CS5810 Assessed Coursework 3

This assignment must be submitted by December 9th, 2022 at 10:00
Feedback will be provided within ten working days of the submission deadline.

Learning outcomes assessed

This coursework will test some concepts of Matlab programming for data analysis, including writing scripts, functions, advanced plotting, manipulating strings and using cell arrays and structure variables.

Instructions

Submit your files through Moodle – follow the link marked "Click here to submit coursework 3" on the Moodle page for CS5810.
The files you submit cannot be overwritten by anyone else, and they cannot be read by any other student. You can, however, overwrite your submission as often as you like, by resubmitting, though only the last version submitted will be kept. Submission after the deadline will be accepted, but it will automatically be recorderd as being late and is subject to College Regulations on late submissions.

IMPORTANT: In this assignment, exercises 1, 2, 3, 4 and 6 require you to write functions, exercise 6 also requires you to write a script. For this assignment you will have to submit:

- A file containing function *myplotarea* required for exercise 1.
- A file containing function *membersplot* required for exercise 2.
- A file containing function *plottrigs* required for exercise 3.
- A file containing function *myplot* required for exercise 4.
- A file containing script *docdistances* required for exercise 5.
- A file containing script *PCAEx6* required for exercise 6.
- A file containing function *subtractMean* required for exercise 6.
- A file containing function *myPCA* required for exercise 6.
- A file containing function *projectData* required for exercise 6.

The following files are **provided** for the exercises:

- A file *pointsEx1.txt* required for exercise 1.
- A file *plot_properties.mat* required for exercise 4.
- A file *pcadata.mat* required for exercise 6.
- A file *pcafaces.mat* required for exercise 6.
- A file containing a function *recoverData.mat* required for exercise 6.
- A file containing a function *displayData.mat* required for exercise 6.
- Six files containing text of *RedRidingHood.txt, PrincessPea.txt, Cinderella.txt, CAFA1.txt, CAFA2.txt and CAFA3.txt* required for exercise 6.

If you will write any other extra function or script as part of your solution for these exercises, you should also submit these. For these extra functions or scripts, you can choose the file name.
A zip file containing a template for each of these files is provided on the course page on Moodle. You need to insert your code for each exercise where indicated and submit them.
Please do not change the above file names in your submission.

All the work you submit should be solely your own work. Coursework submissions are routinely checked for this.

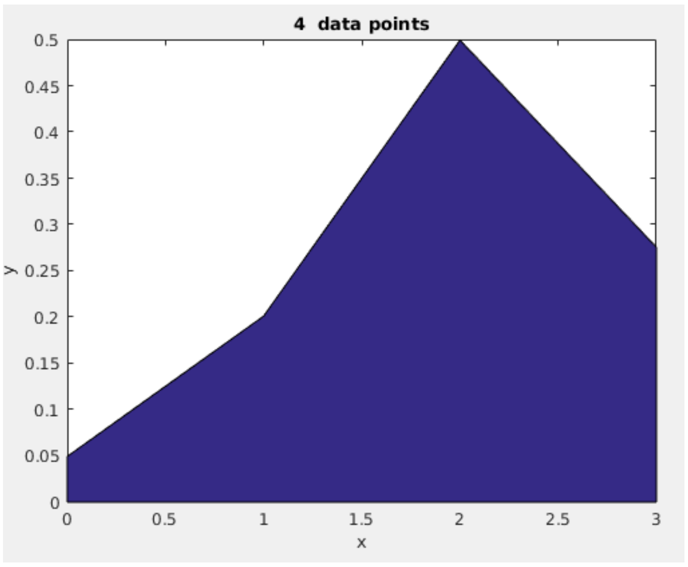
EXERCISE 1 (3 points)

Write a function “myplotarea” that will receive 2 parameters as inputs: the name of a text file and a number *n*. The text file to be used as input to myplotarea is already provided on the Moodle page (file `pointsEx1.txt`). myplotarea will read `pointsEx1.txt` which contains the x and y coordinates for data points and will create an area plot using the first *n* of these points. The format of every line in the file is the letter ‘x’, a space, the x value, a space, the letter ‘y’, space, and the y value. The number of points will be in the plot title. Your function will need to check that the value of *n* is smaller than the total number of points in the file, and provide the user with an appropriate error message in case it is bigger.

