Samasya

Samasya is a mathematics discussion and problem solving club. We discuss a variety of mathematical topics and solve problems as well. We encourage participants to have a look at these problemsbefore the meeting. Discussion, however, will not be limited to these problems. Participants can bring their own problems or mathematical ideas they wish to discuss.

Date: Friday, 11th September
Time: 9:00 p.m.
Venue: OPB LAN Room

Problem 1. A set K of points with integer coordinates in $\mathbb{R}^2$ is said to be connected if for every pair of points $a,\ b \in K$, there exists a finite sequence (of length m) of points $\{a_i\}$ such that $a_1 = a$, $a_m = b$ and $|a_{k+1} - a_k| = 1$ for $1 \leq k < m$. Let $\triangle K = \{a - b \mid a,\ b \in K,\ a \neq b\}$. What is the maximum value of $\triangle K$ when K varies over all connected sets of size $2n + 1$, where $n \in \mathbb{N}$.

Problem 2. Let ABC be a triangle, and let D be the point where the incircle meets BC. Let J_b and J_c be the incentres of the triangles ABD and ACD respectively. Prove that the circumcentre of the triangle AJ_bJ_c lies on the angle bisector of $\angle BAC$

Problem 3. If $a_1, a_2, \ldots, a_n$ are real numbers, prove that:

$$\sum_{i=1}^{n} a_i^2 - \sum_{i=1}^{n} a_i a_{i+1} \leq \left\lfloor \frac{n}{2} \right\rfloor (M - m)^2$$

where $a_{n+1} = a_1$, $M = \max(\{a_i\})$ and $m = \min(\{a_i\})$.

Problem 4. Find all polynomials p with integer coefficients such that the set $S_p = \{p(a) \mid a \in \mathbb{Z}\}$ has a geometric progression.

Problem 5. Consider a grid of n^2 dots where $n \in 2\mathbb{Z}$ through which a closed path goes through. Prove that there exists a pair of adjacent vertices such that if those two vertices are deleted, the part breaks into two disconnected parts, such that each of their lengths is atleast a quarter of the length of the original path.