Time Series Final Project

Samay Jain (s3963844)

2024-06-06

Research Question:

What is the most accurate forecasting model for predicting the gold price over the next 10 months?

Introduction

The project aims to analyze gold price data from January 2000 to May 2024 and forecast gold prices for the next 10 months. To achieve this, ARIMA, SARIMA, and ARMA-GARCH models are fitted to the data. These models are evaluated and compared using statistical tools such as the coefficient test, AIC, BIC, and error metrics to identify the best model for predicting gold prices.

Functions

Seasonal lag function

```
helper <- function(class = c("acf", "pacf"), ...) {

  # Capture additional arguments
  params <- match.call(expand.dots = TRUE)
  params <- as.list(params)[-1]

  # Calculate ACF/PACF values
  if (class == "acf") {
    acf_values <- do.call(acf, c(params, list(plot = FALSE)))
  } else if (class == "pacf") {
    acf_values <- do.call(pacf, c(params, list(plot = FALSE)))
  }

  # Extract values and lags
  acf_data <- data.frame(
    Lag = as.numeric(acf_values$lag),
    ACF = as.numeric(acf_values$acf)
  )

  # Identify seasonal lags to be highlighted
  seasonal_lags <- acf_data$Lag %% 1 == 0

  # Plot ACF/PACF values
  if (class == "acf") {
    do.call(acf, c(params, list(plot = TRUE)))
  } else if (class == "pacf") {
    do.call(pacf, c(params, list(plot = TRUE)))
  }

  # Add colored segments for seasonal lags
  for (i in which(seasonal_lags)) {
    segments(x0 = acf_data$Lag[i], y0 = 0, x1 = acf_data$Lag[i], y1 = acf_data$ACF[i], col =
"red")
  }
}


# seasonal_acf ------------------------------------------------------------

seasonal_acf <- function(...) {
  helper(class = "acf", ...)
}


# seasonal_pacf -----------------------------------------------------------

seasonal_pacf <- function(...) {
  helper(class = "pacf", ...)
}
```

The seasonal lag function highlights the seasonal lags in ACF and PACF plots.

Diagnostic test

```
Diagnostic_test <- function(data, lag = 20,
                            mainacf = "ACF Plot", subacf = "",
                            mainpacf = "PACF Plot", subpacf = "",
                            mainhist = "Histogram", subhist = "",
                            mainqq = "Q-Q Plot", subqq = "",
                            test) {
  if (test == "ACF-PACF") {
    acf(data, lag.max = lag, main = mainacf, sub = subacf)
    pacf(data, lag.max = lag, main = mainpacf, sub = subpacf)
  } else if (test == 'Stationary') {
    acf(data, lag.max = lag, main = mainacf, sub = subacf)
    pacf(data, lag.max = lag, main = mainpacf, sub = subpacf)
    adf_result <- adf.test(data)
    pp_result <- pp.test(data)
    return(list(adf_test = adf_result, pp_test = pp_result))
  } else if (test == 'Normality') {
    hist(data, main = mainhist, sub = subhist)
    qqnorm(data, main = mainqq, sub = subqq)
    qqline(data, col = 2)
    shapiro_result <- shapiro.test(data)
    return(shapiro_result)
  } else if (test == "seasonal ACF-PACF") {
    seasonal_acf(data, lag.max = lag, main = paste(mainacf, "(Seasonal)"), sub = subacf)
    seasonal_pacf(data, lag.max = lag, main = paste(mainpacf, "(Seasonal)"), sub = subpacf)
  } else if (test == 'seasonal Stationary') {
    seasonal_acf(data, lag.max = lag, main = paste(mainacf, "(Seasonal)"), sub = subacf)
    seasonal_pacf(data, lag.max = lag, main = paste(mainpacf, "(Seasonal)"), sub = subpacf)
    adf_result <- adf.test(data)
    pp_result <- pp.test(data)
    return(list(adf_test = adf_result, pp_test = pp_result))
  } else {
    print("Please enter a valid type of test: 'ACF-PACF', 'Stationary', 'Normality', 'seasona
l ACF-PACF', or 'seasonal Stationary'")
  }
}
```

The Diagnostic test function helps to do a stationarity test (ACF, PACF, ADF, PP-test), normality test (Histogram, QQ-plot, Shapiro-Wilk), and seasonal stationarity test (Seasonal ACF-PACF).

Residual analysis

```
residual.analysis <- function(model, std = TRUE,start = 2, shift = 0, class = c("ARIMA","GARC
H","ARMA-GARCH", "garch", "fGARCH")[1]){
  library(TSA)
  library(FitAR)
  library(quantmod)
  if (class == "ARIMA"){
    if (std == TRUE){
      res.model = Lag(rstandard(model),shift)
      res.model = na.omit(res.model)
    }else{
      res.model = Lag(residuals(model),shift)
      res.model = na.omit(res.model)
    }
  }
  par(mfrow=c(3,2))
  plot(res.model,type='o',ylab='Standardised residuals', main="Time series plot of standardis
ed residuals")
  abline(h=0)
  hist(res.model,main="Histogram of standardised residuals")
  qqnorm(res.model,main="QQ plot of standardised residuals")
  qqline(res.model, col = 2)
  acf(res.model,main="ACF of standardised residuals")
  print(shapiro.test(res.model))
  k=0
  if (length(res.model) < 30){
    lagM <- length(res.model) - 1
  } else {
    lagM <- 30
  }
  LBQPlot(res.model, lag.max = lagM, StartLag = k + 1, k = 0, SquaredQ = FALSE)
  par(mfrow=c(1,1))
}
```

Data Exploration

```
data <- read_csv("C:/Users/HP/Downloads/RMIT/Sem 3/Time Series analysis/Gold Futures Historic
al Data.csv")
```

```
# Ordering the date in ascending order
data$Date <- as.Date(data$Date, format = "%m/%d/%Y")
data <- data[order(data$Date), ]
```

The dates are ordered in ascending order to preserve the structure of the data and, ensure that trends, and patterns are accurately captured, leading to a correct model estimation and forecasting.

Converting to time series object

```
data.ts <- ts(data$Price, start = c(2000, 1), end = c(2024, 5), frequency = 12)
```

Descriptive Analysis

- Descriptive statistics
- Time series plot
- Impact of 1st lag gold price on current gold price
- Impact of 2nd lag gold price on current gold price

Descriptive statistics

```
summary(data.ts)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   259.2   575.5  1214.5  1109.6  1564.2  2366.8
```

The descriptive statistics indicate a range of gold price values from $259.2 to $2366.8. The median value of $1214.5 suggests that half the data points are below this value, while the mean of $1109.6 is the average of gold price from (Jan, 2000) to (May, 2024). The first quartile ($575.5) and third quartiles ($1564.2) shows the gold price spread, with the middle 50% of values falling between these points.

Time series plot

```
plot(data.ts, type = "o", pch = 20, ylab = "Gold price", main = "Gold price time series",
     sub=  "Figure 1: Gold price time series")
```

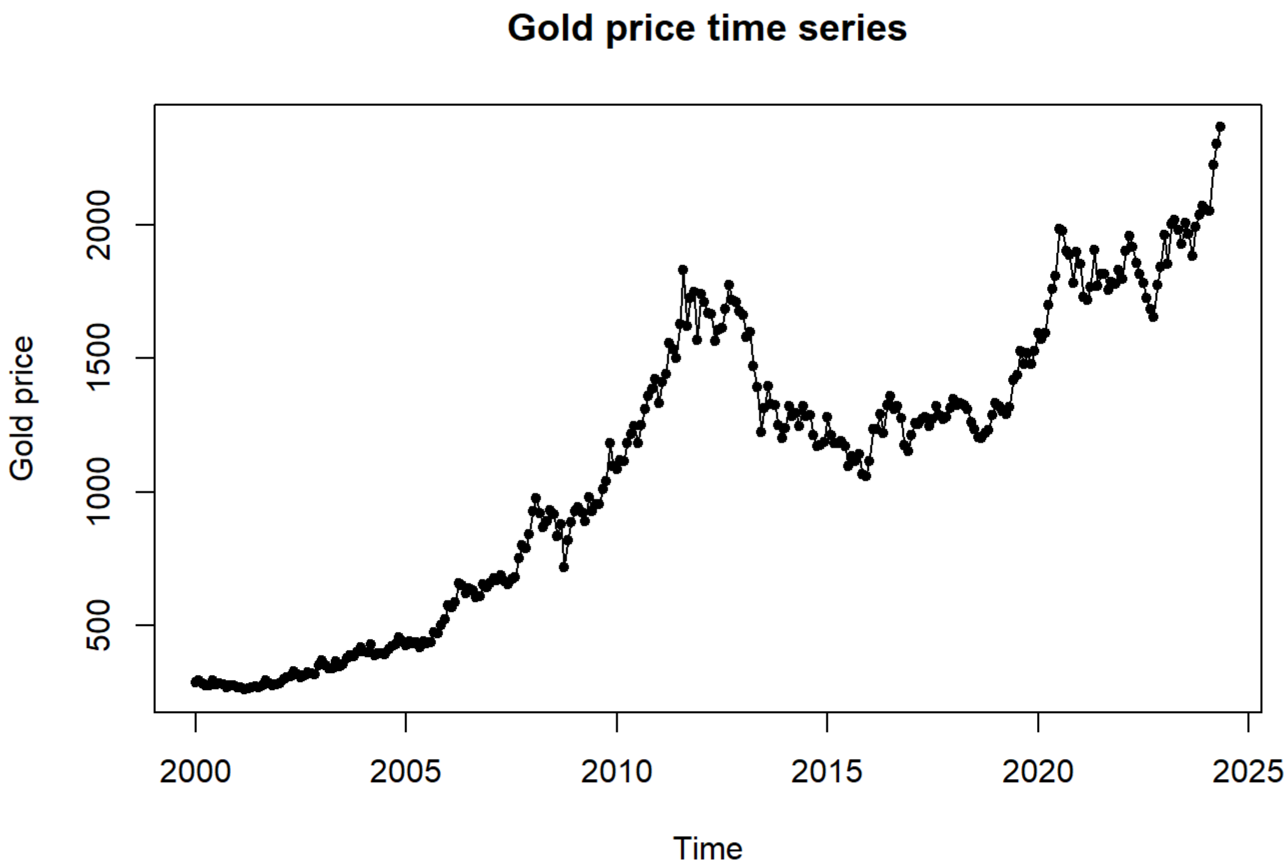


Figure 1: Gold price time series

Interpretation of Gold Price time series plot:

- Trend: The gold price shows a clear upward trend over the entire period from 2000 to 2024.
- Seasonality: From the plot, there is no obvious seasonal pattern observed in the data. The fluctuations and peaks do not appear to follow a regular, repeating pattern over a fixed period.
- Changing Variance: The variance in gold prices increases over time. Early in the series (2001-2005), prices fluctuate within a relatively narrow range. However, as time progresses, the range of fluctuations increases, indicating higher volatility in later years.
- Behavior: The time series plot has consecutive points following each other and indicates auto-regressive behavior, means that current gold prices are influenced by past prices.
- Intervention: No intervention point is observed. However, there are multiple shifts in trend approximately around the years 2008, 2013, and 2019 .

Scatter plot

```
# 1st lag
y = data.ts
x = zlag(data.ts)

# 1st lag plot
plot(y=data.ts, x = zlag(data.ts), ylab = 'Gold_price', xlab = 'Gold price in the previous mo
nth', main = "Scatter plot of neighboring gold price values",
     sub = "Figure 2: Scatter plot of gold price and 1st lag values")
```

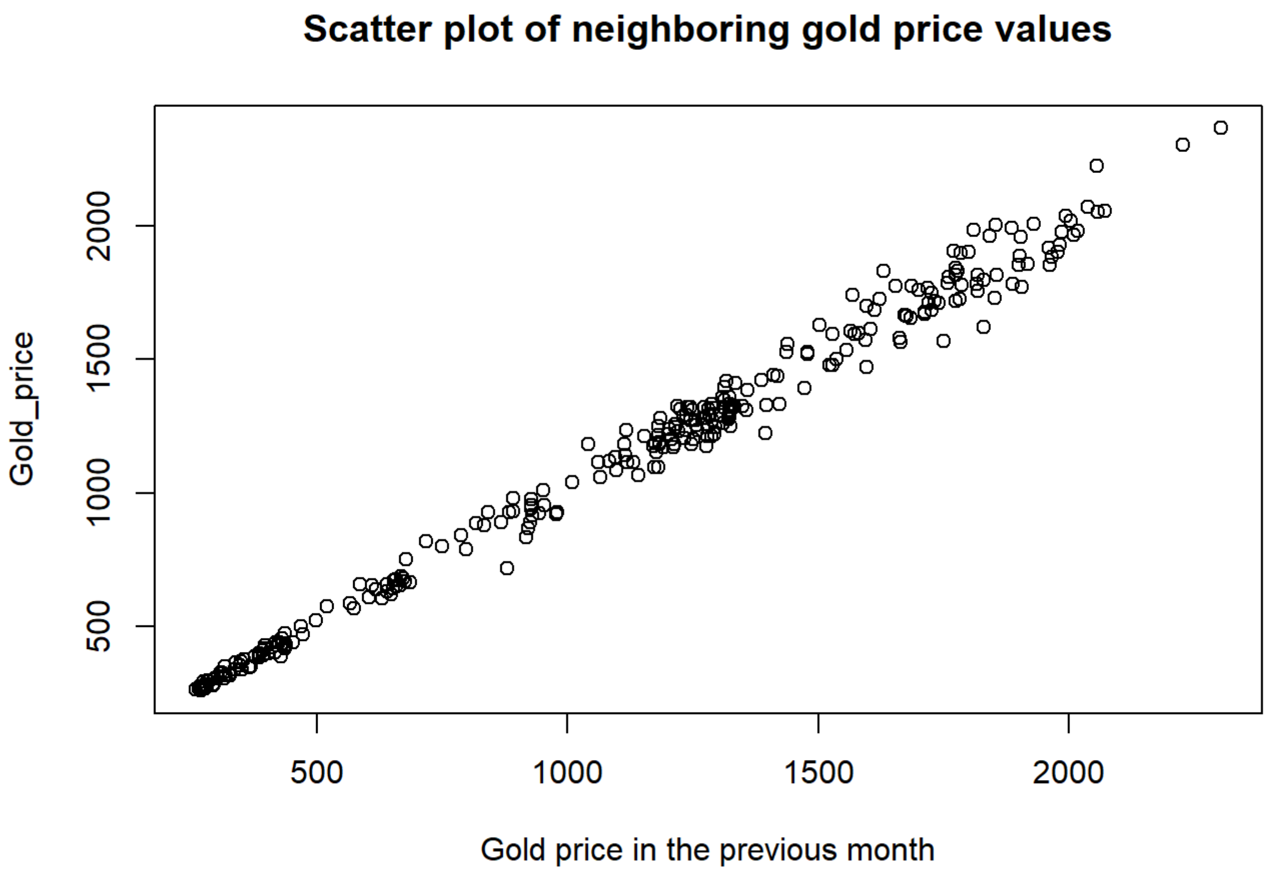


Figure 2: Scatter plot of gold price and 1st lag values

```
# 1st lag correlation
index = 2:length(data.ts)
cor(y[index],x[index])
```

```
## [1] 0.9946506
```

The high correlations observed in the scatter plots from figures 2 indicate a strong relationship between the current month's gold price and its prices in the previous month.

```
# 2nd lag
z = zlag(x)

# 2nd alg plot
plot(y=data.ts, x = zlag(zlag(data.ts)), ylab = 'Gold_price', xlab = 'Gold price in the 2nd p
revious month', main = "Scatter plot of 2nd lag gold price adn gold price values ", sub = "Fi
gure 3: Scatter plot of gold price and 2nd lag values")
```

Figure 3: Scatter plot of gold price and 2nd lag values

```
# 2nd lag correlation
index = 3:length(z)
cor(y[index],z[index])
```

```
## [1] 0.9904694
```

The high correlations observed in the scatter plots from figure 3 indicate a strong relationship between the current month's gold price and its prices in the previous 2 months. Specifically, gold price correlation of (0.9947) with 1st lag and (0.9905) with 2nd lag suggest that the gold price series has strong autoregressive characteristics.

Test of stationary

The following tests are performed to check for the stationarity in the data:

- ACF plot
- PACF plot
- ADF test
- PP test

```
Diagnostic_test(data.ts, lag = 300, mainacf = 'ACF of gold price data', subacf = "Figure 4: A
CF plot of gold price data",mainpacf = 'PACF of gold price data data', subpacf = "Figure 5: P
ACF plot of gold price data", test = "Stationary")
```

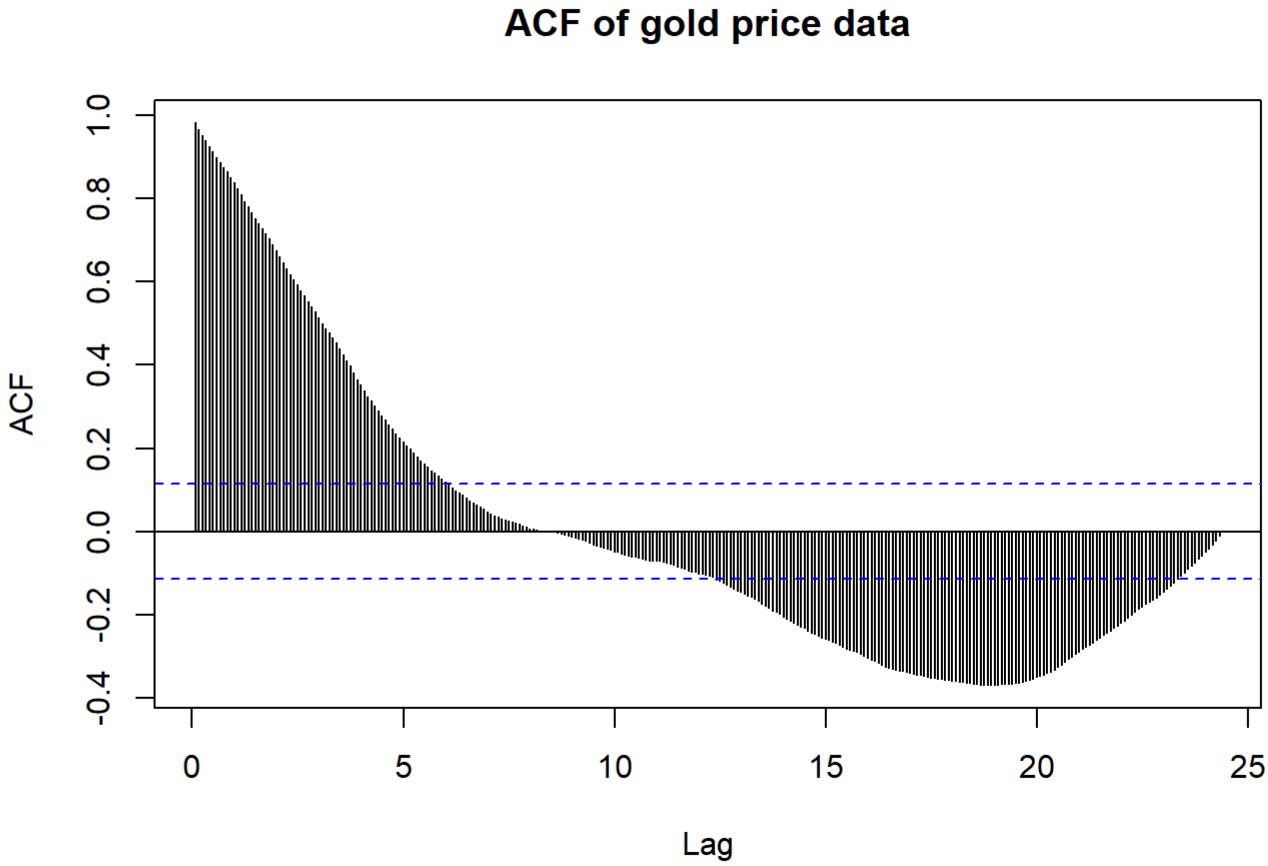


Figure 4: ACF plot of gold price data

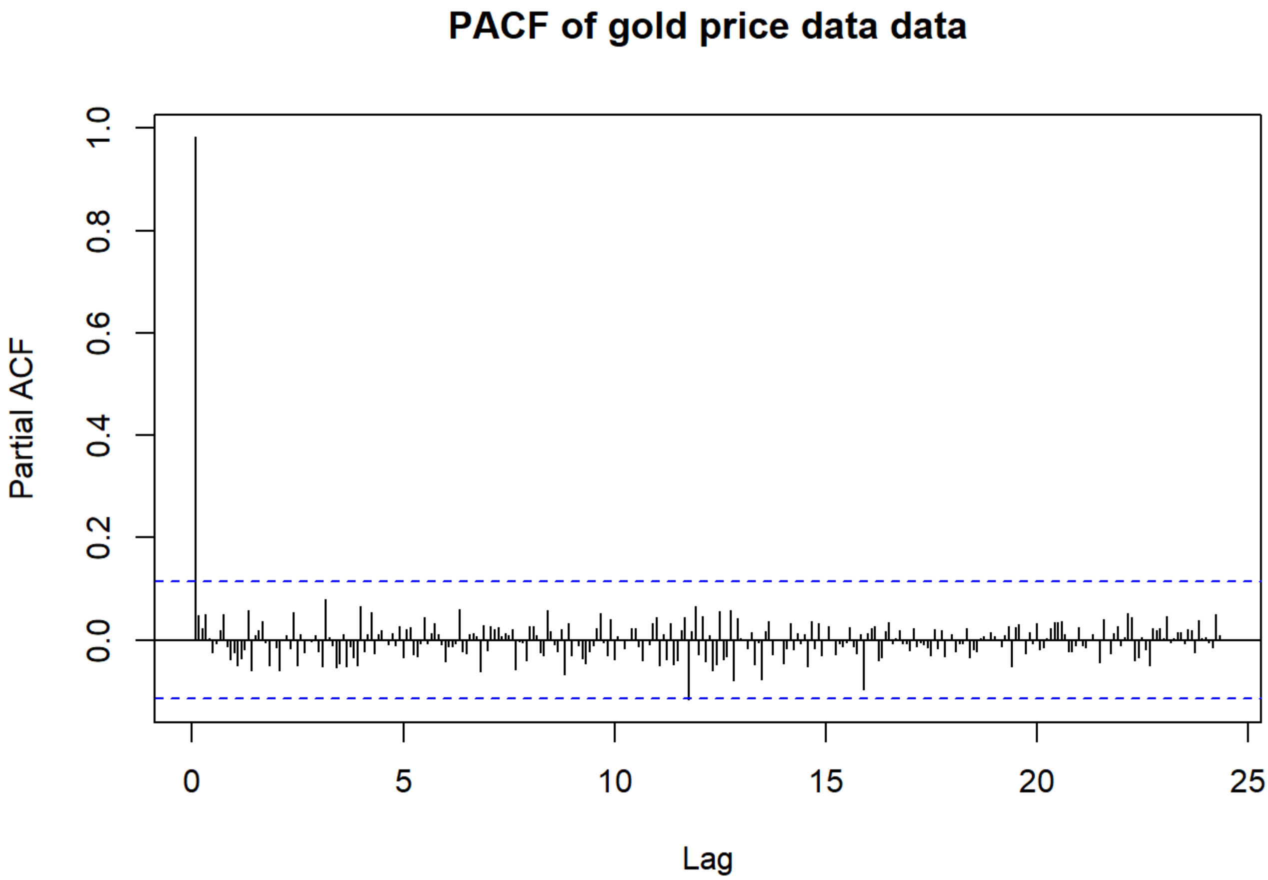


Figure 5: PACF plot of gold price data

```
## $adf_test
##
##  Augmented Dickey-Fuller Test
##
## data:  data
## Dickey-Fuller = -1.7657, Lag order = 6, p-value = 0.6749
## alternative hypothesis: stationary
##
##
## $pp_test
##
##  Phillips-Perron Unit Root Test
##
## data:  data
## Dickey-Fuller Z(alpha) = -7.4695, Truncation lag parameter = 5, p-value
## = 0.6915
## alternative hypothesis: stationary
```

The ACF plot in figure 4 shows a slowly decaying and wave pattern suggesting the presence of trend and seasonality in the series. The PACF plot in figure 5 shows 1st lag highly significant. Both ADF and PP test have a p-value > 0.05, which means that the null hypothesis can not be rejected at 5% significance level, indicating the series in non-stationary. Due to the presence of seasonality in the series, SARIMA models will be fitted.

Test of Normality

The following are checked to test for the normality in the data:

- Histogram
- Normal QQ plot
- Shapiro-Wilk test

```
Diagnostic_test(data.ts, mainhist = "Histogram of gold price series", subhist = "Figure 6: Hi
stogram of gold price series", subqq = "Figure 7: QQ Plot of gold price series", test = "Norm
ality")
```

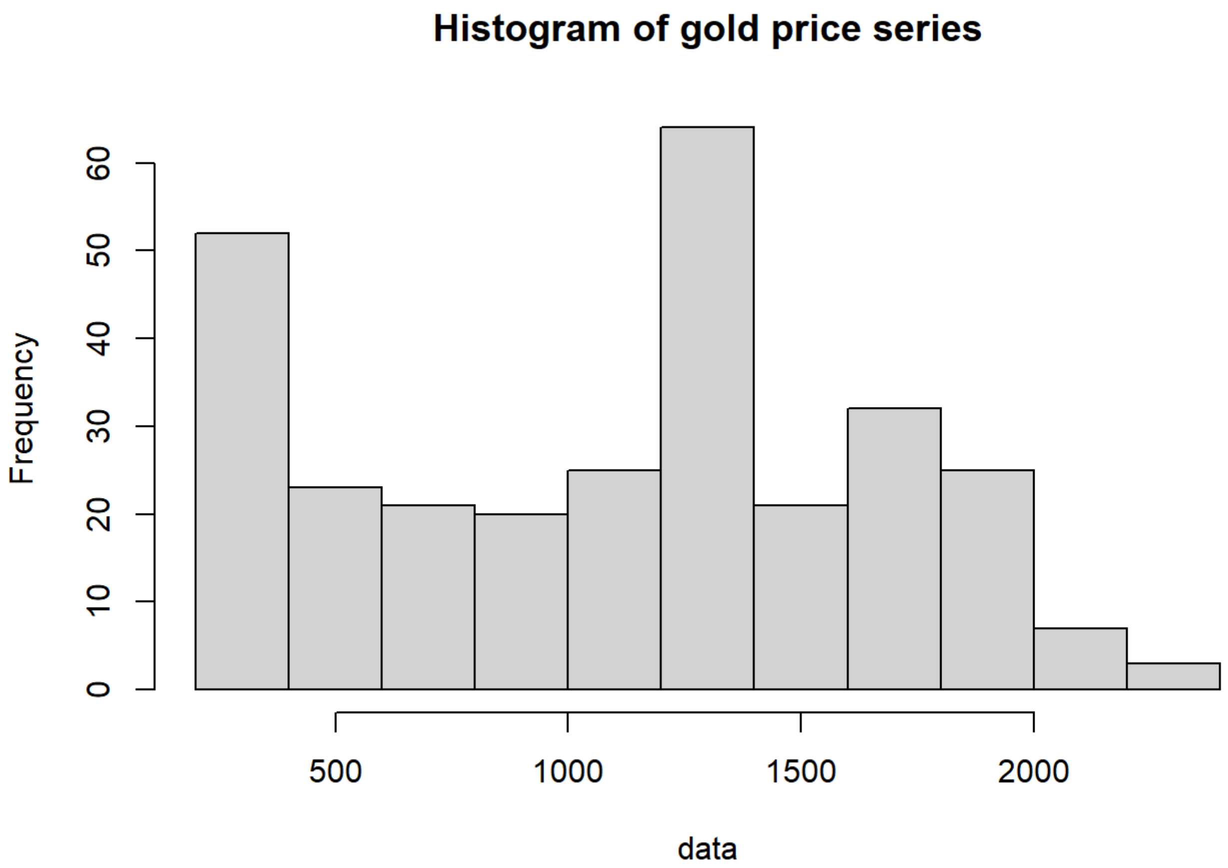


Figure 6: Histogram of gold price series

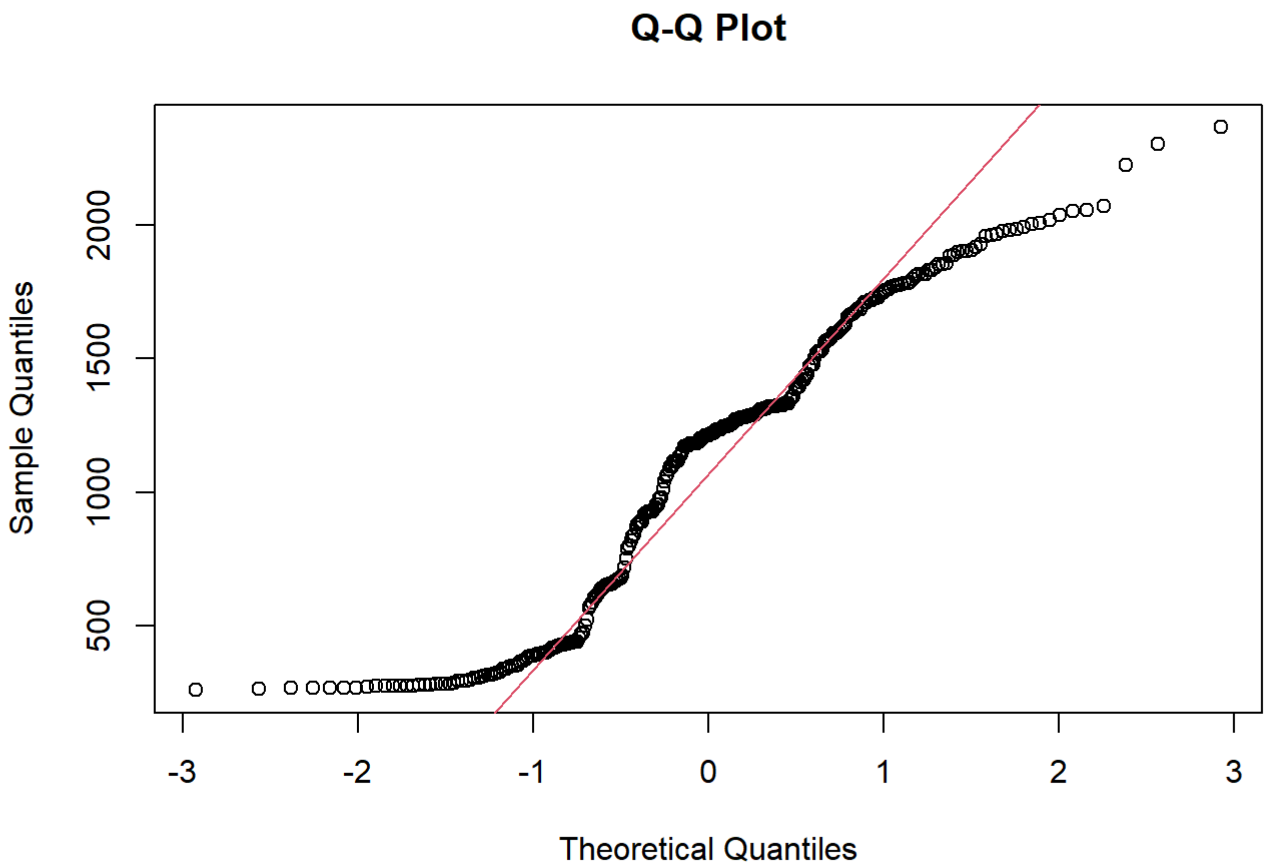


Figure 7: QQ Plot of gold price series

```
##
##  Shapiro-Wilk normality test
##
## data:  data
## W = 0.94027, p-value = 1.659e-09
```

The histogram in figure 6 is not symmetric, and there are outliers present in the series. The QQ-plot in figure 7 shows majority of data points deviate from the normality line and shows presence of outliers in the series. Likewise, the Shapiro test p value is < 0.05 therefore suggesting the data is not normally distributed.

To deal with normality, box transformation is applied, and to remove the trend and attain stationarity, differencing is applied.

