Fall Voltage as a Key Parameter in MPD Thruster Performance Modeling

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Abstract. Magnetoplasmadynamic (MPD) thrusters are a class of electric propulsion devices capable of producing high exhaust velocities and thrust through the control of magnetic fields and electric currents. These thrusters are of particular interest for deep-space missions due to their ability to operate effectively for extended durations. A key aspect of MPD thruster performance is the fall voltage, a voltage drop associated with plasma sheaths at the cathode and anode interfaces that represents energy lost as electrons cross electrode surfaces. In many system-level models, fall voltage is treated as a constant, but this simplification may introduce inaccuracies in predicting performance, particularly at lower discharge currents. This study investigates the sensitivity of MPD thruster performance to fall voltage using a simplified self-field MPD thruster model. By varying fall voltage across a range of 20–60 V while holding other parameters constant, we evaluate impacts on thrust, discharge voltage, exhaust velocity, and efficiency over a discharge current range of 0–1000 A. Results demonstrate that within a self-field J * B scaling framework, fall voltage does not directly influence thrust or exhaust velocity in the electromagnetic framework, it significantly reduces thruster efficiency. A 40 V increase in the fall voltage (20 V to 60 V) produced a reduction of approximately 65–66% in the predicted efficiency in both 200 A and 800 A. These findings underscore the importance of accurately modeling sheath behavior in MPD performance predictions and suggest that constant fall voltage assumptions may lead to substantial efficiency overestimation, particularly in low-current operational regimes.

INTRODUCTION

Magnetoplasmadynamic (MPD) thrusters are a class of electric propulsion devices capable of producing high exhaust velocities and thrust using the control of magnetic fields and electric currents (Hadzihafizovic, 2025). Space missions such as deep space travel and human exploration of Mars are of interest for these thrusters. MPD thrusters, in contrast to chemical thrusters, can function effectively for extended periods of time, supplying the acceleration required for deep-space travel.

The fall voltage, a small voltage drop associated with the plasma sheaths at the cathode and anode, is a key aspect of MPD thruster performance. It represents the energy lost as electrons cross the electrode surfaces. In many system-level models, such as the Modified Mäcker model (Gilland, 2003), fall voltage is treated as a constant. However, this generalization may produce inaccuracies in predicting thrust, exhaust velocity, and efficiency, particularly at lower currents where plasma is only partially ionized.

The main purpose of this study is to investigate the sensitivity of MPD thruster performance to fall voltage. By varying the fall voltage in a simplified model of a self-field MPD thruster, the study evaluates how different assumptions impact thrust, voltage, exhaust velocity, and efficiency. Simulations using Desmos and numerical calculations allow for easy visualization of these effects and give insight into the importance of accurately modeling fall voltage in MPD analysis. The results of this study aim to inform future modeling and the design of MPD thrusters for mission applications.

THEORETICAL BACKGROUND

Thruster Operation

Magnetoplasmadynamic (MPD) thrusters are electric propulsion devices that use electromagnetic forces to accelerate plasma. In a typical MPD thruster, propellant gas is inserted between a central cathode and a surrounding anode. According to Ampère's law, when a large electric current flows through the ionized propellant, it generates a magnetic field (Douglas College Physics, 2016). The current density in the plasma and this magnetic field produce a force that accelerates the plasma out of the thruster, generating thrust.

The mechanism responsible for this acceleration in MPD thruster is the Lorentz force, shown as:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{1}$$

where q represents particle charge, $\mathbf{E}$ represents the electric field, $\mathbf{v}$ represents the particle velocity, and $\mathbf{B}$ represents the magnetic field. This equation describes how electrons and ions respond to electric and magnetic fields at the microscopic level.

However, in a MPD thruster, the plasma consists of an extremely large number of charged particles. Rather than tracking individual particle trajectories, MPD thruster behavior is easier to track using a fluid model of the plasma. When this method is used, the electromagnetic force density can be shown as:

$$\mathbf{f} = \rho_e \mathbf{E} + \mathbf{J} \times \mathbf{B} \tag{2}$$

where ρ_e is the net charge density and $\mathbf{J}$ is the current density.

