# Monetary Economics (Term Paper)

Ibrahim Babatunde Tiamiyu

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**Bayesian Estimation of DSGE Models: A Robustness Analysis.**

**Abstract**

Does public investment alter the DSGE model results in Smets and Wouters (2007)? This study examines whether replacing Fixed Private Domestic Investment with Real Gross Fixed Capital Formation (a broader measure that includes public investment as a proxy for investment) alters the results of the Smets and Wouters (2007) DSGE model. The study shows that technology shocks and price markup shocks remains the dominant drivers of output fluctuations, consistent with the original findings. The results in this study confirms the robustness of the results in Smets and Wouters (2007) and suggests that it effectively captures the key dynamics of both private and public investment in the U.S.A business cycles. (JEL D58, E22, E23, E31, E32)

## 1 INTRODUCTION

Dynamic Stochastic General Equilibrium (DSGE) models have revolutionized macroeconomic theory and policy analysis. By integrating microeconomic foundations with dynamic optimization under uncertainty, these models offer a theoretically consistent approach to understanding economic behavior at both the aggregate and sectoral levels. Initially developed as an extension of Real Business Cycle (RBC) models, DSGE frameworks have expanded to include key elements such as nominal rigidities, financial frictions, and heterogeneous agents, making them essential for contemporary macroeconomic research and policymaking.

The roots of DSGE models can be traced to the Real Business Cycle (RBC) model in Kydland and Prescott (1982), where technology shocks (using a preference-technology model environment) were identified as a major driver of economic cycles. RBC models were groundbreaking in their use of micro-foundations and dynamic optimization. However, they faced criticism for failing to account for nominal rigidities and monetary policy effects.

The New Keynesian DSGE models addressed these shortcomings by incorporating price and wage stickiness, as well as monetary policy rules. Smets and Wouters (2003, 2007) demonstrated the empirical relevance of these models by showing that they outperform traditional econometric approaches, such as Vector Autoregressions (VAR), in macroeconomic forecasting, while using the same Bayesian estimation technique.

The transition from RBC to New Keynesian DSGE models marked a significant milestone by:

- Integrating monetary policy into general equilibrium frameworks,
- Accounting for frictions like sticky prices and wages,
- Providing a foundation for evaluating welfare implications of policy interventions.

This paper contributes to the literature by reassessing the robustness of Smets and Wouters' (2007) findings through the use of a broader measure of investment: Real Gross Fixed Capital Formation, which includes both private and public investment. By extending the investment proxy this study tests whether the conclusions drawn from the original model are sensitive to the specific choice of investment measure. The results show that incorporating public investment does not alter the essential dynamics of the U.S. business cycle captured by Smets and Wouters (2007) in their model, reinforcing the robustness and reliability of their results.

This study contributes to the body of literature on DSGE models by demonstrating that both public and private investments behave similarly in driving macroeconomic fluctuations within the framework of the model. **Therefore, this study is not just an attempt at replicating the Smets and Wouters (2007) paper but to evaluate the "assumption/ judgment call" with regards to choosing a proxy for investment**.

The rest of this paper is organized into four sections: Section 2 reviews the relevant literature, Section 3 discusses the data used and the rationale for selecting an alternative investment proxy, Section 4 presents the estimation results, and Section 5 offers concluding remarks.

## 2 LITERATURE REVIEW

Christiano, Eichenbaum, and Evans (2005) developed a DSGE model featuring moderate nominal rigidities to explain the observed inertia in inflation and persistence in output following a monetary policy shock. The key drivers of their results are staggered wage contracts with an average duration of three quarters and variable capital utilization, which prevent sharp increases in marginal costs after expansionary monetary shocks. Their model successfully replicates hump-shaped responses in output, investment, consumption, and employment, as well as a subdued response in real wages.

A key finding is the dominant role of nominal wage stickiness compared to price stickiness in explaining economic dynamics. While the Calvo pricing framework is adopted as a reduced form for nominal sluggishness, the authors emphasize the need for more structural modeling of wage dynamics to improve the robustness of DSGE models.