Box-Cox Transformation

```
BC <- BoxCox.ar(y=data.ts, lambda=seq(-2, 2, 0.01))
mtext('Figure 8: Box-Cox Transformation: Log-Likelihood vs. Lambda.',line = 4, side = 1, cex
= 0.8)
```

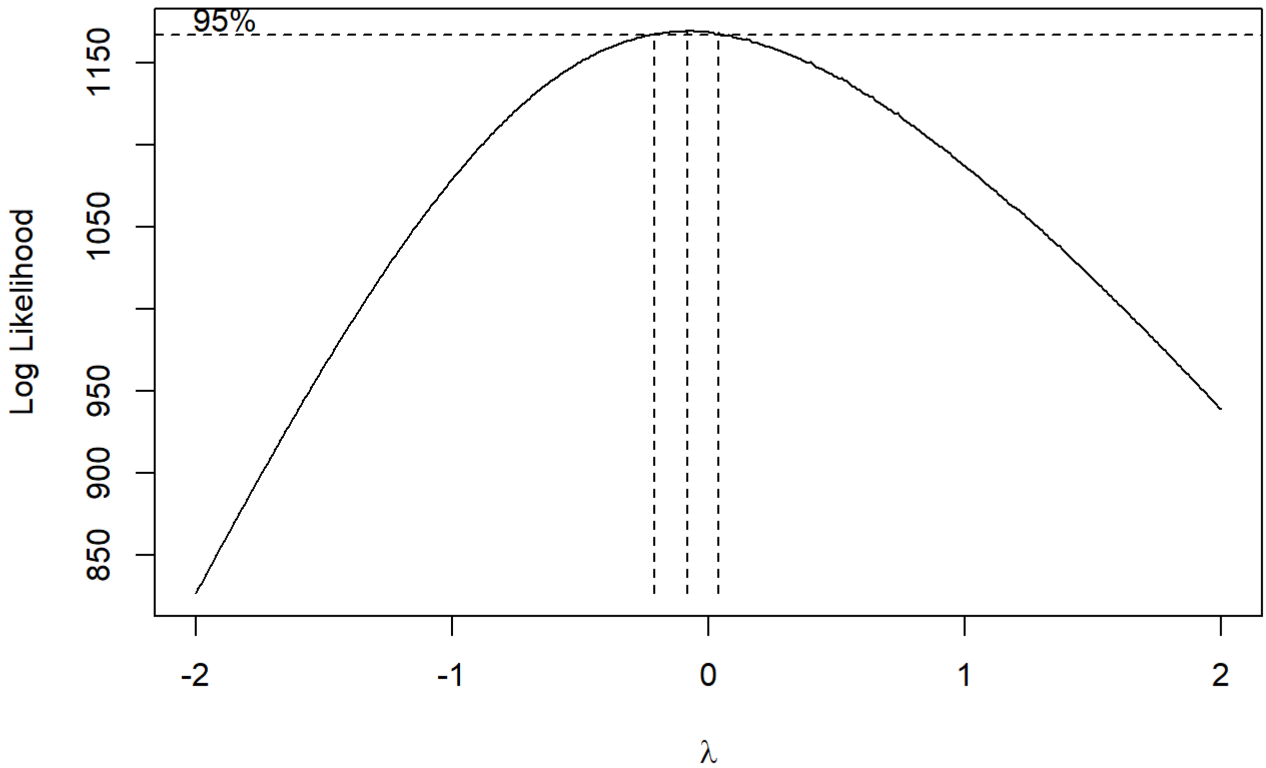


Figure 8: Box-Cox Transformation: Log-Likelihood vs. Lambda.

```
BC$ci
```

```
## [1] -0.21  0.04
```

```
# BoxCox lambda value
lambda <- BC$lambda[which(max(BC$loglike) == BC$loglike)]
lambda
```

```
## [1] -0.08
```

```
BC.data = log(data.ts)
```

From figure 8, it can be observed the lambda value is -0.08 which is close to 0. Therefore, log transformation is applied on the series.

```
# Plot of normalised gold price series
plot(BC.data,type='o', pch = 20, ylab = "Normalised Gold Price", main='Log transformed gold p
rice series', sub = "Figure 9: Log transformed gold price series")
```

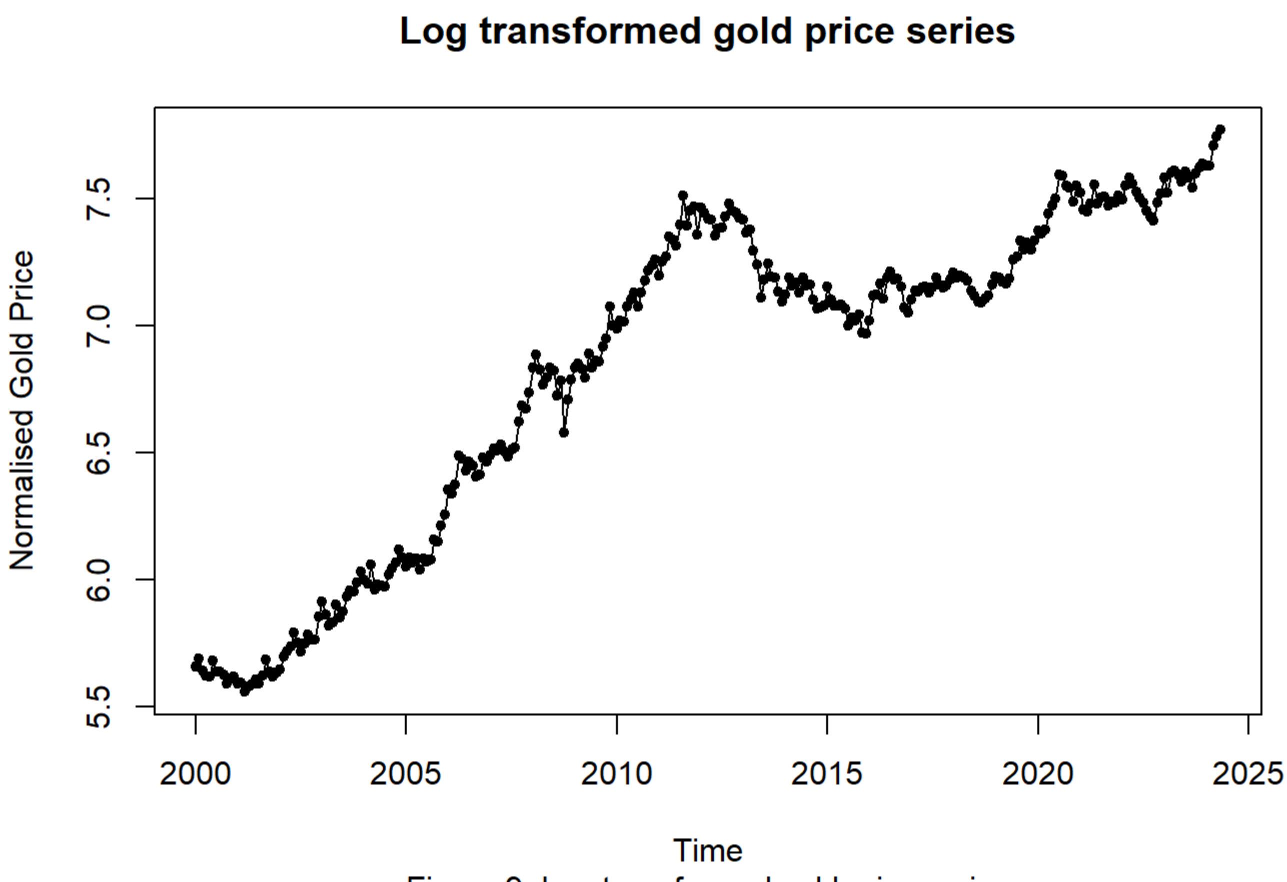


Figure 9: Log transformed gold price series

```
Diagnostic_test(BC.data, subhist = "Figure 10: Histogram of log transformed data", subqq = "F
igure 11: QQ plot of Log transformed gold price series", test = "Normality")
```

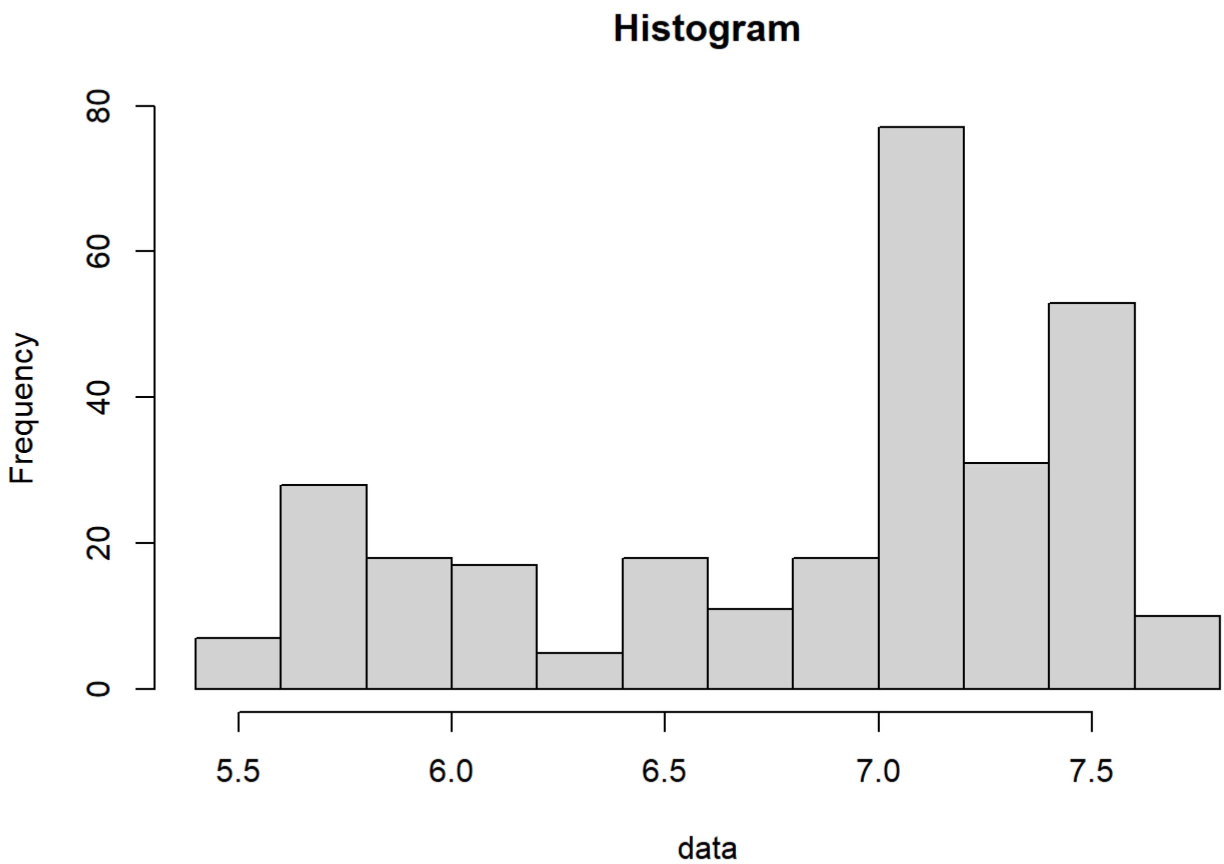


Figure 10: Histogram of log transformed data

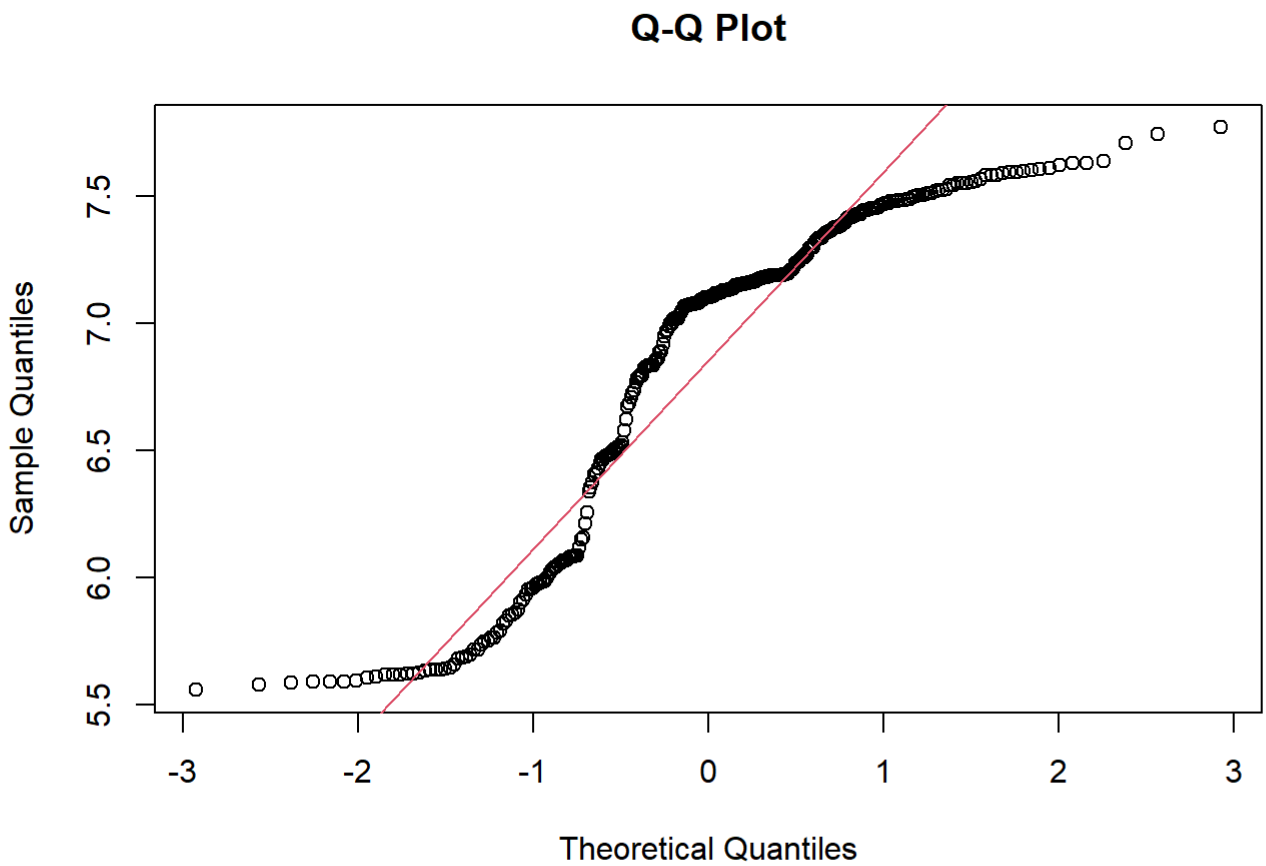


Figure 11: QQ plot of Log transformed gold price series

```
##
##  Shapiro-Wilk normality test
##
## data:  data
## W = 0.87904, p-value = 1.875e-14
```

After applying log transformation on the series, the series did not achieve normality. QQ-plot in figure 11 still shows high deviation of data points with respect to the line of normality. Further, Shapiro-Wilk's test of normality gives a p value < 0.05, and it is worse than that of untransformed series. Therefore, original data is used for further analysis.

Differencing

First differencing

```
diff.ts = diff(data.ts)
plot(diff.ts,type='o',ylab='First difference values', main ="Time series plot of the first di
fference of transformed gold price series.", sub = 'Figure 12: 1st difference series')
```

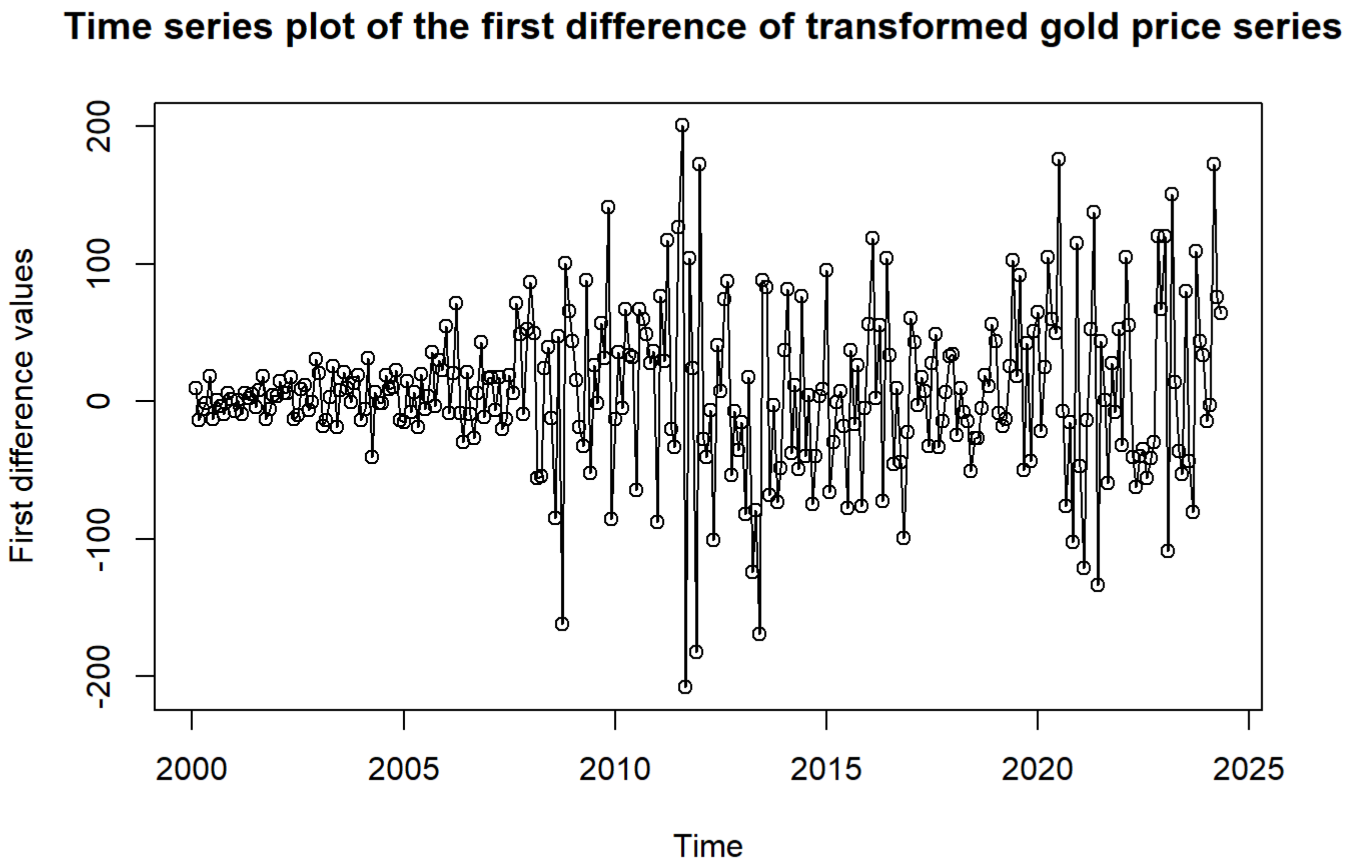


Figure 12: 1st difference series

Figure 12 shows the first difference plot of the gold price series. The plot has a flat mean level indicating no trend, however, the high fluctuations in the differenced series suggest presence of changing variance. McLeod Li test is performed to check for the changing variance in the differenced series.

```
McLeod.Li.test(y=diff.ts, main ="McLeod Li test for differenced series", sub = "Figure 13: Mc
Leod Li test for differenced series")
```

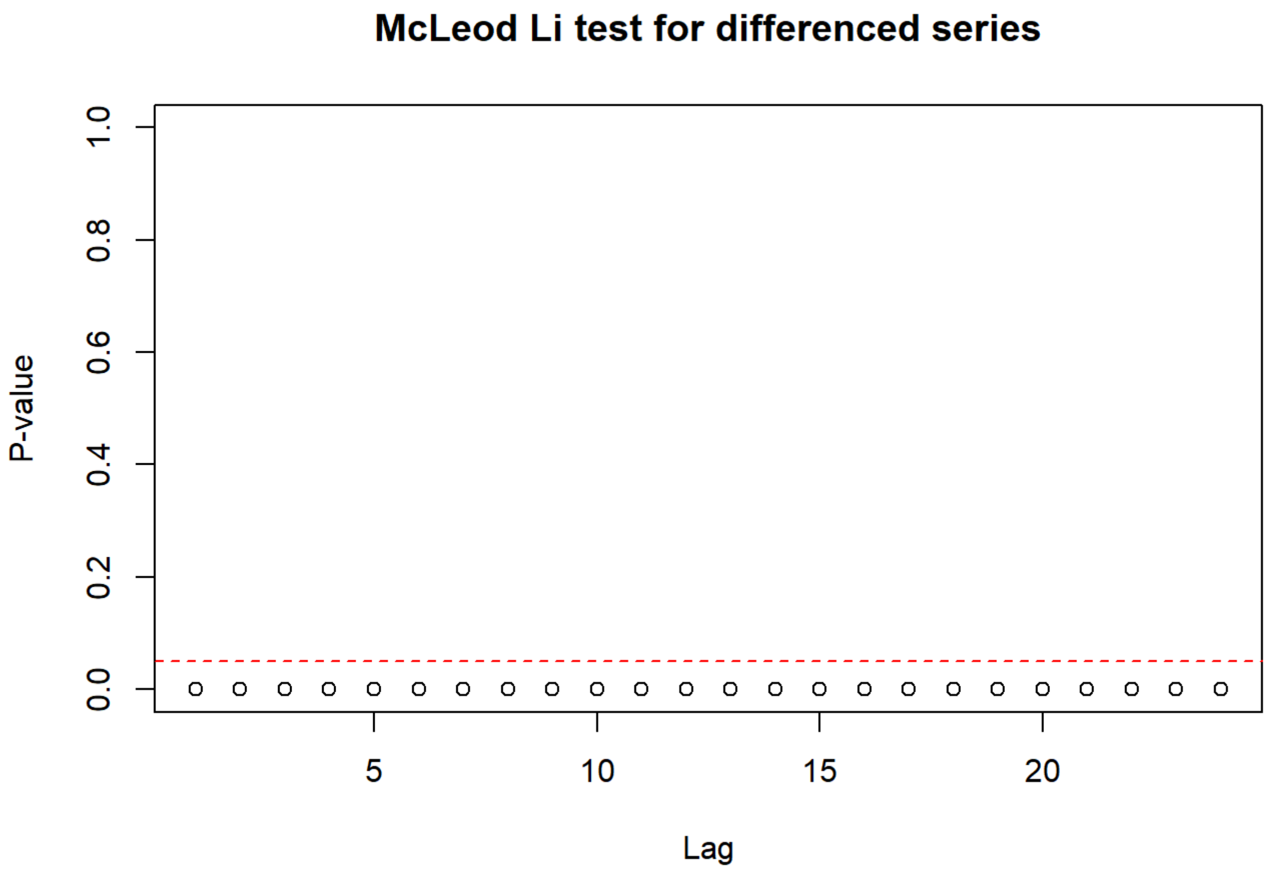


Figure 13: McLeod Li test for differenced series

Figure 13 displays the McLeod Li test for the first difference gold price series. Since the transformation does not help, so MclLeod Li test is applied on first difference oof gold price series. All the lags are beyond the 5% significant level which suggests that the series has changing variance. Due to changing variance, ARMA-GARCH models will be fitted.

First difference series stationary test

```
Diagnostic_test(diff.ts, 100, mainacf = "ACF for First differenced Gold price series", subacf = "Figure 14: ACF plot of first difference gold price time series", mainpacf = "PACF for First differenced Gold price series", subpacf = "Figure 15: PACF plot of first difference gold price time series", test = "Stationary")
```

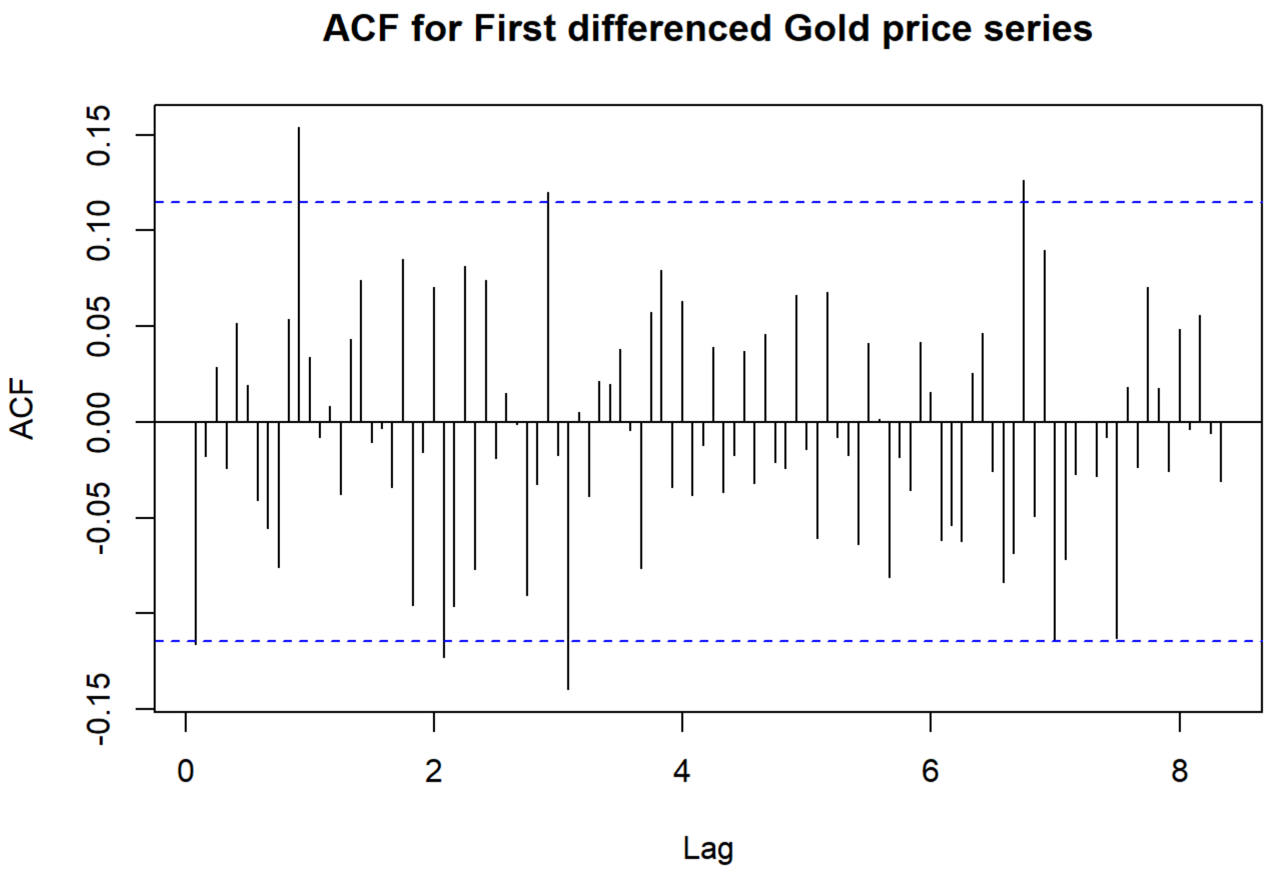


Figure 14: ACF plot of first difference gold price time series

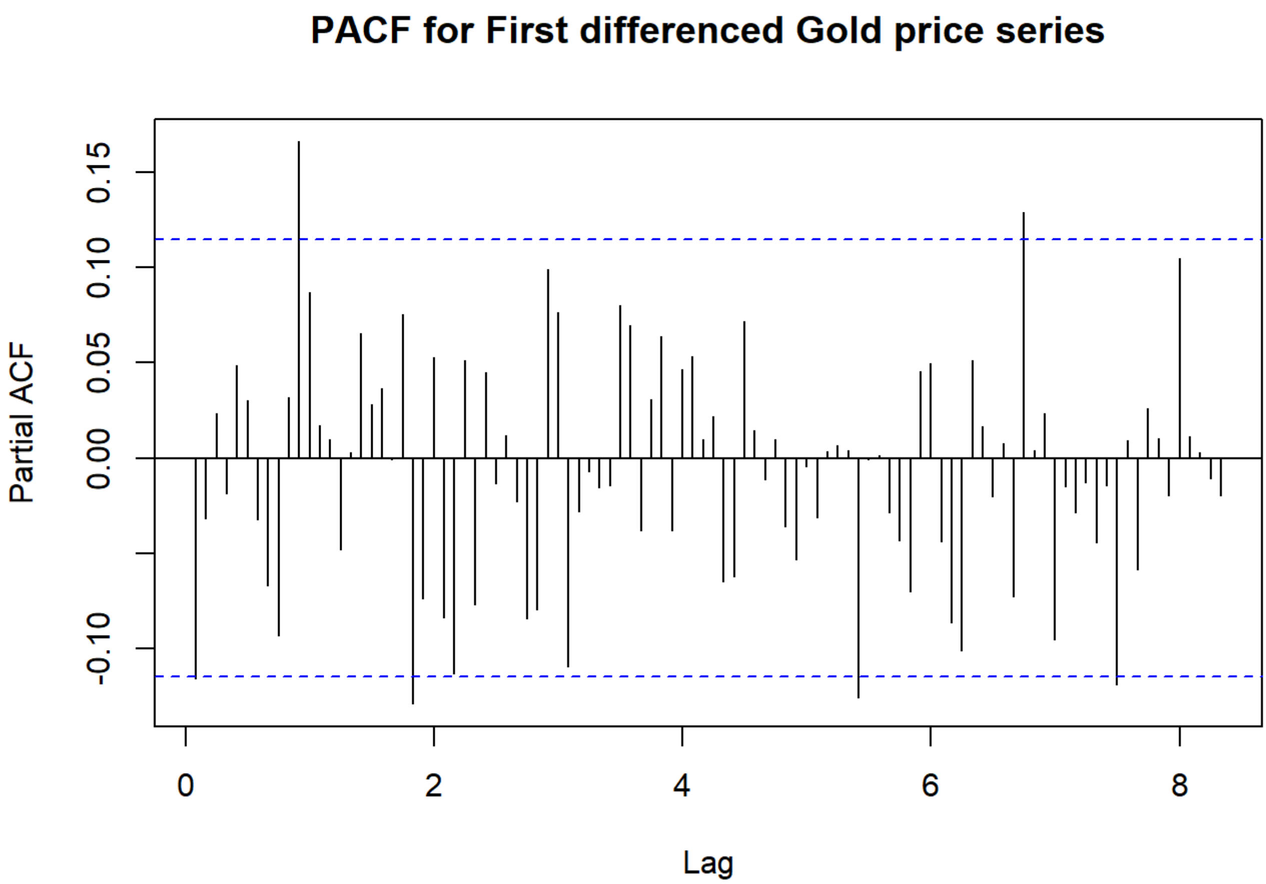


Figure 15: PACF plot of first difference gold price time series

```
## $adf_test
##
##  Augmented Dickey-Fuller Test
##
## data:  data
## Dickey-Fuller = -6.0229, Lag order = 6, p-value = 0.01
## alternative hypothesis: stationary
##
##
## $pp_test
##
##  Phillips-Perron Unit Root Test
##
## data:  data
## Dickey-Fuller Z(alpha) = -322.81, Truncation lag parameter = 5, p-value
## = 0.01
## alternative hypothesis: stationary
```

The ACF plot in figure 14 doesn't show any pattern, the PACF plot in figure 15 doesn't show any highly significant lag suggesting stationary in the series.

ADF and PP test have a p-vale < 0.05, suggesting the null hypothesis can be rejected at 5% significance level, indicating stationary.

Stationarity is achieved by first difference of gold price series. Consequently, for ARIMA(p,d,q) models, 1st difference will be used.

ARIMA models

The following 3 methods are used to identify potential models.

- Significant lags from ACF and PACF plots
- EACF plot
- BIC table

Significant lags in ACF and PACF plot

From figure 14, ACF shows 1 significant lag (late lags are ignored), so q = 1. From figure 15, PACF shows 1 lag significant (late lags are ignored), so p = 1

Model identified from ACF and PACF plot of the differenced series:

- ARIMA(1,1,1)

EACF plot

```
eacf(diff.ts, ar.max = 5, ma.max = 5)
```

```
## AR/MA
##   0 1 2 3 4 5
## 0 o o o o o o
## 1 x o o o o o
## 2 x o o o o o
## 3 x o o o o o
## 4 x x x x o o
## 5 x x x x o o
```

The top-left “o” identified in the EACF plot is (0,0)

Neighbor models:

- ARIMA(0,1,0)
- ARIMA(0,1,1)
- ARIMA(1,1,1)

There are no AR/MA subsets in ARIMA(0,1,0), so is not considered for analysis.

BIC plot

```
res = armasubsets(y=diff.ts,nar=4,nma=4,y.name='p',ar.method='ols')
```

```
## Reordering variables and trying again:
```

```
plot(res)
mtext('Figure 16:BIC Table',line = 4, side = 1, cex = 0.8)
```

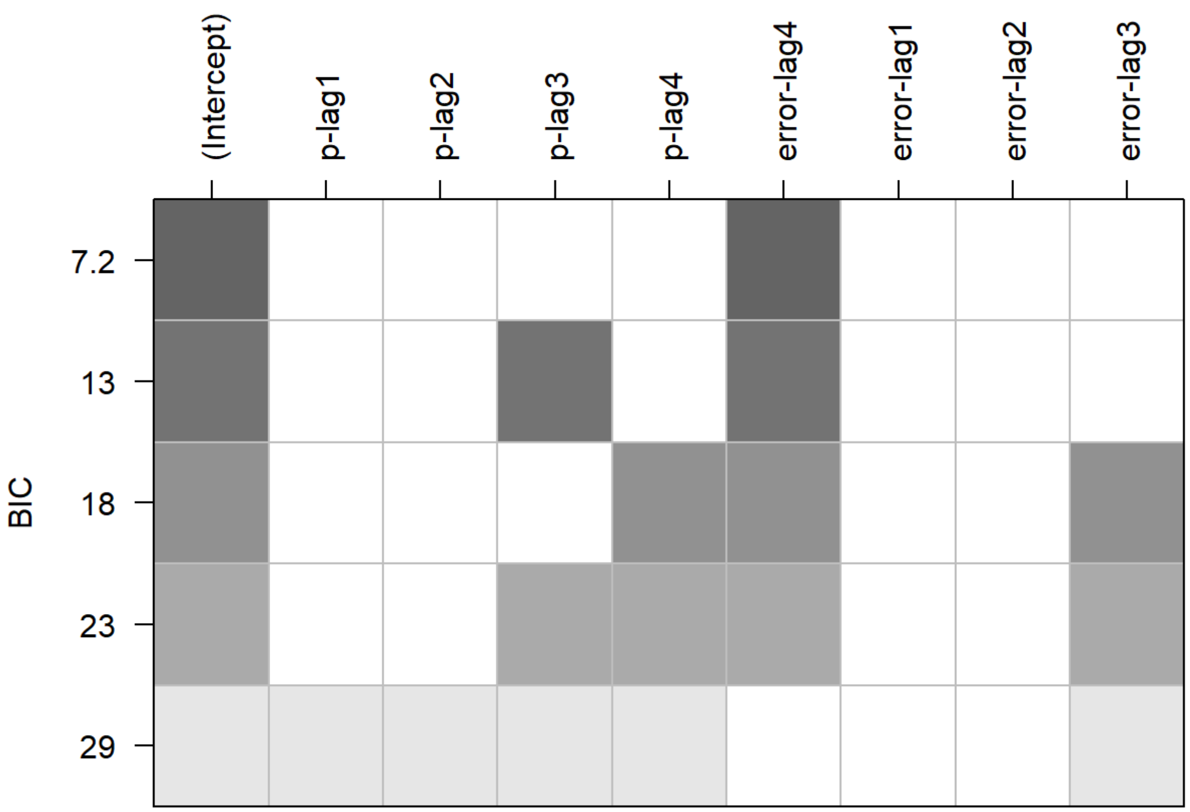


Figure 16:BIC Table

From BIC table following model the top model has the lowest BIC value(7.2):

- ARIMA(0,1,4)

Parameter Estimation

ARIMA(0,1,1)

```
model.011 = Arima(data.ts,order=c(0,1,1),method='ML')
coeftest(model.011)
```

```
##
## z test of coefficients:
##
##      Estimate Std. Error z value Pr(>|z|)
## ma1 -0.100260   0.057853  -1.733  0.08309 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
model.011css = Arima(data.ts,order=c(0,1,1),method='CSS')
coeftest(model.011css)
```

```
##
## z test of coefficients:
##
##      Estimate Std. Error z value Pr(>|z|)
## ma1 -0.100595   0.057945 -1.7361  0.08255 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

No coefficients are significant for model.011 at 5% significance level.

ARIMA(1,1,1)

```
model.111 = Arima(data.ts,order=c(1,1,1),method='ML')
coeftest(model.111)
```

```
##
## z test of coefficients:
##
##      Estimate Std. Error z value Pr(>|z|)
## ar1 -0.030240   0.433075 -0.0698   0.9443
## ma1 -0.071377   0.431113 -0.1656   0.8685
```

```
model.111css = Arima(data.ts,order=c(1,1,1),method='CSS')
coeftest(model.111css)
```

```
##
## z test of coefficients:
##
##      Estimate Std. Error z value Pr(>|z|)
## ar1 -0.047353   0.434847 -0.1089   0.9133
## ma1 -0.052988   0.435171 -0.1218   0.9031
```

No coefficients are significant for model.111

ARIMA(0,1,4)

```
model.014 = Arima(data.ts,order=c(0,1,4),method='ML')
coeftest(model.014)
```

```
##
## z test of coefficients:
##
##       Estimate Std. Error z value Pr(>|z|)
## ma1 -0.1022941  0.0593224 -1.7244  0.08464 .
## ma2 -0.0054222  0.0598917 -0.0905  0.92786
## ma3  0.0415385  0.0596254  0.6967  0.48602
## ma4  0.0113208  0.0662584  0.1709  0.86434
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
model.014css = Arima(data.ts,order=c(0,1,4),method='CSS')
coeftest(model.014css)
```

```
##
## z test of coefficients:
##
##       Estimate Std. Error z value Pr(>|z|)
## ma1 -0.1026921  0.0594365 -1.7278  0.08403 .
## ma2 -0.0055445  0.0601135 -0.0922  0.92651
## ma3  0.0420180  0.0599702  0.7006  0.48352
## ma4  0.0116994  0.0668821  0.1749  0.86114
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

No coefficients are significant for model.014

Model.011 is the best model based in coefficient testing.