In MPD thrusters, the charge density is approximately zero (quasineutral plasma), causing the force to simplify to (Jahn, 1968, pp. 240–245; Francis, 2016):

$$\mathbf{f} \approx \mathbf{J} \times \mathbf{B} \tag{3}$$

In self-field MPD thrusters, the discharge current flowing between the cathode and anode generates a magnetic field. The product of the current density and this magnetic field is a force directed along the thruster, accelerating plasma downstream and generating thrust. This interaction represents the Lorentz force acting upon individual charged particles and forms the basis for thrust production.

Performance Metrics

Many performance metrics are commonly used to evaluate MPD thrusters. The most important quantities for this study are thrust, exhaust velocity, mass flow rate, voltage, and efficiency.

Thrust (T) represents the force produced by the thruster as plasma is accelerated.

Mass flow rate ($\dot{m}$) describes the amount of mass passing through the thruster per unit of time (NASA Glenn Research Center, n.d.).

Exhaust velocity (u_e) is the velocity at which plasma exits the thruster and is related to thrust by:

$$u_e = \frac{T}{\dot{m}} \tag{4}$$

This shows that higher thrust correlates to a higher exhaust velocity (for a fixed mass flow rate).

Thruster efficiency (η) is defined as the fraction of input power converted into directed kinetic energy of the exhaust (Jahn, 1968; Choueiri, 1998):

$$\eta = \frac{Tu_e}{VI} \tag{5}$$

where V is the total discharge voltage, and I is the current. Efficiency is very sensitive to voltage losses, making accurate modeling of voltage essential for MPD thruster performance.

MODELING FRAMEWORK

Simplified Self-Field MPD Thruster Model

To investigate the sensitivity of MPD thruster performance to fall voltage, a simplified self-field MPD thruster model will be used. Self-field MPD thrusters rely on the magnetic field generated by the discharge current from the thruster, without the use of external magnetic fields to accelerate plasma. This allows thrust to be modeled as a function of discharge current and plasma properties, making it appropriate for analysis.

The total discharge voltage across an MPD thruster can be expressed as the sum of multiple contributions:

$$V = V_{\text{EM}} + V_{\text{ohmic}} + V_{\text{fall}} \tag{6}$$

where V_{EM} represents the electromagnetic acceleration voltage, V_{ohmic} represents resistive (ohmic) losses within the plasma, and V_{fall} represents the combined cathode and anode fall voltage.

In many MPD models, V_{fall} is treated as a constant. However, experimental evidence suggests that fall voltage may depend on discharge current, plasma density, or other parameters. This study isolates the effect of V_{fall} by treating it as a variable parameter within a fixed thruster model.

Electromagnetic Thrust Model

In a self-field MPD thruster, thrust is primarily generated through $\mathbf{J} \times \mathbf{B}$. Under simplifying assumptions, the total thrust can be approximated as scaling with the square of the discharge current:

$$T = KI^2 \tag{7}$$

where I is the discharge current, and K is a proportionality constant that mainly depends on thruster geometry and magnetic field distribution.

This current–thrust relationship arises from the fact that both the current density ($\mathbf{J}$) and the self-induced magnetic field ($\mathbf{B}$) scale with current. While more detailed models will include geometric integrals and plasma non-uniformities, this form captures the main electromagnetic behavior and is commonly used in simple MPD performance modeling.

Exhaust Velocity and Mass Flow

Using the definition of exhaust velocity,

$$u_e = \frac{T}{\dot{m}} = \frac{KI^2}{\dot{m}} \tag{8}$$

For the purposes of this study, the mass flow rate ($\dot{m}$) is treated as constant. This isolates the effect of fall voltage on performance metrics without introducing other variables.

Formulating Efficiency

Thruster efficiency can be written as:

$$\eta = \frac{Tu_e}{VI} \tag{9}$$

Substituting the expressions for thrust, exhaust velocity, and voltage:

$$\eta = \frac{(KI^2)(KI^2/\dot{m})}{\dot{m}(aI + V_{\text{fall}})} \tag{10}$$

This simplifies to:

$$\eta = \frac{K^2I^3}{\dot{m}(aI + V_{\text{fall}})} \tag{11}$$

This equation shows that fall voltage directly reduces efficiency by increasing the denominator without contributing to thrust. The effect increases as I decreases, which motivates this study focused on V_{fall}.