Smets and Wouters (2007) presented a dynamic stochastic general equilibrium (DSGE) model of the U.S. economy that is estimated using a Bayesian likelihood approach with seven key macroeconomic time series. Their model incorporates various real and nominal frictions, along with seven distinct structural shocks, to capture the complexities of economic fluctuations. One of the significant findings of the study is that their DSGE model is capable of competing with Bayesian Vector Autoregression (BVAR) models in out-of-sample forecasting. This suggests that the structural model provides valuable insights and enhances the forecasting of key macroeconomic variables, particularly at business cycle frequencies.

The study focused on understanding the sources of business cycle fluctuations, examining the relationship between output and inflation, and investigating the effects of productivity shocks on labor supply. A central feature of their analysis is the identification of several critical structural shocks, including technology and wage markup shocks, which are found to significantly drive output and inflation movements. Smets and Wouters argue that despite the model's highly stylized nature, its ability to fit U.S. macroeconomic data convincingly and outperform standard VAR models supports the use of DSGE models in business cycle analysis.

An and Schorfheide (2007) provided a comprehensive review of Bayesian methods used to estimate and evaluate Dynamic Stochastic General Equilibrium (DSGE) models. They discuss the estimation of linearized DSGE models, which has been a dominant approach in empirical macroeconomics, and explore methods for model evaluation based on Bayesian model checking and posterior odds comparisons. The paper also contrasts DSGE models with Vector Autoregressions (VARs) and addresses the development of techniques for the non-linear estimation of DSGE models, particularly focusing on second-order perturbation methods. The study highlights the importance of Bayesian estimation in the empirical macroeconomics literature, with an emphasis on the growing use of non-linear estimation methods enabled by advances in computational technology. The study illustrated the application of these tools by applying them to artificial data generated from both correctly specified and misspecified linearized DSGE models, as well as a model solved with a second-order perturbation approach. The results from the study was a depiction of the potential advantages of non-linear methods over the traditional linearized approaches.

Justiniano, Primiceri, and Tambalotti (2010) investigate the origins of business cycle fluctuations through an estimated New Neoclassical Synthesis model of the U.S. economy. Their central finding is that shocks to the marginal efficiency of investment (MEI) are the primary drivers of movements in output, hours worked, and investment at business cycle frequencies. The transmission of these shocks is facilitated by imperfect competition and endogenous markups, while neutral technology shocks play a secondary role, particularly in explaining the co-movement of output and consumption.

Interestingly, labor supply shocks, while significant at very low frequencies, contribute negligibly to business cycle dynamics, challenging their micro-foundational appeal. The study note that the volatility of investment shocks exceeds the observed variability in the relative price of investment, suggesting that factors such as capital accumulation frictions—potentially tied to financial sector inter-mediation may play a critical role.

Leeper, Walker, and Yang (2010) analyze the macroeconomic effects of government investment within a neoclassical growth model, focusing on two critical factors: implementation delays and expected fiscal adjustments to deficit-financed spending. They find that delays in the construction of public capital can lead to small or even negative short-run effects on labor and output, with government investment being potentially contractionary in the short term. Over longer horizons, the fiscal adjustment instruments used to stabilize government debt play a significant role in determining the overall effects of government investment on growth.

A key finding of the study is that the productivity of public capital is pivotal: even moderate levels of productivity in public investment can lead to positive long-run output multipliers. However, the effectiveness of government investment as a fiscal stimulus depends on its productivity and the implementation delays involved.

In Guerron-Quintana (2010), the study emphasizes the critical role of observable variables in the estimation of Dynamic Stochastic General Equilibrium (DSGE) models. The paper argues that the choice of observables is more important than previously thought, as both the inclusion and exclusion of specific variables can significantly affect the model's forecasting accuracy and parameter estimates. Using too many observables can lead to model misspecification, although including measurement errors helps alleviate this problem. Excluding important variables like real wages, consumption, or inflation can distort parameter estimates, particularly those related to sticky contracts in prices and wages, as well as habit formation. The study recommends a specific set of seven observables—output, consumption, investment, real wages, total labor, interest rates, and inflation—as the most suitable for DSGE estimation.