AIC and BIC values

The AIC and BIC scores are calculated using “ML” methods.

```
aic_table =AIC(model.011, model.111, model.014)
bic_table =BIC(model.011, model.111, model.014)

sorted_aic_table <- aic_table[order(aic_table$AIC), ]
sorted_bic_table <- bic_table[order(bic_table$BIC), ]

sorted_aic_table
```

```
##           df      AIC
## model.011  2 3200.211
## model.111  3 3202.208
## model.014  5 3205.699
```

```
sorted_bic_table
```

```
##           df      BIC
## model.011  2 3207.564
## model.111  3 3213.238
## model.014  5 3224.083
```

From the AIC and BIC score, model.011 is the best model.

Error measures

```
Smodel.011 = accuracy(summary(model.011))
Smodel.111 = accuracy(summary(model.111))
Smodel.014 = accuracy(summary(model.014))

headers <- c("ME", "RMSE", "MAE", "MPE", "MAPE", "MASE", "ACF1")

merged_table <- data.frame(rbind
    ( c(Smodel.011), c(Smodel.111),c(Smodel.014)))
model_names <- c("model.011", "model.111","model.014")

rownames(merged_table) <- model_names
colnames(merged_table) <- headers
merged_table
```

```
##                 ME     RMSE      MAE       MPE     MAPE      MASE        ACF1
## model.011 7.865242 57.52952 41.24505 0.6799457 3.656716 0.2946539 -0.01928541
## model.111 7.852654 57.52924 41.25099 0.6787429 3.657211 0.2946964 -0.01795060
## model.014 7.547298 57.47852 41.28719 0.6491016 3.657498 0.2949550 -0.01645941
```

The error metrics shows very close values for all the models.

Based on coef_test, AIC, BIC, and error metrics, model.011 is the best model.

Over-parameterisation

Parameter tuning is done to identify any further potential models.

The following models will be tested under parameter tuning:

- ARIMA(0,1,2)
- ARIMA(1,1,1)

We have already tested the ARIMA(1,1,1) model and concluded that it wasn't a significant model.

Parameter estimation is conducted on ARIMA(0,1,2) model.

ARIMA(0,1,2)

```
model.012 = Arima(data.ts,order=c(0,1,2),method='ML')
coeftest(model.012)
```

```
##
## z test of coefficients:
##
##       Estimate Std. Error z value Pr(>|z|)
## ma1 -0.1010029  0.0589617 -1.7130  0.08671 .
## ma2  0.0043249  0.0586188  0.0738  0.94119
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
model.012css = Arima(data.ts,order=c(0,1,2),method='CSS')
coeftest(model.012css)
```

```
##
## z test of coefficients:
##
##       Estimate Std. Error z value Pr(>|z|)
## ma1 -0.1013471  0.0590650 -1.7159  0.08619 .
## ma2  0.0043658  0.0588095  0.0742  0.94082
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

No coefficients are significant for model.012 at 5% level. Hence, we can ignore this model.

ARIMA models gives us model.011 as the best option, however, we will try different approaches to get the best model.

SARIMA models

ACF and PACF plot

Plotting ACF(figure 3) and PACF(figure 4) of gold price series again

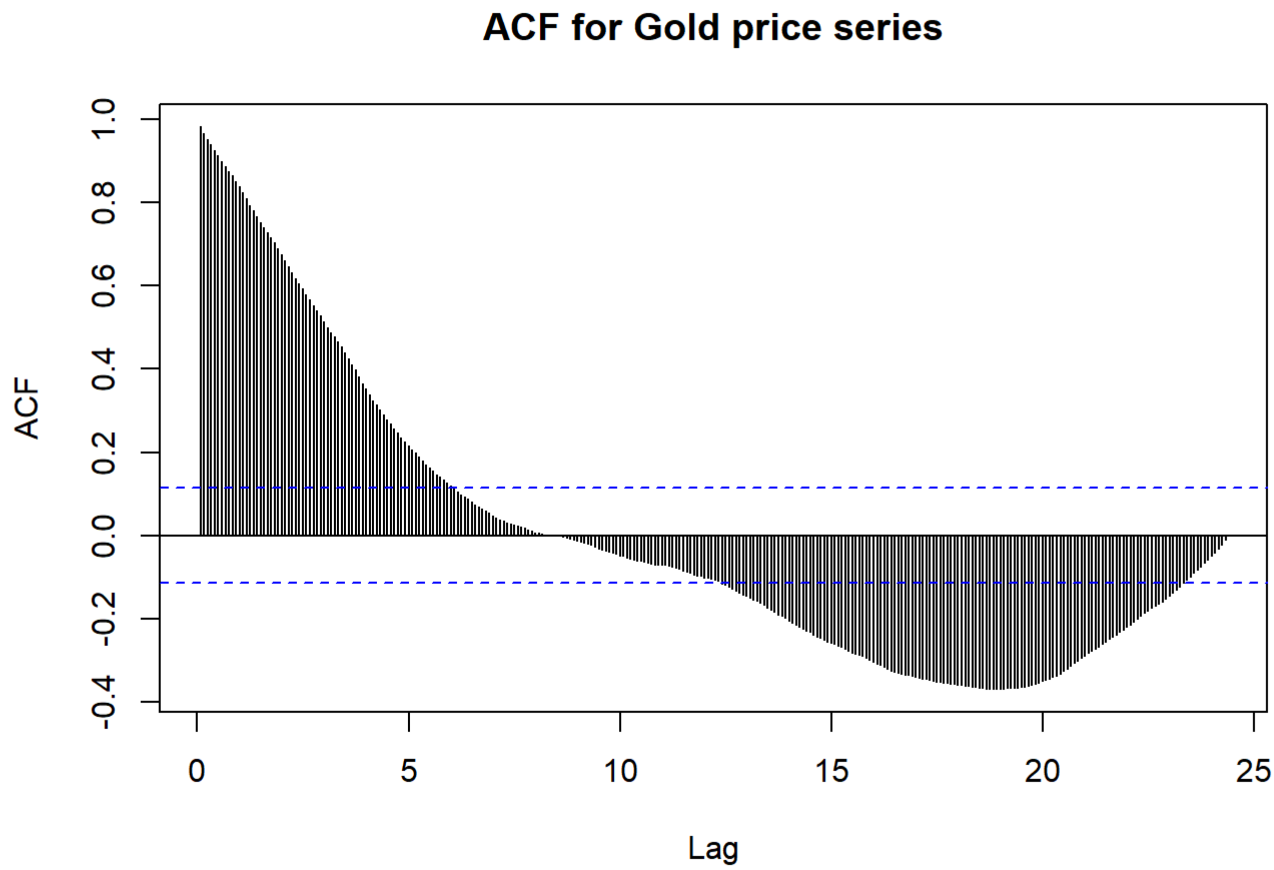


Figure 3: ACF plot of gold price time series

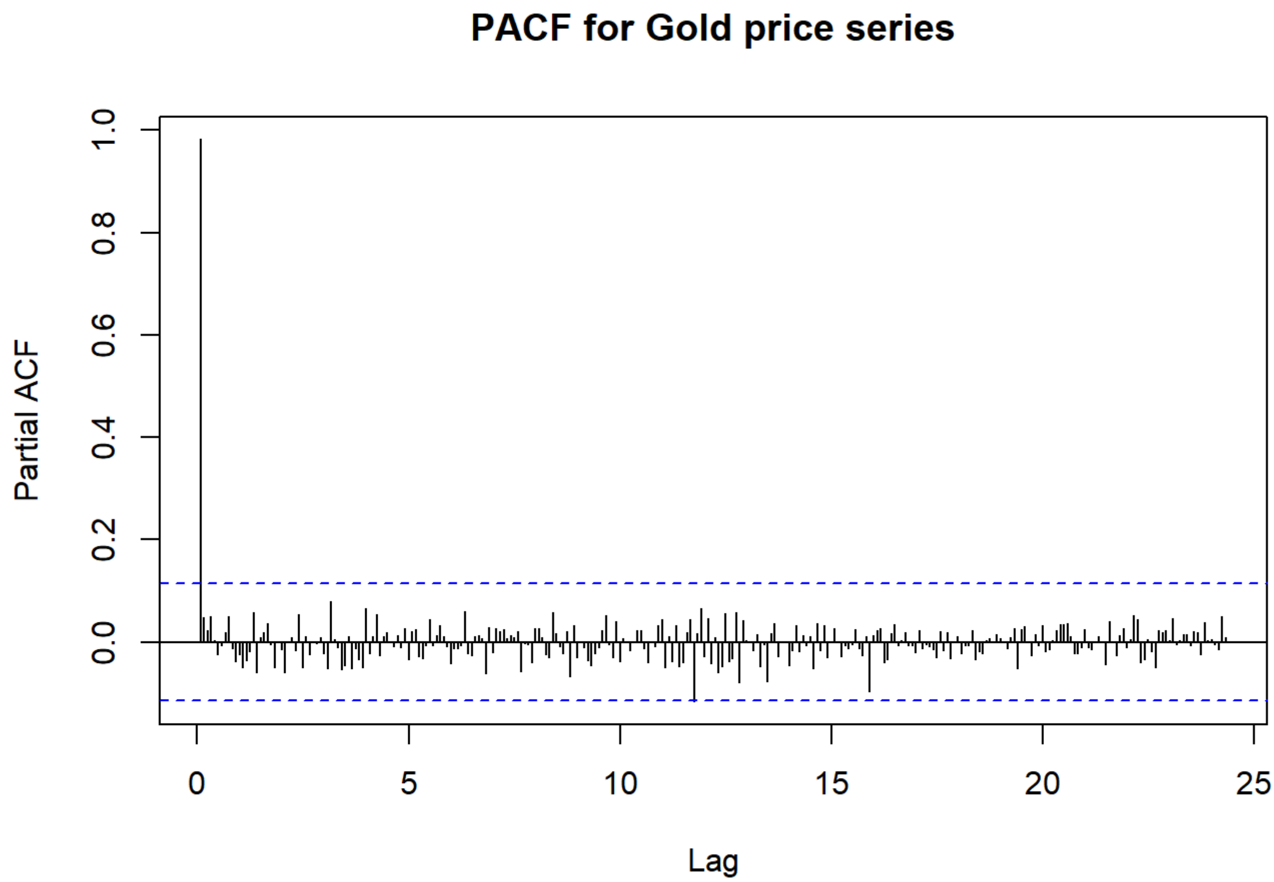


Figure 4: PACF plot of gold price time series

The ACF plots in figure 3 revealed presence of trend and seasonality, SARIMA models are fitted on the series.

To remove the trend, 1st seasonal difference is applied

Seasonal Difference (D=1)

```
# Plain model fit with 1st differencing
m1.gold = Arima(data.ts,order=c(0,0,0), seasonal=list(order=c(0,1,0), period=12))

# m1 residuals
res.m1 = rstandard(m1.gold)
plot(res.m1,xlab='Time',ylab='Residuals', main="Time series plot of the residuals.", sub = "F
igure 17: Time series of residuals")
```

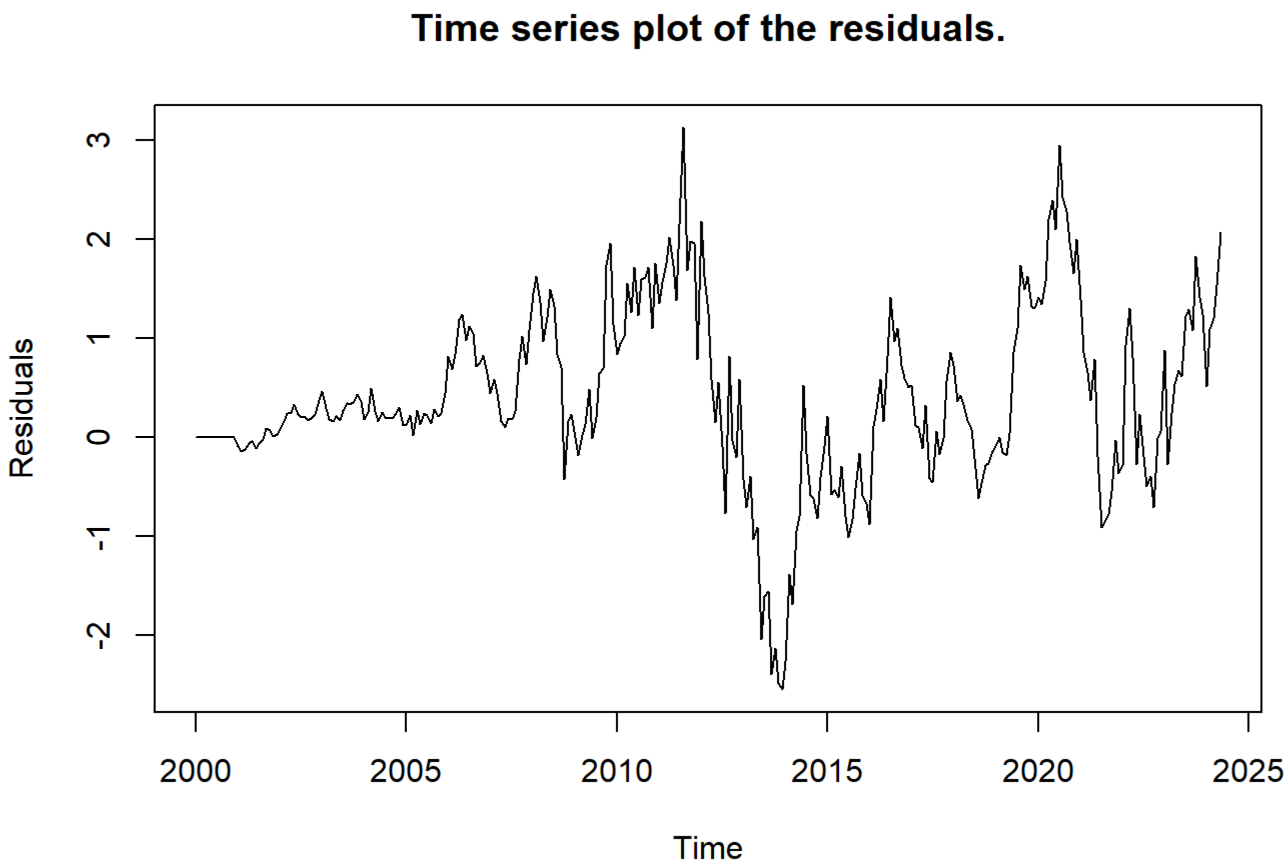


Figure 17: Time series of residuals

```
Diagnostic_test(res.m1, 48, mainacf = "The sample ACF of the residuals", subacf = "Figure 18:
Sample ACF plot of residulas", mainpacf = "The sample PACF of the residuals", subpacf = "Figu
re 19: Sample PACF plot of residuals", test = "seasonal Stationary")
```

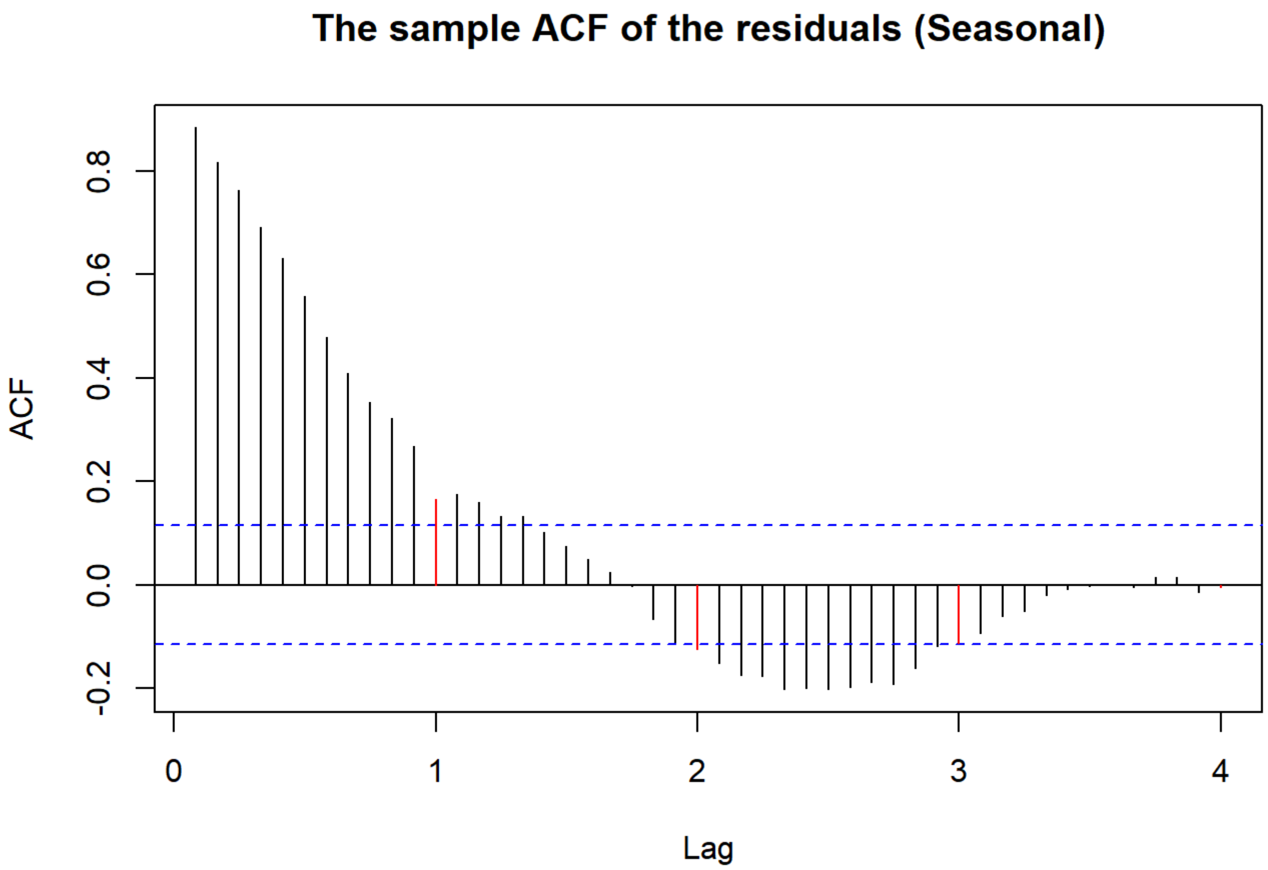


Figure 18: Sample ACF plot of residulas

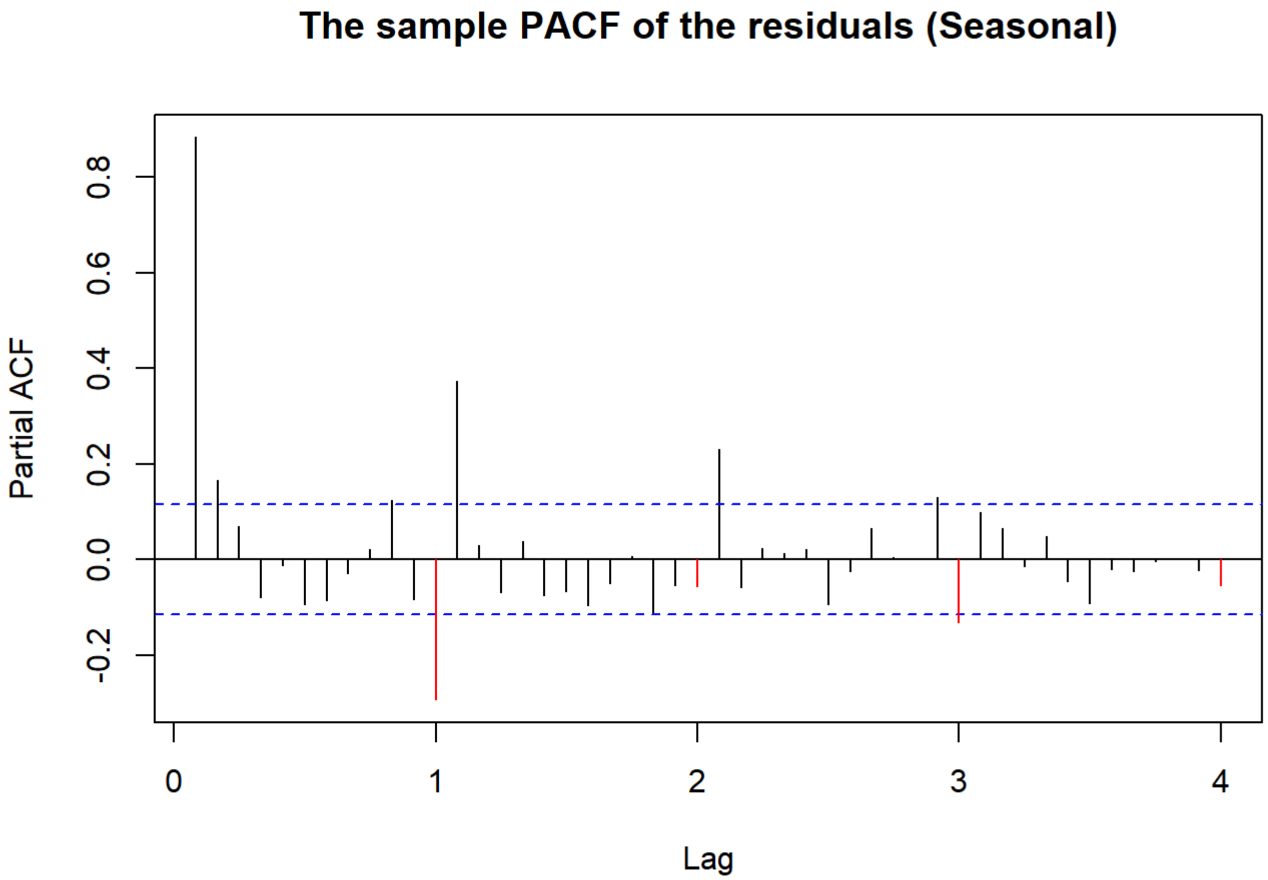


Figure 19: Sample PACF plot of residuals

```
## $adf_test
##
##  Augmented Dickey-Fuller Test
##
## data:  data
## Dickey-Fuller = -3.5707, Lag order = 6, p-value = 0.03631
## alternative hypothesis: stationary
##
##
## $pp_test
##
##  Phillips-Perron Unit Root Test
##
## data:  data
## Dickey-Fuller Z(alpha) = -28.533, Truncation lag parameter = 5, p-value
## = 0.01
## alternative hypothesis: stationary
```

The first seasonal difference residual plot(Figure 17) still has trend and fluctuations.

The 1st seasonal difference fitted series has a trend, but, the series is stationary as suggested by ADF and PP test. The PACF plot has 1st seasonal lag significant, so P = 1. The ACF plot has 1st, and 2nd seasonal lag significant, so, Q=2 is applied to in the model to get rid of seasonal component.

Seasonal parameter ((P,Q) = (1,2))

```
m2.gold = Arima(data.ts,order=c(0,0,0),seasonal=list(order=c(1,1,2), period=12,  method="M
L"))

res.m2 = rstandard(m2.gold)

plot(res.m2,xlab='Time',ylab='Residuals',main="Time series plot of the residuals.",sub = "Fig
ure m2.1: Time series of residuals")
```

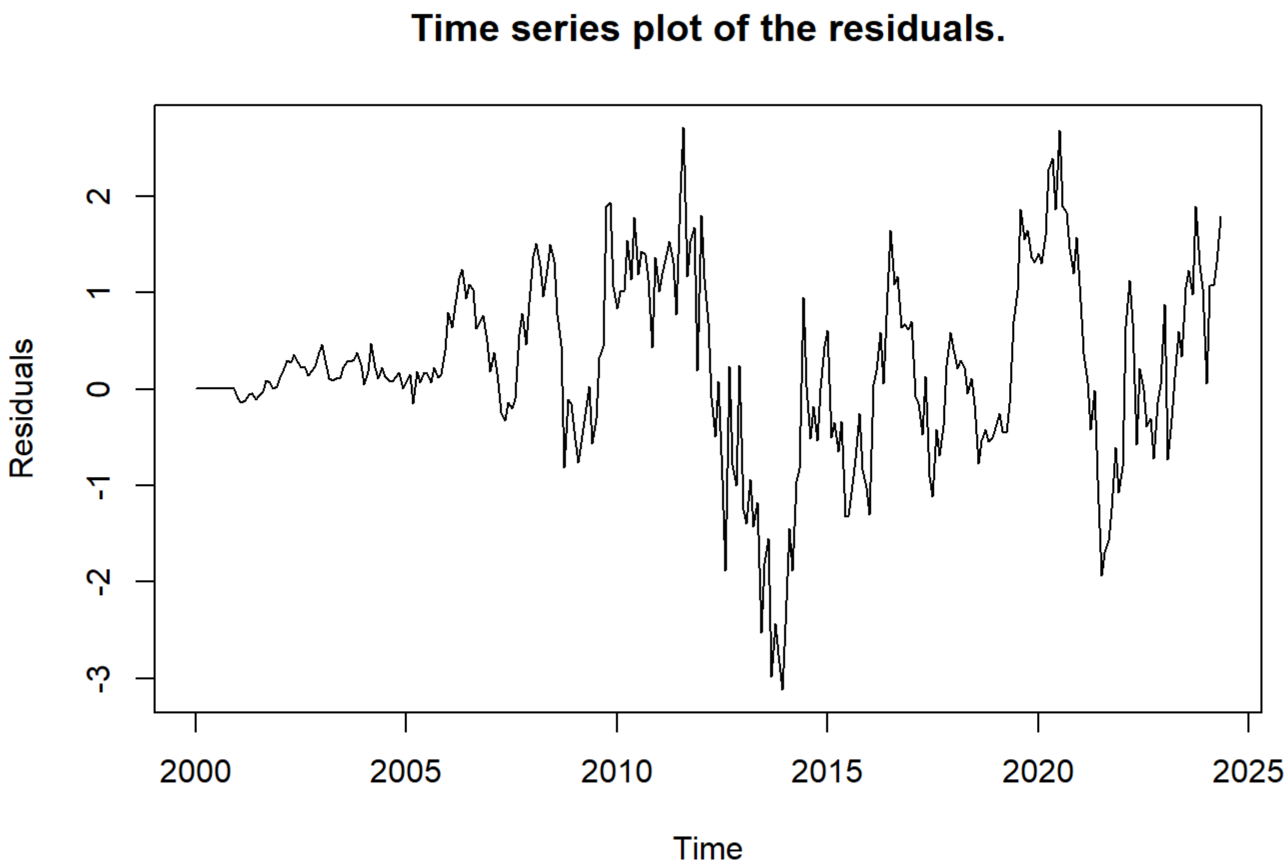


Figure m2.1: Time series of residuals

```
Diagnostic_test(res.m2, 48, mainacf = "The sample ACF of the residuals", subacf = "Figure 20:
Sample ACF plot of residulas", mainpacf = "The sample PACF of the residuals", subpacf = "Figu
re 21: Sample PACF plot of residuals", test = "seasonal ACF-PACF")
```

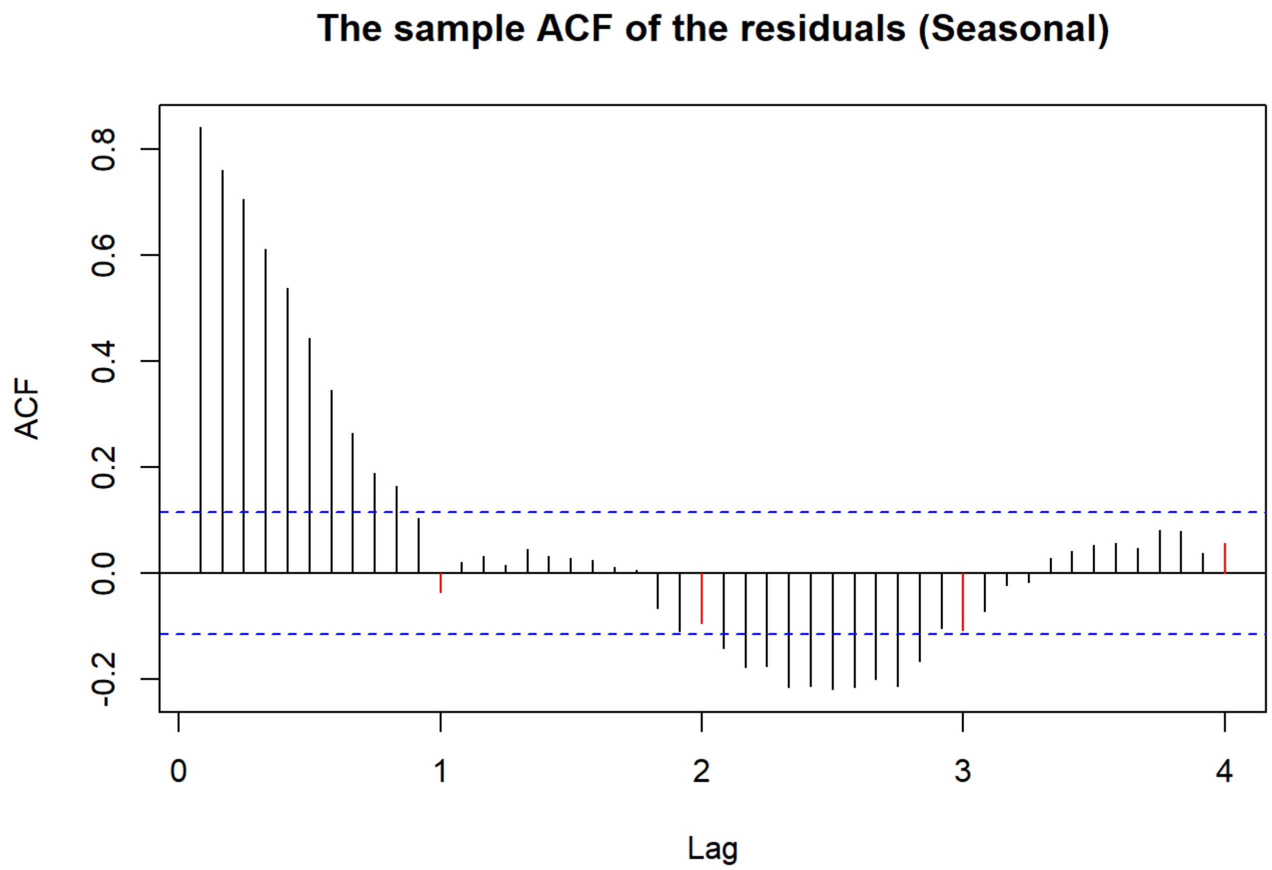


Figure 20: Sample ACF plot of residulas

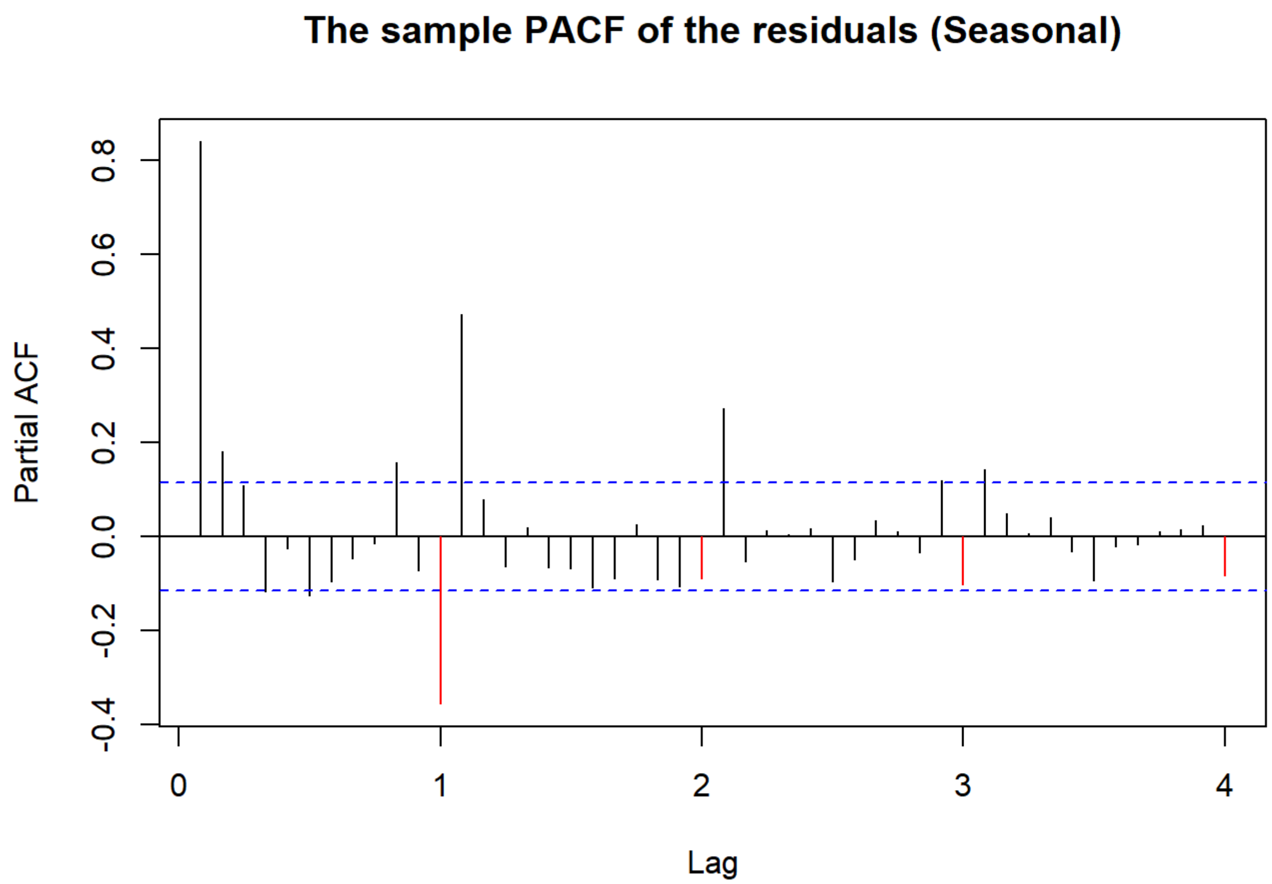


Figure 21: Sample PACF plot of residuals

The seasonal parameter fitted residual plot(Figure m2.1) still has trend and fluctuations.

There is a high auto correlation at the first lag of PACF, and nearly all the auto correlations are significant before the first seasonal lag in ACF. So, first ordinary difference is applied to remove this trend.

Ordinal difference (d=1)