For example, the function call:

```
myplotarea('pointsEx1.txt',4)
```

will produce the following Figure:



EXERCISE 2 (4 points)

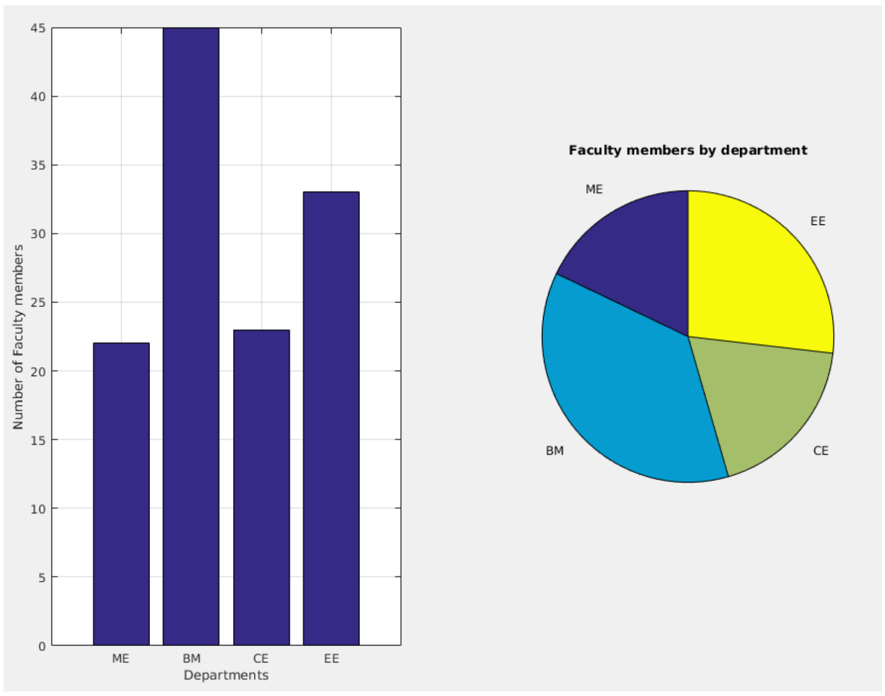
The number of faculty members in each department at a certain College of Engineering is:

```
ME   22
BM   45
CE   23
EE   33
```

Write a function “membersplot” that will receive as input 2 strings taken from the set {bar, barh, pie} and will produce a figure containing 2 subplots, where each subplot contains the plot of the data as indicated by the strings. Make sure that you have appropriate titles, labels, and legends on your plots. For example, the call:

```
membersplot('bar', 'pie')
```

will produce the following Figure:



EXERCISE 3   (4 points)

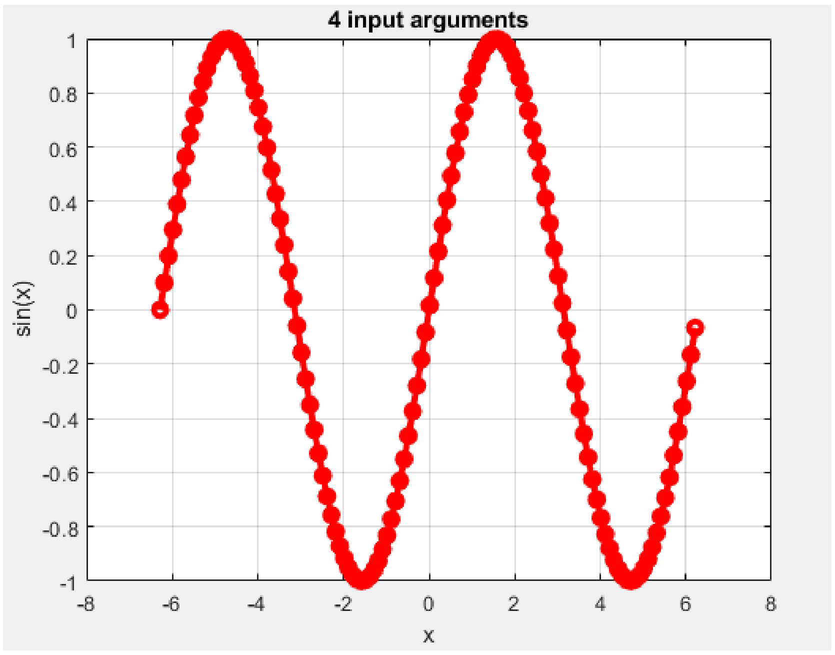
Write a function “plottrigs” that will plot either the *sin* or the *cos* in the range from -2 π to 2 π by joining points calculated at intervals of 0.1. The function will receive as input the strings ‘*sin*’ or ‘*cos*’. The default of the plots will be a black line, of thickness equal to 1, where points are marked by small dots. However, the function can have a variable number of further input parameters:

