

Samasya

Samasya is a mathematics discussion and problem solving club. We discuss a variety of mathematical topics and solve problems as well. We encourage participants to have a look at these problems before the meeting. Discussion, however, will not be limited to these problems. Participants can bring their own problems or mathematical ideas they wish to discuss.

Date: 16th October, 2015

Time: 9:00 p.m.

Venue: OPB LAN Room

Problem 1. Let S be a subset of the power set of \mathbb{N} such that for any $A, B \in S$, either $A \subset B$ or $B \subset A$. Can S be an uncountable set?

Problem 2. Let $d \leq n$, where d and n are positive integers and d is even. How many subsets of $\{1, 2, \dots, n\}$ exist such that any two have symmetric difference of size at most d ? Symmetric difference of two sets A and B is $(A \setminus B) \cup (B \setminus A)$.

Problem 3. An ordered field \mathbb{F} is said to have the intermediate value property if for every continuous function $f : (a, b) \mapsto \mathbb{F}$, $f(a) < 0$ and $f(b) > 0$, there exists a $c \in (a, b)$, such that $f(c) = 0$. Show that an ordered field with the intermediate value property has the least upper bound property.

Problem 4. Let $1, n_1, n_2, \dots$ be an increasing sequence of integers such that none of them are prime and any two distinct elements are co-prime. Show that the sum $\sum_{j=1}^{\infty} \frac{1}{n_j}$ converges.

Problem 5. Show that there are arbitrarily large n for which $n^4 + 1$ has a prime divisor larger than $2n$.