Homework 5

Problem 1

(a) Use Desmos or a graphing calculator (or whatever) to graph the following function:

$$f(t) = 3sin\,(8t)cos\,(2t)$$

(i) What is the period of one "beat", i.e. one bump in the "envelope" of the function?
(ii) How does the period compare to the period of the two sinusoidal functions which are being multiplied together?

(b) Use a trig identity to write $f(t)$ as a *sum* of two sinusoidal functions.

(c) Calculate the Fourier transform of $f(t)$, i.e. $\tilde{f}(\omega)$

(d) Sketch a graph of the Fourier transform $\tilde{f}(\omega)$ as a function of ω, clearly indicating where the function is zero and at what frequencies the function is non-zero. When the function is non-zero, what is the function's value?

(e) What frequencies would be nonzero in the Fourier transform of $g(t) = 10sin(3t)\,cos\,(5t)$?

(f) Calculate the Fourier transform of $\delta(t - 4)$. (Hint: this uses the "selection" property of the delta function when used in the integrand of an integral.) How can you describe it?

Problem 2

Find $\tilde{f}(k)$, the Fourier transform of the function $f(x)$ given by:

$f(x) = e^{-bx}$ for all $x \geq 0$, where b is a positive constant;

$f(x) = 0$ for all $x < 0$.

Problem 3

Use the definition of the Fourier transform to show the following:

(a) Show that if $f(x)$ is an odd, real-valued function, then the Fourier transform of $f(x)$ written as $C(k)$ must be a purely imaginary-valued function. (Hint: use Euler's formula to expand the integrand in the integral expression for $C(k)$.)

(b) If the function $g(t)$ has Fourier transform $\tilde{g}(\omega)$, show that the Fourier transform of $g(t - s)$ for some constant parameter s is $e^{-i\omega s}\tilde{g}(\omega)$. Use that result to explain how time-shifting a function changes its Fourier transform: does it change the magnitude, the phase, or both?

Problem 4

Find the real-valued Fourier series representation (that is, the one involving cosines and sines) of the function $f(x) = \left|sin\left(\frac{\pi}{L}x\right)\right|$, where the vertical bars mean to take the absolute value. In other words, find all the coefficients, including a_0. (Hint: use trig sum formulas, similar to what Morin does in section 3.1.) In the case of the a_n coefficients, your calculation will produce two terms which are added, each involving n. Combine those terms with a common denominator; you should find that a_n is proportional to $\frac{1}{(4n^2-1)}$.