- If a second argument is passed, it specifies a colour for the plot (according to the one-letter coding employed by plot; e.g. ‘r’ stands for ‘red’ and ‘b’ for ‘blue’);
- If a third argument is passed, it specifies line width for the plot;
- If a fourth argument is passed, it specifies a marker symbol (according to the one-letter coding employed by plot; e.g. ‘o’ stands for the ‘circle’ and ‘s’ stands for the ‘square’).

The plot title will include the total number of arguments passed to the function. For example, the call:

```
plottrigs('sin', 'r', 3, 'o')
```

will produce the following Figure:



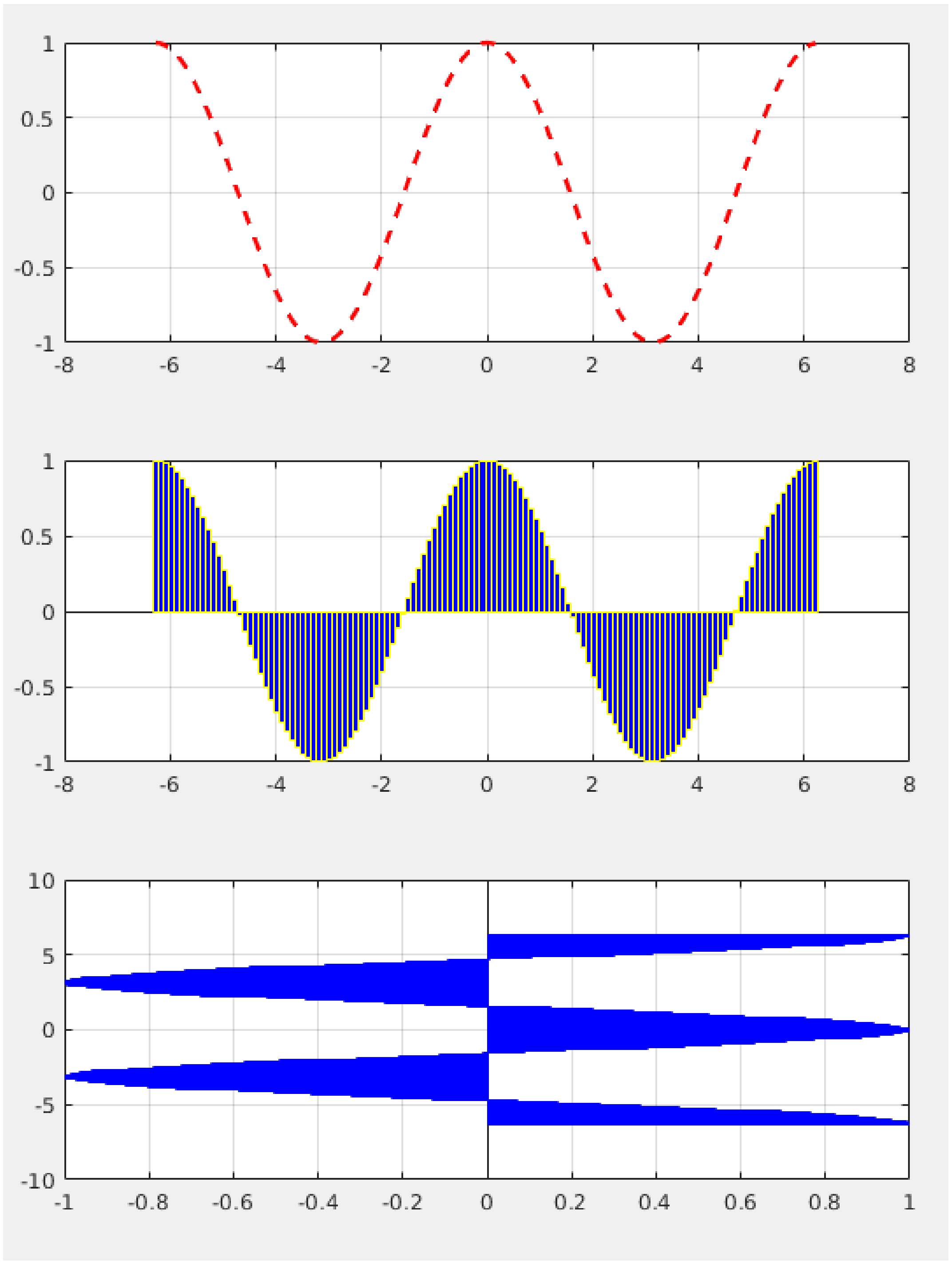
(**Hint**: the Matlab function `eval` can be useful here. Also you will need `varargin` to be able to pass an input argument list of variable length)

