9.3 THE RANK-SUM TEST PROCEDURE

LEARNING OBJECTIVES

- Explain the purpose of the rank-sum test by describing its role as an alternative to the two-sample t-test.
- Perform the rank-sum test by ranking combined sample data and computing the test statistic.
- Evaluate the assumptions of the rank-sum test by determining whether the data meet the necessary conditions for its application.
- Interpret the results of the rank-sum test by comparing the test statistic to critical values or p-values.

PARAMETRIC VS. NON-PARAMETRIC METHODS

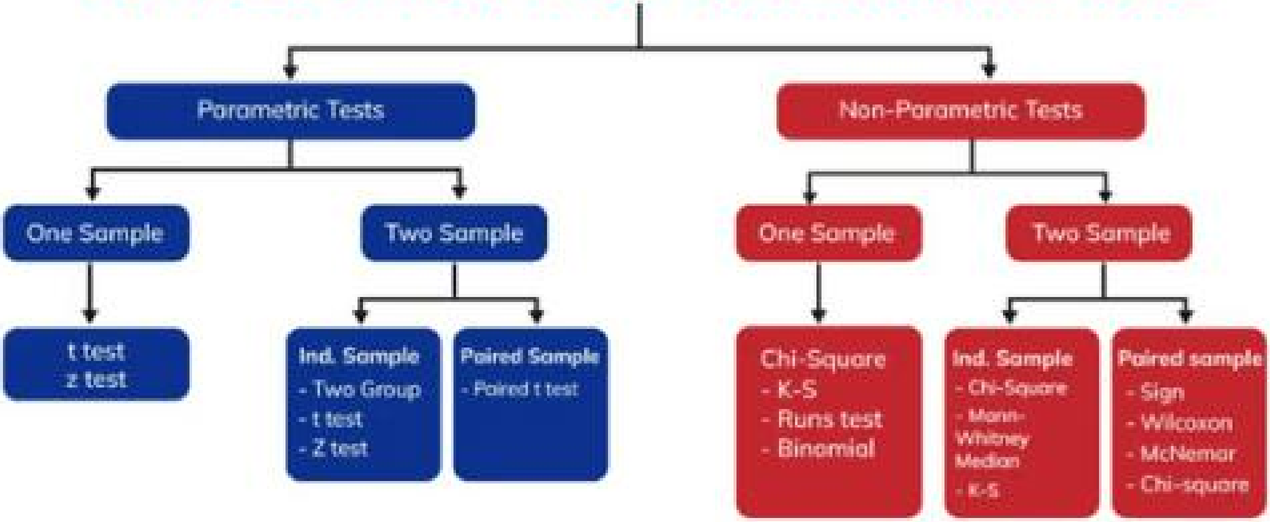
Parametric

- Assume a specific probability distribution (e.g., Normal, Exponential).
- Require estimation of fixed parameters (mean, variance, etc.).
- More powerful when assumptions hold but less robust if violated.

Non-parametrics

- Do not assume a specific probability distribution.
- Often rank-based or resampling methods.
- More flexible but sometimes less powerful.

Parametric & Non-Parametric Test



WHAT IS THE RANK-SUM TEST?

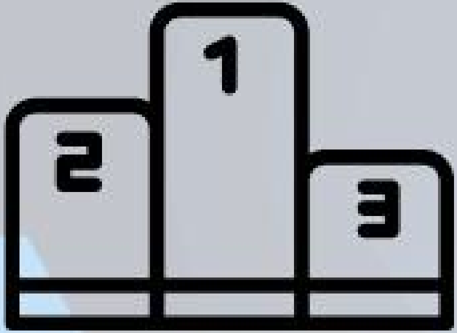
- Also known as the Mann-Whitney-Wilcoxon (MWW) test
- A nonparametric alternative to comparing two population medians
- Suitable for both small and large samples
- Does not require normality assumption

FORMULATING THE HYPOTHESES

- **Null Hypothesis (H_0)**: The two independent samples come from populations with the same median, i.e.,
$H_0 : \tilde{\mu}_1 = \tilde{\mu}_2$

- **Alternative Hypothesis (H_A)**: The populations have different medians. Depending on the test type, we have:
  - **Two-tailed test:**
$H_A : \tilde{\mu}_1 \neq \tilde{\mu}_2$
  - **Left-tailed test:**
$H_A : \tilde{\mu}_1 < \tilde{\mu}_2$
  - **Right-tailed test:**
$H_A : \tilde{\mu}_1 > \tilde{\mu}_2$

- This test is **nonparametric**, meaning it does not assume normality but instead relies on ranks.



WHY USE RANKS INSTEAD OF RAW DATA?

- The test is based on ranking all observations from both samples together
- The sum of ranks in each sample provides a test statistic
- Works well even when population distributions are non-normal or skewed

RANKING THE DATA

1. Combine observations from both samples
2. Arrange them in increasing order
3. Assign ranks accordingly

If ties exist, assign mid-ranks

Notation in Ranking

Implementation of the rank-sum test procedure begins by *ranking* the data, a process that consists of the following steps:

- Combine the observations, $X_{11}, \ldots, X_{1n_1}$ and $X_{21}, \ldots, X_{2n_2}$, from the two samples into an overall set of $N = n_1 + n_2$ observations.
- Arrange the combined set of observations from smallest to largest.
- For each observation X_{ij}, define its **rank** R_{ij} to be the position that X_{ij} occupies in this ordered arrangement.

EXAMPLE OF RANKING PROCESS

Table 9-1 Illustration of the ranking process

Original Data						
X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	X_{24}
0.03	−1.42	−0.25	−0.77	−2.93	0.48	−2.38
Ordered Observations						
X_{22}	X_{24}	X_{12}	X_{21}	X_{13}	X_{11}	X_{23}
−2.93	−2.38	−1.42	−0.77	−0.25	0.03	0.48
Ranks of the Data						
R_{11}	R_{12}	R_{13}	R_{21}	R_{22}	R_{23}	R_{24}
6	3	5	4	1	7	2

```
from scipy.stats import rankdata

# Compute ranks
ranks = rankdata(data)
```