

### Project 3 written

1) For any vector  $\vec{v}$ , let  $\hat{\vec{v}} = \frac{\vec{v}}{\|\vec{v}\|}$  (The vector unit length normalized to)

$$\text{Let } \hat{e}_3 = \hat{t} \quad (+z; \text{ into screen})$$

$$\hat{e}_2 = \hat{u} - (\hat{u} \cdot \hat{t}) \cdot \hat{t}$$

$$\hat{e}_2 = \hat{e}_1^\perp$$

$$\hat{e}_1 = \hat{t} \times \hat{e}_2$$

in retrospect I would have done this cross-product with  $\hat{u}$  instead of  $\hat{e}_2$ , which would have made calculating  $\hat{e}_2$  nicer as well

This is a left-handed frame

+x to the right

+y up

+z in

2)  $\vec{q} = Q - C$  (vector from C to Q)

$$d = \vec{q} \cdot \hat{e}_3 \quad (\text{depth from camera into screen})$$

$$\vec{q}' = \vec{q} \cdot \frac{\|\hat{e}_3\|}{d} = \frac{\vec{q}}{d} \quad (\text{vector from C to Q'})$$

$$Q' = C + \vec{q}'$$

are these also correct?

$$x = (\theta - Q') \cdot \hat{e}_1$$

$$y = (\theta - Q') \cdot \hat{e}_2$$

$$x = \vec{q}' \cdot \hat{e}_1$$

$$y = \vec{q}' \cdot \hat{e}_2$$

$$3) \vec{p} = P_1 - P_0$$

In order to intersect the line - not line segment -  $P_0 P_1$ ,  $Q_0$  and  $Q_1$  must be on opposite sides

$$\vec{v} = \vec{p}^\perp$$

$$a' = ((Q_0 - P_0) \cdot \vec{v})$$

$$b' = ((Q_1 - P_0) \cdot \vec{v})$$

$$\vec{a} = a' \cdot \vec{v}$$

$$\vec{b} = b' \cdot \vec{v}$$

$$\vec{a} + \vec{b} \text{ must equal } 0$$

$$\text{aka } \text{sign}((Q_0 - P_0) \cdot \vec{v}) \neq \text{sign}((Q_1 - P_0) \cdot \vec{v})$$

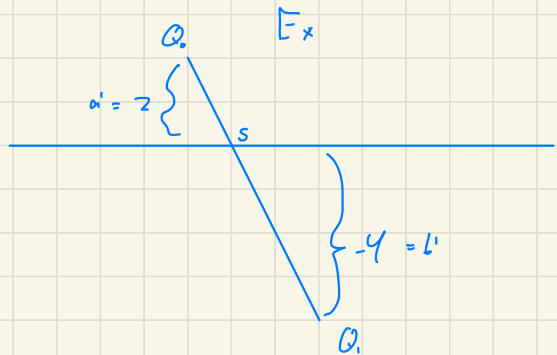
$$S = (1-s)Q_0 + sQ_1 \quad \text{when} \quad s = \frac{a'}{b' - a'}$$

$S$  must be between  $P_0$  and  $P_1$

$$\vec{z} = S - P_0$$

$$0 < \|\vec{z}\| < \|\vec{p}\|$$

$$\Rightarrow 0 < \frac{\|\vec{z}\|}{\|\vec{p}\|}$$



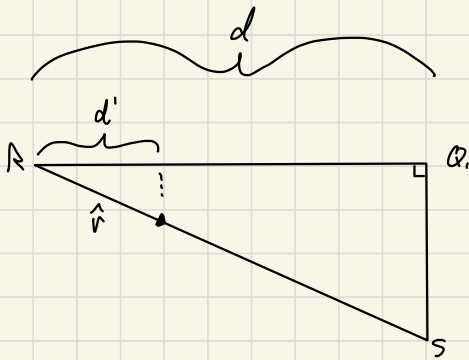
4) Let  $\vec{q}_2 = Q_2 - Q_1$   
 $\vec{q}_3 = Q_3 - Q_1$

The vector normal to the triangle is thus  $\vec{q}_1 \times \vec{q}_2$

$$\hat{n} = \hat{q}_1 \times \hat{q}_2$$

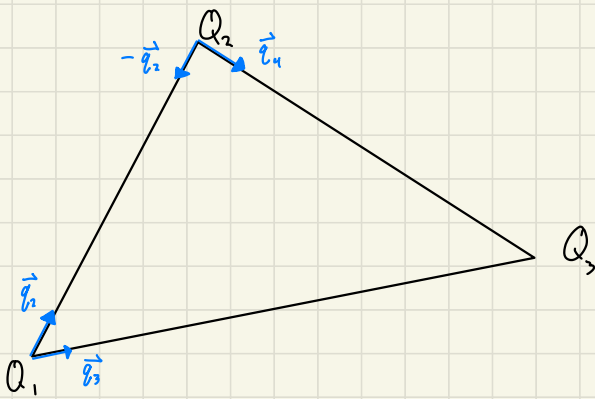
Distance from A to the plane is

$$d = (Q_1 - R) \cdot \hat{n} = \|Q_1 - R\|$$



$$S = R + \frac{d}{d'} \hat{r}$$

$S$  is on the plane, but maybe not a point of intersection



$S$  is in  $\Delta Q_1 Q_2 Q_3$

iff

$\{ (S - Q_1) \text{ is within } \angle \vec{q}_2 \vec{q}_3$

and

$(S - Q_2) \text{ is within } \angle -\vec{q}_2 \vec{q}_4$

$$\Rightarrow \text{sign}(\vec{q}_2 \times (S - Q_1)) = \text{sign}(\vec{q}_2 \times \vec{q}_3)$$

$$\text{and} \\ \text{sign}(\vec{q}_3 \times (S - Q_1)) = \text{sign}(\vec{q}_3 \times \vec{q}_2)$$

$$\text{sign}(-\vec{q}_2 \times (S - Q_2)) = \text{sign}(-\vec{q}_2 \times \vec{q}_4)$$

$$\text{and} \\ \text{sign}(\vec{q}_4 \times (S - Q_2)) = \text{sign}(\vec{q}_4 \times \vec{q}_2)$$