EXERCISE 4 (4 points)

Write a function “myplot” that receives two input vectors, containing the coordinates of points to be plotted. The function will load the `plot_properties.mat` file (provided on the Moodle page), containing an array of structs that specifies 3 different types of plots together with their properties, and will make the 3 corresponding plots as subplots of the same figure. For example, the following call:

```
myplot(-2*pi:0.1:2*pi,cos(-2*pi:0.1:2*pi))
```

will produce the following Figure:



EXERCISE 5 (10 points)

In this exercise you will write a script called "docdistances" that will calculate distances between pairs of text documents. These distances will be based on a vanilla-version of term frequency–inverse document frequency (tf-idf). Your script will calculate the distances between 6 documents: 3 documents are synopsis of fairy tales (Red riding hood, the Princess and the pea and Cinderella); the other 3 documents are the abstract of papers related to protein function prediction (identified as CAFA1, CAFA2 and CAFA3). You will find these documents on the Moodle page (the files name are: "RedRidingHood.txt", "PrincessPea.txt", "Cinderella.txt", "CAFA1.txt", "CAFA2.txt", "CAFA3.txt").

Important: in this exercise you are NOT allowed to use the Matlab functions `tfidf` and `wordCloudCounts`.

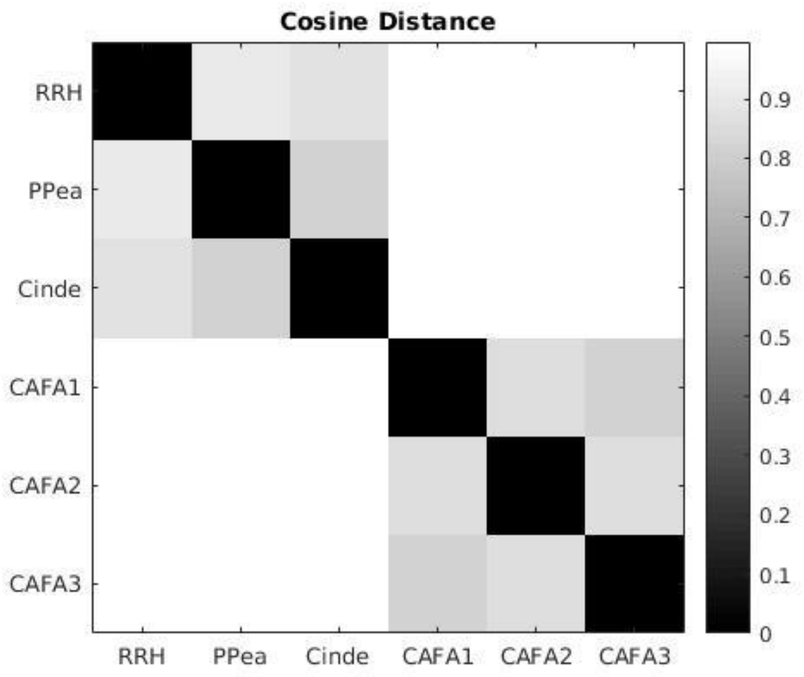
Your script will:

1. For each document, calculate its tf-idf vector.
   The tf-idf vector of a document is a vector whose length is equal to the total number of different terms (words) which are present in the corpus (in this case, the corpus is the entire set of 6 documents). Each term is assigned a specific element of the vector, which is in the same position for the tf-idf vector of every document. For a given document d, the vector element corresponding to term t is calculated as the product of 2 values:

   a) **Term frequency**: the number of times that term t appears in document d
   b) **Inverse document frequency**: the log base 10 of the inverse fraction of the documents that contain the term, i.e.

$$log_{10} \frac{number\ of\ documents\ in\ the\ corpus}{number\ of\ documents\ where\ term\ t\ appears}$$

2. Calculate the cosine distance between every pair of tf-idf vectors representing each document. (This is equal to 1 minus the cosine of the angle between the 2 vectors.)
3. Collect these distances into a 6x6 matrix where the value in the *(i,j)* element contains the distance between document *i* and document *j*. Then make a figure that displays the matrix. Your Figure should look similar to the Figure below (here I have used `imagesc`, set the `colormap` to `gray` and added the `colorbar`).