Canova, Ferroni, and Mathes (2014) propose two methods for selecting observable variables in the estimation of singular DSGE models. These methods are designed for situations where a researcher is reluctant to include a large number of potentially non-structural shocks or measurement errors in the decision rules. The proposed methods focus on optimizing parameter identification and minimizing the informational discrepancy between singular and non-singular models. The criteria provided are easy to implement and can effectively rank different combinations of observables. The results indicate that the best combinations for parameter identification and information content in the model typically include output, consumption, and investment, which help identify both inter-temporal and intra-temporal links, essential for correctly measuring income and substitution effects. Interestingly, combining interest rate and inflation together in the estimation worsens identification and leads to a greater loss of information. The study also highlights the trade-offs between including hours or labor productivity with output, emphasizing that the best models selected by the methods perform well in capturing the dynamics of the singular DSGE model. On the other hand, using excessive observables or measurement errors can distort parameter estimates and undermine inference. The paper offers practical suggestions for applied researchers and demonstrates that the methods improve model performance while avoiding complications from over-fitting.

Blanchard and Perotti (2002) examine the dynamic effects of government spending and tax shocks on U.S. economic activity, focusing on the postwar period. The study employs a mixed structural VAR/event study approach, using institutional information about the tax and transfer systems to identify the automatic responses of taxes and spending to economic activity. This approach helps infer the impact of fiscal shocks. Their results reveal a positive effect of government spending shocks on output, while tax shocks tend to have a negative effect on economic activity.

In Kamps (2005), the dynamic effects of public capital on economic outcomes are analyzed using vector auto-regressive (VAR) methodology, which treats all variables as endogenous and accounts for feedback effects from output to input variables. This approach contrasts with the traditional production function method and allows for a more comprehensive understanding of public capital's role in the economy.

The results show that, for most OECD countries, shocks to public capital lead to significant positive effects on output, suggesting that public investment has a beneficial impact on economic performance. However, contrary to the high returns often found in production function analyses, Kamps' findings indicate that there is little evidence for "supernormal" returns to public capital, possibly because the production function approach fails to capture the feedback from output to public capital. The study also reveals that, in the long run, public capital and private capital are complementary, but in the short run, their relationship can vary: in some countries, they are short-run substitutes, while in others, they are complements. Regarding employment, the research finds no significant evidence that additional public capital directly fosters job creation, with employment responses being statistically insignificant across most countries. This suggests that while public capital may drive output growth, its effect on employment is less clear.

# 3 Methodology

## 3.1 Data

This study utilized seven (7) macroeconomic time-series data in the estimation of the required equations.

- Real GDP
- Personal Consumption Expenditure
- Hours Worked (Non-farm Business Sector)

- Hourly Compensation (Non-farm Business Sector; Incorporating Civilian Employment)
- Implicit Price Deflator of GDP
- Federal Funds Rate

All the above are in-line with the Data in Smets and Wouters (2007), except for the use of another proxy for Investment; **Real Gross Fixed Capital Formation**.

### JUSTIFICATION FOR THE CHANGE OF THE PROXY FOR INVESTMENT

The Fixed Private Domestic Investment used as a proxy for investment in Smets and Wouters (2007) focuses only on private-sector fixed investments within the domestic economy. It does not capture government investments and, thereby might not picture the true broader economic activity related to capital formation.

However, the proxy used for investment in this study, Real Gross Fixed Capital Formation, is relatively more comprehensive as a measure of investment. It incorporates the investment in tangible assets by all the agents in the economy. This is particularly important in a setting where government investment in fixed assets could influence the business cycle in the economy (a fiscal policy perspective).