```
m3.gold = Arima(data.ts, order=c(0,1,0), seasonal= list(order=c(1,1,2), period=12,  method="M
L"))

res.m3 = rstandard(m3.gold)
plot(res.m3,xlab='Time',ylab='Residuals',main="Time series plot of the residuals.", sub = "Fi
gure m3.1: Time series of residuals")
```

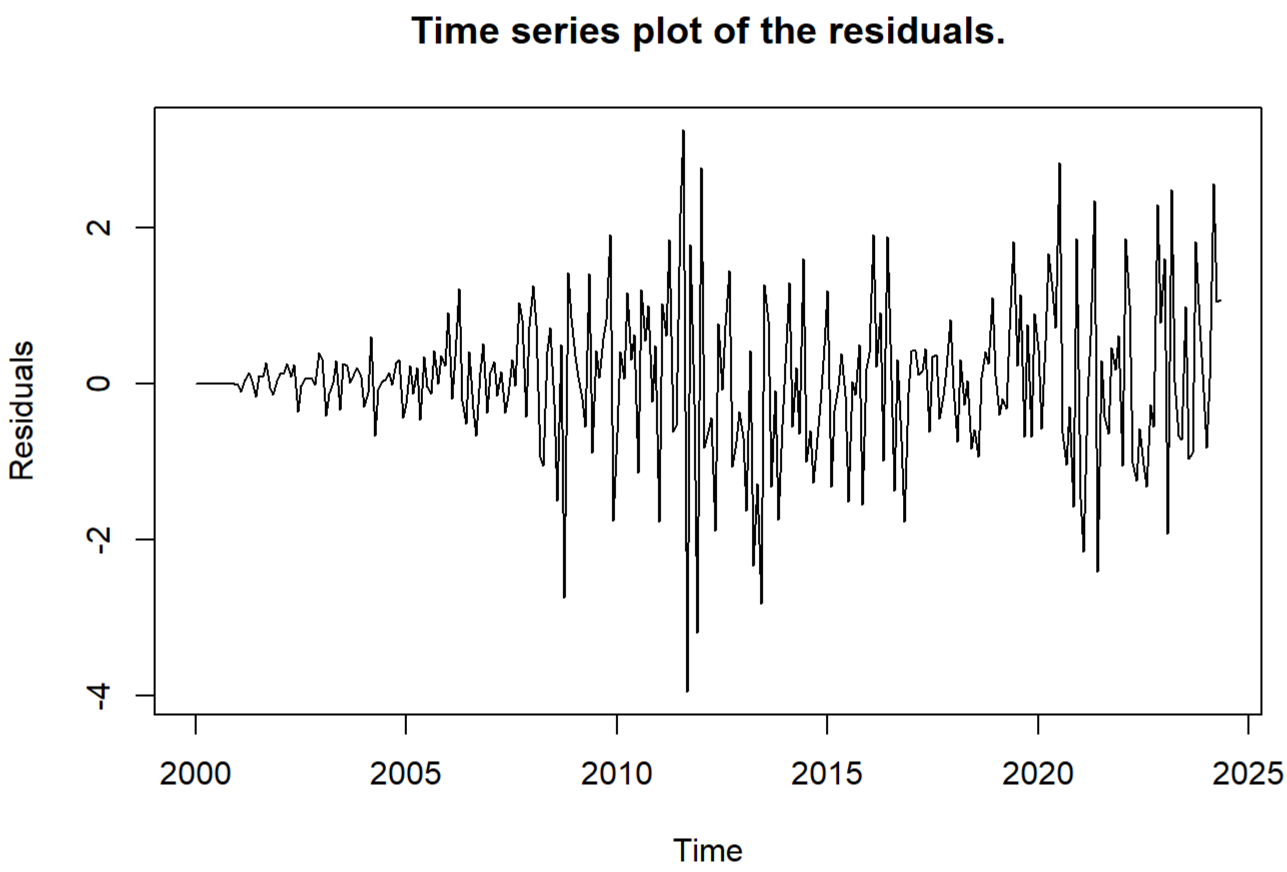


Figure m3.1: Time series of residuals

```
Diagnostic_test(res.m3, 48, mainacf = "The sample ACF of the residuals", subacf = "Figure 22:
Sample ACF plot of residulas", mainpacf = "The sample PACF of the residuals", subpacf = "Figu
re 23: Sample PACF plot of residuals", test = "seasonal ACF-PACF")
```

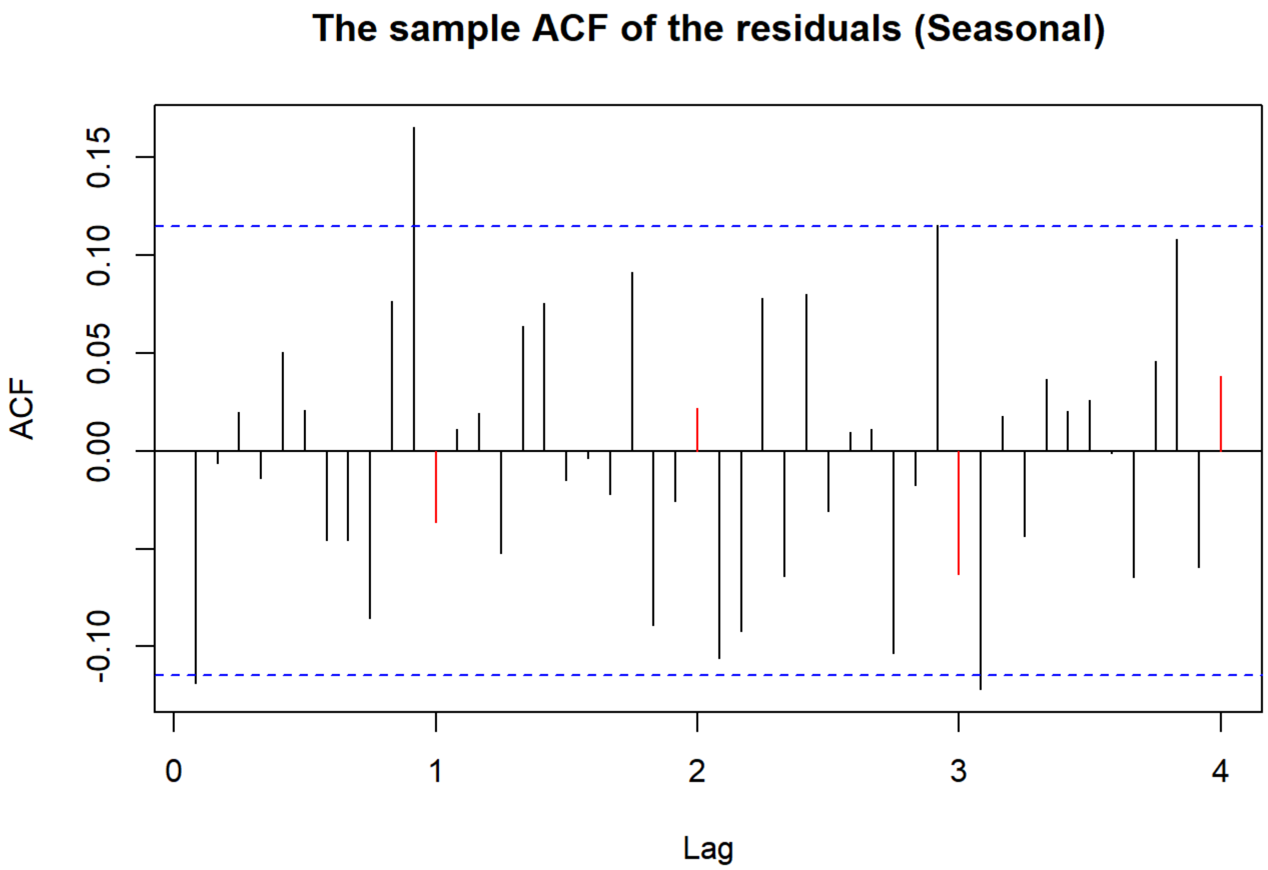


Figure 22: Sample ACF plot of residulas

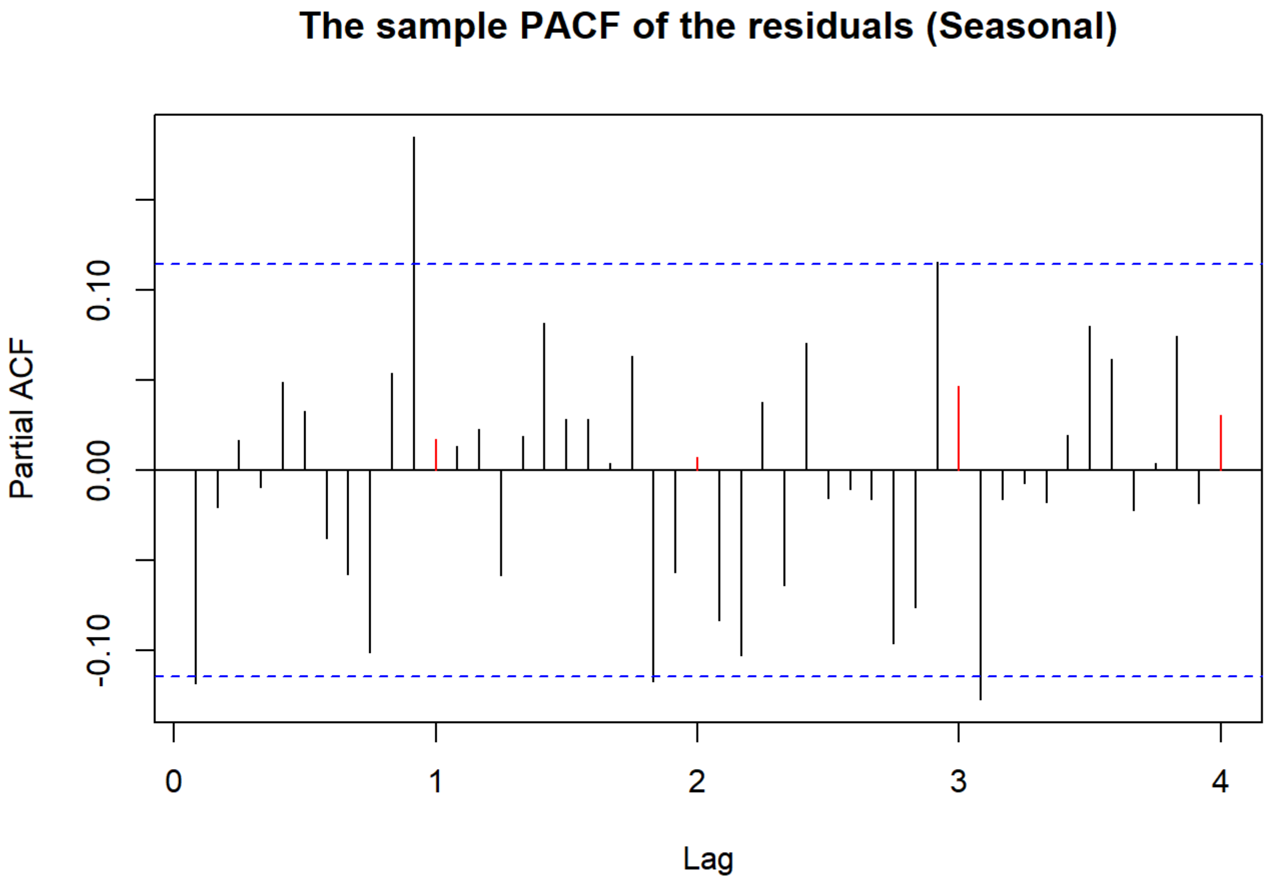


Figure 23: Sample PACF plot of residuals

The first ordinal difference residual plot(Figure m3.1) has a flat mean level but has high fluctuations.

After applying 1st ordinal differencing, there is no trend. There are 2 significant lags in ACF, q = 2. There are 2 significant lags in PACF, p = 2

Ordinal parameter (p,q) = (2,2)

```
m4.gold = Arima(data.ts, order=c(2,1,2),seasonal=list(order=c(1,1,2), period=12,  method="M
L"))

res.m4 = rstandard(m4.gold)
plot(res.m4,xlab='Time',ylab='Residuals', main="Time series plot of the residuals.", sub = "F
igure m4.1: Time series of residuals")
```

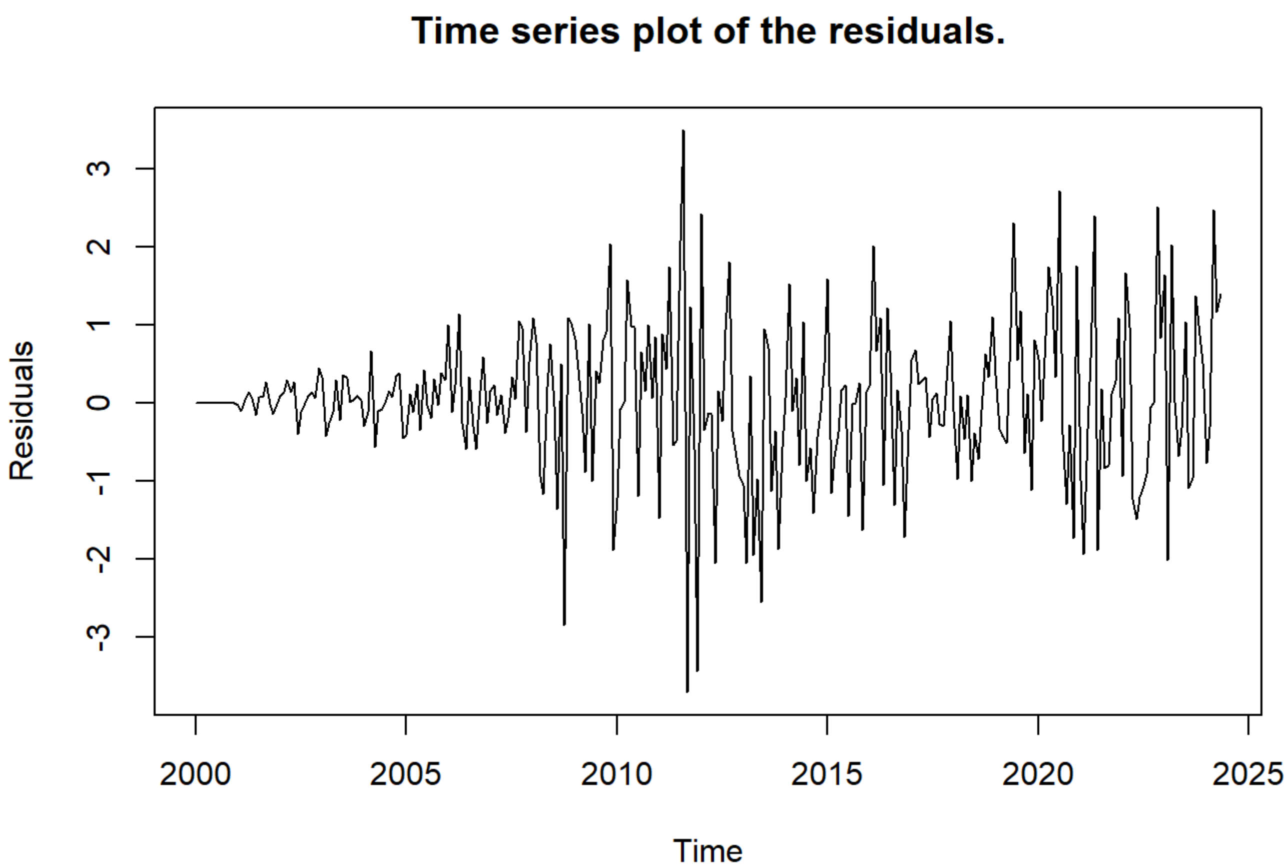


Figure m4.1: Time series of residuals

```
Diagnostic_test(res.m4, 48, "The sample ACF of the residuals", "Figure 24: Sample ACF plot of
residulas", "The sample PACF of the residuals", "Figure 25: Sample PACF plot of residuals", t
est = "seasonal ACF-PACF")
```

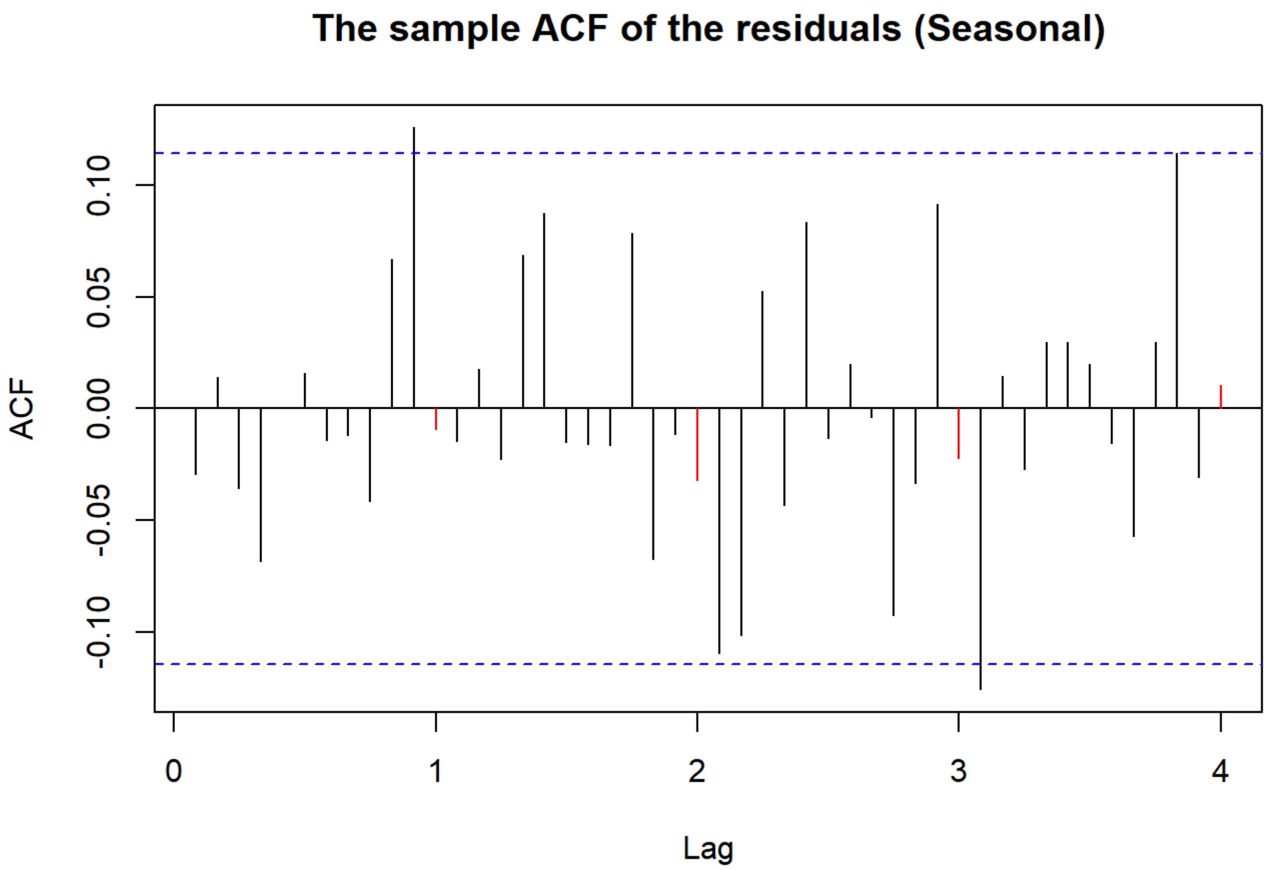


Figure 24: Sample ACF plot of residulas

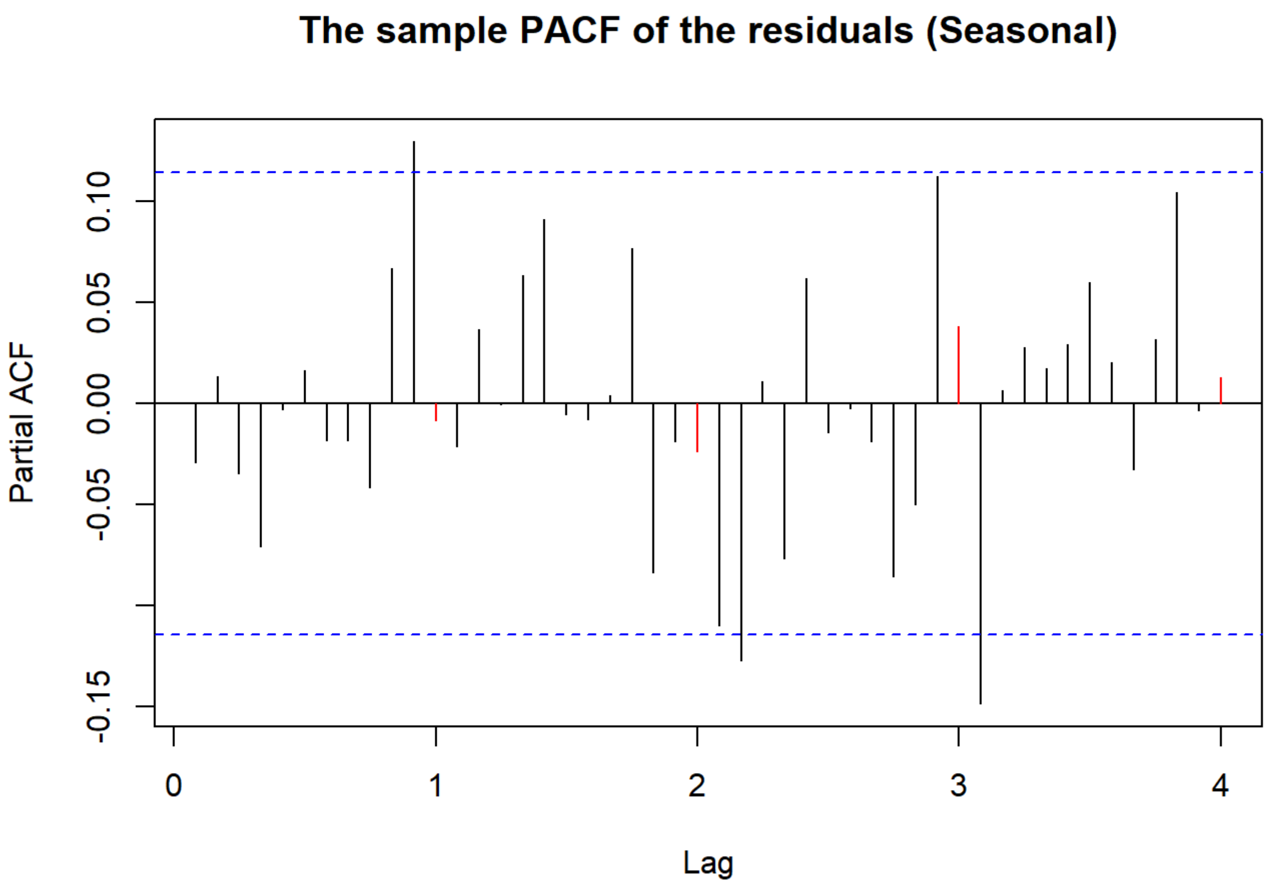


Figure 25: Sample PACF plot of residuals

The ordinal fitted parameter residual plot(Figure m4.1) has a flat mean level but has high fluctuations.

From the ACF and PACF plots of the residuals, SARIMA(2,1,2)x(1,1,2)_12 is the identified as a potential model.

EACF

```
eacf(res.m3)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x o o o o o o o o o x  o  o  o
## 1 x o o o o o o o o o x  o  o  o
## 2 x o o o o o o o o o x  o  o  o
## 3 x o o o o o o o o o x  o  o  o
## 4 x x x x o o o o o o x  o  o  o
## 5 x x o x o o o o o o o  o  o  o
## 6 x x o x o o o o o o x  o  o  o
## 7 x o x x o o o o o o x  o  o  o
```

From the EACF plot following models are identified:

- SARIMA(0,1,1)x(1,1,2)_12

Neighbor models:

- SARIMA(0,1,2)x(1,1,2)_12
- SARIMA(1,1,1)x(1,1,2)_12
- SARIMA(1,1,2)x(1,1,2)_12

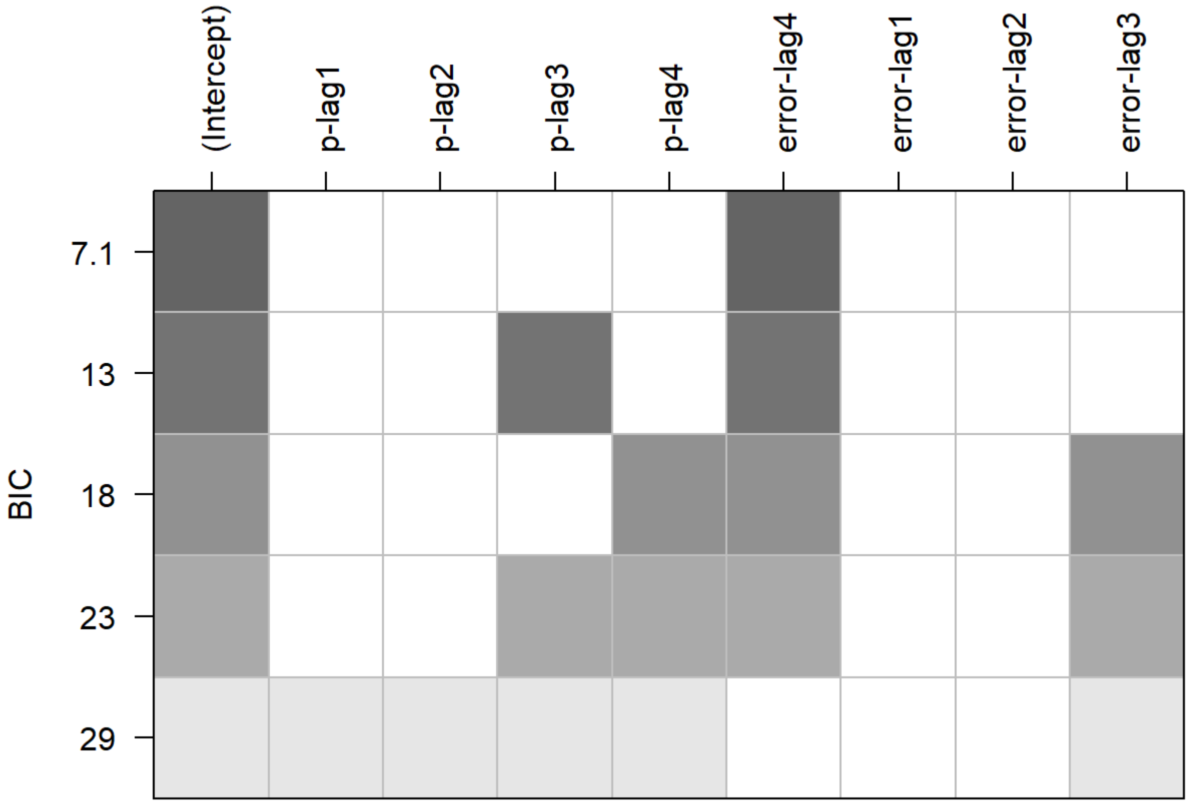
BIC

```
bic_table = armasubsets(y=res.m3,nar=4,nma=4,y.name='p',ar.method='ols')
```

```
## Warning in leaps.setup(x, y, wt = wt, nbest = nbest, nvmax = nvmax, force.in =
## force.in, : 3 linear dependencies found
```

```
## Reordering variables and trying again:
```

```
plot(bic_table)
```



From the BIC table following model is identified: - SARIMA(0,1,4)x(1,1,2)_12

Parameter Estimation

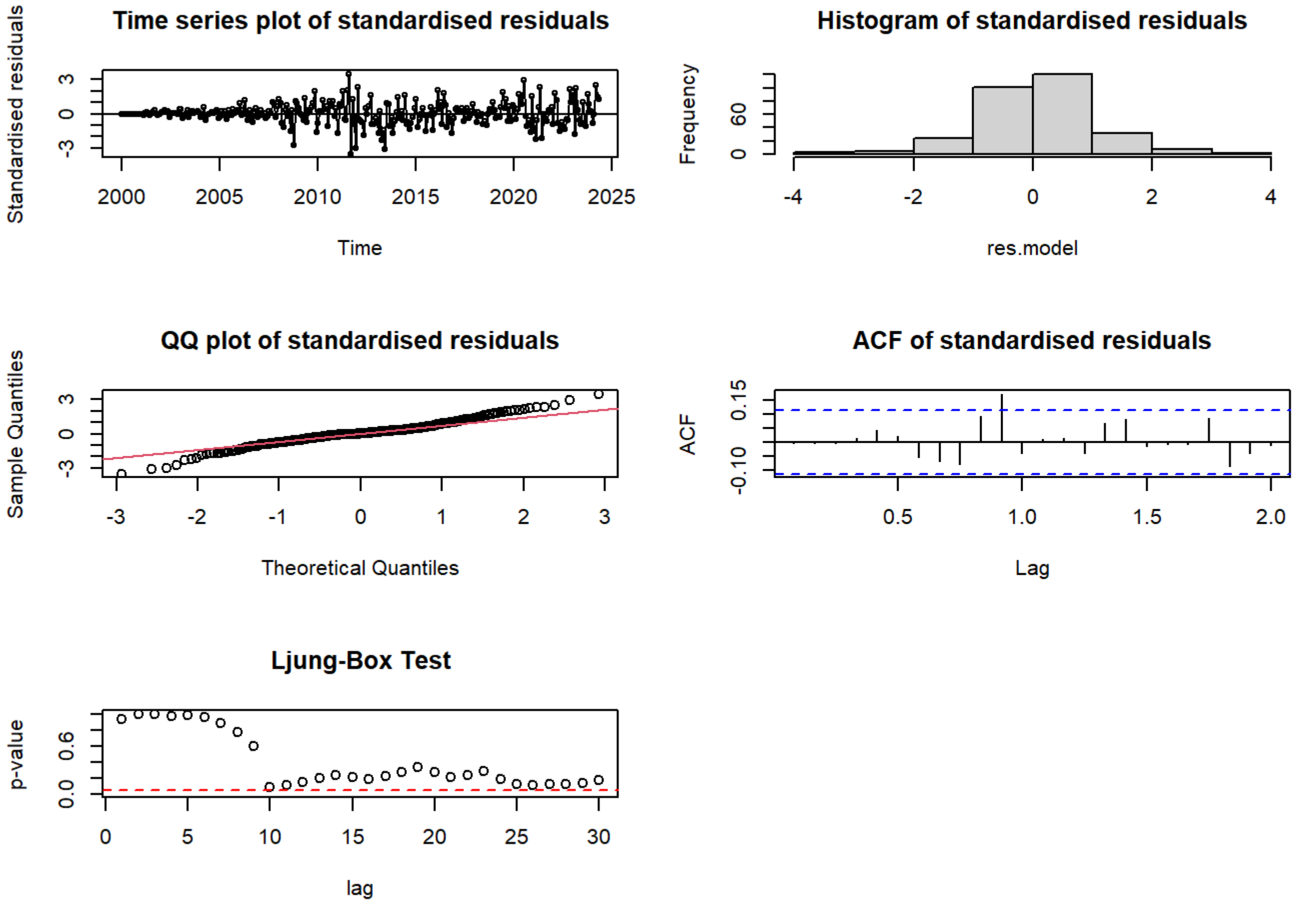
SARIMA(2,1,2)x(1,1,2)_12

```
m_212_112_12 = Arima(data.ts, order=c(2,1,2), seasonal=list(order=c(1,1,2), period=12), metho
d = "ML")
coeftest(m_212_112_12)
```

```
##
## z test of coefficients:
##
##       Estimate Std. Error z value  Pr(>|z|)
## ar1  -0.74926    0.76657 -0.9774  0.328359
## ar2  -0.21582    0.61478 -0.3510  0.725555
## ma1   0.62663    0.77771  0.8057  0.420390
## ma2   0.12220    0.62078  0.1969  0.843939
## sar1  0.76815    0.28441  2.7008  0.006916 **
## sma1 -1.67331    0.35934 -4.6567 3.214e-06 ***
## sma2  0.67412    0.32241  2.0909  0.036540 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
residual.analysis(model = m_212_112_12)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.97099, p-value = 1.212e-05
```



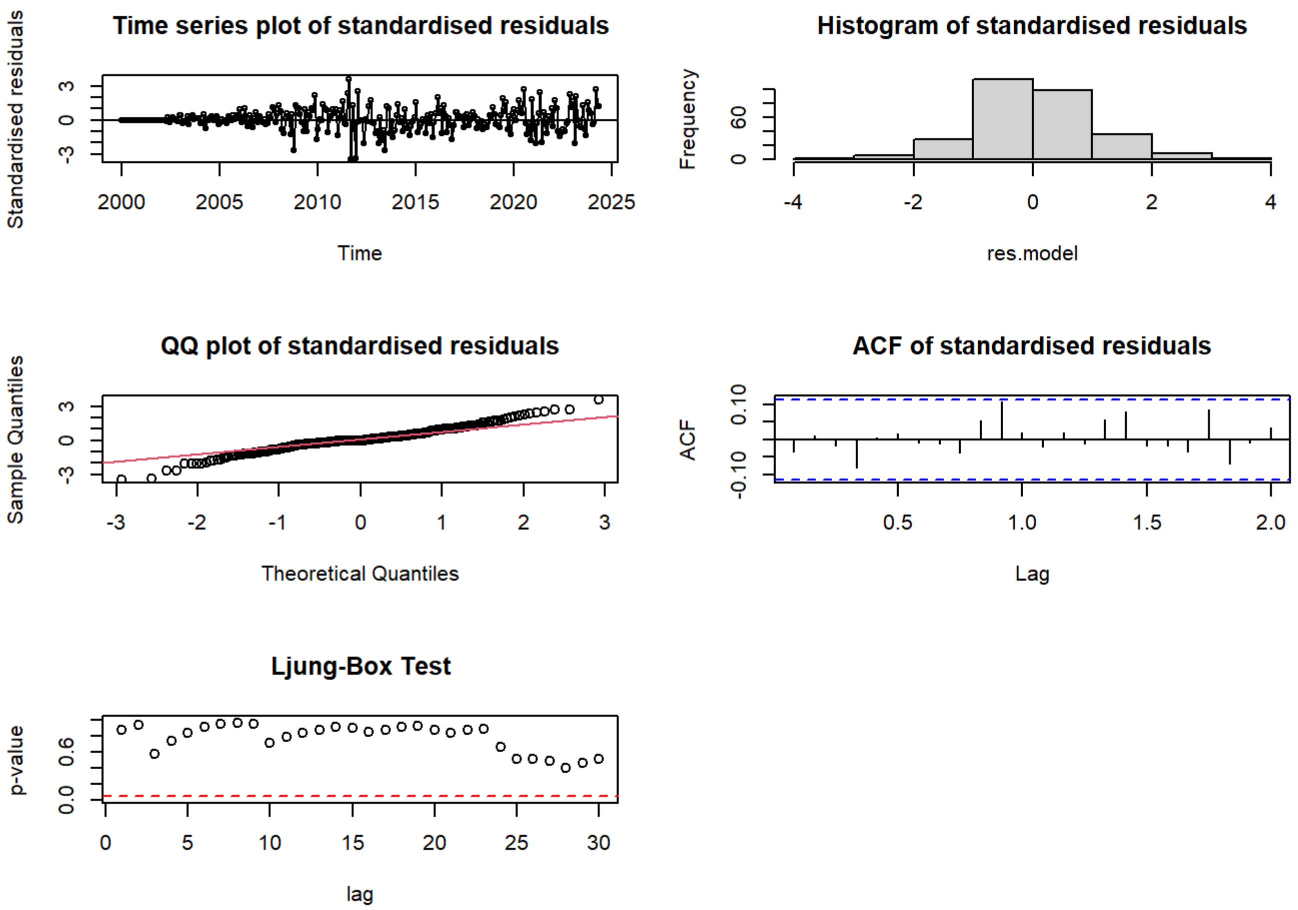
For SARIMA(2,1,2)x(1,1,2)_12 “ML” model, all seasonal coefficients are significant whereas, ordinal coefficients are insignificant. The residual plot has a flat mean level indicating no trend. The histogram is symmetric and indicates outliers. Most of the residuals follows the line of normality but their are some deviation at the tails. Shapiro-Wilk test has a p-value < 0.05, indicating non-normality. Ljung-Box test shows all lags above the significance level, therefore, no auto-correlation is present in the residuals.