It is interesting to note that the 2 types of documents form 2 clear groups: the synopsis of fairy tales are more similar to each other than they are to scientific papers. Also, the "Princess and the pea" is more similar to "Cinderella" than to "Red Riding Hood", and this makes sense as the "Princess and the pea" and "Cinderella" have more elements in common …

(**Hint**: the Matlab functions `textscan` and `pdist` can be useful for this exercise)

EXERCISE 6

In this exercise you will explore Principal Components Analysis (PCA). The exercise is divided in 2 parts. In the first part, you will work with a small toy dataset of artificial data, developing functions needed for carrying out PCA. In the second part, you will work with a larger dataset of real world data (images of faces of famous people), and you will be able to re-use the functions you wrote for the small toy dataset on a larger scale.

Note that this is a good approach to use when implementing algorithms: make sure that they work well on small datasets and then apply them to your real problem. *Remember that the implementation for the small dataset needs to be efficient, otherwise its execution time will be very large when applied to the larger dataset*.

For both datasets, you will:

1. Calculate the principal components of your data
2. Project your high dimensional data onto (a smaller space defined by) a few principal components
3. Recover your original high dimensional data by re-projecting back the projected data onto the original space.

Note also that this exercise is about implementing PCA yourself, so you cannot use any of the different functions available in Matlab which implement it (e.g. `pca`). You will have to calculate the principal components through the eigendecomposition of the covariance matrix of the data (you will need to use the

Matlab function `eig` for the eigendecomposition. Note that there is a similar function in Matlab called `eigs`, but it returns only the first 6 eigenvalues, so you should not use it).

Part 1 (8 points)

You will write a script called `PCAEx6` that will work on a small dataset of 50 random points, Gaussian distributed, in 2D. These are contained in the file `pcadata.mat`, available on the Moodle page. Your script will:

1. Load the datapoints contained in the file `pcadata.mat`. Let us call X this initial set of datapoints. X has size 50x2 as there are 50 points in 2 dimensions.

2. Create Figure 1. In this figure, plot the points as blue circles in 2D, in a figure whose axis are set in the range xmin = 0, xmax=7, ymin=2, ymax=8

3. Call a function called `subtractMean` that you will also write and include in the submission. The function receives a dataset (i.e. a matrix) as the only input argument, and returns two arguments: a dataset obtained from the input dataset by subtracting its mean (that is, from each column you need to subtract the mean of that column); and the mean of the input dataset (that is, a vector of 2 components containing the mean of the 2 columns of your dataset). *Your script will run this function on your dataset X, and obtain a new dataset Xmu and the mean of X, which we shall call mu.*

4. Call a function called `myPCA` that you will also write and include in the submission. This function receives a dataset (i.e. a matrix) as the only input argument, and returns two arguments: the first is a matrix in which the columns are the principal components (i.e. the eigenvectors of the covariance matrix of the dataset) and the second is the set of corresponding eigenvalues. The eigenvectors will need to be ordered according to the size of their corresponding eigenvalues, in decreasing order; that is, the first column will be the eigenvector corresponding to the largest eigenvalue, the second column will be the eigenvector corresponding to the second largest eigenvalue, and so on.
   Your script will run this function on your dataset Xmu, and obtain a matrix U of principal components and a vector S with their corresponding eigenvalues.
   [**HINT**: the principal components are the eigenvectors of the covariance matrix of the data. So to implement this step you will need to use the Matlab function `cov` for calculating the covariance of the data, and the function `eig`, for calculating eigenvalues and eigenvectors.]

