Samasya Solutions

These are sketches of solutions to the problems discussed at Samasya on the date mentioned below. Some problems are without a solution; that probably means we were unable to solve them. If you think you have a solution for one of those unsolved problems, feel free to contact the site maintainer.

Date: 4^th^ September, 2015

Solution 1 (Closed)**.** Countability argument: algebraic numbers are countable, real numbers are not, ergo, there exist uncountably many transcendental numbers.

Solution 2 (Semi-open)**.** Showing the function is an isomorphism is standard symbol chasing. For $\mathbb{Q}^+$, construct the isomorphism by mapping the $(2k+1)^{\text{th}}$ prime to the reciprocal of the $2k^{\text{th}}$ prime and map the $2k^{\text{th}}$ prime to the $(2k+1)^{\text{th}}$ prime.

For $\mathbb{C}^\times$, one would try $f(x) = x^i$, but that does not work out. However, no proof was found for the fact that no such function exists.

Solution 3 (Open)**.** This problem was not discussed.

Solution 4 (Closed)**.** It's sufficient to prove for an irrational a. Consider the sequence $\{na\}$, where $n \in \mathbb{N}$ and $\{\cdot\}$ is the greatest integer function. One needs to show that this sequence is dense in $(0, 1)$. That is shown using the pigeon hole principle. The statement of the problems follows through after this.

Solution 5 (Closed)**.** The problem essentially asks for integer solutions in y and z for the following inequality:

$$x \cdot 10^y \leq 2^z < (x+1) \cdot 10^y$$

Taking $\log_2$ throughout, one gets an inequation of the form $n - ma < \epsilon$, which has an integer solution, by problem 4.

Solution 6 (Closed)**.** (To fill in the details) Riemann-Zeta