# Activity 3 - Samuel Barr (18038795)

In this hypothetical game two cleaning robots A and B are given the option to act cooperatively in their cleaning, sharing information about the debris they find as the explore the beach. They are also given the option to work independently, not sharing any information with the other agent.

In this scenario, the payout for each agent represents the amount of cleaning done by the respective agent. If both agents act cooperatively, they both achieve a very high payout ($a$) as they can coordinate their actions efficiently. If one agent acts cooperatively and the other independently, the independent agent receives a high payment ($b$) and the cooperative agent receives very low payout ($c$). This is due to the independent agent taking advantage of the information from the other agent. If both agents act independently, they both receive a low payout ($d$).

This information can be presented in a normal form game shown in figure 1. The values for $a$, $b$, $c$ and $d$ can be assumed to be any number as long as $a > b \geq c > d$. This is an example of the Stag Hunt game, a cooperative game where each player must cooperate in order to receive the maximum payoff.

*Figure 1*

| | B | | |
|---|---|---|---|
| | | Cooperative | Independent |
| A | Cooperative | 10, 10 | 2, 8 |
| | Independent | 8, 2 | 5, 5 |

In this game there are two pure-strategy Nash Equilibria, when both agents decide to cooperate and when they both decide to act independently. We know this as if agent B chooses to cooperate and A knows this, agent A will always pick to cooperate as well. When agent B chooses to act independently, agent A will also choose to act independently. As the game is symmetrical, this also applies to agent B. Both agents will always choose to make the same choice as their opponent if they know what they will decide, making this a cooperation game.

The game also has one mixed-strategy Nash Equilibrium, depending on the payoffs. Using the example shown in figure 1, we can calculate that the mixed strategy for both players as the game is symmetrical. The probability of coopering is $\sigma_C = 3/5$ and the probability of acting independently is $\sigma_I = 2/5$. This gives an expected payoff of 6.8 for both players.

$$EU_C = EU_I$$

$$EU_C = \ \sigma_C(10) + 3(1 - \sigma_C)$$

$$EU_I = \ \sigma_C(8) + 5(1 - \sigma_C)$$

$$\sigma_C = 3/5 \ \ \sigma_I = 2/5$$

Therefore, we know that it is in the best interest of each agent to cooperate. If this is impossible for some reason such as the other player being irrational, they should cooperate with 60% probability, and act independently with 40% probability. This gives an expected payoff of 6.8 per round.