5. Add to Figure 1 the plot of the 2 principal components, in which the first component (corresponding to the largest eigenvalue) is red, and the second one is green. Your Figure 1 should look similar to Figure A below. Also print out on the command window the coordinates of the top eigenvector.
   [**HINT**: you can use the command `line` to draw the eigenvector. Do not forget that the eigenvectors were calculated from data from which the mean had been subtracted. So you will need to add the mean to the eigenvectors in order to make the plot.]

6. Call a function called `projectData` that you will also write and include in the submission. The function receives 3 arguments: a dataset (i.e. a matrix), a set of eigenvectors and a positive integer k. It provides as the only output argument a dataset obtained by projecting the input dataset onto the first k eigenvectors. Note that the first k eigenvectors are the k eigenvectors corresponding to the k largest eigenvalues.
   Your script will run this function on your dataset Xmu, using the eigenvectors in U and with k equal to 1 and obtain a matrix Z of projected data.
   [**Hint**: here the projection of a datapoint onto an eigenvector can be obtained by calculating the dot product between the point and the vector, since the eigenvectors you obtain from Matlab have unit norm].

7. Print in the command window the projection of the first 3 points in your dataset, i.e. `Z(1:3, :)`.

8. Call a function `recoverData` provided on the Moodle page. The function receives 4 arguments: a dataset (i.e. a matrix), a set of eigenvectors, a positive integer k and a vector mu. It provides as the only output argument a dataset obtained by projecting back your points onto the original space. Using the variable names described above the call in your script call would be:

   ```
   Xrec = recoverData(Z, U, K, mu)
   ```

   Note that this function will work correctly only if the eigenvectors in U are ordered according to the size of their corresponding eigenvalues, in decreasing order, as explained in point 4.
   Your script will run the above line obtaining in Xrec the recovered datapoints.

9. Create Figure 2. In this figure, plot the points as blue circles in 2D, in a figure whose axis are set in the range xmin = 0, xmax=7, ymin=2, ymax=8. Then add the recovered points as red stars. (Using the variable names described above your recovered points will be contained in the variable Xrec). Your Figure 2 should look similar to the Figure B below.

