mechanical oscillators: Energy is converted between kinetic and potential.

Electrical oscillators: Energy is converted between electric ($\vec{E}$ field) & magnetic ($\vec{B}$ field).

① **capacitor:** device that stores energy in an electric field.

→ Ex: parallel plate capacitor.

Each small volume of space (dV) with an electric field $\vec{E}$ stores a small amount of electric energy (dU_E):

$$\underset{\substack{\uparrow \\ \text{Big } U \\ \text{energy}}}{dU_E} = \underbrace{\frac{1}{2}\epsilon_0 |\vec{E}|^2}_{u_E} dV \quad , \quad u_E = \frac{1}{2}\epsilon_0 |\vec{E}|^2$$

→ electric energy density of free space; unit: $\frac{J}{m^3}$.

↳ small u → energy density.

The total energy stored is:

"big U" → $$U_E = \int_{R^3} \frac{1}{2}\epsilon_0 |\vec{E}|^2 dV = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV.$$

② **Inductor:** A device that stores energy in magnetic field.

→ Ex: A simple solenoid.

$$\therefore dU_B = \frac{1}{2\mu_0}|\vec{B}|^2 dV \quad ; \quad u_B = \frac{1}{2\mu_0}|\vec{B}|^2.$$

$$\therefore\ U_B = \int_{\mathbb{R}^3} \frac{1}{2\mu_0} |\vec{B}|^2 \, dV = \frac{1}{2} L I^2 .$$

↳ self inductance.

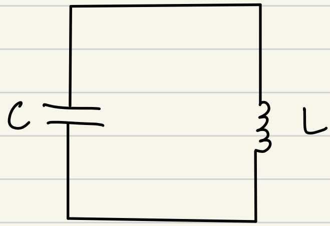
<u>Voltage rules:</u>

capacitor: $|V_C| = \left| \frac{1}{C} \cdot Q \right|$

Inductor: $|V_L| = \left| -\frac{d\Phi_B}{dt} \right| = \left| -\frac{d}{dt}(LI) \right| = \left| L\frac{dI}{dt} \right| .$

<u>LC oscillator:</u>

→ Simplest electric oscillator. Energy exchanged between electric and magnetic.



from voltage loop rule:

$$V_C + V_L = 0$$

$$\Rightarrow \frac{q}{C} + L\frac{dI}{dt} = 0$$

$$\Rightarrow \frac{q}{C} + L\frac{d^2q}{dt^2} = 0$$

$$\therefore\ \frac{1}{LC} q + \frac{d^2 q}{dt^2} = 0 .$$ → equation of the SHO.

$$\therefore\ q(t) = q_0 e^{i(\omega_0 t + \delta)} \ ; \quad \omega_0 \equiv \frac{1}{\sqrt{LC}} .$$

→ q_0 & δ are determine by initial conditions.

$$\therefore \quad I(t) = \dot{q}(t) = (i\omega_0)(q_0 e^{i(\omega_0 t + \delta)})$$
$$= i\omega_0 q(t).$$

$\hookrightarrow$ current has a phase shift of 90° compared to charge.

Energy:

$$U_E = \frac{1}{2C} q^2 = \frac{1}{2C}\left[q_0 \cos(\omega_0 t + \delta)\right]^2$$
$$= \frac{q_0^2}{2C} \cos^2(\omega_0 t + \delta).$$

$$U_B = \frac{1}{2} L I^2 = \frac{1}{2} L \left[\mathrm{Re}\left(q_0 i\omega_0 e^{i(\omega_0 t + \delta)}\right)\right]^2$$
$$= \frac{1}{2} L \left[-\omega_0 \sin(\omega_0 t + \delta)\, q_0\right]^2$$
$$= \frac{1}{2} q_0^2 \omega_0^2 L \sin^2(\omega_0 t + \delta)$$
$$= \frac{q_0^2}{2C} \sin^2(\omega_0 t + \delta).$$

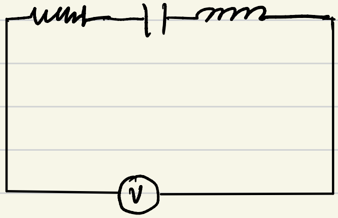
$\therefore$ total energy: $U_E + U_B = \frac{q_0^2}{2C}\left[\cos^2(\omega_0 t + \delta) + \sin^2(\omega_0 t + \delta)\right]$
$$= \underline{\underline{\frac{q_0^2}{2C}}}$$

Note:

mass with no friction $\longrightarrow$ LC circuit

$\downarrow$

with damping & driving $\longrightarrow$ LCR with power supply.



$$V = V_d \cos(\omega t)$$

$$\therefore \quad iR + L\frac{di}{dt} + \frac{q}{C} = V_d \cos(\omega t)$$

$$\Rightarrow \quad R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{C} = V_d \cos(\omega t)$$

$$\Rightarrow \quad \ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{LC}q = \frac{V_d}{L}\cos(\omega t).$$

$$\Rightarrow \quad \ddot{q} + \gamma\dot{q} + \omega_0^2 q = V\cos(\omega t)$$

where $\gamma = \dfrac{R}{L}$; $\omega_0^2 = \dfrac{1}{LC}$; $V = \dfrac{V_d}{L}$.

$$q(t) = \underbrace{q_n(t)}_{\text{transient}} + \underbrace{q_p(t)}_{\text{steady state}}$$

$$\therefore \quad q(t) = Q_n e^{-\gamma t/2} \cos(\omega_n t + \theta) + Q_p \cos(\omega t + \phi).$$

ω_n ↳ $\sqrt{\omega_0^2 - \gamma^2/4}$

$$\therefore \quad Q_p = \frac{V}{\sqrt{(\omega_0^2 - \omega_f^2)^2 - \gamma^2 \omega_f^2}}$$

$$\tan(\phi) = \frac{-\gamma\omega_f}{\omega_0^2 - \omega_f^2}$$

$$\langle P_{driving} \rangle = \frac{1}{2} b (\omega_f Q_p)^2 = \frac{1}{2} R (\omega_f Q_p)^2$$

$$= \frac{1}{2} R \omega_f^2 \frac{V^2}{(\omega_0^2 - \omega_f^2)^2 + \frac{R^2}{L^2} \omega_f^2}$$