```
m_212_112_12CSS = Arima(data.ts, order=c(2,1,2), seasonal=list(order=c(1,1,2), period=12), me
thod = "CSS")
coeftest(m_212_112_12CSS)
```

```
##
## z test of coefficients:
##
##       Estimate Std. Error  z value Pr(>|z|)
## ar1   1.203065   0.084336  14.2651   <2e-16 ***
## ar2  -0.830350   0.087463  -9.4938   <2e-16 ***
## ma1  -1.310747   0.051232 -25.5846   <2e-16 ***
## ma2   0.946876   0.056345  16.8049   <2e-16 ***
## sar1 -0.422156   0.605564  -0.6971   0.4857
## sma1 -0.561639   0.585456  -0.9593   0.3374
## sma2 -0.374496   0.554813  -0.6750   0.4997
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
residual.analysis(model = m_212_112_12CSS)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.97, p-value = 8.531e-06
```



For SARIMA(2,1,2)x(1,1,2)_12 “CSS” model, all seasonal coefficients are insignificant whereas, ordinal coefficients are significant. The residual plot has a flat mean level indicating no trend. The histogram is symmetric and indicates outliers. Most of the residuals follows the line of normality but their are some deviation at the tails. Shapiro-Wilk test has a p-value < 0.05, indicating non-normality. Ljung-Box test shows all lags above the significance level, therefore, no auto-correlation is present in the residuals.

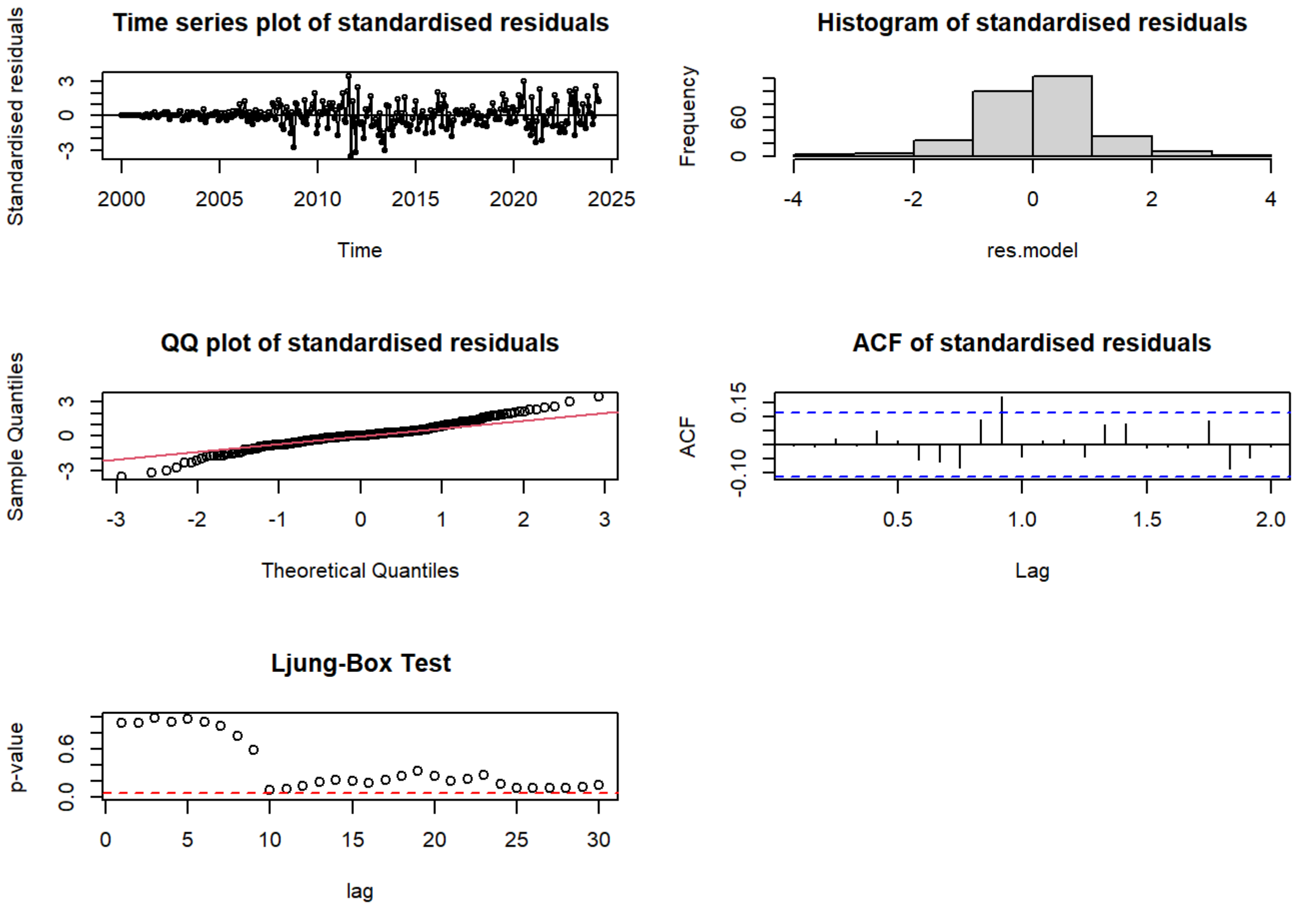
SARIMA(0,1,1)x(1,1,2)_12

```
m_011_112_12 = Arima(data.ts,order=c(0,1,1),seasonal=list(order=c(1,1,2), period=12),method =
"ML")
coeftest(m_011_112_12)
```

```
##
## z test of coefficients:
##
##       Estimate Std. Error z value  Pr(>|z|)
## ma1  -0.12441    0.05988 -2.0776 0.0377481 *
## sar1  0.75479    0.31216  2.4179 0.0156085 *
## sma1 -1.65389    0.49154 -3.3647 0.0007662 ***
## sma2  0.65704    0.40822  1.6095 0.1075006
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
residual.analysis(model = m_011_112_12)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.96954, p-value = 7.248e-06
```



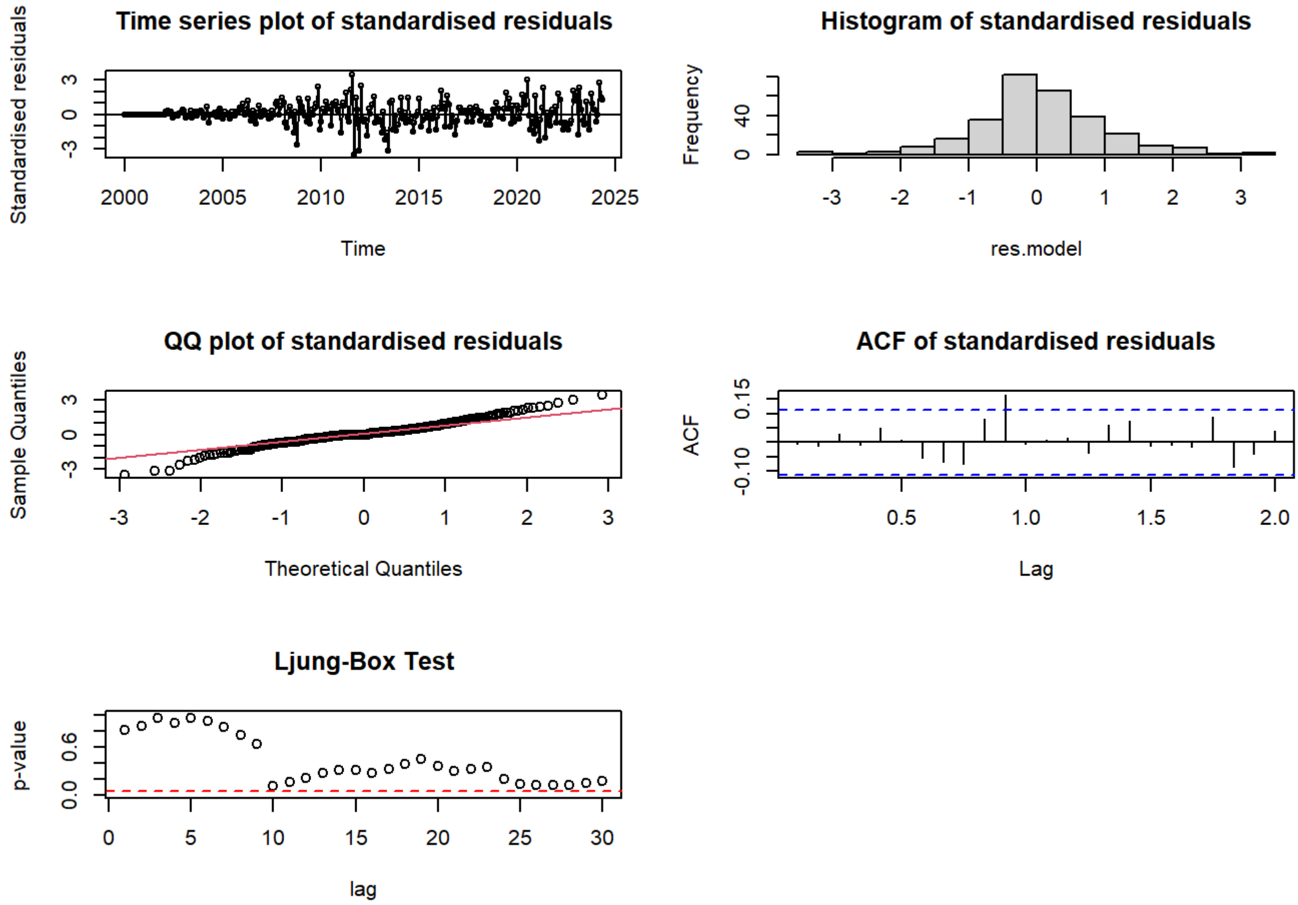
For SARIMA(0,1,1)x(1,1,2)_12 “ML” model, all coefficient except “sma2”, are significant. The residual plot has a flat mean level indicating no trend. The histogram is symmetric and indicates outliers. Most of the residuals follows the line of normality but their are some deviation at the tails. Shapiro-Wilk test has a p-value < 0.05, indicating non-normality. Ljung-Box test shows all lags above the significance level, therefore, no auto-correlation is present in the residuals.

```
m_011_012_12CSS = Arima(data.ts,order=c(0,1,1),seasonal=list(order=c(1,1,2), period=12),metho
d = "CSS")
coeftest(m_011_012_12CSS)
```

```
##
## z test of coefficients:
##
##       Estimate Std. Error z value Pr(>|z|)
## ma1  -0.127333   0.060858 -2.0923 0.036413 *
## sar1  0.087110   0.398916  0.2184 0.827144
## sma1 -0.997808   0.385440 -2.5887 0.009633 **
## sma2  0.038891   0.365215  0.1065 0.915195
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
residual.analysis(model = m_011_012_12CSS)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.97198, p-value = 1.736e-05
```



For SARIMA(0,1,1)x(1,1,2)_12 "CSS" model, only "ma1" and "sma1" are significant, whereas, "sar1" and "sma2" are insignificant. The residual plot has a flat mean level indicating no trend. The histogram is symmetric and indicates outliers. Most of the residuals follows the line of normality but their are some deviation at the tails. Shapiro-Wilk test has a p-value < 0.05, indicating non-normality. Ljung-Box test shows all lags above the significance level, therefore, no auto-correlation is present in the residuals.

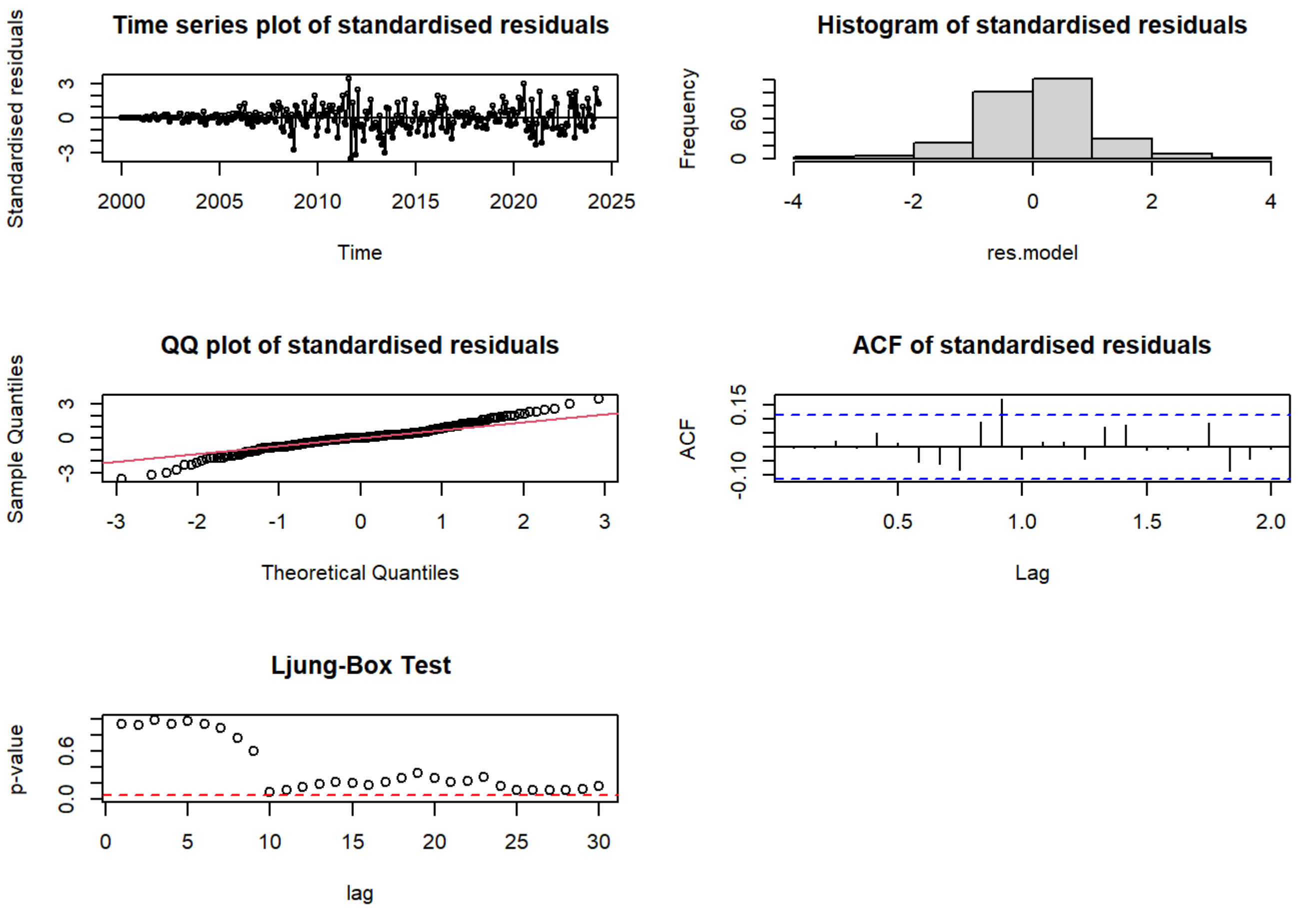
SARIMA(0,1,2)x(1,1,2)_12

```
m_012_112_12 = Arima(data.ts,order=c(0,1,2),seasonal=list(order=c(1,1,2), period=12),method =
"ML")
coeftest(m_012_112_12)
```

```
##
## z test of coefficients:
##
##        Estimate Std. Error z value  Pr(>|z|)
## ma1  -0.1240004  0.0605497 -2.0479  0.040569 *
## ma2  -0.0014879  0.0599799 -0.0248  0.980209
## sar1  0.7760672  0.2791549  2.7801  0.005435 **
## sma1 -1.6821929  0.3672676 -4.5803 4.643e-06 ***
## sma2  0.6825690  0.3234879  2.1100  0.034856 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
residual.analysis(model = m_012_112_12)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.96946, p-value = 7.068e-06
```



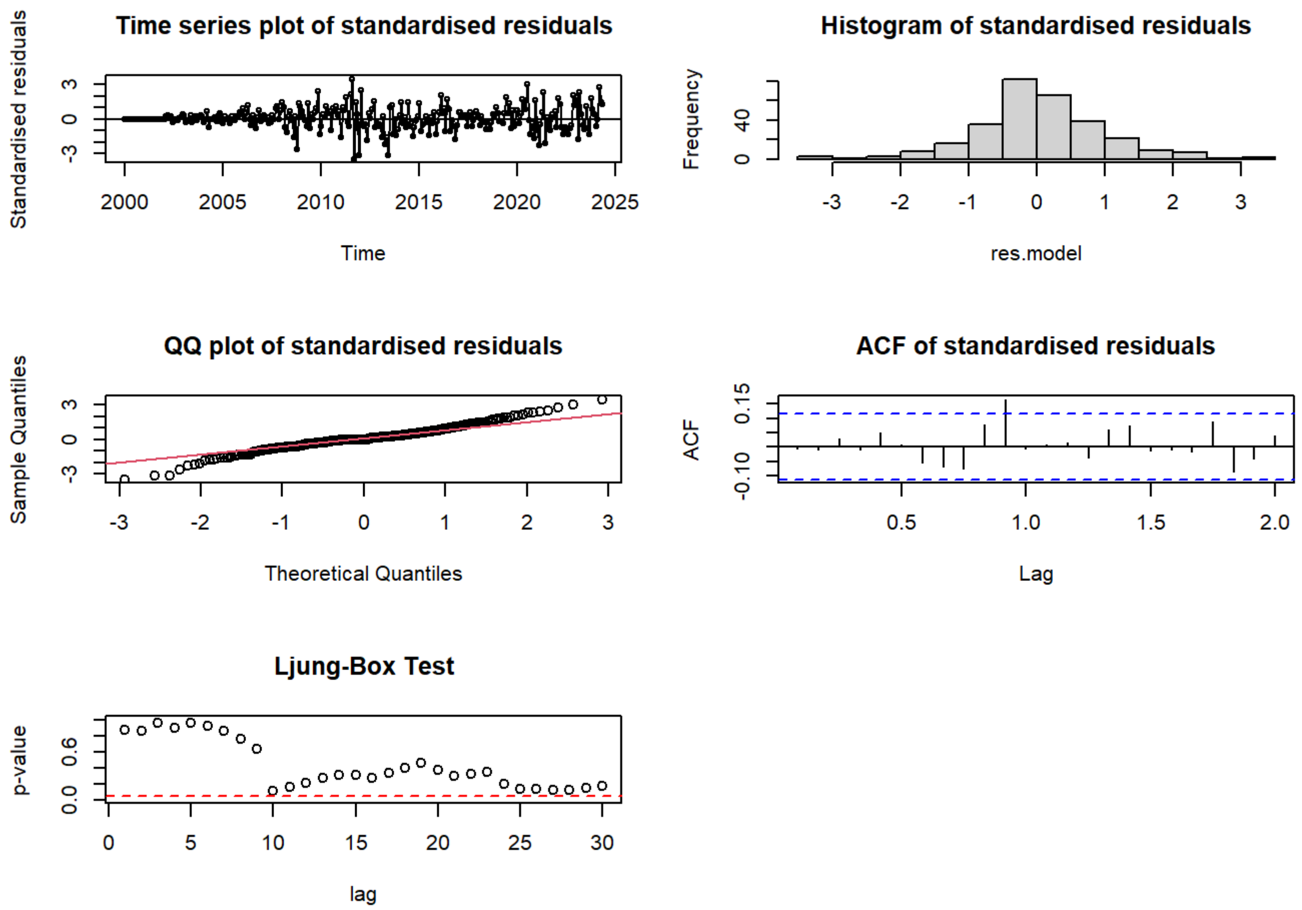
For SARIMA(0,1,2)x(1,1,2)_12 “ML” model, all coefficients except “ma2” are significant. The residual plot has a flat mean level indicating no trend. The histogram is symmetric and indicates outliers. Most of the residuals follows the line of normality but their are some deviation at the tails. Shapiro-Wilk test has a p-value < 0.05, indicating non-normality. Ljung-Box test shows all lags above the significance level, therefore, no auto-correlation is present in the residuals.

```
m_012_112_12CSS = Arima(data.ts,order=c(0,1,2),seasonal=list(order=c(1,1,2), period=12),metho
d = "CSS")
coeftest(m_012_112_12CSS)
```

```
##
## z test of coefficients:
##
##        Estimate Std. Error z value Pr(>|z|)
## ma1  -0.1266208  0.0614870 -2.0593  0.03946 *
## ma2  -0.0042727  0.0607099 -0.0704  0.94389
## sar1  0.0864579  0.4007471  0.2157  0.82919
## sma1 -0.9970637  0.3871374 -2.5755  0.01001 *
## sma2  0.0380059  0.3668397  0.1036  0.91748
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
residual.analysis(model = m_012_112_12CSS)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.97182, p-value = 1.638e-05
```



For SARIMA(0,1,2)x(1,1,2)_12 “CSS” model, only “ma1” and “sma1” are significant, whereas, “sar1” and “sma2” are insignificant. The residual plot has a flat mean level indicating no trend. The histogram is symmetric and indicates outliers. Most of the residuals follows the line of normality but their are some deviation at the tails. Shapiro-Wilk test has a p-value < 0.05, indicating non-normality. Ljung-Box test shows all lags above the significance level, therefore, no auto-correlation is present in the residuals.

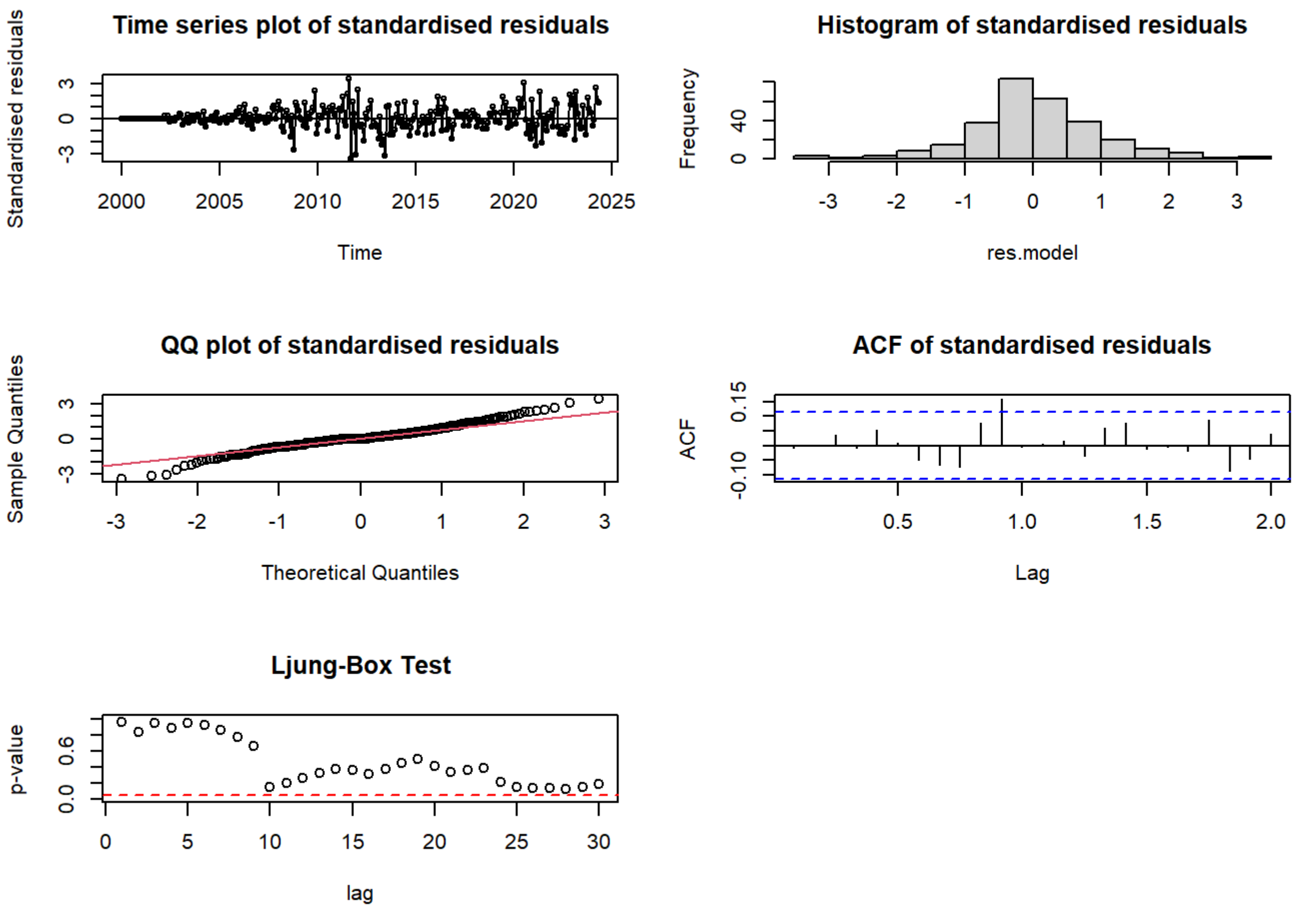
SARIMA(1,1,1)x(1,1,2)_12

```
m_111_112_12 = Arima(data.ts,order=c(1,1,1),seasonal=list(order=c(1,1,2), period=12),method =
"CSS")
coeftest(m_111_112_12)
```

```
##
## z test of coefficients:
##
##       Estimate Std. Error z value Pr(>|z|)
## ar1   0.146881   0.371661  0.3952  0.69269
## ma1  -0.276334   0.354457 -0.7796  0.43563
## sar1  0.094398   0.365935  0.2580  0.79643
## sma1 -1.004664   0.355303 -2.8276  0.00469 **
## sma2  0.042400   0.337063  0.1258  0.89990
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
residual.analysis(model = m_111_112_12)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.97155, p-value = 1.486e-05
```



For SARIMA(1,1,1)x(1,1,2)_12 “CSS” model, only “sma1” is significant. The residual plot has a flat mean level indicating no trend. The histogram is symmetric and indicates outliers. Most of the residuals follows the line of normality but their are some deviation at the tails. Shapiro-Wilk test has a p-value < 0.05, indicating non-normality. Ljung-Box test shows all lags above the significance level, therefore, no auto-correlation is present in the residuals.

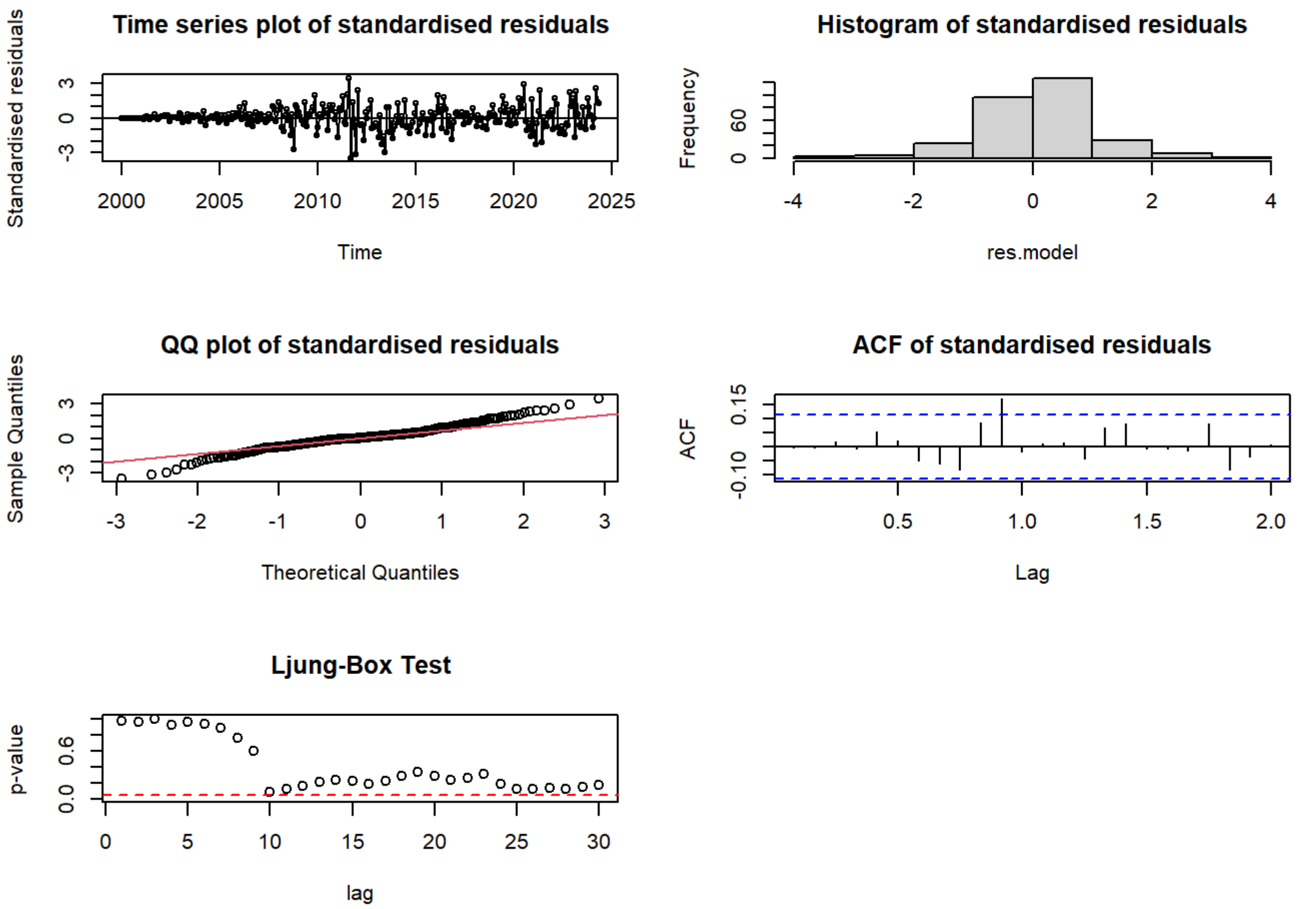
##SARIMA(1,1,2)x(1,1,2)_12

```
m_112_112_12 = Arima(data.ts,order=c(1,1,2),seasonal=list(order=c(1,1,2), period=12),method =
"ML")
coeftest(m_112_112_12)
```

```
##
## z test of coefficients:
##
##        Estimate Std. Error z value Pr(>|z|)
## ar1   -0.085334        NaN     NaN      NaN
## ma1   -0.034553        NaN     NaN      NaN
## ma2   -0.014057        NaN     NaN      NaN
## sar1  -0.327372        NaN     NaN      NaN
## sma1  -0.556799        NaN     NaN      NaN
## sma2  -0.294625        NaN     NaN      NaN
```

```
residual.analysis(model = m_112_112_12)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.96974, p-value = 7.772e-06
```



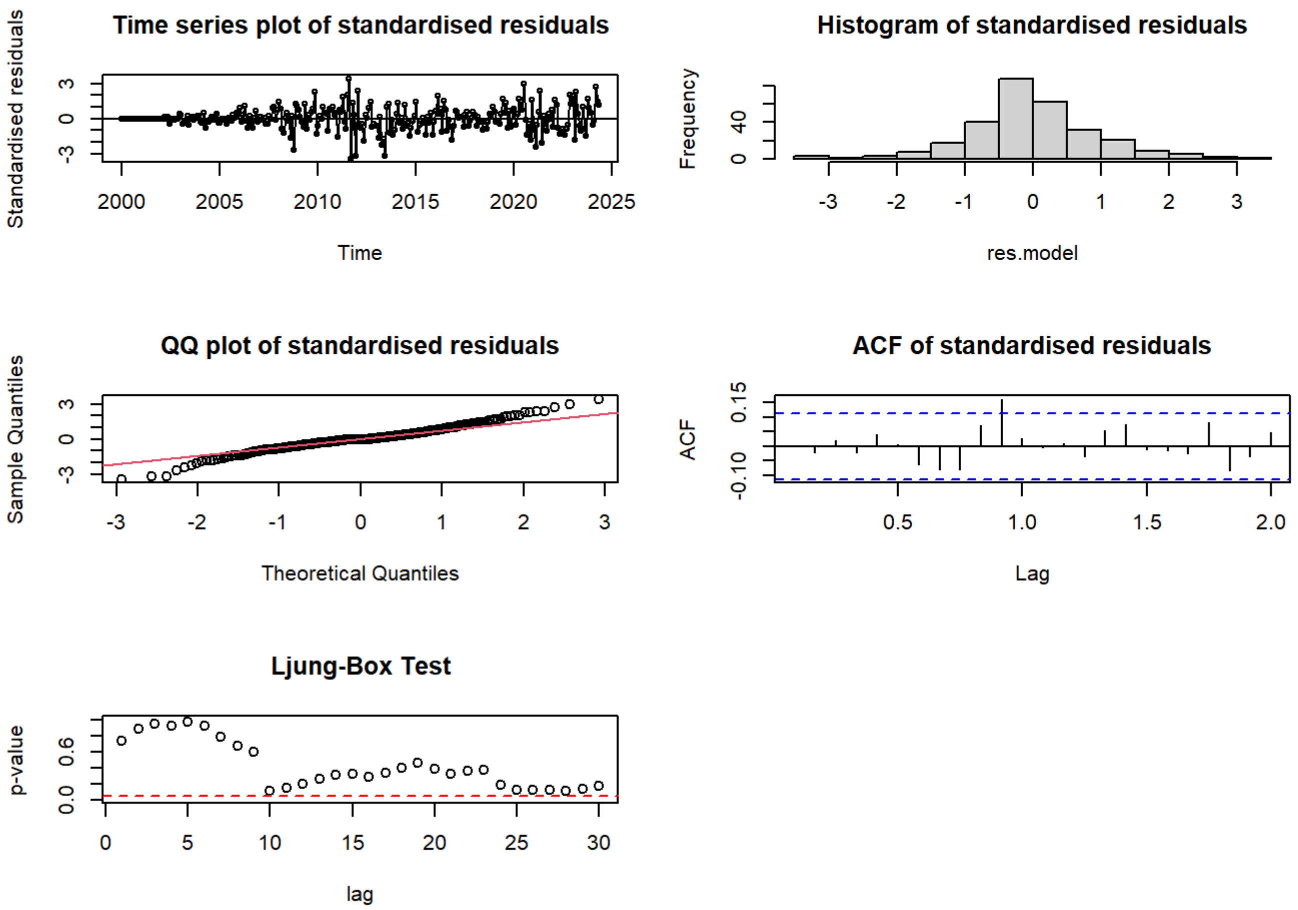
We observe NaN values in the coefficient test. This could be because of 0 values in the model.