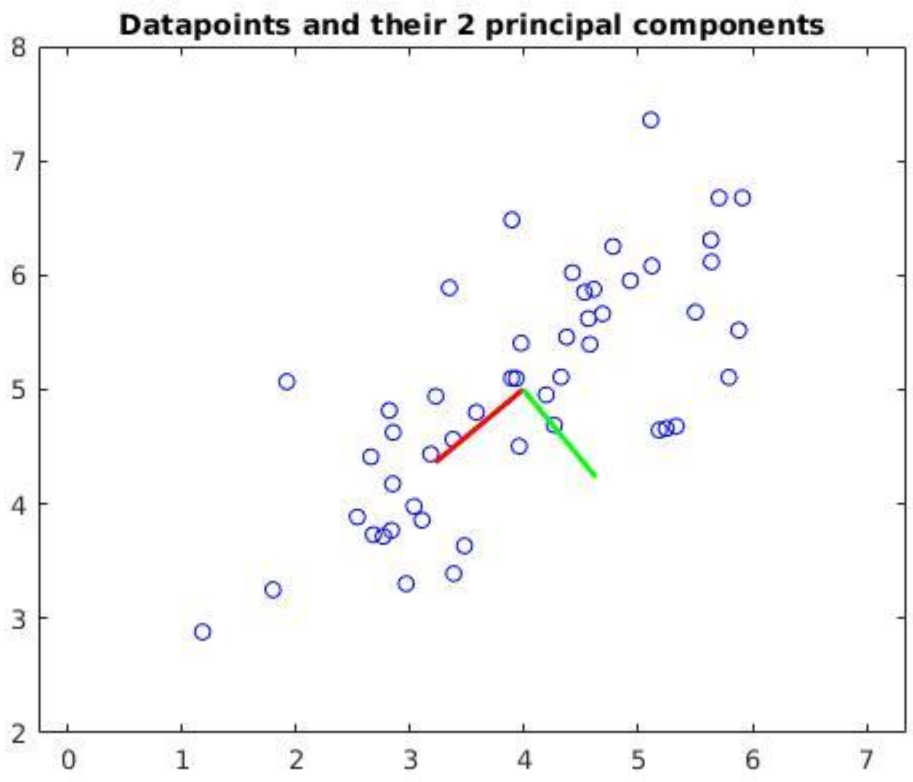


Figure A

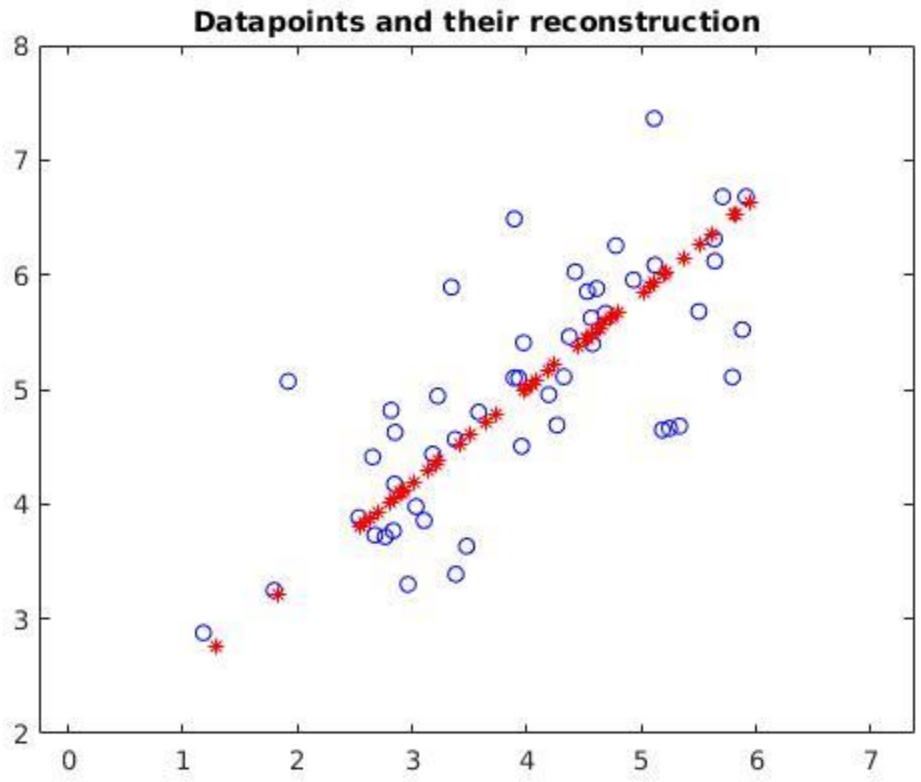


Figure B

Part 2 (7 points)

In this part of the exercise you will repeat on a larger real world dataset exactly the same analysis that you have already performed on the small toy dataset. Your script will use the functions you have written for the small toy dataset. Your script will work on a dataset of 5000 images of faces of famous people, taken from a public repository. Each image is a 32x32 matrix of pixel which has been linearized into a vector of size 1024. These images are contained in the file `pcafaces.mat`, available on the Moodle page.

1. Load the datapoints contained in the file `pcafaces.mat`. Let us call X this initial set of datapoints. X has size 5000x1024, as there are 5000 faces, each represented by a vector of pixels of size 1024. (This is a large dataset so it might take a few seconds, depending on your machine).

2. Create Figure 3. Use the function `displayData` provided on the Moodle page to display the first 100 images in the datasets. Using the variable names described above the call in your script call would be:

   ```
   displayData(X(1:100, :))
   ```

   Your Figure 3 should look similar to the Figure C below.

3. Subtract the mean from X using your function `subtractMean`

4. Project the data onto the first 200 principal components using your function `projectData`

5. Recover your images back onto the original space using the function `recoverData` (which is provided on the Moodle page).

6. Create Figure 4. This figure will contain 2 subplots. The first subplot will display the first 100 images of the original data (that is, it will be the same as Figure 3). The second subplot will display the first

100 images of the reconstructed data – in this way you will be able to compare how good your reconstruction is! Your Figure 4 should look similar to the Figure D below.