# 4 Results

## 4.1 HISTORICAL DECOMPOSITION OF OUTPUT

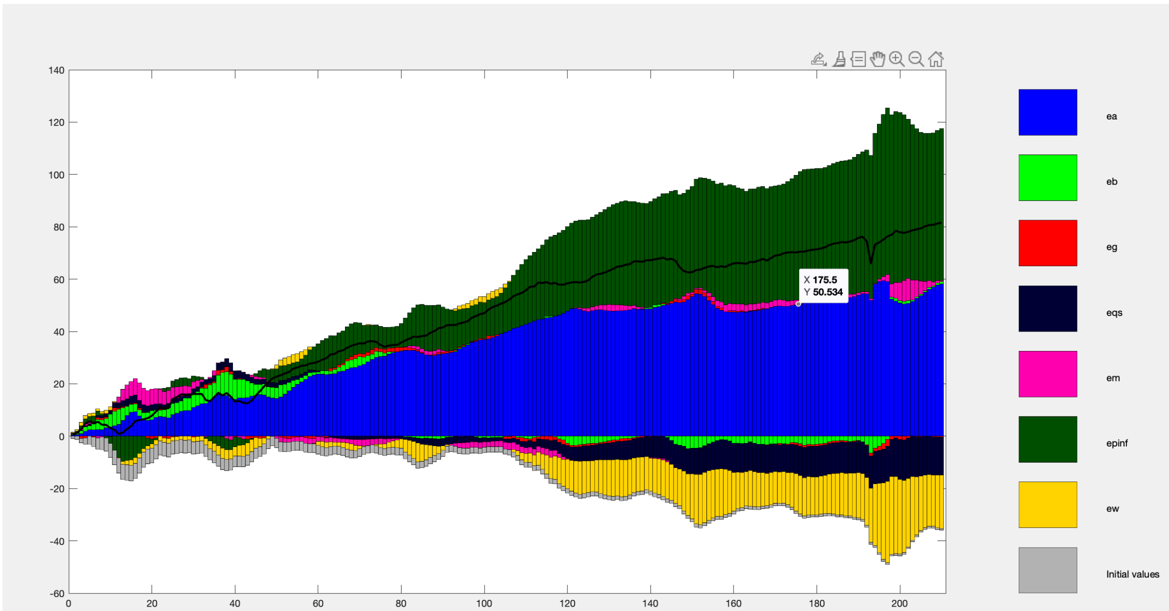


Figure 1: The historical decomposition of output

The importance of the graph above is to depict the historical decomposition of output to attribute fluctuations to the various shocks identified in the model.

This graph reveals that technology shocks is the largest contributor to output fluctuations, consistent with the findings in Smets and Wouters (2007). This highlights the significant role of productivity in shaping the business cycle, where increases in technological efficiency lead to higher output and improved economic conditions.

In addition to technology shocks, the "historical decomposition of output" graph depicts that price-mark shocks play a substantial role in driving output variability. Particularly after some years of relatively lower contribution. This underscores the importance of nominal rigidities and market imperfections in the DSGE model, where changes in markup and pricing behavior can significantly affect real output, as outlined in the work of Smets and Wouters (2007).

Wage markup shocks, though not as impactful as technology or price markup shocks, is still worthy of analysis. They can affect labor costs and, consequently, production decisions and consumption. Wage markup shocks reflects the bargaining power between workers and firms or the costs of adjusting wages in response to shocks. Therefore, this result could suggests some rigidity or frictions in the labor market that are affecting economic performance.

In addition to the relatively more obvious shocks (technology, price markup, and wage markup shocks, respectively), the other shocks contribute to output fluctuations at different weights. This reflects the complexities of the model and the broader range of forces at play in the economy.