```
m_112_112_12CSS = Arima(data.ts,order=c(1,1,2),seasonal=list(order=c(1,1,2), period=12),metho
d = "CSS")
coeftest(m_112_112_12CSS)
```

```
##
## z test of coefficients:
##
##       Estimate Std. Error z value Pr(>|z|)
## ar1   0.908585   0.343082  2.6483 0.008090 **
## ma1  -1.042586   0.302885 -3.4422 0.000577 ***
## ma2   0.130596   0.062684  2.0834 0.037216 *
## sar1 -0.361433   2.085287 -0.1733 0.862396
## sma1 -0.585736   1.897328 -0.3087 0.757537
## sma2 -0.364041   1.788012 -0.2036 0.838665
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
residual.analysis(model = m_112_112_12CSS)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.97274, p-value = 2.298e-05
```



For SARIMA(1,1,2)x(1,1,2)_12 “CSS” model, all ordinal components are significant, whereas, all seasonal components are insignificant. The residual plot has a flat mean level indicating no trend. The histogram is symmetric and indicates outliers. Most of the residuals follows the line of normality but their are some deviation at the tails. Shapiro-Wilk test has a p-value < 0.05, indicating non-normality. Ljung-Box test shows all lags above the significance level, therefore, no auto-correlation is present in the residuals.

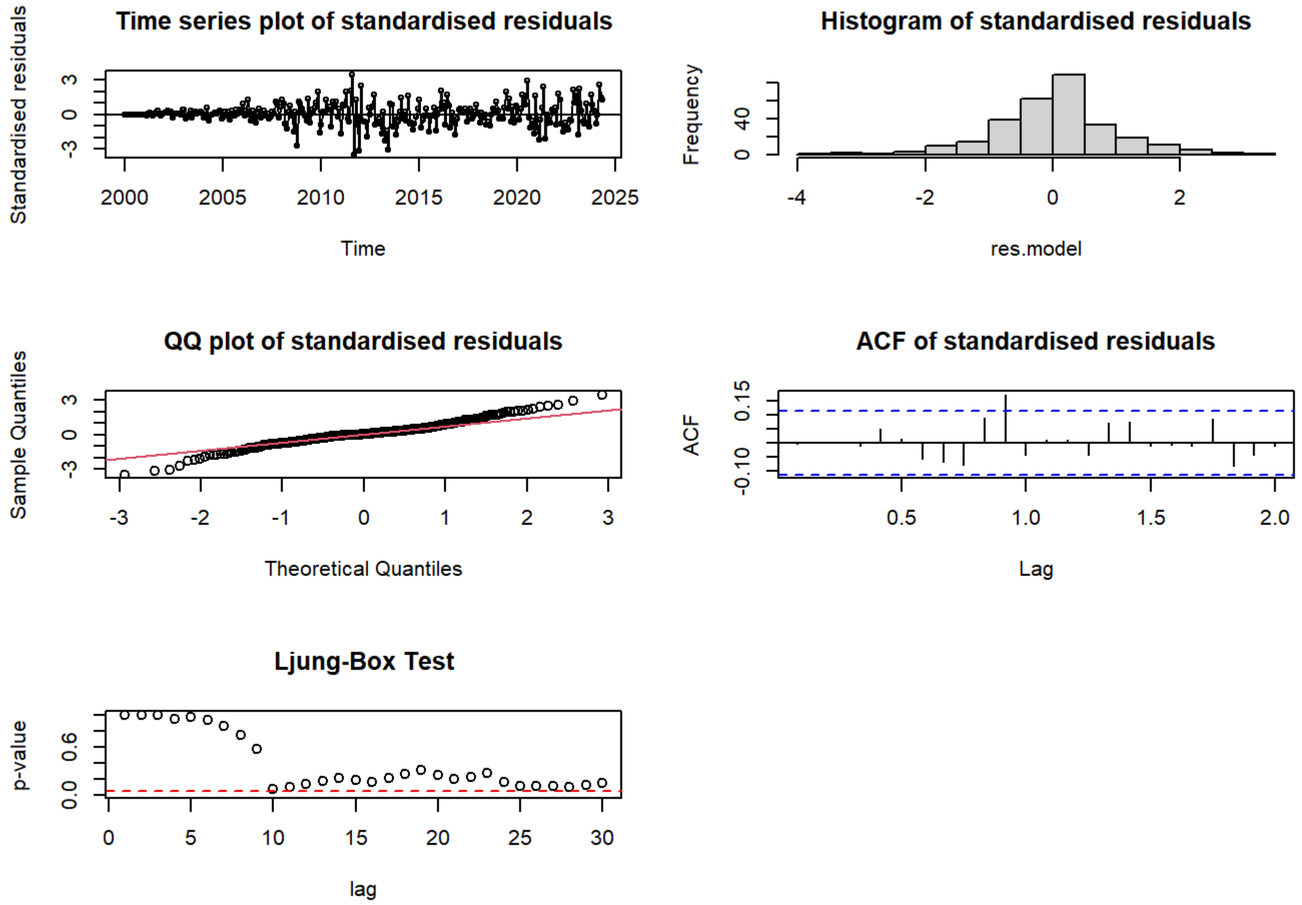
##SARIMA(0,1,4)x(1,1,2)_12

```
m_014_112_12 = Arima(data.ts,order=c(0,1,4),seasonal=list(order=c(1,1,2), period=12),method =
"ML")
coeftest(m_014_112_12)
```

```
##
## z test of coefficients:
##
##        Estimate Std. Error z value  Pr(>|z|)
## ma1  -0.1250362  0.0607214 -2.0592   0.03948 *
## ma2  -0.0067193  0.0609401 -0.1103   0.91220
## ma3   0.0258773  0.0621805  0.4162   0.67729
## ma4   0.0042836  0.0669249  0.0640   0.94897
## sar1  0.7505003  0.2959207  2.5362   0.01121 *
## sma1 -1.6525416  0.4069906 -4.0604 4.899e-05 ***
## sma2  0.6539994  0.3508248  1.8642   0.06230 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
residual.analysis(model = m_014_112_12)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.97101, p-value = 1.221e-05
```



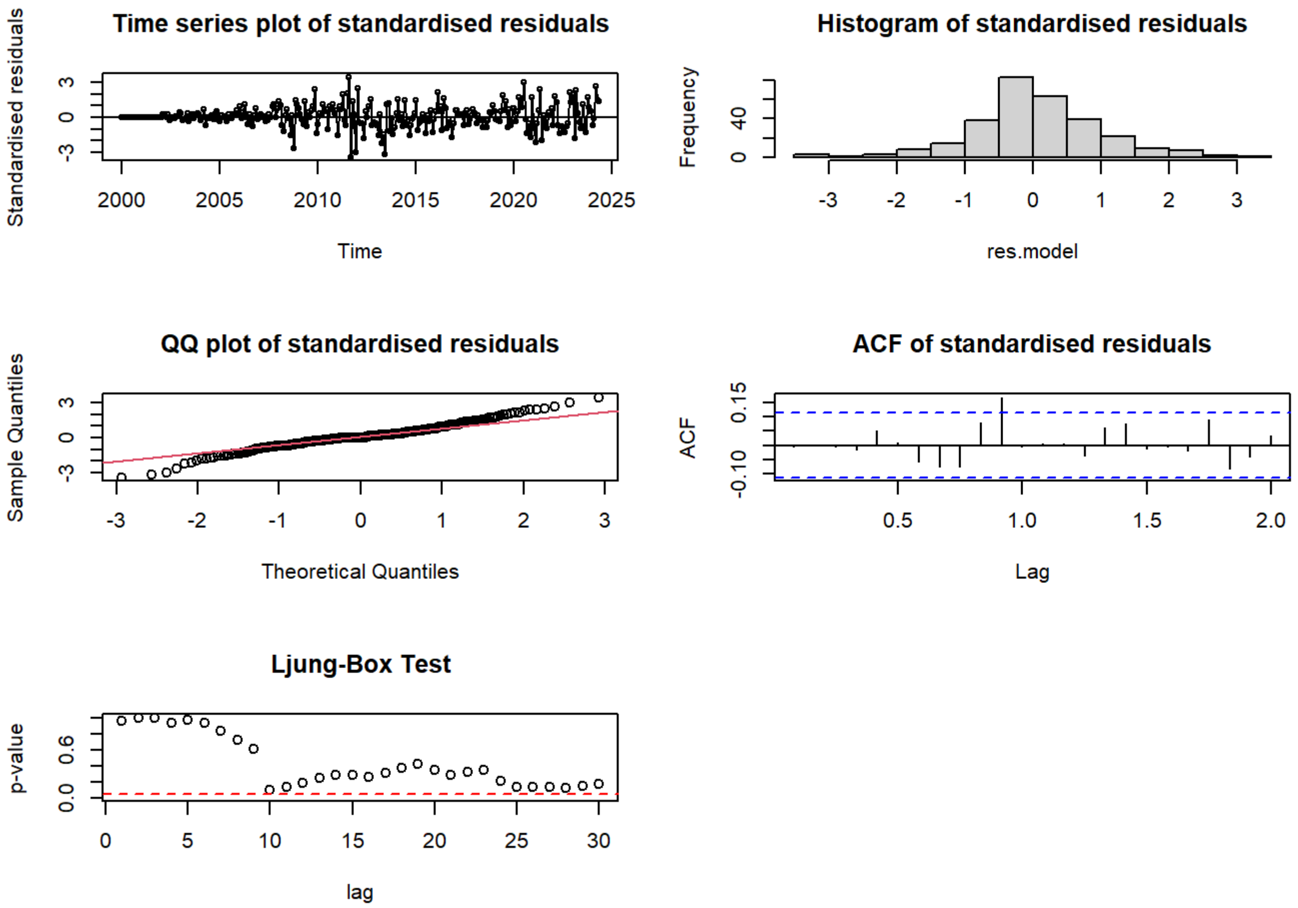
For SARIMA(0,1,4)x(1,1,2)_12 “ML” model, only “ma1”, “sar1”, and “sma1” are significant. The residual plot has a flat mean level indicating no trend. The histogram is symmetric and indicates outliers. Most of the residuals follows the line of normality but their are some deviation at the tails. Shapiro-Wilk test has a p-value < 0.05, indicating non-normality. Ljung-Box test shows all lags above the significance level, therefore, no auto-correlation is present in the residuals.

```
m_014_112_12CSS = Arima(data.ts,order=c(0,1,4),seasonal=list(order=c(1,1,2), period=12),metho
d = "CSS")
coeftest(m_014_112_12CSS)
```

```
##
## z test of coefficients:
##
##       Estimate Std. Error z value Pr(>|z|)
## ma1  -0.127928   0.061647 -2.0752 0.037970 *
## ma2  -0.013038   0.061584 -0.2117 0.832330
## ma3   0.039836   0.064433  0.6182 0.536412
## ma4   0.003782   0.067957  0.0557 0.955619
## sar1  0.076672   0.392715  0.1952 0.845209
## sma1 -0.983590   0.381432 -2.5787 0.009918 **
## sma2  0.022072   0.363463  0.0607 0.951577
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
residual.analysis(model = m_014_112_12CSS)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.97368, p-value = 3.252e-05
```



For SARIMA(0,1,4)x(1,1,2)_12 “CSS” model, only “ma1” and “sma1” are significant. The residual plot has a flat mean level indicating no trend. The histogram is symmetric and indicates outliers. Most of the residuals follows the line of normality but their are some deviation at the tails. Shapiro-Wilk test has a p-value < 0.05, indicating non-normality. Ljung-Box test shows all lags above the significance level, therefore, no auto-correlation is present in the residuals.

Based on coefficient testing and residual analysis, SARIMA(0,1,1)x(1,1,2)_12 is the best model option.

Goodness of fit

AIC and BIC values

The AIC and BIC scores are calculated using “ML” methods. SARIMA_111_112_12 is not a compatible model for “ML” method. Therefore, it is excluded from the matrix.

```
aic_table =AIC(m_212_112_12,m_011_112_12,m_012_112_12,m_112_112_12, m_014_112_12)
bic_table =BIC(m_212_112_12,m_011_112_12,m_012_112_12,m_112_112_12, m_014_112_12)

sorted_aic_table <- aic_table[order(aic_table$AIC), ]
sorted_bic_table <- bic_table[order(bic_table$BIC), ]

sorted_aic_table
```

```
##              df      AIC
## m_011_112_12  5 3107.079
## m_012_112_12  6 3109.077
## m_112_112_12  7 3112.029
## m_212_112_12  8 3112.815
## m_014_112_12  8 3112.904
```

```
sorted_bic_table
```

```
##              df      BIC
## m_011_112_12  5 3125.253
## m_012_112_12  6 3130.885
## m_112_112_12  7 3137.472
## m_212_112_12  8 3141.893
## m_014_112_12  8 3141.983
```

Based on AIC and BIC scores, SARIMA(0,1,1)x(1,1,2)_12 is the best model option.

Error measures

```
Smodel.011 = accuracy(summary(m_212_112_12))
Smodel.012 = accuracy(summary(m_011_112_12))
Smodel.111 = accuracy(summary(m_012_112_12))
Smodel.112 = accuracy(summary(m_111_112_12))
Smodel.014 = accuracy(summary(m_112_112_12))
Smodel.212 = accuracy(summary(m_014_112_12))




headers <- c("ME", "RMSE", "MAE", "MPE", "MAPE", "MASE", "ACF1")

merged_table <- data.frame(rbind
    ( c(Smodel.011), c(Smodel.012),c(Smodel.111),c(Smodel.112),c(Smodel.014),(Smodel.212)))
model_names <- c("model.011", "model.012", "model.111","model.112","model.014","model.212")

rownames(merged_table) <- model_names
colnames(merged_table) <- headers
merged_table
```

```
##                  ME     RMSE      MAE       MPE     MAPE      MASE         ACF1
## model.011 2.565955 56.41334 40.08606 0.2467946 3.442390 0.2863742 -0.002880773
## model.012 2.594659 56.59559 40.14168 0.2506678 3.447710 0.2867715 -0.002502538
## model.111 2.594967 56.42091 40.01146 0.2509124 3.436849 0.2858412 -0.002436669
## model.112 3.597293 57.65522 40.92263 0.2496347 3.467470 0.2923506 -0.008512861
## model.014 2.645187 57.82150 40.99477 0.2560791 3.520331 0.2928660 -0.002645721
## model.212 2.555405 56.44648 40.15336 0.2450779 3.445569 0.2868549 -0.002032425
```

The error measures have very close values for all models.

In conclusion, we use SARIMA(0,1,1)x(1,1,2)_12) model.

Over-parameterisation

Parameter tuning is done to identify any further potential models.

The following models will be tested under parameter tuning:

- SARIMA(0,1,2)x(1,1,2)_12
- SARIMA(1,1,1)x(1,1,2)_12

We have already tested the SARIMA(0,1,2)x(1,1,2)_12, and SARIMA(1,1,1)x(1,1,2)_12 model and concluded that they weren't significant models.

Model Specification ARMA x GARCH Part 1

Since our time series data had changing variance we had to consider ARMA x GARCH model for making our predictions. After applying the log transformation with the first order of differencing the following time series plot is displayed.

```
r.gold <- diff(log(data.ts))*100
plot(r.gold, ylab = "Gold price", main ="Return series for Gold price",sub = "Figure 26: Retu
rn series for Gold price")
```

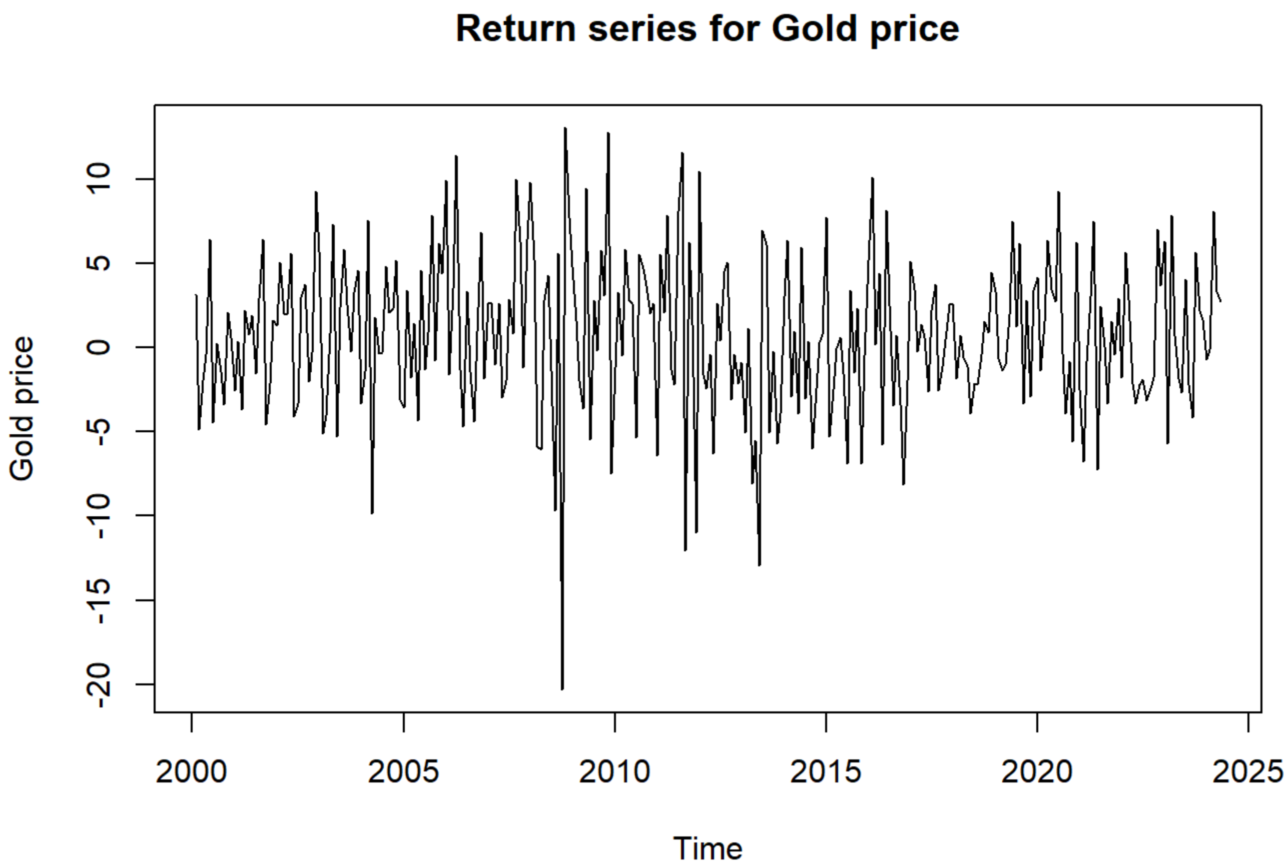


Figure 26: Return series for Gold price

It is clear from the Figure 26 that there is a changing variance in the return series. To verify this we will conduct the McLeod Li test on the return series.

```
McLeod.Li.test(y=r.gold,main ="McLeodLi Test for Changing Variance", sub = "Figure 27: McLeod
Li Test for Changing Variance")
```

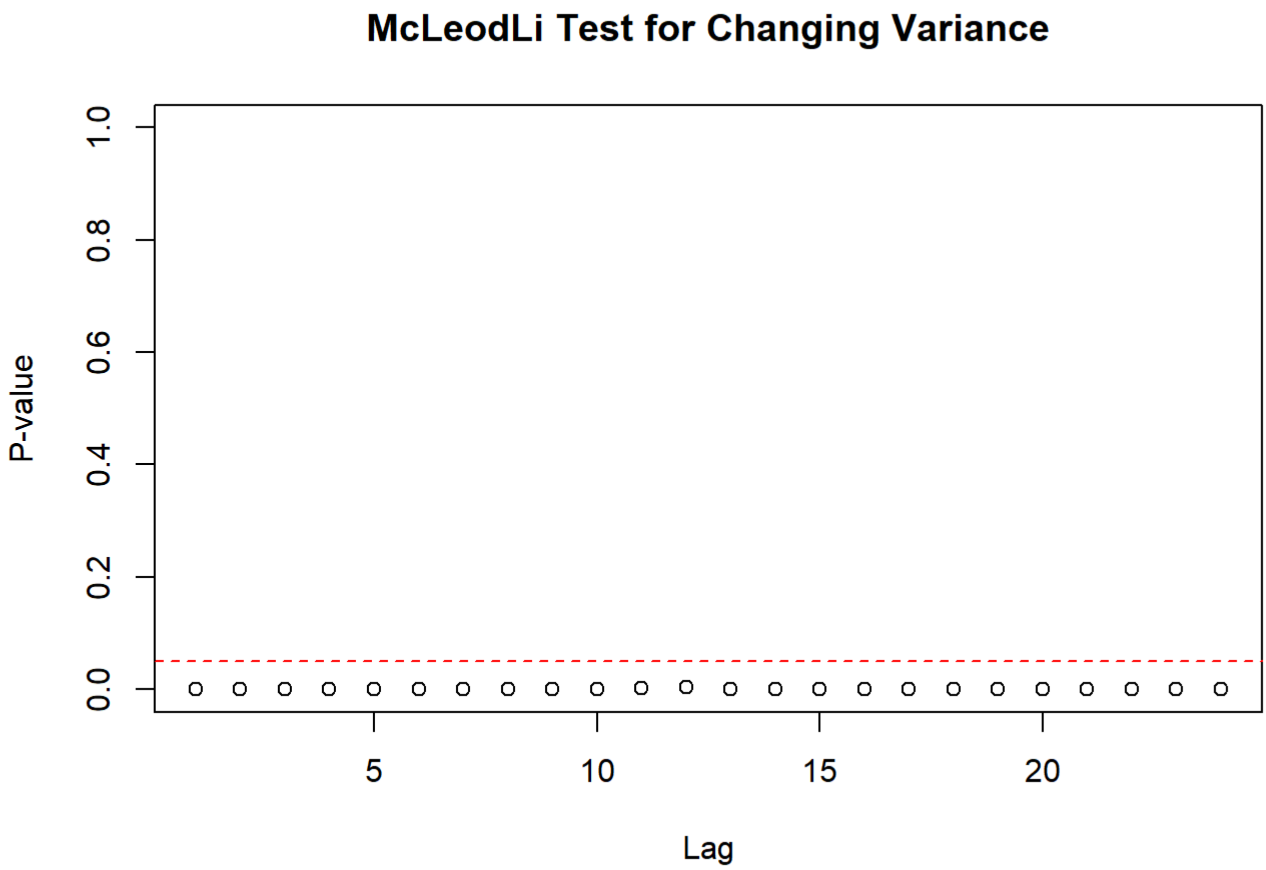


Figure 27: McLeodLi Test for Changing Variance

From the above Figure 27, the p-value at each lag is below the significance level of 0.05 that means there is changing variance in the return series.

Stationary test

In the return series, the volatility is obvious and there is no sense of trend or seasonality. The stationarity of the return series is confirmed by the ADF test. To support the ADF test, we will go on with displaying the ACF and PACF plots.

```
Diagnostic_test(r.gold, 48, mainacf = "ACF plot for return series", subacf = "Figure 28: ACF plot for return series", mainpacf ="PACF plot for return series",subpacf = "Figure 29: PACF plot for return series", test = "Stationary")
```

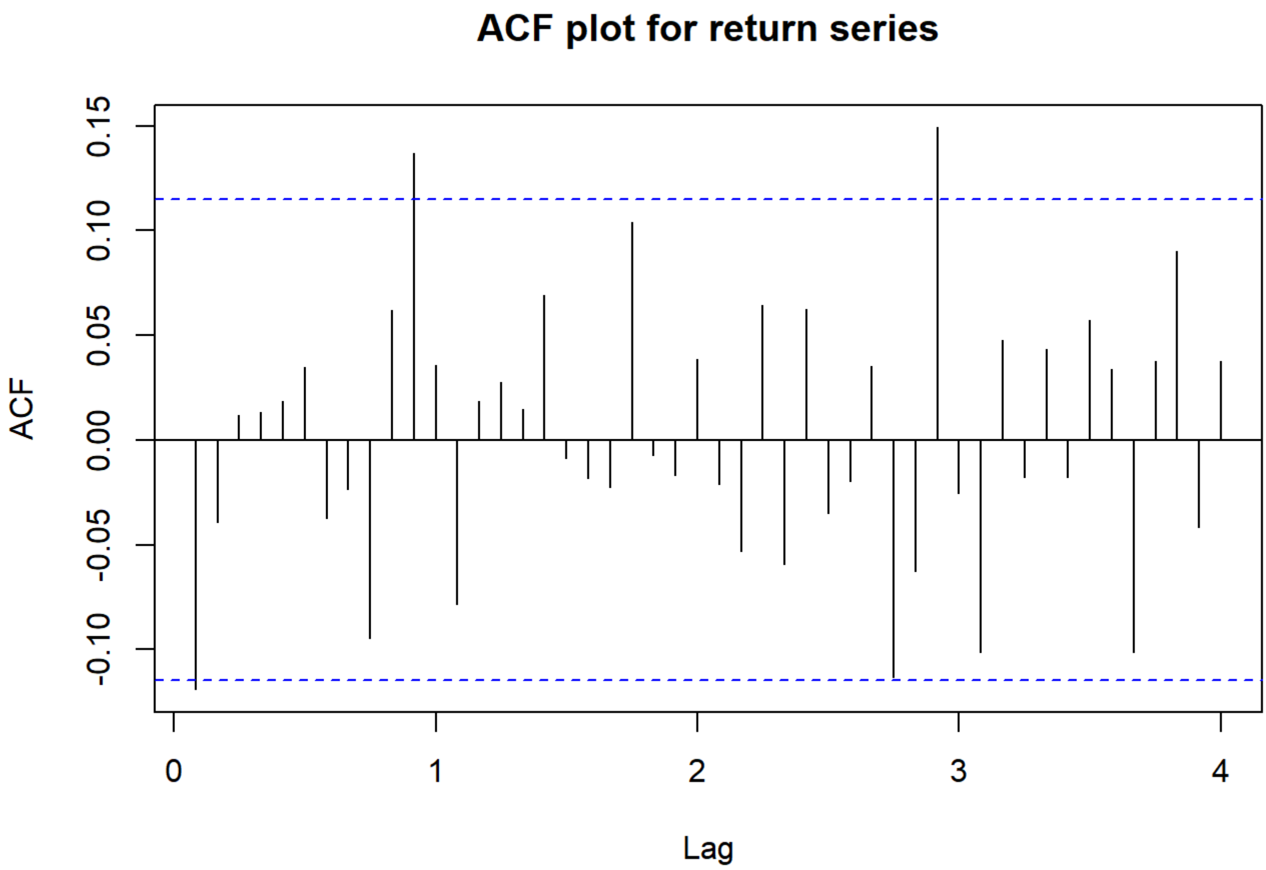


Figure 28: ACF plot for return series

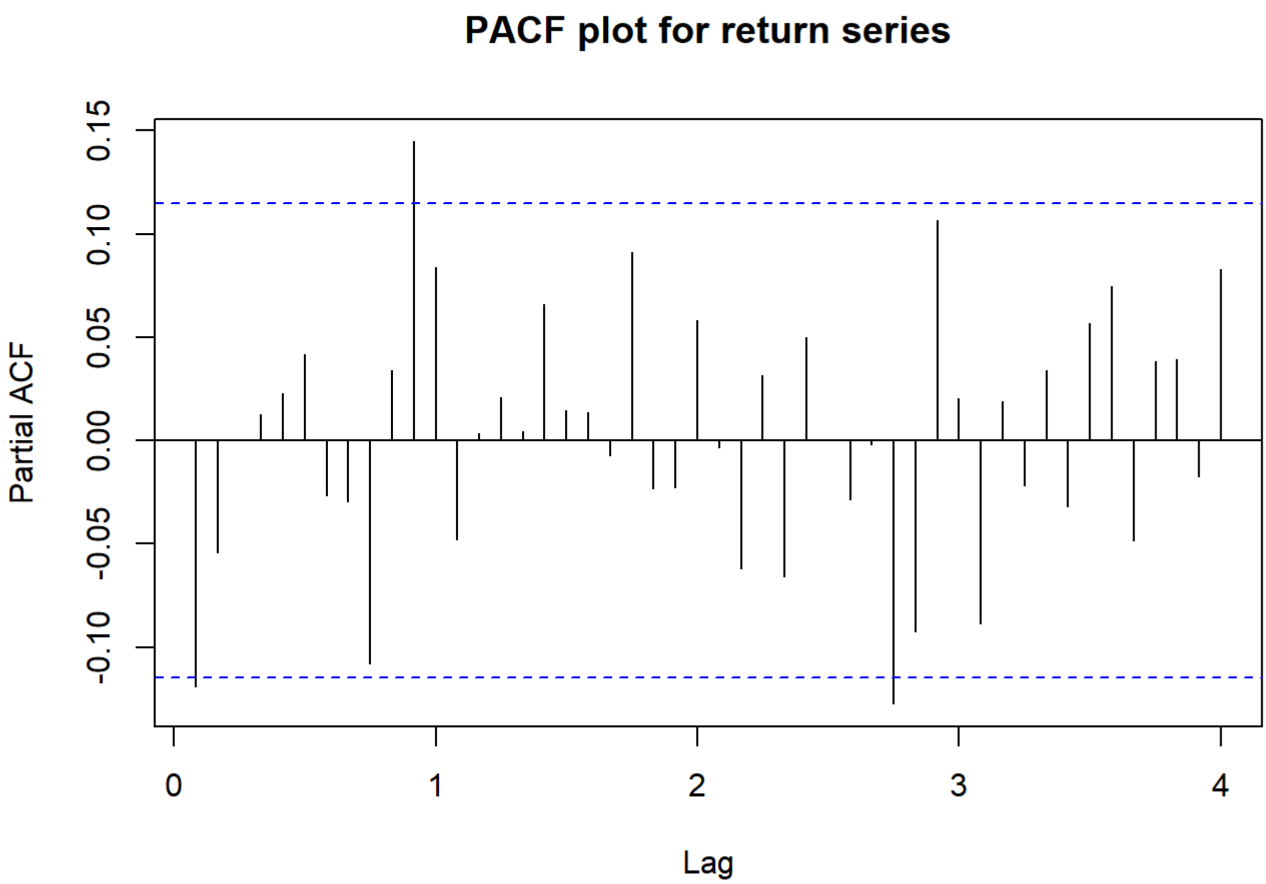


Figure 29: PACF plot for return series

```
## $adf_test
##
##  Augmented Dickey-Fuller Test
##
## data:  data
## Dickey-Fuller = -6.2687, Lag order = 6, p-value = 0.01
## alternative hypothesis: stationary
##
##
## $pp_test
##
##  Phillips-Perron Unit Root Test
##
## data:  data
## Dickey-Fuller Z(alpha) = -317.97, Truncation lag parameter = 5, p-value
## = 0.01
## alternative hypothesis: stationary
```

The PACF plot in figure 29 shows the 1st lag significant indicating the value of p=1. Whereas, the ACF plot in figure 28 shows 1st lag is significant indicating the value of q=1.

In total ACF and PACF Plots propose 1 set of possible models: {ARMA(1,1)}

EACF ARMA

```
eacf(r.gold)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x o o o o o o o o o x  o  o  o
## 1 x o o o o o o o o o o  o  o  o
## 2 o x o o o o o o o o o  o  o  o
## 3 o o o o o o o o o o o  o  o  o
## 4 x o o x o o o o o o o  o  o  o
## 5 x o o x o o o o o o o  o  o  o
## 6 x x x x o o o o o o o  o  o  o
## 7 x o x x o o o o o o o  o  o  o
```

The top-left “o” is identified in the EACF plot is at (0,1). From the EACF plot following models are identified:

- ARMA(0,1)

Neighbor models:

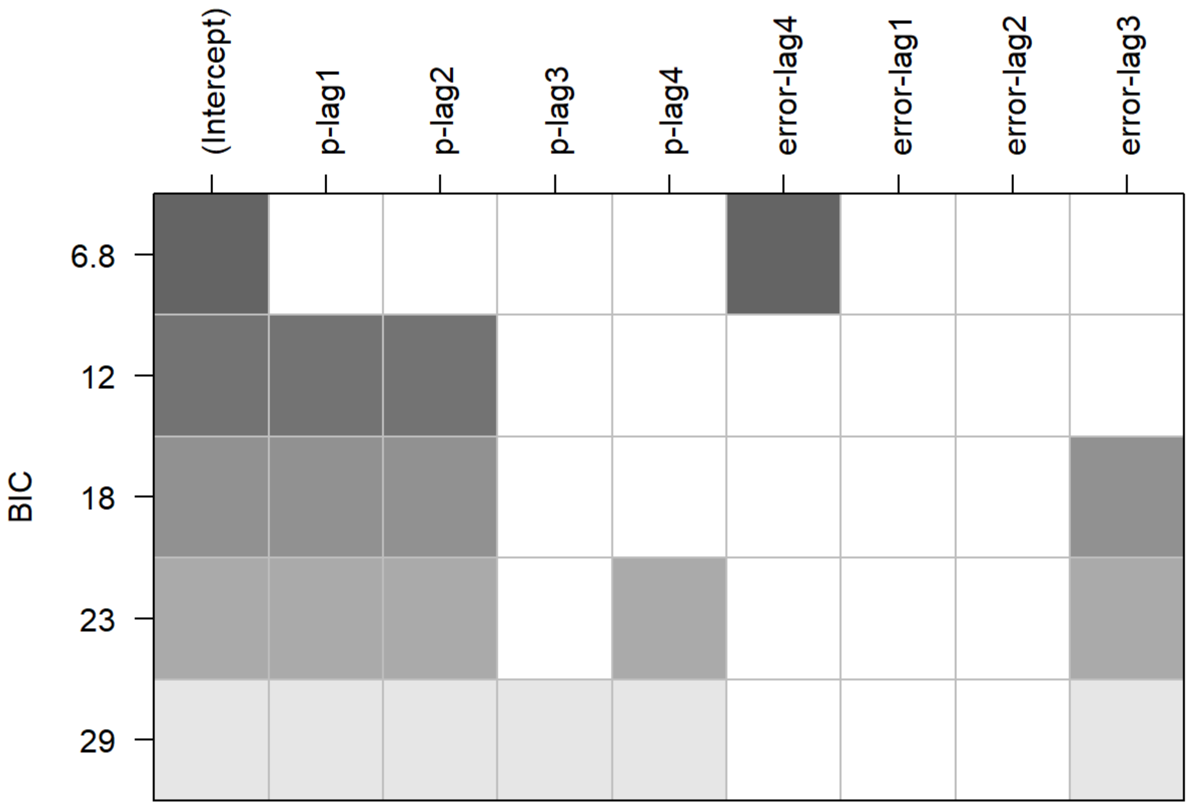
- ARMA(0,2)
- ARMA(1,1)
- ARMA(1,2)

BIC ARMA

```
par(mfrow=c(1,1))
bic_table = armasubsets(y=r.gold,nar=4,nma=4,y.name='p',ar.method='ols')
```

