Question 1

Table 1: Effect of model/promoter parameter on mRNA concentration

Parameters	10% increase			50% increase		
	Low I	**Medium I**	**High I**	**Low I**	**Medium I**	**High I**
BNIDing function, K	0	0	0	0	0	0
n	0	0	0	-	-	0
W_1	0	0	0	+	+	0
W_2	0	0	0	0	0	0
Elongation constant, e_x	0	0	+	+	+	+
length of gene, L_j	0	-	-	-	-	-
initiation rate constant, k_I	0	SI	+	+	+	+
cell volume	0	0	0	0	0	0
total/frac. RNAP available	SI	+	+	+	+	+
gene concentration, G_j	0	0	0	0	0	0
saturation constant, K_{XP}	0	0	0	0	0	0
doubling time	0	SI	+	SI	SI	+
half life	+	+	+	+	+	+
specific growth rate, μ	0	0	0	SD	-	-
degradation constant, k_D	SD	SD	-	-	-	-
dry mass/cell	0	SD	-	-	-	-

*mRNA concentration at medium I read @ 0.05 mM
~ no/insignificant change
SI = slight increase
SD = slight decrease

Question 2

Table 2: Reference for model/promoter parameters

Parameter (symbol)	**Value (unit)**	**Source**
Transcription Elongation constant (e_{TX})	42 nt/s	BNID 108487
Translation elongation rate (e_{TL})	14 aa/s	BNID 108487
Rate constant for initiation (k_I)	0.024 s^{-1}	BNID 105135
Saturation constant ($K_{X,P}$)	0.0225 µM	BNID 105135
RNAP in *E. coli* ($R_{X,P}$)[1]	8000 copies/cell	BNID 101441
RNAP activity[1]	24%	BNID 101441
Number of ribosomes[1]	45100 rib/cell	BNID 101441
Ribosome activity[1]	80%	BNID 101441
Protein half-life	20 hr	BNID 111930
E. coli cell volume	0.58 μm^3	BNID 114928
Median half-life of *E. coli* mRNA ($t_{1/2}$)	~4 min	BNID 104324
Dry mass of *E. coli*	280 fg/cell	BNID 103904

[1] value @ $t_{doubling}$ = 0.5 hr

Assumptions made for the purpose of modeling:

- The active RNAP is allocated for transcription of each gene type equally
- The active ribosome is allocated to each translation of each protein type equally
- BNIDing of only RNAP to promoter leads to negligible transcription ($W_{1X} = W_{2X} = W_{3X} = 0.0$)
- BNIDing functions (K_D) and weight factors for inducer are equal for all mRNA ($W_{1I} = W_{12} = W_{13} = W_{23} = W_{32} = 300.0$)
- Translation initiation rate constant ($k_{TL,I}$) and saturation constant (K_{XP}) are same as transcription initiation ($k_{TX,I}$) and elongation (K_{XP}) rate constants.

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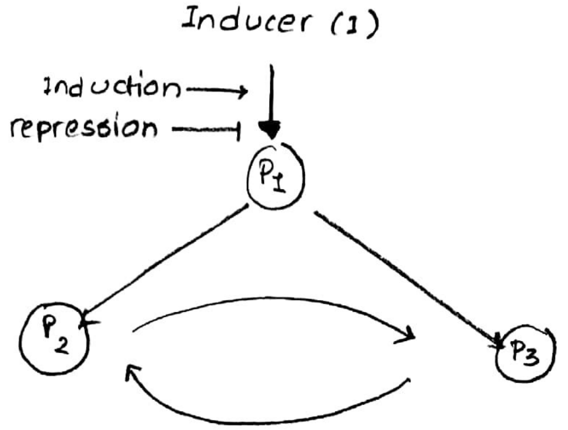
Question 2 (cont'd)

(a)



Material balance:

$$\underbrace{\begin{bmatrix}\dot m_1\\ \dot m_2\\ \dot m_3\\ \dot p_1\\ \dot p_2\\ \dot p_3\end{bmatrix}}_{\dot{\tilde{x}}} = \overbrace{\begin{bmatrix}-(k_d^m+\mu) & 0 & 0 & 0 & 0 & 0\\ 0 & -(k_d^m+\mu) & 0 & 0 & 0 & 0\\ 0 & 0 & -(k_d^m+\mu) & 0 & 0 & 0\\ 0 & 0 & 0 & -(k_d^p+\mu) & 0 & 0\\ 0 & 0 & 0 & 0 & -(k_d^p+\mu) & 0\\ 0 & 0 & 0 & 0 & 0 & -(k_d^p+\mu)\end{bmatrix}}^{A}\underbrace{\begin{bmatrix}m_1\\ m_2\\ m_3\\ p_1\\ p_2\\ p_3\end{bmatrix}}_{\tilde{x}} + \underbrace{\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}}_{S}\underbrace{\begin{bmatrix}TX_1\\ TX_2\\ TX_3\\ TL_1\\ TL_2\\ TL_3\end{bmatrix}}_{\tilde{r}}$$

In eq^n form,

$$\left.\begin{aligned}\frac{dm_i}{dt} &= TX_i - (k_d^m+\mu)\,m_i\\ \frac{dp_i}{dt} &= TL_i - (k_d^p+\mu)\,p_i\end{aligned}\right\} i=1,2,3$$

- $TX_i = r_{x_i} \cdot u_i$

$$u_1 = \frac{W_1 + W_2 f_I}{1 + W_1 + W_2 f_I} \quad ; \quad f_I = \frac{I^n}{K^n + I^n}$$

$$u_2 = \frac{W_2 + W_{12} f_{12} + W_{32} f_{32}}{1 + W_2 + W_{12} f_{12} + W_{32} f_{32}} \quad ; \quad f_{12} = \frac{P_1^n}{K^n + P_1^n} \quad ; \quad f_{32} = \frac{P_3^n}{K^n + P_3^n}$$

$$u_3 = \frac{W_3 + W_{13} f_{13} + W_{23} f_{23}}{1 + W_3 + W_{13} f_{13} + W_{23} f_{23}} \quad ; \quad f_{23} = \frac{P_2^n}{K^n + P_2^n} \quad ; \quad f_{13} = \frac{P_2^n}{K^n + P_2^n}$$

Assumptions, $W_1 = W_2 = W_3 = 0.0$

$K_1 = K_2 = K_3 = K = 0.3\,mM$

$n = 1.5$

$$r_{x_i} = k_{E_i} \cdot R_{x,T} \cdot \frac{G_p}{(K_{xp} \tau_i) + (\tau_i + 1) G_p}$$

G_p, $R_{x,T}$, $K_{xp} \rightarrow$ Table 2

$$k_{E,i} = \langle k_E \rangle \frac{L_x}{L_{x,i}}$$

$$\tau_i = \frac{k_{E_i}}{k_I}$$

$\rightarrow$ McClure

$$\langle k_E \rangle = e_x / L_x$$

Table 2

- $TL_i = r_{L_i} \cdot \cancel{u_i} \rightarrow 1.$

$$r_{L_i} = k_{E,TL_i} \cdot R_{L,T} \cdot \frac{m_i}{(K_{xp} + \tau_{L_i}) + (\tau_{L_i} + 1) m_i}$$

$R_{L,T}$, $K_{xp} \rightarrow$ Table 2

$$k_{E,TL_i} = \langle k_{E,TL} \rangle \frac{L_L}{L_{L_i}}$$

$$\tau_{L_i} = \frac{k_{E_i}}{k_I}$$

$$\langle k_{E,TL} \rangle = e_L / L_L$$

Table 2

- (b) & (c) $\rightarrow$ codes and plots are attached.