## 4.2 IMPULSE RESPONSE FUNCTIONS

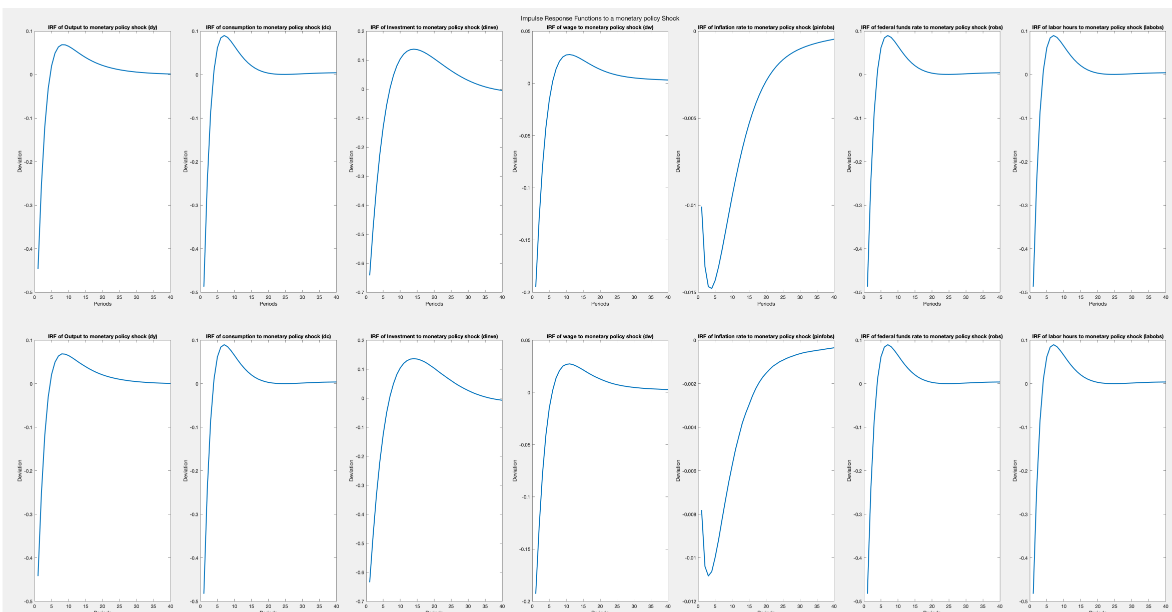


Figure 2: IRFs to a Monetary Policy Shock

A monetary policy shock, typically represented by an unexpected change in the nominal interest rate, has a direct and significant impact on economic activity. The IRF depicts that both the posterior mean and posterior median distributions move similarly and show a clear pattern of economic responses. The consistent behavior in the posterior distributions indicates that the model estimates the response to monetary policy shocks with high precision.

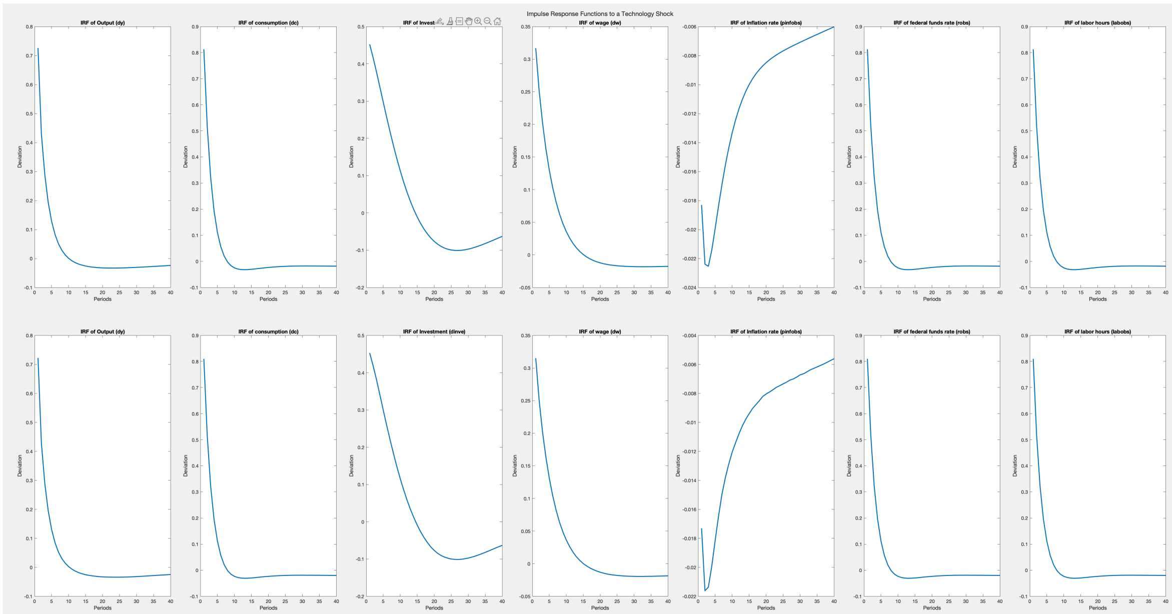


Figure 3: IRFs to a Technology Shock

A positive technology shock typically represents an improvement in productivity or technological advancements, leading to a rise in output and other economic variables. The positive response in both the mean and median indicates that technology shocks drive productivity gains, contributing to sustained economic growth. The consistency of the results reinforces the idea that technological progress is a key driver of output growth in the DSGE framework.

