Samasya

Samasya is a mathematics discussion and problem solving club. We discuss a variety of mathematical topics and solve problems as well. We encourage participants to have a look at these problemsbefore the meeting. Discussion, however, will not be limited to these problems. Participants can bring their own problems or mathematical ideas they wish to discuss.

Date: 2nd October, 2015
Time: 9:00 p.m.
Venue: OPB LAN Room

Problem 1. Let sd be a function from $\mathbb{N}$ to $\mathbb{N}$ such that $sd(x)$ is the sum of digits of x when written in base 10. What is the value of $sd(sd(sd(4444^{4444})))$?

Problem 2. Let $\{a_n\}$ and $\{b_n\}$ be two sequences of real numbers. Also, suppose that $\{a_n\}$ is a subsequence of $\{b_n\}$ and $\{b_n\}$ is a subsequence of $\{a_n\}$. Do there exists such sequences if $\{a_n\} \neq \{b_n\}$? What if $\{a_n\}$ does not converge? But if $\{a_n\}$ does converge, and there exists a sequence $\{b_n\}$ with the mentioned property of being a subsequence of $\{a_n\}$ and vice versa, is it true that the two sequences must be the same?

Problem 3. A function f from $\mathbb{R}$ to $\mathbb{R}$ is said to have the intermediate value property if for all a, $b \in \mathbb{R}$ such that $a < b$, and for all c between $f(a)$ and $f(b)$, there exists an $x \in (a, b)$ such that $f(x) = c$. It turns out that even if a function has the intermediate value property, it need not be continuous. To show such a function is continuous, one must put additional constraints on it. It turns out that if the set $S_r = \{x \ : \ f(x) = r\}$ is closed for all rational numbers r, then the function f does turn out to be continuous. Prove it.

The past problems and solutions are available at samasya.github.io.