Assumptions and Limitations

To focus the analysis on the influence of fall voltage, the following assumptions and limitations will be made:

- The plasma is quasineutral, meaning the ionized gas remains neutral by balancing out the positive and negative charges.
- Mass flow rate is constant and independent from discharge current.
- Thruster geometry remains fixed.
- A range of 0–1000 A is applied to preserve realistic parameters.

While these limits and assumptions limit the model's applicability to real thrusters, they allow for clear isolation of fall voltage effects and will provide useful insight for future applications.

METHODOLOGY

This study evaluates MPD thruster performance sensitivity to fall voltage by varying V_{fall} across a range while holding other parameters constant. Performance metrics include thrust, discharge voltage, exhaust velocity, and efficiency as functions of discharge current.

The equations are implemented in Desmos, allowing for visualization of how changes in fall voltage directly affect performance. Multiple fall voltage values are plotted to directly compare their impact across the same current range.

Model parameters are set as: $K = 2 \times 10^{-6}$ N/A^2, $\dot{m} = 0.02$ kg/s, and $a = 0.002$ Ω. Fall voltage values of 20 V, 40 V, and 60 V are evaluated over a discharge current range of 0–1000 A.

RESULTS

The simplified self-field MPD thruster model was evaluated over a discharge current range of 0–1000 A for three assumed fall voltages: 20 V, 40 V, and 60 V.

Thrust Behavior

Thrust increases quadratically with discharge current, consistent with the relationship $T = KI^2$. As expected, thrust is independent of fall voltage in this model. All cases produce identical thrust curves, confirming that fall voltage does not directly influence electromagnetic thrust generation.

This result validates that fall voltage acts as an electrical loss mechanism rather than a thrust-producing contribution.

Discharge Voltage

The total discharge voltage increases linearly with current for each case, with vertical offsets corresponding to the selected fall voltages. This means that increasing fall voltage shifts the voltage–current curve upward without changing its slope.

This confirms that fall voltage increases total electrical input power at every current level.

Exhaust Velocity

Since exhaust velocity is directly proportional to thrust under constant mass flow rate ($u_e = T/\dot{m}$), it also increases quadratically with discharge current. Similar to thrust, exhaust velocity remains unaffected by fall voltage assumptions.

Efficiency Trends

Thruster efficiency exhibits strong sensitivity to fall voltage, particularly at low discharge currents.

At lower current levels, the constant fall voltage term dominates the denominator of the efficiency expression:

$$\eta = \frac{K^2 I^3}{\dot{m}(aI + V_{\text{fall}})} \tag{12}$$

As a result:

- Higher fall voltage significantly reduces efficiency in the low-current regime.
- The efficiency curves for different fall voltages diverge most strongly at low current.
- At higher currents, the term aI becomes large compared to V_{fall}, reducing the relative impact of fall voltage.

Overall, this indicates that assuming a constant fall voltage may introduce modeling error when analyzing MPD performance in partially ionized or low-current regimes.

Quantitative Analysis

To quantify the impact of fall voltage on efficiency, values of 200 A and 800 A are evaluated.

The efficiency expression derived earlier is:

$$\eta = \frac{K^2 I^3}{\dot{m}(aI + V_{\text{fall}})} \tag{13}$$

Using the selected parameters:

- $K = 2 \times 10^{-6}$ N/A^2
- $\dot{m} = 0.02$ kg/s
- $a = 0.002\ \Omega$

First, compute the constant term:

$$K^2 = (2 \times 10^{-6})^2 = 4 \times 10^{-12} \tag{14}$$

Substituting constants:

$$\dot{m}(aI + V_{\text{fall}}) = 0.02(0.002I + V_{\text{fall}}) = 0.00004I + 0.02V_{\text{fall}} \tag{15}$$

The efficiency expression simplifies to:

$$\eta = \frac{4 \times 10^{-12} I^3}{0.00004I + 0.02V_{\text{fall}}} \tag{16}$$

Case 1: $I = 200$ A

Step 1: Numerator

$$I^3 = (200)^3 = 8 \times 10^6 \tag{17}$$

$$4 \times 10^{-12} \times 8 \times 10^6 = 32 \times 10^{-6} = 3.2 \times 10^{-5} \tag{18}$$

