# CSE291E-FA22

## Problem 1: Deform a shape

In this problem, we will practice part of what we learned in the image-to-3D lecture for shape deformation.

### 1. Laplacian

Given a mesh $M = (V, E, F)$, we assume that the adjacency matrix is $A \in \mathbb{R}^{n \times n}$, $D \in \mathbb{R}^{n \times n}$ is a diagonal matrix where $D[i, i]$ is the degree of the $i$-th vertex. The Laplacian matrix is defined as $L = D - A$.

Prove that:

(a) $\sum_{(i,j)\in E} \|x_i - x_j\|^2 = x^T L x$ for $x \in \mathbb{R}^n$. [1pt]

(b) $L \in \mathbb{S}^n_+$, i.e., $L$ is a symmetric and positive semi-definite matrix. [1pt]

(c) For the data matrix $P \in \mathbb{R}^{n \times 3}$ where each row corresponds to a point in $\mathbb{R}^3$, denote the columns of $P$ as $P = [x, y, z]$ and rows of $P$ as $P = [p_1^T; p_2^T; \dots; p_n^T]$, show that $\sum_{(i,j)\in E} \|p_i - p_j\|^2 = x^T L x + y^T L y + z^T L z$. (hint: Use the conclusion from 1(a)) [1pt]

## a)

Let us consider in the graph. Then $\sum_{i,j\in E} ||x_i - x_j|^2$ will contain the term $x_i^2\ D[i, i]$ times, $x_j^2\ D[j, j]$ terms and the term $x_i x_j$ will apear once with weight $-A[i, j]$ and once with weight $-A[j, i]$ but since $A$ is symmetric, this will just occur with weight -2. So the portion of $\sum_{i,j\in E} ||x_i - x_j|^2$ containing $x_i, x_j$ will be $D[i, i]x_i^2 + D[j, j]x_j^2 - 2A[i, j]x_i x_j$ which can be rearraged as $\begin{bmatrix} x_i & x_j \end{bmatrix} \begin{bmatrix} D[i,i] & -A[i,j] \\ -A[j,i] & D[j,j] \end{bmatrix} \begin{bmatrix} x_i \\ x_j \end{bmatrix}$. So if we consider all points $x_1, x_2, x_3, \dots, x_n$, using the above pattern, we get

$$\sum_{i,j\in E} ||x_i - x_j||^2 = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix}^T (D - A) \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix} = x^T L x$$

## b)

Consider the LHS of the above property, the LHS of the equation is always positive and 0 only if the graph is empty. Hence $\sum_{i,j\in E} ||x_i - x_j||^2 \geq 0$. Hence now we have $x^T L x \geq 0$ and 0 only if $x = 0$. Hence $L \in S^n_+$ i.e. $L$ is a positive-semidefinite matrix.

## c)

Let $p_i^T = \begin{bmatrix} x_i & y_i & z_i \end{bmatrix}$ and $P$ be the collection of all points i.e. $P = \begin{bmatrix} p_1^T \\ p_2^T \\ . \\ . \\ p_n^T \end{bmatrix}$. Now

$\sum_{i,j\in E} ||p_i - p_j||^2 = \sum_{i,j\in E}(||x_i - x_j||^2 + ||y_i - y_j||^2 + ||z_i - z_j||^2) = \sum_{i,j\in E}(|x_i - x_j||^2) + \sum_{i,j\in E}(||y_i - y_j||^2) + \sum_{i,j\in E}(||z_i - z$
$\implies \sum_{i,j\in E} ||p_i - p_j||^2 = x^T L x + y^T L y + z^T L z$
using the part a result along each coordinate.

### 2. Normalized Laplacian

Normalized Laplacian is defined as the normalized version of the Laplacian matrix above:

$$L_{norm} = D^{-1} L$$

(a) Prove that the sum of each row of $L_{norm}$ is 0. [1pt]

## a)

Observe that $[A]_i$ is the connectivity of the $i^{th}$ vertex to all other vertices, so sum of all elements in the $i^{th}$ row of $A$ defines the total number of connections of the $i^{th}$ vertex which is the degree of the $i^{th}$ vertex and is equal to $D[i, i]$. Hence we have $D[i, i] = \sum_{j=1}^n A_{ij}$, and since $D$ is a diagonal matrix, we can write $D[i, i] = \sum_{j=1}^n D_{ij}$, hence we get
$\sum_{j=1}^n D_{ij} = \sum_{j=1}^n A_{ij} \implies \sum_{j=1}^n (D_{ij} - A_{ij}) = 0 \implies \sum_{j=1}^n [D^{-1} L]_i = 0\ \forall i$

(b) The difference between a vertex $x$ and the average position of its 1-ring neighborhood is a quantity that provides interesting geometric insight of the shape. It can be shown that,

$$x - \frac{1}{|N(x)|} \sum_{y_i \in N(x)} y_i \approx H\vec{n}\Delta A$$

for a good mesh, where $N(x)$ is the 1-ring neighorhood vertices of $x$ by the mesh topology, $H = \frac{1}{2}(\kappa_{min} + \kappa_{\max})$ is the mean curvature at $x$ (in the sense of the underlying continuous surface being approximated), $\vec{n}$ is the surface normal vector at $x$, and $\Delta A$ is a quantity proportional to the total area of the 1-ring fan (triangles formed by $x$ and vertices along the 1-ring).