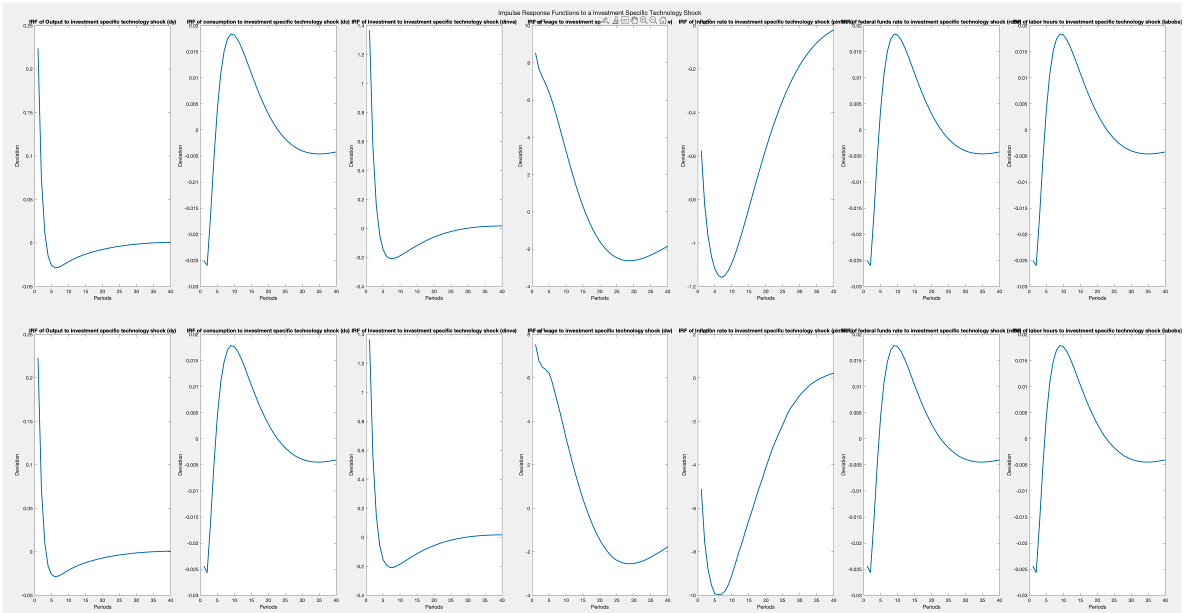


Figure 4: IRFs to an Investment-Specific Technology Shock

An investment-specific technology shock typically refers to changes that improve the efficiency of investment, such as lower costs or more productive capital goods. The consistency in the results of both posterior distributions suggests that investment-specific technology shocks play a crucial role in determining the economy's investment behavior. The positive response in output emphasizes how technological improvements in investment can drive long-term economic growth.

