Samasya

Samasya is a mathematics discussion and problem solving club. We discuss a variety of mathematical topics and solve problems as well. We encourage participants to have a look at these problemsbefore the meeting. Discussion, however, will not be limited to these problems. Participants can bring their own problems or mathematical ideas they wish to discuss.

Date: 6$^{\text{th}}$ November, 2015
Time: 9:00 p.m.
Venue: OPB LAN Room

Problem 1. Given a balance scale, and N coins, such that $N-1$ have the same weight, but one is either heavier or lighter (you do not know which), what is the minimum number of weighings W you need to make to determine the odd coin? Conversely, given that you can weigh at most W times, what is the maximum number of coins N from which you can pick out the faulty coin?

Problem 2. For which integers $n > 2$ does the set of positive integers less than and relatively prime to n and greater than 1 constitute an arithmetic progression?

Problem 3. Given a positive natural number n, a partition of n is a non-decreasing strictly positive sequence of integers $\{a_1, a_2, \dots a_m\}$ such that $\sum_{i=1}^{m} a_i = n$. Given a particular partition of P, let $A(P)$ be the number of ones that appear in P, and let $B(P)$ be the number of distinct elements in the partition. For example, given the partition $P = \{1, 1, 1, 1\}$ of 4, $A(P) = 4$ and $B(P) = 1$. Let the set $S = \{P_1, P_2, \dots P_k\}$ be all the partitions of a given n. Show that

$$\sum_{i=1}^{k} A(P_i) = \sum_{i=1}^{k} B(P_i)$$

Problem 4. If a and b are two positive irrational numbers such that $\frac{1}{a} + \frac{1}{b} = 1$, then show that the sequence $\{a, b, 2a, 2b, 3a, 3b, \dots\}$ contains every natural number exactly once.

Problem 5 (Group Theory)**.** Let G be a finite abelian group. It is known that for all positive n, the number of solutions to $x^n = e$ is at most n, where e is the identity element of the group. Show that the group is cyclic, i.e. there exists an element a in the group such that every element of the group is of the form a^m, where m is an integer.

The past problems and solutions are available at samasya.github.io.