

# Samasya

Samasya is a mathematics discussion and problem solving club. We discuss a variety of mathematical topics and solve problems as well. We encourage participants to have a look at these problems before the meeting. Discussion, however, will not be limited to these problems. Participants can bring their own problems or mathematical ideas they wish to discuss.

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**Date:** 6<sup>th</sup> November, 2015

**Time:** 9:00 p.m.

**Venue:** OPB LAN Room

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**Problem 1.** Given a balance scale, and  $N$  coins, such that  $N - 1$  have the same weight, but one is either heavier or lighter (you do not know which), what is the minimum number of weighings  $W$  you need to make to determine the odd coin? Conversely, given that you can weigh at most  $W$  times, what is the maximum number of coins  $N$  from which you can pick out the faulty coin?

**Problem 2.** For which integers  $n > 2$  does the set of positive integers less than and relatively prime to  $n$  and greater than 1 constitute an arithmetic progression?

**Problem 3.** Given a positive natural number  $n$ , a partition of  $n$  is a non-decreasing strictly positive sequence of integers  $\{a_1, a_2, \dots, a_m\}$  such that  $\sum_{i=1}^m a_i = n$ . Given a particular partition of  $P$ , let  $A(P)$  be the number of ones that appear in  $P$ , and let  $B(P)$  be the number of distinct elements in the partition. For example, given the partition  $P = \{1, 1, 1, 1\}$  of 4,  $A(P) = 4$  and  $B(P) = 1$ . Let the set  $S = \{P_1, P_2, \dots, P_k\}$  be all the partitions of a given  $n$ . Show that

$$\sum_{i=1}^k A(P_i) = \sum_{i=1}^k B(P_i)$$

**Problem 4.** If  $a$  and  $b$  are two positive irrational numbers such that  $\frac{1}{a} + \frac{1}{b} = 1$ , then show that the sets  $\{\lfloor na \rfloor : n \in \mathbb{N}\}$  and  $\{\lfloor nb \rfloor : n \in \mathbb{N}\}$  together contain every natural number exactly once.

**Problem 5 (Group Theory).** Let  $G$  be a finite abelian group. It is known that for all positive  $n$ , the number of solutions to  $x^n = e$  is at most  $n$ , where  $e$  is the identity element of the group. Show that the group is cyclic, i.e. there exists an element  $a$  in the group such that every element of the group is of the form  $a^m$ , where  $m$  is an integer.