## 4.3 MEAN OF THE SHOCKS

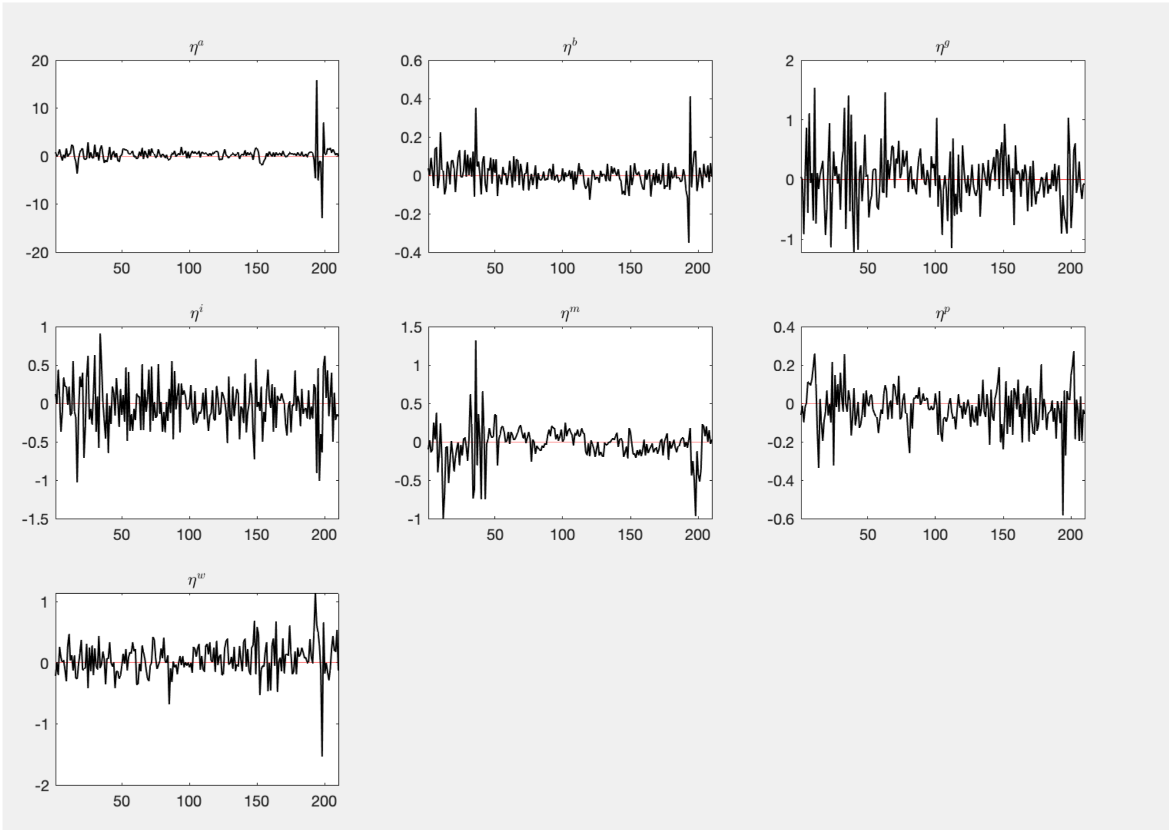


Figure 5: Mean of the Shocks

The graph depicting the mean responses of the shocks shows that all of the shocks converge to zero over time. This result suggests that the effects of the various shocks are transitory in nature within the context of the model. In other words, while the shocks initially have an impact on economic variables such as output, investment, and consumption, their influence gradually diminishes as the economy returns to its steady state. This pattern of convergence to zero reflects the model's assumption that, after the initial adjustment period, the economy's long-run equilibrium is unaffected by temporary fluctuations caused by these shocks. The fact that the responses tend to zero reinforces the notion that the model is capturing short-term dynamics effectively, with shocks having temporary effects that do not persist indefinitely. This result is consistent with typical DSGE models, where shocks are assumed to only affect the economy in the short term, allowing the system to self-correct and return to its long-run path.

# 5 Conclusion and Policy Recommendations

The findings of this study suggests that technology and price-markup shocks play a dominant role in driving output fluctuations in the U.S. economy. This has important policy implications for enhancing economic stability and mitigating the adverse effects of such shocks. Policymakers should prioritize measures aimed at fostering productivity growth, such as investments in innovation, research and development, and education, which can strengthen the economy's capacity to absorb and respond to technological shocks. Additionally, addressing price rigidities through policies that promote competition and price flexibility in goods and labor markets could help dampen the impact of price-markup shocks on output and inflation dynamics. By targeting these key sources of macroeconomic fluctuations, policymakers can improve economic resilience and ensure more stable growth over the business cycle. The robustness of the results in Smets and Wouters' (2007), even when a broader investment proxy (Real Gross Fixed Capital Formation) is used, strengthens the findings of their dynamic stochastic general equilibrium (DSGE) model in capturing macroeconomic dynamics.

