Samasya

Samasya is a mathematics discussion and problem solving club. We discuss a variety of mathematical topics and solve problems as well. We encourage participants to have a look at these problemsbefore the meeting. Discussion, however, will not be limited to these problems. Participants can bring their own problems or mathematical ideas they wish to discuss.

Date: 18$^{\text{th}}$ September, 2015
Time: 9:00 p.m.
Venue: OPB LAN Room

Problem 1. Does there exist a function f from $\mathbb{R}$ to $\mathbb{R}$ such that for any non-empty open interval $(a, b) \subset \mathbb{R}$, the restriction of f to (a, b) is onto on $\mathbb{R}$, i.e. $\forall y \in \mathbb{R}$ there exists an $x \in (a, b)$ such that $f(x) = y$.

Problem 2. It is a known fact that the set of rationals $\mathbb{Q}$ is countable. Hence, one can enumerate the rationals in a sequence, such that every rational number appears in the sequence exactly once. Does there exist an enumeration of the rationals in a sequence $q_1, q_2, \ldots$ such that $\sum(q_i - q_{i+1})^2$ converges?

Problem 3. Form a "triangle" with 10 blocks in its top row, 9 blocks in the next row, etc., until the bottom row has one block. Each row is centered below the row above it. Color the blocks in the top row red, white and blue in any way. Use these two rules to color the remaining rows of the triangle:

- If two consecutive blocks have the same color, the block between them in the row below also has the same color.
- If two consecutive blocks have different colors, the block between them in the row below has the third color.

Is there a nice way of directly computing the color of the block in the bottom row, given the top row, without actually computing the colors of the rows in between?

Problem 4. Given a polygon of area A, show that one can, using finitely many straight edge cuts and joins, cut up the polygon and reassemble it into any other polygon of area A.

Problem 5. Most people believe the derivatives of differentiable functions are continuous without ever having seen the proof for it, or having proven it themselves. It's actually not true. Construct an example to show the fact. Furthermore, show that even though the derivative may not be continuous, it still has the intermediate value property, i.e. if $f'(x) = a$ and $f'(y) = b$, then $\forall c \in (a, b)$, there exists a $z \in (x, y)$ such that $f(z) = c$.

Problem 6. There are 432 equidistant points placed on a circle, with each point having one of four colours (red, blue, green and yellow), and there are 108 points of each color. Show that there exist four congruent triangles, one with all red points, one with all blue points, one with all green points and one with all yellow points.

The past problems and solutions are available at samasya.github.io.