For fun, you can experiment and check the quality of the reconstructed faces for different number of principal components…

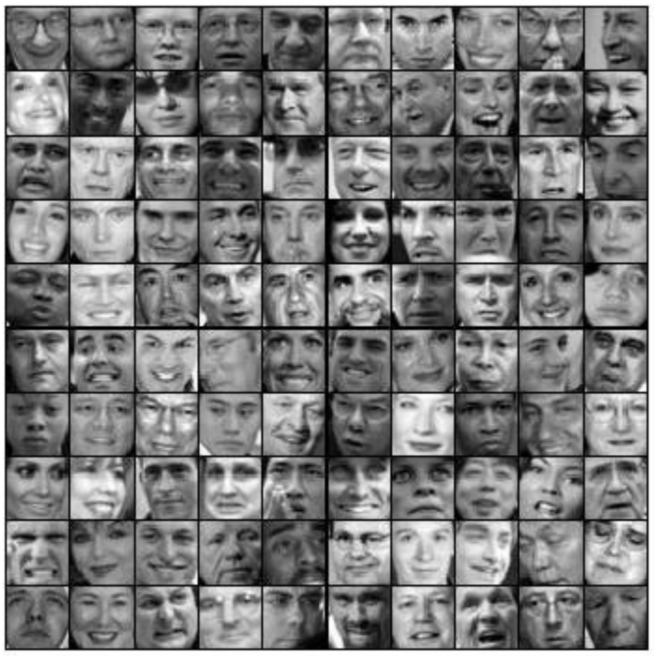


Figure C

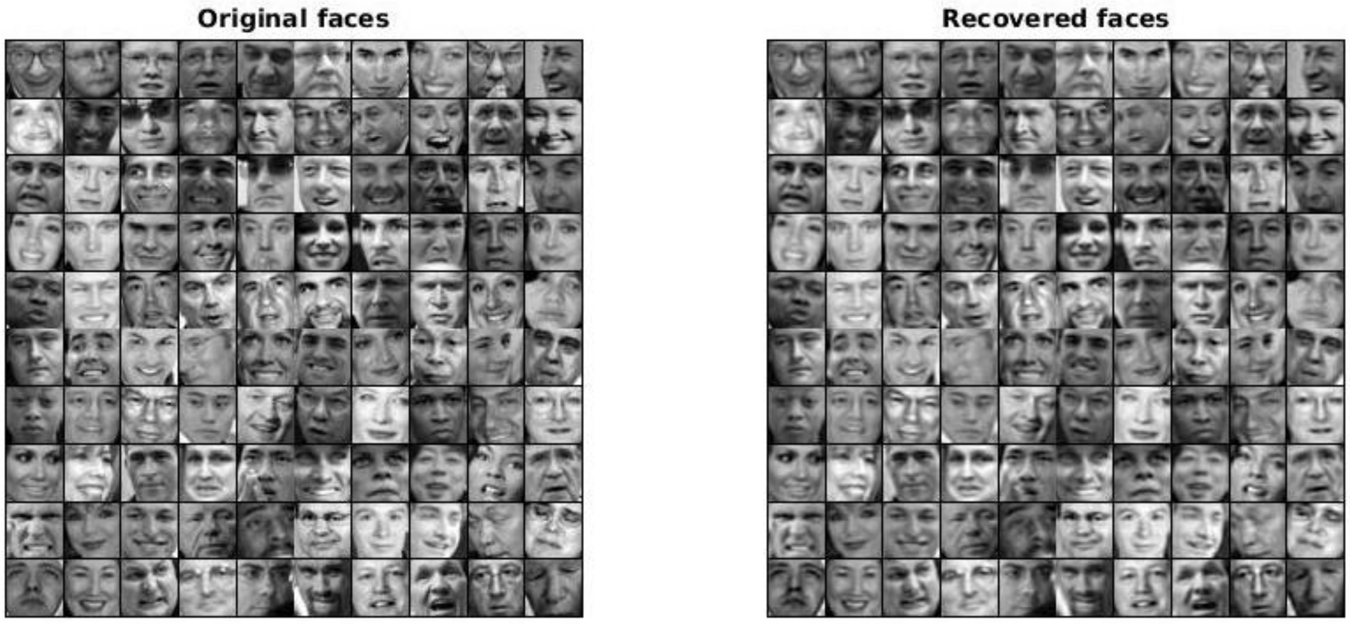


Figure D

Marking Criteria

This coursework is assessed and mandatory and is worth 40% of your total final grade for this course.
In order to obtain full marks for each question, you must answer it correctly but also completely, based on the contents taught in this course.
It will be important to provide input and output parameters to the functions as requested in the exercises.
Unless explicitly specified in the exercise, avoid using the “input” function or printing outputs on the command window.
Marks will be given for writing elegant, compact, vectorised code, avoiding the use of “loops” (*for* or *while* loops) where possible, and including comments.