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# 6 APPENDIX/ Estimation Results

| parameters | prior mean | post. mean | 90% HPD interval | | prior | pstdev |
|---|---|---|---|---|---|---|
| crhoa | 0.500 | 0.9760 | 0.9567 | 0.9959 | beta | 0.2000 |
| crhob | 0.500 | 0.7678 | 0.6537 | 0.8609 | beta | 0.2000 |
| crhog | 0.500 | 0.9858 | 0.9767 | 0.9977 | beta | 0.2000 |
| crhoqs | 0.500 | 0.5270 | 0.4081 | 0.6472 | beta | 0.2000 |
| crhoms | 0.500 | 0.3798 | 0.2602 | 0.4999 | beta | 0.2000 |
| crhopinf | 0.500 | 0.9624 | 0.9212 | 0.9993 | beta | 0.2000 |
| crhow | 0.500 | 0.8939 | 0.8610 | 0.9296 | beta | 0.2000 |
| cmap | 0.500 | 0.8970 | 0.7837 | 0.9899 | beta | 0.2000 |
| cmaw | 0.500 | 0.1264 | 0.0223 | 0.2228 | beta | 0.2000 |
| csadjcost | 4.000 | 9.6647 | 8.0791 | 11.2091 | norm | 1.5000 |
| csigma | 1.500 | 0.7954 | 0.7134 | 0.8769 | norm | 0.3750 |
| chabb | 0.700 | 0.7523 | 0.6912 | 0.8118 | beta | 0.1000 |
| cprobw | 0.500 | 0.6463 | 0.5793 | 0.7103 | beta | 0.1000 |
| csigl | 2.000 | 0.7677 | 0.2500 | 1.2724 | norm | 0.7500 |
| cprobp | 0.500 | 0.9242 | 0.8996 | 0.9500 | beta | 0.1000 |
| cindw | 0.500 | 0.5286 | 0.2972 | 0.7646 | beta | 0.1500 |
| cindp | 0.500 | 0.3697 | 0.0943 | 0.7382 | beta | 0.1500 |
| czcap | 0.500 | 0.7780 | 0.6519 | 0.9055 | beta | 0.1500 |
| cfc | 1.250 | 1.3778 | 1.1994 | 1.5499 | norm | 0.1250 |
| crpi | 1.500 | 1.8539 | 1.5315 | 2.1682 | norm | 0.2500 |
| crr | 0.750 | 0.8642 | 0.8241 | 0.9032 | beta | 0.1000 |
| cry | 0.125 | 0.1281 | 0.0623 | 0.2132 | norm | 0.0500 |
| crdy | 0.125 | 0.1045 | 0.0503 | 0.1588 | norm | 0.0500 |
| constepinf | 0.625 | 0.7045 | 0.5120 | 0.8882 | gamm | 0.1000 |
| constebeta | 0.250 | 0.1439 | 0.0563 | 0.2286 | gamm | 0.1000 |
| constelab | 0.000 | 2.7884 | 0.4459 | 5.2732 | norm | 2.0000 |
| ctrend | 0.400 | 0.6843 | 0.5912 | 0.7884 | norm | 0.1000 |
| cgy | 0.500 | 0.0874 | 0.0553 | 0.1191 | norm | 0.2500 |
| calfa | 0.300 | 0.1698 | 0.1326 | 0.2078 | norm | 0.0500 |

Figure 6: Parameter Estimates

| standard deviation of shocks | prior mean | post. mean | 90% HPD interval | | prior | pstdev |
|---|---|---|---|---|---|---|
| ea | 0.100 | 1.9196 | 1.7345 | 2.0948 | invg | 2.0000 |
| eb | 0.100 | 0.1700 | 0.1164 | 0.2400 | invg | 2.0000 |
| eg | 0.100 | 0.4856 | 0.4448 | 0.5255 | invg | 2.0000 |
| eqs | 0.100 | 0.4111 | 0.3268 | 0.4937 | invg | 2.0000 |
| em | 0.100 | 0.2602 | 0.2279 | 0.2912 | invg | 2.0000 |
| epinf | 0.100 | 0.1070 | 0.0730 | 0.1448 | invg | 2.0000 |
| ew | 0.100 | 0.2308 | 0.1907 | 0.2717 | invg | 2.0000 |

Figure 7: Standard Deviation of Shocks