Step 2: Denominator for each fall voltage

- 20 V: $0.00004(200) + 0.02(20) = 0.08 + 0.4 = 0.408$
- 40 V: $0.08 + 0.8 = 0.808$
- 60 V: $0.08 + 1.2 = 1.208$

Step 3: Efficiencies

- 20 V: $\eta = 3.2 \times 10^{-5}/0.408 = 7.84 \times 10^{-5}$
- 40 V: $\eta = 3.2 \times 10^{-5}/0.808 = 3.96 \times 10^{-5}$
- 60 V: $\eta = 3.2 \times 10^{-5}/1.208 = 2.65 \times 10^{-5}$

Percent Reduction (20 V $\rightarrow$ 60 V):

$$\frac{7.84 \times 10^{-5} - 2.65 \times 10^{-5}}{7.84 \times 10^{-5}} \approx 0.66 \approx 66\% \text{ decrease} \tag{19}$$

Case 2: $I = 800$ A

Step 1: Numerator

$$I^3 = (800)^3 = 5.12 \times 10^8 \tag{20}$$

$$4 \times 10^{-12} \times 5.12 \times 10^8 = 20.48 \times 10^{-4} = 0.002048 \tag{21}$$

Step 2: Denominator for each fall voltage

Linear term: $0.00004(800) = 0.32$

- 20 V: $0.32 + 0.4 = 0.432$
- 40 V: $0.32 + 0.8 = 0.832$
- 60 V: $0.32 + 1.2 = 1.232$

Step 3: Efficiencies

- 20 V: $\eta = 0.002048/0.432 = 0.00474$

- 40 V: $\eta = 0.002048/0.832 = 0.00246$
- 60 V: $\eta = 0.002048/1.232 = 0.00166$

Percent Reduction:

$$\frac{0.00474 - 0.00166}{0.00474} \approx 0.65 \approx 65\% \text{ decrease} \tag{22}$$

CONCLUSIONS

This study investigated the influence of fall voltage on the predicted performance of a simplified self-field magnetoplasmadynamic (MPD) thruster model. By isolating fall voltage as a variable parameter and holding thruster scaling, mass flow rate, and voltage coefficients constant, the analysis evaluated its impact on thrust, exhaust velocity, discharge voltage, and efficiency.

The results demonstrate that fall voltage does not affect thrust or exhaust velocity within the simplified electromagnetic framework, as thrust is governed solely by the $\mathbf{J} \times \mathbf{B}$ interaction and scales quadratically with discharge current. However, fall voltage directly increases total discharge voltage and therefore electrical input power.

Quantitative analysis revealed that an increase of fall voltage from 20 V to 60 V produced approximately a 65–66% reduction in predicted efficiency at both 200 A and 800 A. This reduction occurs because fall voltage contributes to input power without contributing to electromagnetic thrust production. Fall voltage remains a significant portion of total discharge voltage across the examined current range.

These findings indicate that assuming constant or underestimated fall voltage values in MPD performance models may lead to large overprediction of thruster efficiency. The sensitivity observed in this study suggests that accurate modeling of sheath behavior and plasma interactions is critical, particularly in regimes where electromagnetic voltage contributions are comparable in magnitude to fall voltage.

It is important to note that this study employs a simplified analytic model with constant mass flow rate, voltage scaling, and quasineutral plasma assumptions. Real MPD thrusters exhibit nonlinear plasma dynamics, electrode erosion effects, and current-dependent sheath behavior that may amplify or alter the trends observed here. Therefore, while the model gives insight into sensitivity structure, further investigation using experimental data and nonlinear plasma models is necessary to refine predictive accuracy.

Overall, this study demonstrates that fall voltage is not just a minor electrical loss term, but a dominant parameter in MPD efficiency modeling. Future work should focus on incorporating current-dependent fall voltage behavior and validated sheath models to improve performance predictions for high-power electric propulsion systems.

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Acknowledgments

The author used AI-based language assistance tools to refine wording and improve clarity of presentation. All modeling, derivations, and analytical conclusions were independently developed.