```
## Reordering variables and trying again:
```

```
plot(bic_table)
```



From the BIC table following model is identified: - ARMA(0,4)

Parameter Estimation ARMA

Combining all the possible models from ACF, PACF, EACF and BIC table, we get

- ARMA(0,4)
- ARMA(0,1)
- ARMA(0,2)
- ARMA(1,1)
- ARMA(1,2)

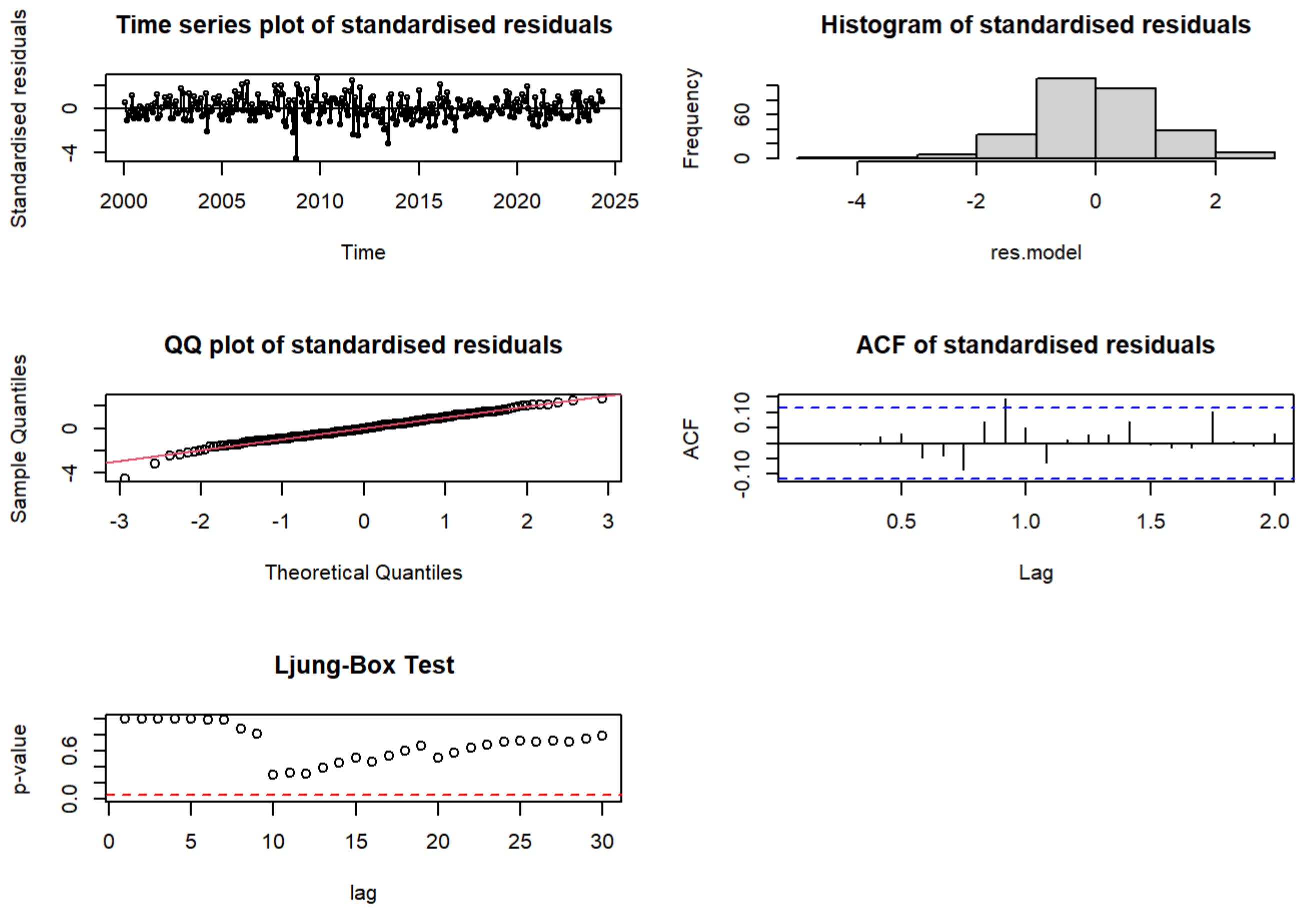
ARMA (0,4)

```
m.04 = Arima(r.gold,order=c(0,0,4),method='ML')
coeftest(m.04)
```

```
##
## z test of coefficients:
##
##            Estimate Std. Error z value Pr(>|z|)
## ma1       -0.126663   0.058535 -2.1639 0.030473 *
## ma2       -0.039129   0.059176 -0.6612 0.508462
## ma3        0.018187   0.058836  0.3091 0.757238
## ma4        0.025408   0.061579  0.4126 0.679899
## intercept  0.722343   0.237584  3.0404 0.002363 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
residual.analysis(model=m.04, std = TRUE, class = "ARIMA")
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.98749, p-value = 0.01248
```



For ARMA(0,4) “ML” model, only “ma1” is significant, whereas, all components are insignificant. The residual plot has a flat mean level indicating no trend. The histogram is left skewed and indicates outliers. Most of the residuals follows the line of normality but their are some deviation at the left tail. Shapiro-Wilk test has a p-value < 0.05, indicating non-normality. Ljung-Box test shows all lags above the significance level, therefore, no auto-correlation is present in the residuals.

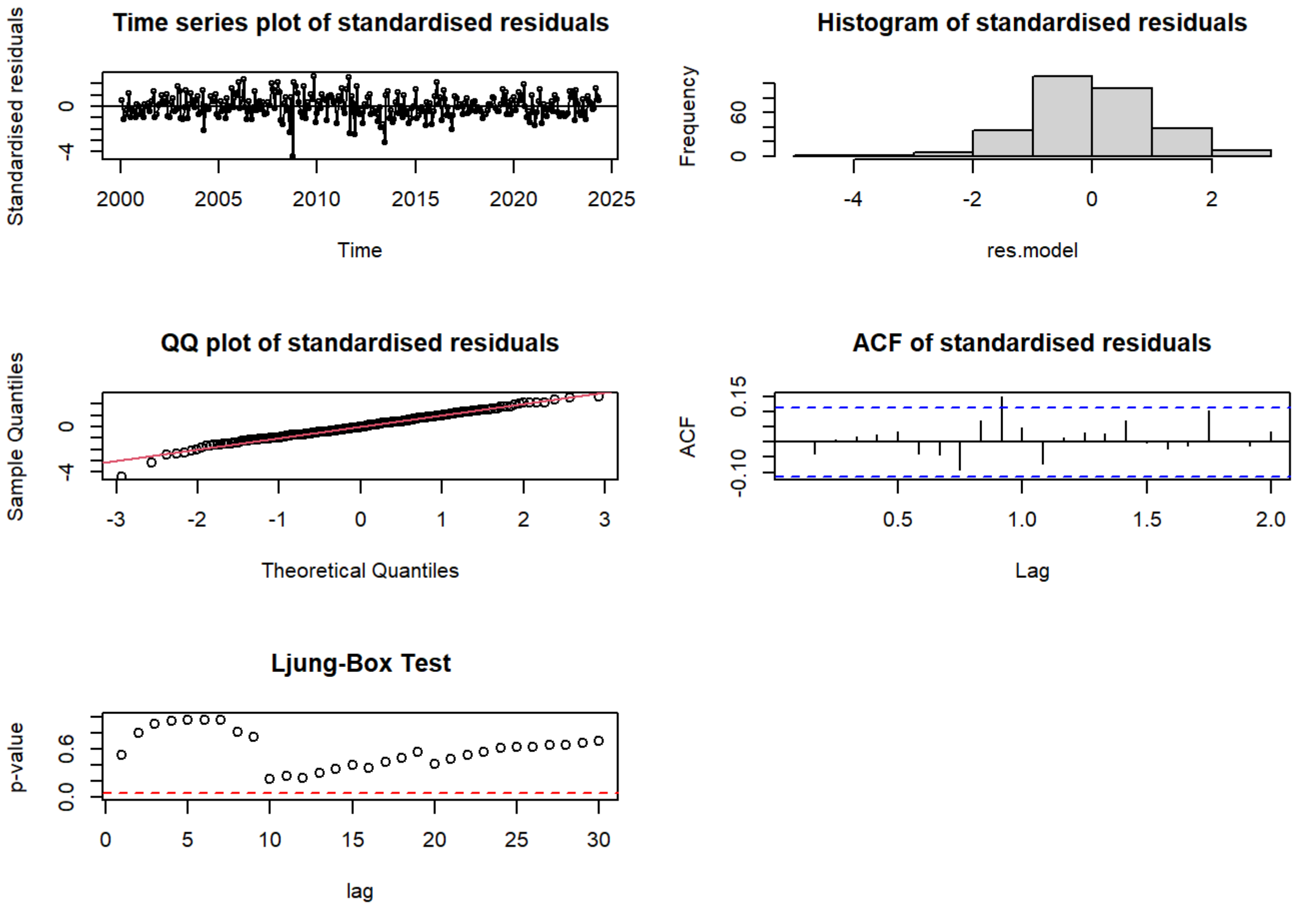
ARMA (0,1)

```
m.01 = Arima(r.gold,order=c(0,0,1),method='ML')
coeftest(m.01)
```

```
##
## z test of coefficients:
##
##            Estimate Std. Error z value Pr(>|z|)
## ma1       -0.130280   0.060008 -2.1711 0.029927 *
## intercept  0.721647   0.235722  3.0614 0.002203 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
residual.analysis(model=m.01, std = TRUE, class = "ARIMA")
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.98886, p-value = 0.02437
```



For ARMA(0,1) “ML” model had all coefficient significant. The residual plot has a flat mean level indicating no trend. The histogram is left skewed and indicates outliers. Most of the residuals follows the line of normality but their are some deviation at the left tail. Shapiro-Wilk test has a p-value < 0.05, indicating non-normality. Ljung-Box test shows all lags above the significance level, therefore, no auto-correlation is present in the residuals.

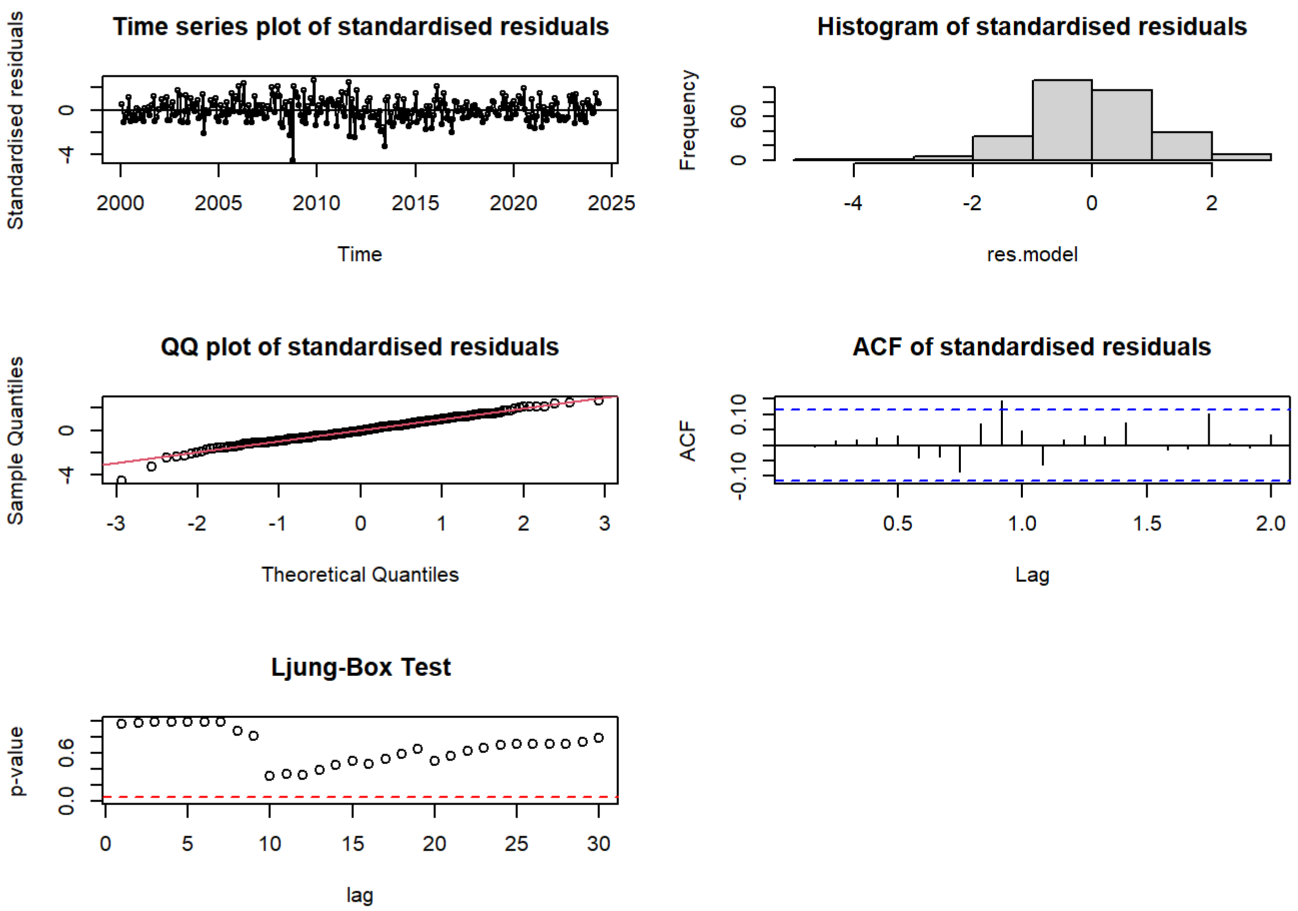
ARMA (0,2)

```
m.02 = Arima(r.gold,order=c(0,0,2),method='ML')
coeftest(m.02)
```

```
##
## z test of coefficients:
##
##             Estimate Std. Error z value Pr(>|z|)
## ma1        -0.124433   0.058572 -2.1244 0.033634 *
## ma2        -0.034710   0.057458 -0.6041 0.545784
## intercept   0.721616   0.227820  3.1675 0.001538 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
residual.analysis(model=m.02, std = TRUE, class = "ARIMA")
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.98752, p-value = 0.01268
```



For ARMA(0,2) “ML” model had “ma1” coefficient significant. The residual plot has a flat mean level indicating no trend. The histogram is left skewed and indicates outliers. Most of the residuals follows the line of normality but their are some deviation at the left tail. Shapiro-Wilk test has a p-value < 0.05, indicating non-normality.

Ljung-Box test shows all lags above the significance level, therefore, no auto-correlation is present in the residuals.

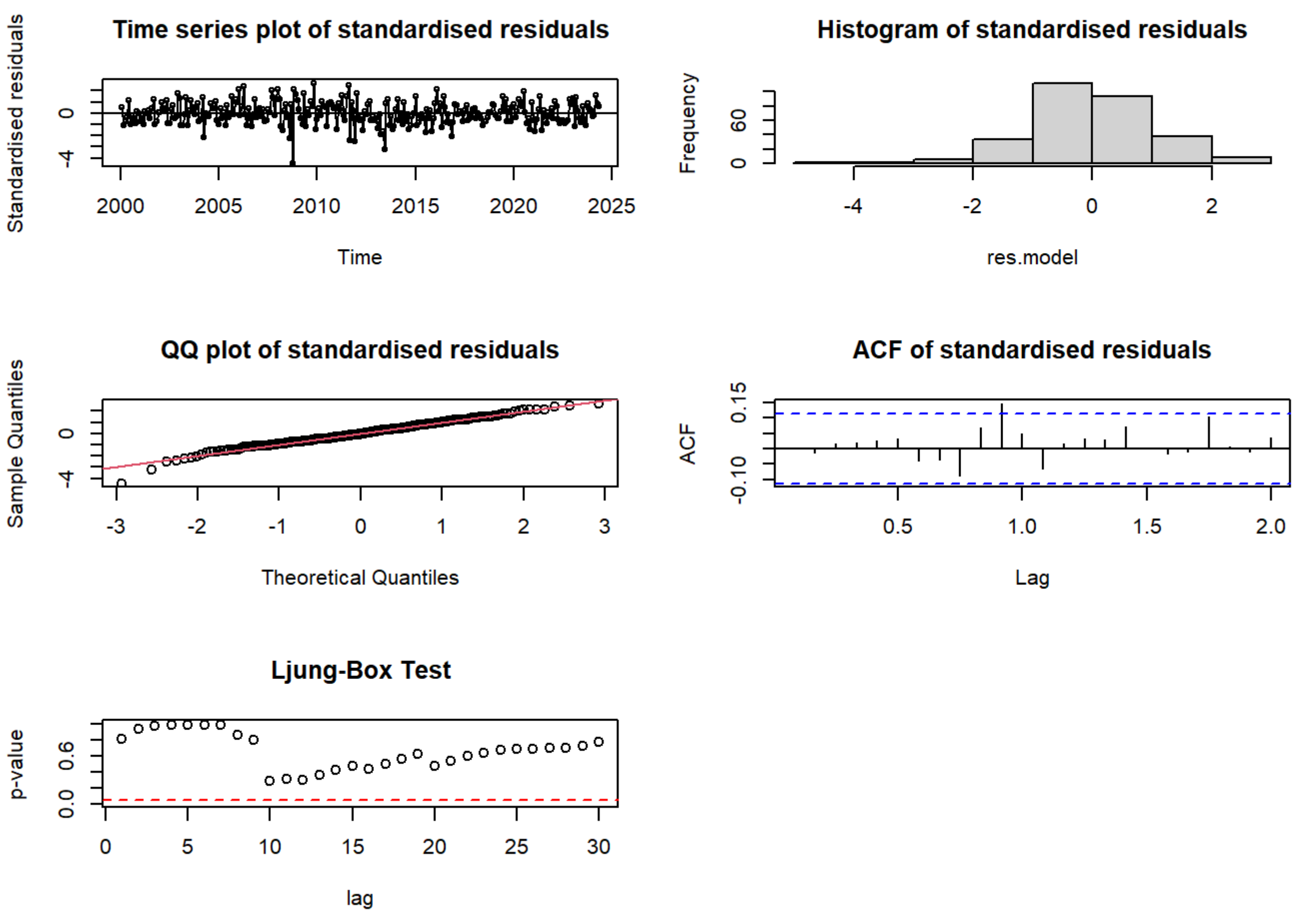
ARMA (1,1)

```
m.11 = Arima(r.gold,order=c(1,0,1),method='ML')
coeftest(m.11)
```

```
##
## z test of coefficients:
##
##           Estimate Std. Error z value Pr(>|z|)
## ar1        0.18421    0.34410  0.5354 0.592407
## ma1       -0.31117    0.33132 -0.9392 0.347640
## intercept  0.72144    0.22881  3.1530 0.001616 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
residual.analysis(model=m.11, std = TRUE, class = "ARIMA")
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.98793, p-value = 0.01546
```



For ARMA(1,1) “ML” model had no coefficient significant. The residual plot has a flat mean level indicating no trend. The histogram is left skewed and indicates outliers. Most of the residuals follows the line of normality but their are some deviation at the left tail. Shapiro-Wilk test has a p-value < 0.05, indicating non-normality. Ljung-Box test shows all lags above the significance level, therefore, no auto-correlation is present in the residuals.

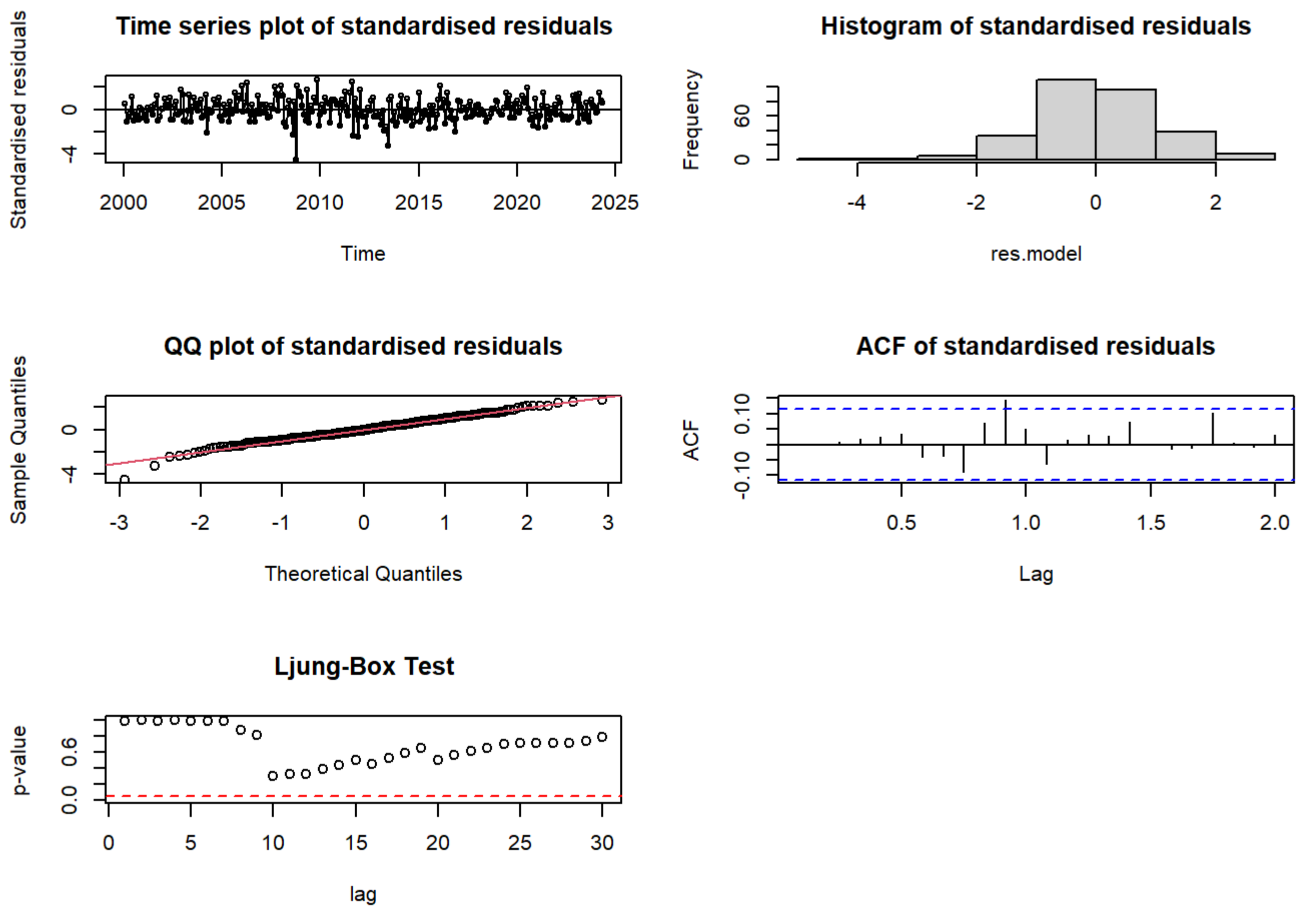
ARMA (1,2)

```
m.12 = Arima(r.gold,order=c(1,0,2),method='ML')
coeftest(m.12)
```

```
##
## z test of coefficients:
##
##            Estimate Std. Error z value Pr(>|z|)
## ar1       -0.178454   1.086952 -0.1642 0.869591
## ma1        0.053723   1.085102  0.0495 0.960513
## ma2       -0.059109   0.149284 -0.3959 0.692144
## intercept  0.721397   0.228644  3.1551 0.001604 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
residual.analysis(model=m.12, std = TRUE, class = "ARIMA")
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.98749, p-value = 0.01246
```



For ARMA(1,2) “ML” model had no coefficient significant. The residual plot has a flat mean level indicating no trend. The histogram is left skewed and indicates outliers. Most of the residuals follows the line of normality but their are some deviation at the left tail. Shapiro-Wilk test has a p-value < 0.05, indicating non-normality. Ljung-Box test shows all lags above the significance level, therefore, no auto-correlation is present in the residuals.

To conclude, ARMA(0,1) had all the coefficients significant. Therefore we can say ARMA(0,1) is the most suited model.

Model Specification ARMA x GARCH Part 2

```
abs.r.res.gold = abs(rstandard(m.01))
sq.r.res.gold = rstandard(m.01)^2
```

ACF and PACF absolute

```
Diagnostic_test(abs.r.res.gold, mainacf="ACF plot for abs residual return series", subacf = "Figure 30: ACF plot for abs residual return series", mainpacf = "PACF plot for abs residual return series", subpacf = "Figure 31: PACF plot for abs residual return series", test = "ACF-PACF")
```

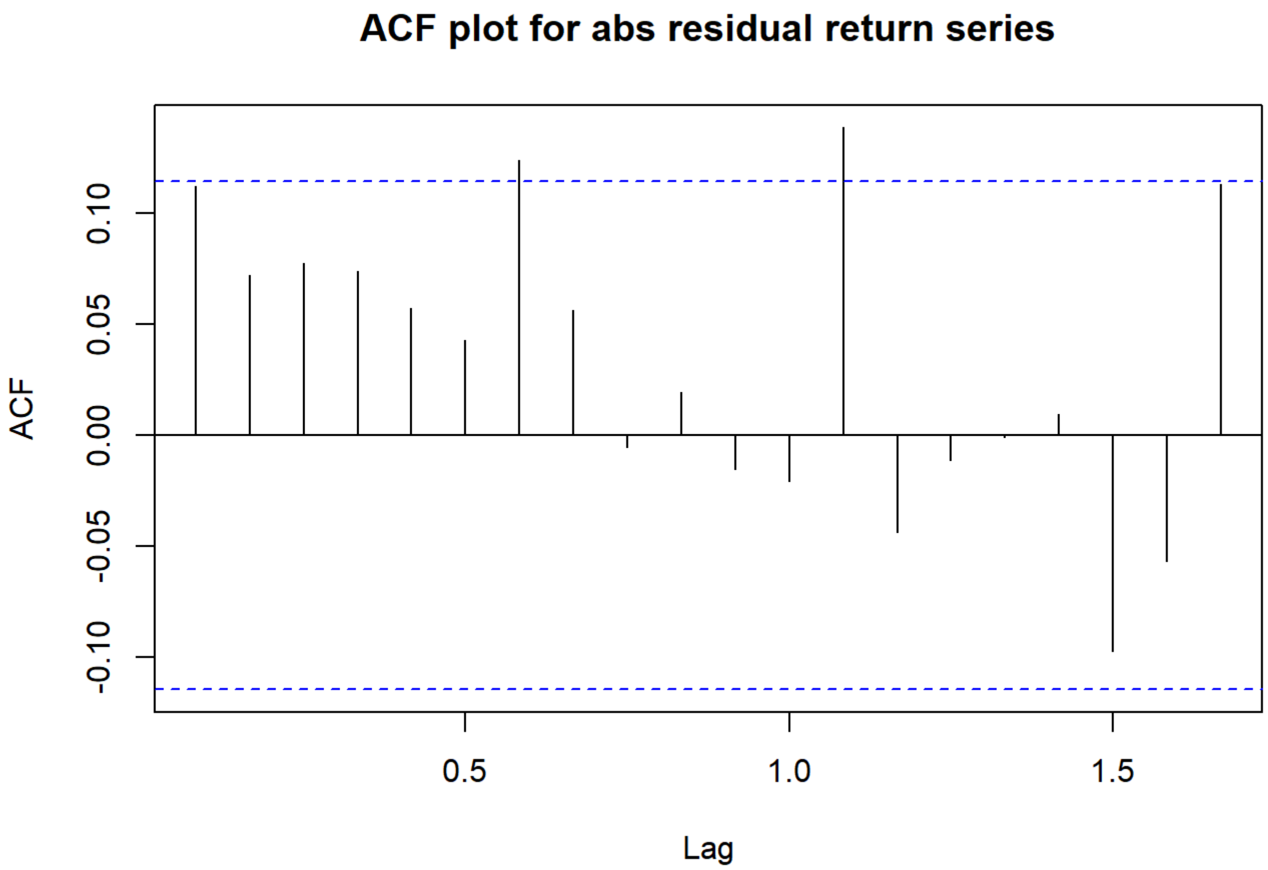


Figure 30: ACF plot for abs residual return series

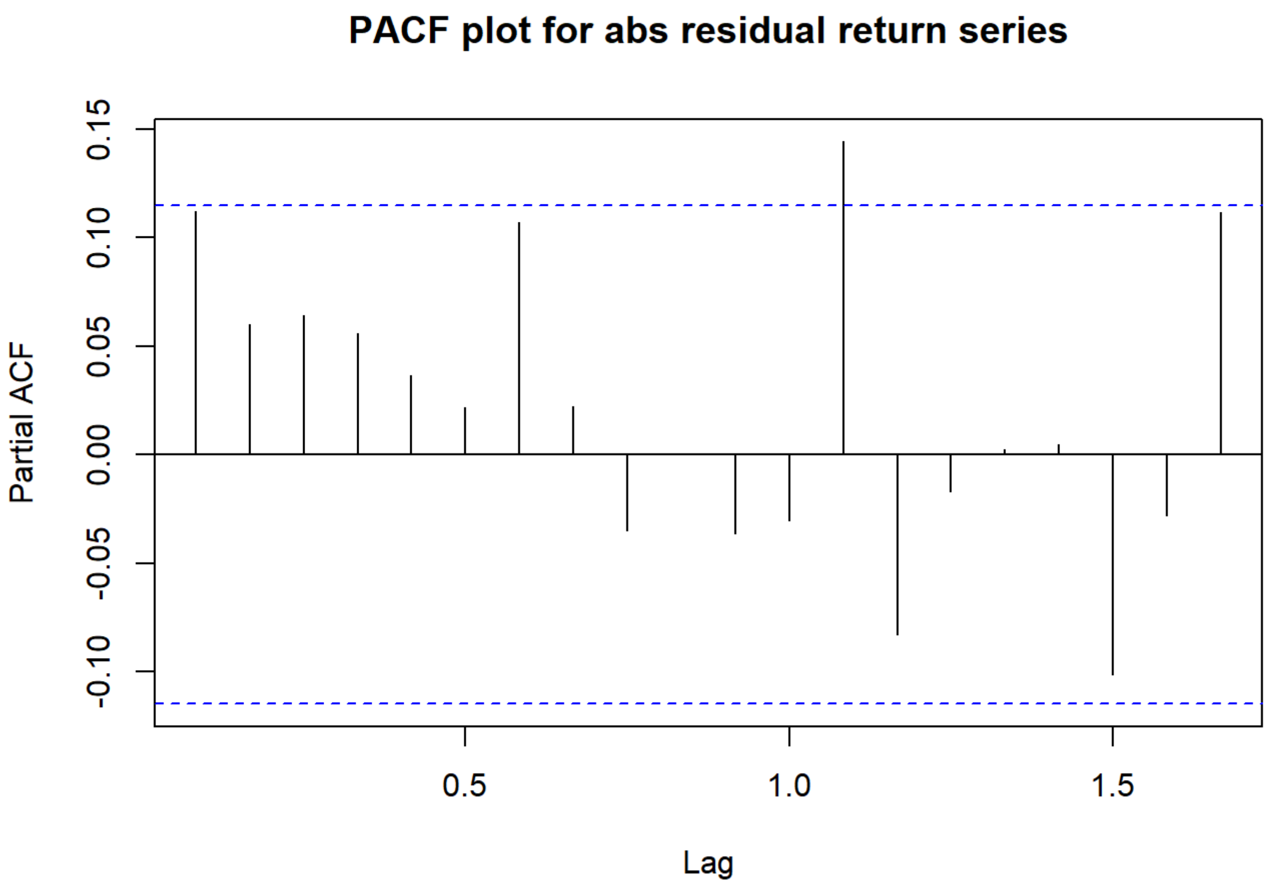


Figure 31: PACF plot for abs residual return series

To get the value of (p,q), first 5 lags are considered from ACF and PACF plot

From Figure 30 ACF plot, there are no significant lags, q = 0 and from Figure 31 PACF plot, there are no lags significant, so p = 0.

max(p,q) = 0 q = 0 max(p,q = 0) does not lead to any models.

EACF abs garch

```
eacf(abs.r.res.gold)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 o o o o o o x o o o o  o  x  o
## 1 x o o o o o o o o o o  o  o  o
## 2 x x o o o o o o o o o  o  o  o
## 3 x x x o o o o o o o o  o  o  o
## 4 x o o o o o o o o o o  o  o  o
## 5 x o o o o o o o o o o  o  o  o
## 6 x x x x o x o o o o o  o  o  o
## 7 x x x x o x x o o o o  o  o  o
```

Top left “o” is in (0,0) - max(p,q) = 0 and q=0 => max(p,q = 0) = 0; does not lead any models.

Neighbor models:

- max(p,q) = 0 and q = 1 => max(p,q = 1) = 0; does not lead any models.
- max(p,q)= 1 and q = 1 => max(p,q = 1) = 1, hence p = 0 or 1

{GARCH(0,1) and GARCH(1,1)}

ACF and PACF squared

