of time.)

(d) For this system, how long (in seconds) will it take the amplitude of oscillations to be reduced by a factor of 2, i.e. to be half as large? Compare your answer to what you got in part b.

Problem 3

An (un-driven) under-damped oscillator oscillates at a frequency which is slightly lower than $\omega_0 = \sqrt{k/m}$. However, it is only a little lower if the damping is small.

(a) Find out what damping would make the oscillation frequency 10% lower than ω_0 . Express the damping γ as some multiple of ω_0 . (Hint: that multiple will be between 0.5 and 1.)

(b) What is the Q value corresponding to the damping in part a? In other words, the Q that would make the oscillation frequency 10% lower than ω_0 ?

(c) If, instead, $Q = 6.0$, how much would the oscillation frequency differ from ω_0 ? Express this as a percentage. (Hint: it is some value less than 2%.)

Problem 4

An object of mass 0.2kg is hung from a spring whose spring constant is 80N/m. The object is subject to a resistive force, -bv, where v is the velocity.

(a) Set up the differential equation of motion for the free oscillations of the system.

(b) If the damped frequency is $\frac{\sqrt{3}}{2}$ of the undamped frequency, what is the value of the constant b?

(c) What is the Q of the system, and by what factor is the amplitude of the oscillation reduced after 10 complete cycles?

Problem 5

According to classical electromagnetic theory an accelerated electron radiates energy at the rate Ke^2a^2/c^3 where K = $6*10^9$ Nm^2/C^2, e = electronic charge C, a = instantaneous acceleration m/s^2, and c = speed of light m/s.

(a) If an electron were oscillating along a straight line with frequency ν (Hz) and amplitude A, how much energy would it radiate away during 1 cycle? Assume that the motion is described adequately by x = A sin $2\pi\nu$t during any one cycle.
(b) What is the Q of this oscillator?

(c) Putting for ν a typical optical frequency (i.e., for visible light) estimate numerically the approximate Q and the "half-life" of the radiating system.