```
Diagnostic_test(sq.r.res.gold, mainacf="ACF plot for squared residual return series", subacf
= "Figure 32: ACF plot for squared residual return series", mainpacf = "PACF plot for squared
residual return series", subpacf = "Figure 33: PACF plot for squared residual return series",
test = "ACF-PACF")
```

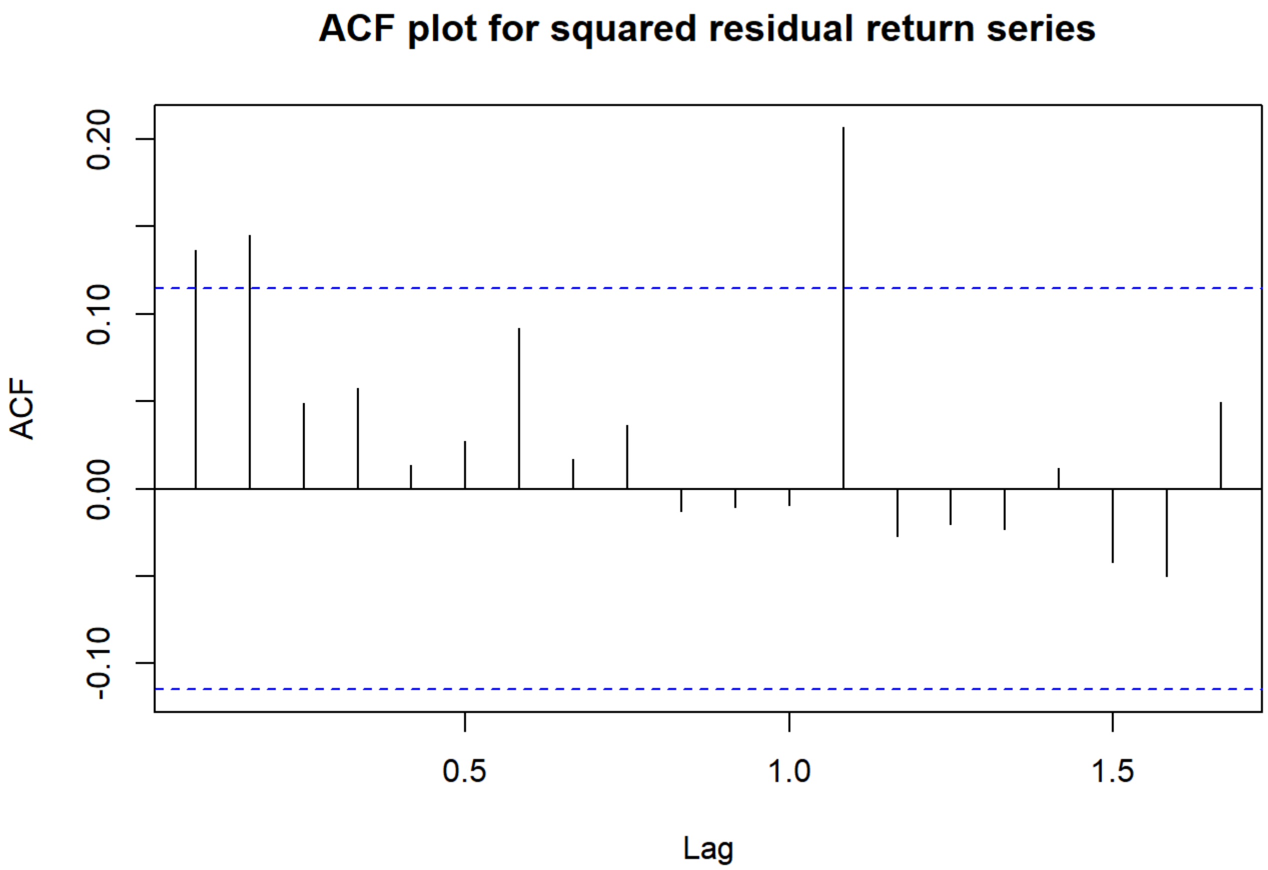


Figure 32: ACF plot for squared residual return series

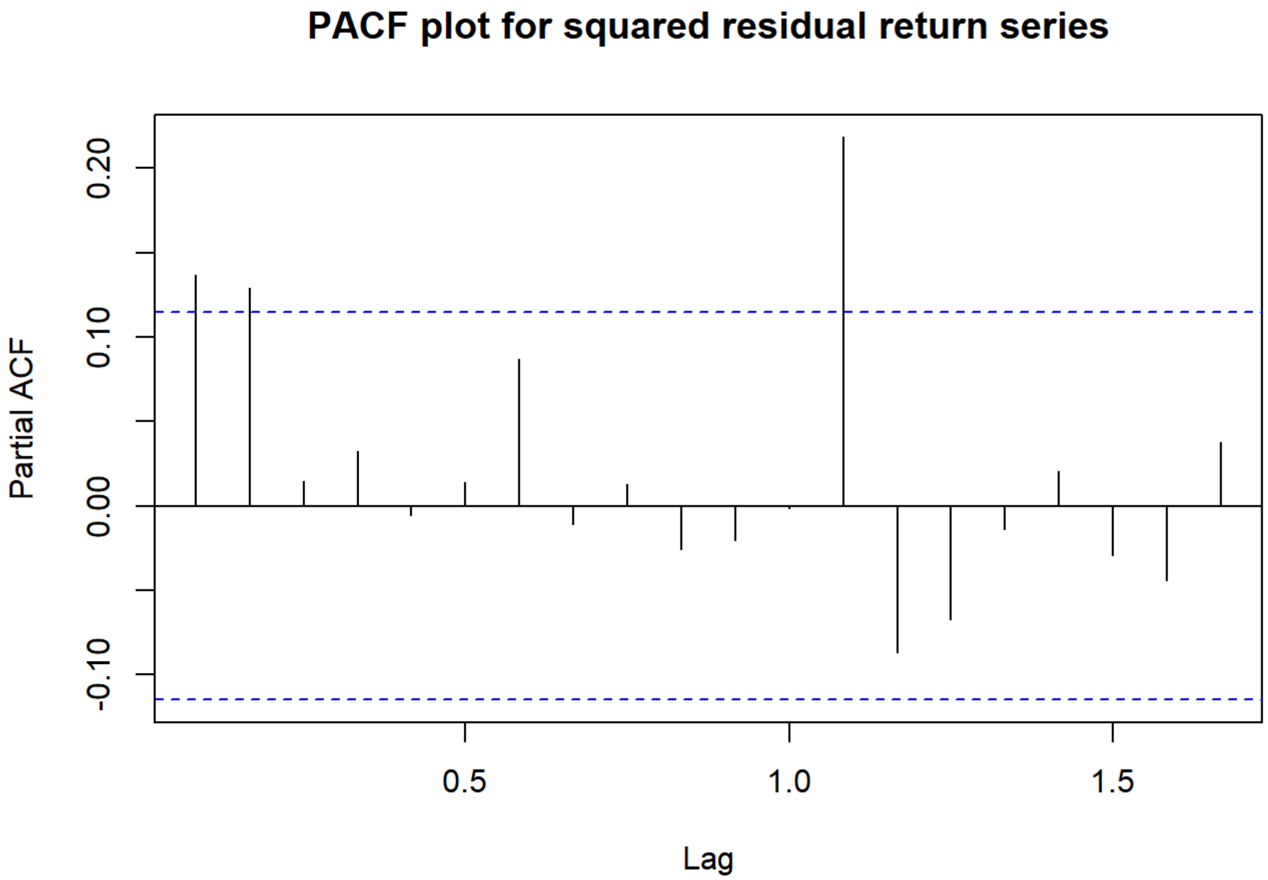


Figure 33: PACF plot for squared residual return series

From Figure 32 ACF plot we get q = 2 and from Figure 33 PACF plot we get p =2.

max(p,q) = 2 q = 2 max(p,q = 2), we get p = 0,1,2

GARCH {0,2},GARCH {1,2} and GARCH {2,2}

EACF sqaured

```
eacf(sq.r.res.gold)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x o o o o o o o o o  o  x  o
## 1 x o o o o o o o o o o  o  x  x
## 2 o x o o o o o o o o o  o  x  x
## 3 x x o o o o o o o o o  o  x  o
## 4 x x o o o o o o o o o  o  x  o
## 5 x o x o x o o o o o o  o  x  o
## 6 x o x o x x o o o o o  o  x  o
## 7 x x x x o x o o o o o  o  x  o
```

- max(p,q) = 0 and q = 2 => max(p,q = 2) = 0; does not lead any models.

Neighbor models:

- max(p,q) = 0 and q = 3 => max(p,q = 3) = 0; does not lead any models.
- max(p,q) = 1 and q = 2 => max(p,q = 2) = 1; does not lead any models.
- max(p,q) = 1 and q = 3 => max(p,q = 3) = 1; does not lead any models

GARCH {0,2} GARCH {1,2} GARCH {2,2}

Parameter Estimation ARMA x GARCH

```
m.01.01<- fGarch::garchFit(~ arma(0,1)+garch(1,0),
data = r.gold, trace=F)
m.01.11<- fGarch::garchFit(~ arma(0,1)+garch(1,1),
data = r.gold, trace=F)
m.01.02<- fGarch::garchFit(~ arma(0,1)+garch(2,0),
data = r.gold, trace=F)
m.01.12<- fGarch::garchFit(~ arma(0,1)+garch(2,1),
data = r.gold, trace=F)
m.01.22<- fGarch::garchFit(~ arma(0,1)+garch(2,2),
data = r.gold, trace=F)
```

```
summary(m.01.01)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  fGarch::garchFit(formula = ~arma(0, 1) + garch(1, 0), data = r.gold,
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(0, 1) + garch(1, 0)
## <environment: 0x000002b5d6d5ec48>
##  [data = r.gold]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##        mu        ma1      omega     alpha1
##  0.617973  -0.061223  18.513131   0.134143
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu       0.61797     0.25263    2.446   0.0144 *
## ma1     -0.06122     0.06945   -0.881   0.3781
## omega   18.51313     2.03017    9.119   <2e-16 ***
## alpha1   0.13414     0.07917    1.694   0.0902 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -859.1102    normalized:  -2.942158
##
## Description:
##  Thu Jun  6 19:08:23 2024 by user: HP
##
##
## Standardised Residuals Tests:
##                                  Statistic      p-Value
##  Jarque-Bera Test   R    Chi^2  14.0913851 0.0008711533
##  Shapiro-Wilk Test  R    W       0.9893178 0.0306262931
##  Ljung-Box Test     R    Q(10)   5.0977723 0.8845516951
##  Ljung-Box Test     R    Q(15)  13.9437475 0.5297996750
##  Ljung-Box Test     R    Q(20)  16.1940736 0.7045135769
##  Ljung-Box Test     R^2  Q(10)   9.0240701 0.5298207696
##  Ljung-Box Test     R^2  Q(15)  27.3561308 0.0259651604
##  Ljung-Box Test     R^2  Q(20)  30.0591929 0.0689001850
##  LM Arch Test       R    TR^2    8.4759943 0.7469152657
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 5.911714 5.962080 5.911345 5.931889
```

No coefficients are significant for m.01.01

```
summary(m.01.11)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  fGarch::garchFit(formula = ~arma(0, 1) + garch(1, 1), data = r.gold,
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(0, 1) + garch(1, 1)
## <environment: 0x000002b5d5554cb0>
##  [data = r.gold]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##       mu       ma1     omega    alpha1     beta1
##  0.68780  -0.08278   1.98535   0.11590   0.79422
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu       0.68780     0.23419    2.937  0.00332 **
## ma1     -0.08278     0.06371   -1.299  0.19382
## omega    1.98535     1.69370    1.172  0.24112
## alpha1   0.11590     0.05476    2.117  0.03430 *
## beta1    0.79422     0.11338    7.005 2.48e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -854.7267    normalized:  -2.927146
##
## Description:
##  Thu Jun  6 19:08:23 2024 by user: HP
##
##
## Standardised Residuals Tests:
##                                   Statistic   p-Value
##  Jarque-Bera Test   R    Chi^2   0.8945865 0.6393564
##  Shapiro-Wilk Test  R    W       0.9941019 0.3158700
##  Ljung-Box Test     R    Q(10)   4.2274411 0.9365039
##  Ljung-Box Test     R    Q(15)  11.7499547 0.6978485
##  Ljung-Box Test     R    Q(20)  12.9923204 0.8777133
##  Ljung-Box Test     R^2  Q(10)   4.0367199 0.9456753
##  Ljung-Box Test     R^2  Q(15)  20.9957574 0.1369649
##  Ljung-Box Test     R^2  Q(20)  27.2772419 0.1276427
##  LM Arch Test       R    TR^2    5.3563064 0.9450076
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 5.888539 5.951497 5.887966 5.913758
```

Only 2 coefficients are significant for m.01.11

```
summary(m.01.02)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  fGarch::garchFit(formula = ~arma(0, 1) + garch(2, 0), data = r.gold,
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(0, 1) + garch(2, 0)
## <environment: 0x000002b5d3cf0770>
##  [data = r.gold]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##        mu        ma1      omega     alpha1     alpha2
##  0.703328  -0.082857  16.474510   0.101305   0.124140
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu       0.70333     0.24036    2.926  0.00343 **
## ma1     -0.08286     0.06959   -1.191  0.23381
## omega   16.47451     2.12282    7.761 8.44e-15 ***
## alpha1   0.10131     0.06976    1.452  0.14644
## alpha2   0.12414     0.07417    1.674  0.09420 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -856.9624    normalized:  -2.934803
##
## Description:
##  Thu Jun  6 19:08:23 2024 by user: HP
##
##
## Standardised Residuals Tests:
##                                  Statistic   p-Value
##  Jarque-Bera Test   R    Chi^2   0.9695397 0.6158389
##  Shapiro-Wilk Test  R    W       0.9962201 0.7135513
##  Ljung-Box Test     R    Q(10)   5.1626066 0.8800543
##  Ljung-Box Test     R    Q(15)  13.3966096 0.5716930
##  Ljung-Box Test     R    Q(20)  15.2553057 0.7616173
##  Ljung-Box Test     R^2  Q(10)   6.9489060 0.7302596
##  Ljung-Box Test     R^2  Q(15)  21.5439469 0.1203316
##  Ljung-Box Test     R^2  Q(20)  27.6226388 0.1186376
##  LM Arch Test       R    TR^2    7.1048214 0.8506094
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 5.903852 5.966810 5.903278 5.929070
```

No coefficients are significant for m.01.02

```
summary(m.01.12)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  fGarch::garchFit(formula = ~arma(0, 1) + garch(2, 1), data = r.gold,
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(0, 1) + garch(2, 1)
## <environment: 0x000002b5ce3e81d8>
##  [data = r.gold]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##        mu        ma1      omega     alpha1     alpha2      beta1
##  0.720678  -0.091151   3.042020   0.084049   0.077411   0.701344
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu       0.72068     0.23287    3.095  0.00197 **
## ma1     -0.09115     0.06322   -1.442  0.14934
## omega    3.04202     2.55682    1.190  0.23414
## alpha1   0.08405     0.06664    1.261  0.20721
## alpha2   0.07741     0.10537    0.735  0.46256
## beta1    0.70134     0.17864    3.926 8.63e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -854.5314    normalized:  -2.926477
##
## Description:
##  Thu Jun  6 19:08:23 2024 by user: HP
##
##
## Standardised Residuals Tests:
##                                 Statistic    p-Value
##  Jarque-Bera Test   R    Chi^2   0.3496448 0.83960612
##  Shapiro-Wilk Test  R    W       0.9958064 0.62562284
##  Ljung-Box Test     R    Q(10)   4.3902078 0.92803215
##  Ljung-Box Test     R    Q(15)  11.9511476 0.68272299
##  Ljung-Box Test     R    Q(20)  13.2328564 0.86717108
##  Ljung-Box Test     R^2  Q(10)   3.8771917 0.95271704
##  Ljung-Box Test     R^2  Q(15)  21.6361673 0.11770494
##  Ljung-Box Test     R^2  Q(20)  28.9631969 0.08848655
##  LM Arch Test       R    TR^2    5.0209708 0.95727485
##
## Information Criterion Statistics:
```

```
##       AIC      BIC      SIC     HQIC
## 5.894050 5.969600 5.893228 5.924313
```

Only 1 coefficient is significant for m.01.12

```
summary(m.01.22)
```

```
## 
## Title:
##  GARCH Modelling 
## 
## Call:
##  fGarch::garchFit(formula = ~arma(0, 1) + garch(2, 2), data = r.gold, 
##     trace = F) 
## 
## Mean and Variance Equation:
##  data ~ arma(0, 1) + garch(2, 2)
## <environment: 0x000002b5d9dbf830>
##  [data = r.gold]
## 
## Conditional Distribution:
##  norm 
## 
## Coefficient(s):
##        mu        ma1      omega     alpha1     alpha2      beta1      beta2  
##  0.716377  -0.089948   3.655391   0.087388   0.106455   0.362793   0.277344  
## 
## Std. Errors:
##  based on Hessian 
## 
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)   
## mu       0.71638     0.23356    3.067  0.00216 **
## ma1     -0.08995     0.06381   -1.410  0.15868   
## omega    3.65539     3.57465    1.023  0.30650   
## alpha1   0.08739     0.06837    1.278  0.20121   
## alpha2   0.10645     0.12822    0.830  0.40640   
## beta1    0.36279     1.25619    0.289  0.77273   
## beta2    0.27734     1.04574    0.265  0.79085   
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Log Likelihood:
##  -854.5153    normalized:  -2.926422 
## 
## Description:
##  Thu Jun  6 19:08:23 2024 by user: HP 
## 
## 
## Standardised Residuals Tests:
##                                 Statistic   p-Value
##  Jarque-Bera Test   R    Chi^2   0.2522403 0.8815089
##  Shapiro-Wilk Test  R    W       0.9957120 0.6056971
##  Ljung-Box Test     R    Q(10)   4.4582808 0.9243146
##  Ljung-Box Test     R    Q(15)  11.9297594 0.6843378
##  Ljung-Box Test     R    Q(20)  13.2251157 0.8675177
##  Ljung-Box Test     R^2  Q(10)   3.8941063 0.9519976
##  Ljung-Box Test     R^2  Q(15)  20.4326163 0.1559575
##  Ljung-Box Test     R^2  Q(20)  27.8302268 0.1134796
##  LM Arch Test       R    TR^2    5.0417624 0.9565695
## 
## Information Criterion Statistics:
```

```
##      AIC      BIC      SIC     HQIC
## 5.900789 5.988931 5.899676 5.936095
```

No coefficients are significant for m.01.22

Summary analysis gave ARMA(0,1) X GARCH(1,1) as best model for forecast.

Forecasting

ARIMA forecast:

```
m5_011 = Arima(data.ts,order=c(0,1,1),
                          lambda = 0, method = "ML")
preds1 = forecast(m5_011, lambda = 0, h = 10)
preds1
```

```
##          Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## Jun 2024       2359.698 2221.531 2506.458 2151.695 2587.808
## Jul 2024       2359.698 2175.361 2559.655 2083.682 2672.276
## Aug 2024       2359.698 2139.554 2602.492 2031.457 2740.975
## Sep 2024       2359.698 2109.459 2639.622 1987.919 2801.007
## Oct 2024       2359.698 2083.116 2673.002 1950.077 2855.360
## Nov 2024       2359.698 2059.480 2703.679 1916.340 2905.629
## Dec 2024       2359.698 2037.916 2732.287 1885.739 2952.781
## Jan 2025       2359.698 2018.004 2759.248 1857.632 2997.457
## Feb 2025       2359.698 1999.448 2784.856 1831.572 3040.107
## Mar 2025       2359.698 1982.030 2809.328 1807.227 3081.060
```

```
plot(preds1, sub = "Figure 34: Forecast from ARIMA model")
```

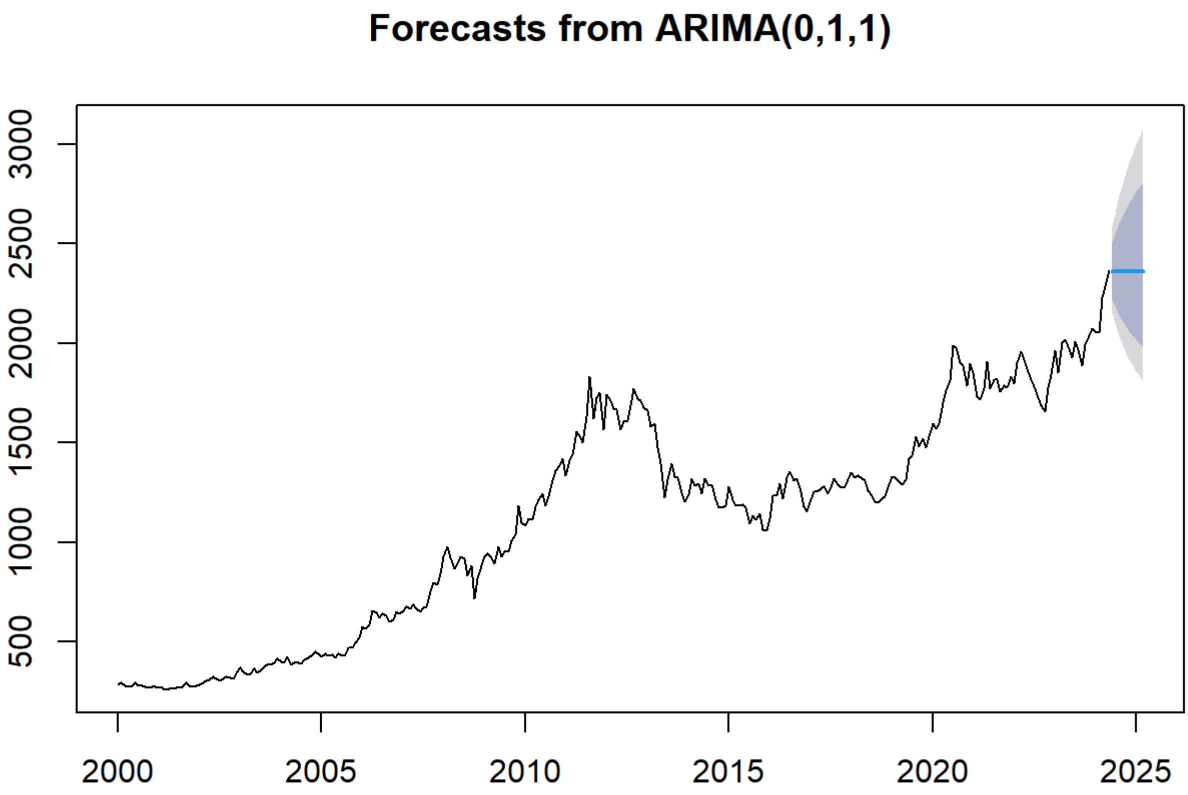


Figure 34: Forecast from ARIMA model

The best ARIMA model is ARIMA(0,1,1). The forecast from this model shows consistent values over the next 10 months.

ARMA x GARCH forecast:

```
spec <- ugarchspec(variance.model = list(model = "sGARCH",
garchOrder = c(1, 1)
),
mean.model = list(armaOrder = c(0, 1)))
model_401_11_2 <- ugarchfit(spec = spec, data = r.gold,
solver = "hybrid",
solver.control = list(trace=0))
frc <- ugarchforecast(model_401_11_2,n.ahead=10,data=r.gold)
frc
```

```
##
## *------------------------------------*
## *       GARCH Model Forecast         *
## *------------------------------------*
## Model: sGARCH
## Horizon: 10
## Roll Steps: 0
## Out of Sample: 0
##
## 0-roll forecast [T0=May 2024]:
##       Series Sigma
## T+1  0.4943 4.167
## T+2  0.6953 4.218
## T+3  0.6953 4.265
## T+4  0.6953 4.307
## T+5  0.6953 4.344
## T+6  0.6953 4.378
## T+7  0.6953 4.409
## T+8  0.6953 4.437
## T+9  0.6953 4.462
## T+10 0.6953 4.485
```

```
plot(frc, which = 1, sub = "Figure 35: Forecast of ARMA-GARCH series")
mtext('Figure 35: Forecast of ARMA-GARCH series',line = 4, side = 1, cex = 0.8)
```

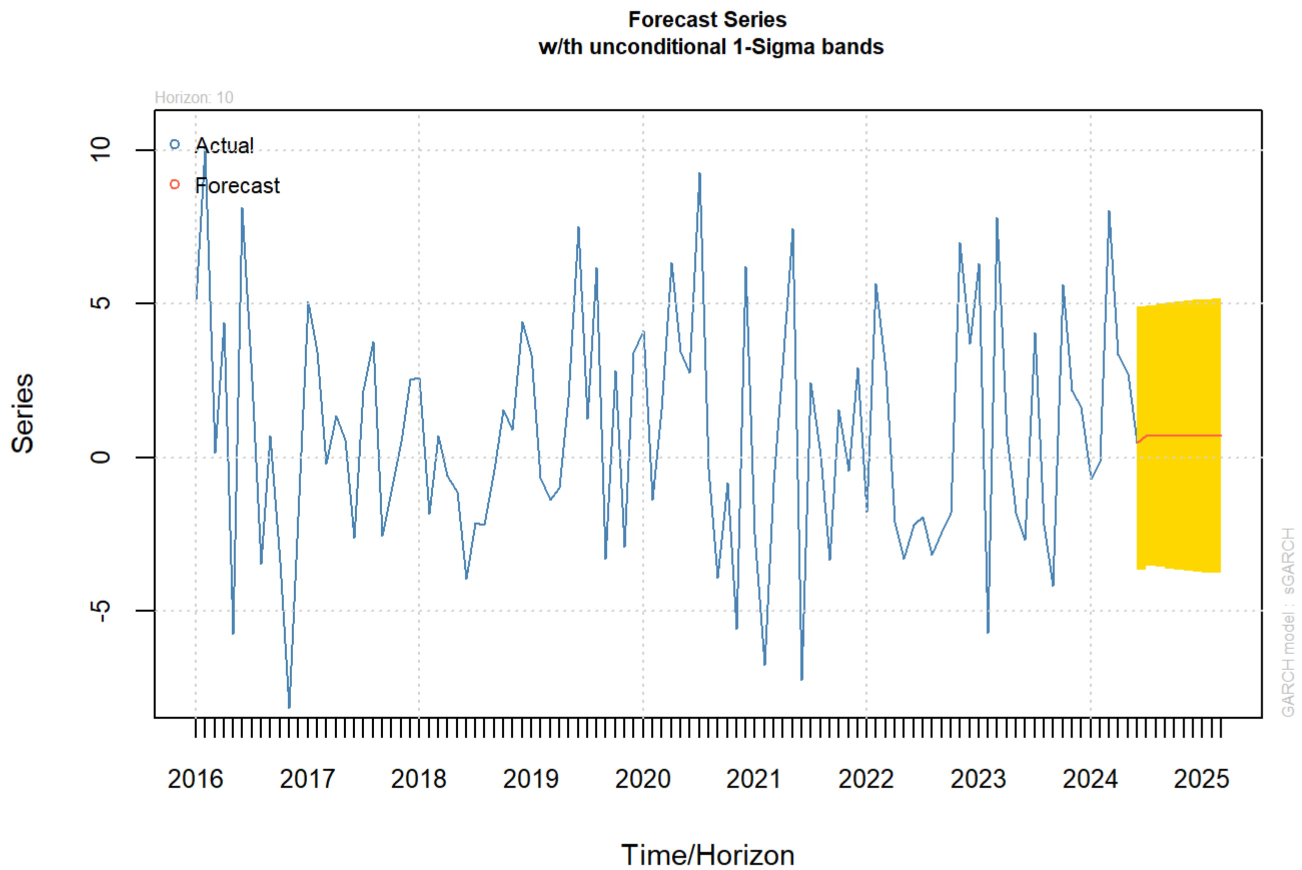


Figure 35: Forecast of ARMA-GARCH series

```
plot(frc, which = 3, sub = "Figure 36: Forecast of ARMA-GARCH sigma")
mtext('Figure 36: Forecast of ARMA-GARCH sigma',line = 4, side = 1, cex = 0.8)
```

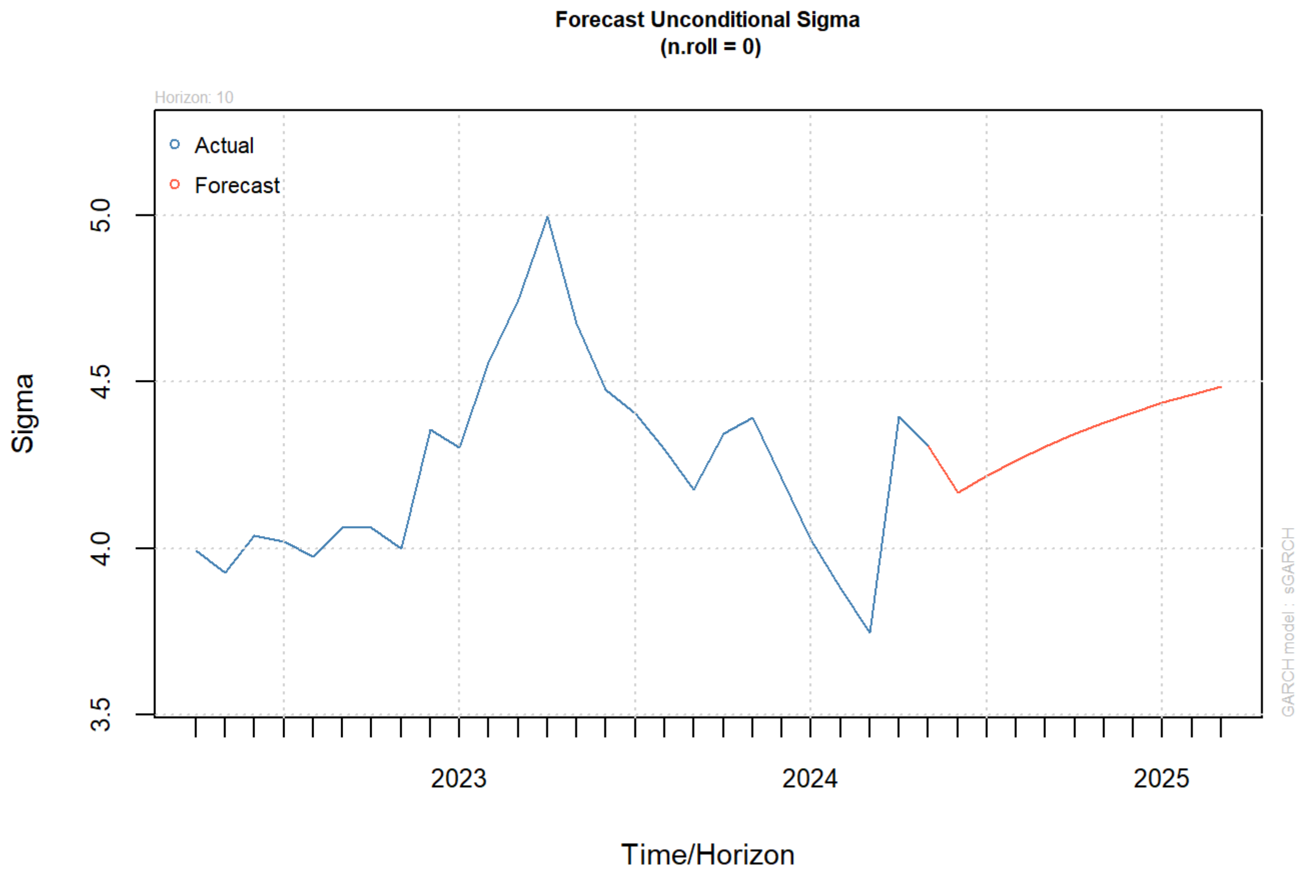


Figure 36: Forecast of ARMA-GARCH sigma

Based on the ARMA-GARCH model, the forecast for the next 10 months is consistent, indicating stable predicted values. However, the high variance within the yellow highlighted area suggests significant uncertainty in the forecast, underscoring the potential for considerable deviation from the central trend.

The ARMA-GARCH model's forecast indicates a rising trend in volatility over the next 10 months. While the historical sigma values indicates fluctuations, the forecast suggests that volatility will increase consistently. This rising trend in forecasted volatility highlights the growing uncertainty and potential risk in the series during the forecast period.

SARIMA forecast:

```
m_011_112_12 = Arima(data.ts,order=c(0,1,1),seasonal=list(order=c(1,1,2), period=12), lambda
= 0, method = "ML")
preds1 = forecast(m_011_112_12, lambda = 0, h = 10)
preds1
```

```
##          Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## Jun 2024       2340.072 2200.317 2488.704 2129.746 2571.169
## Jul 2024       2363.698 2179.262 2563.742 2087.527 2676.404
## Aug 2024       2386.897 2166.255 2630.013 2057.834 2768.581
## Sep 2024       2373.474 2125.122 2650.849 2004.352 2810.574
## Oct 2024       2375.637 2101.590 2685.420 1969.557 2865.442
## Nov 2024       2405.892 2105.147 2749.602 1961.474 2951.004
## Dec 2024       2446.021 2118.690 2823.925 1963.535 3047.066
## Jan 2025       2499.756 2144.846 2913.393 1977.845 3159.388
## Feb 2025       2508.555 2133.329 2949.778 1957.981 3213.946
## Mar 2025       2538.021 2140.261 3009.703 1955.593 3293.912
```

```
plot(preds1, main = "Forecast from SARIMA_011_112_12", sub = "Figure 37: Forecast of SARIMA s
eries")
```

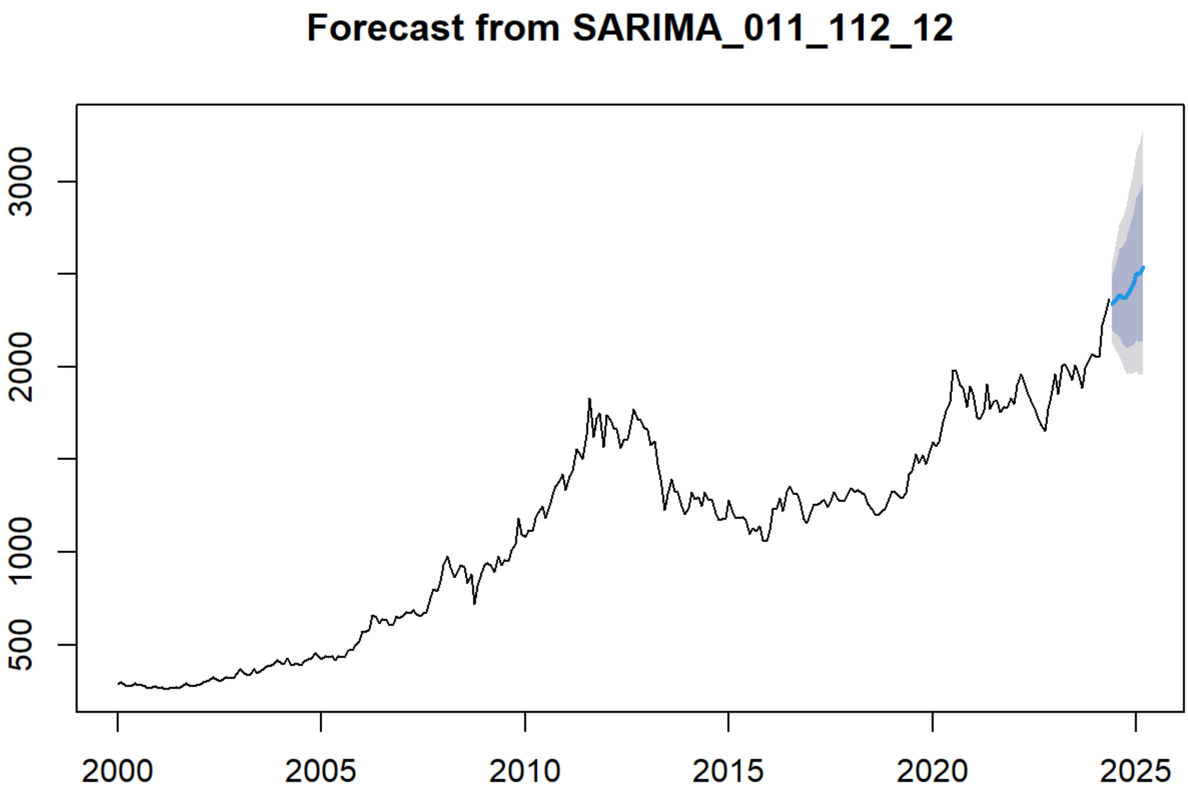


Figure 37: Forecast of SARIMA series

The SARIMA(0,1,1)(1,1,2)[12] model forecasts a continued upward trend over the next 10 months, with increasing uncertainty as time progresses.

Conclusion

Based on the analysis, both ARIMA and ARMA-GARCH models did not get significant results, making their forecasts unreliable. In contrast, the SARIMA model proved to be the most effective, as evidenced by residual analysis, goodness of fit metrics (AIC, BIC), error metrics, and significant coefficients. Specifically, the SARIMA((0,1,1)(1,1,2)_12 model emerged as the best fit.

The forecast from the SARIMA model indicates an upward trend for the next 10 months.

Reference

[1] Demirhan, H 2024, lecture and lab notes, Time Series, RMIT University, Melbourne

[2] Tran L, Pham, T 2024, seasonality function, Time Series, RMIT University, Melbourne