Define $\Delta p_i := p_i - \frac{1}{|N(p_i)|} \sum_{p_j \in N(p_i)} p_j$. Prove that $\Delta p_i = [L_{norm} P]_i$, where $P$ and $p_i$ are defined as in 1(c), and $[X]_i$ is to access the $i$-th row of $X$. [1pt]

## b)

Consider a graph $(V, E, F)$. Consider a vertex $p_i$. The 1-ring neighbourhood of $p_i$ is simply the edges origination from $p_i$. The adjacency matrix's $i^{th}$ row indicates which vertices are connected to $p_i$ and hence $|N(p_i)| = D[i,i]$ and $\sum_{p_j \in N(p_i)} p_j = [A]_i P = [AP]_i$ . Now also $D$ is a diagonal matrix, hence $\frac{1}{D[i,i]}[AP]_i = [D^{-1}AP]_i$. Now $p_i = [P]_i = [IP]_i = [D^{-1}DP]_i$. Therefore we can finally write
$\Delta p_i := p_i - \frac{1}{|N(p_i)|} \sum_{p_j \in N(p_i)} p_j = [D^{-1}DP]_i - [D^{-1}AP]_i = [D^{-1}DP - D^{-1}AP]_i = [D^{-1}(D-A)P]_i = [D^{-1}LP]_i = [L_{norm}P]_i$

### 3. Shape Deformation (extra credit)

Please load the source.obj and target.obj files using the trimesh library of Python, and optimize to deform the vertices of the source.obj to match target.obj. Plot the source object, target object, and deformed object. [5pt]

In [1]:
```
import trimesh
import numpy as np
import torch
import torch.nn as nn
from torch.optim import Adam
from chamfer import Chamfer_distance_torch
import matplotlib.pyplot as plt
import open3d
from tqdm import tqdm
source = trimesh.load('source.obj')
target = trimesh.load('target.obj')
```

```
Jupyter environment detected. Enabling Open3D WebVisualizer.
[Open3D INFO] WebRTC GUI backend enabled.
[Open3D INFO] WebRTCWindowSystem: HTTP handshake server disabled.
```

In [2]:
```
source_vertex = source.vertices
```

In [3]:
```
source_tensor = torch.tensor(np.array(source_vertex)[None,:,:], requires_grad = True, dtype = torch.float32)
source_tensor.dim()
```

Out[3]: 3

In [4]:
```
target_tensor = torch.tensor(np.array(target.vertices)[None,:,:], dtype = torch.float32)
target_tensor.shape
```

Out[4]: torch.Size([1, 1502, 3])

In [5]:
```
target_vertex = target.vertices
```

In [6]:
```
def plot_mesh(mesh,color, title):
    fig = plt.figure()
    ax = fig.add_subplot(111, projection='3d')
    ax.plot_trisurf(mesh.vertices[:, 0], mesh.vertices[:,1], triangles=mesh.faces, Z=mesh.vertices[:,2], cma
    plt.title(title)
    plt.show()
```

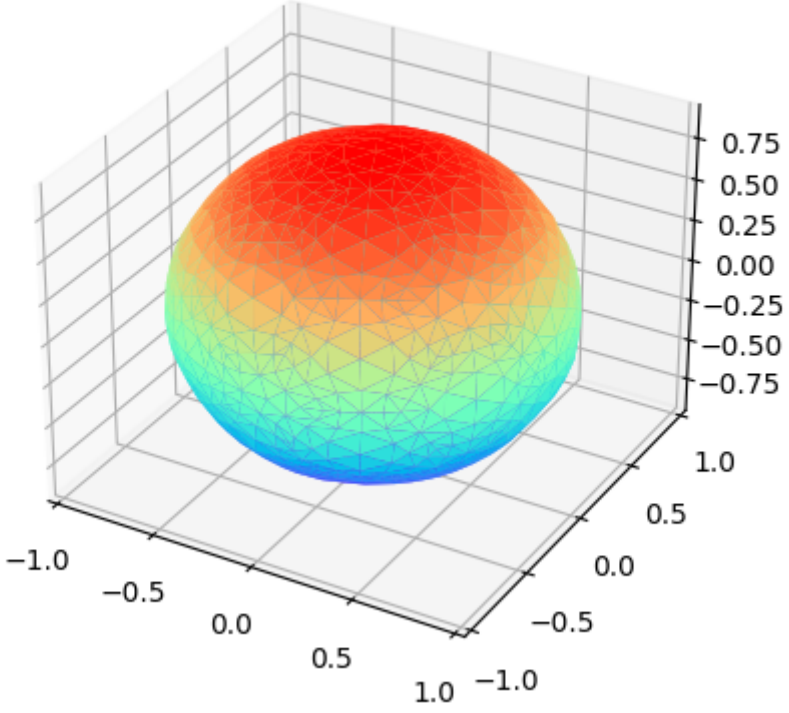
In [7]:
```
from matplotlib.cm import get_cmap
NUM_OBJECTS = 79
cmap = get_cmap('rainbow', NUM_OBJECTS)
COLOR_PALETTE = np.array([cmap(i)[:3] for i in range(NUM_OBJECTS + 3)])
COLOR_PALETTE = np.array(COLOR_PALETTE * 255, dtype=np.uint8)
COLOR_PALETTE[-3] = [119, 135, 150]
COLOR_PALETTE[-2] = [176, 194, 216]
COLOR_PALETTE[-1] = [255, 255, 225]
```

### Wthout using neural network (Directly optimising the source vertex)

In [8]:
```
chamfer_losses = []
iterations = 1000

# Instantiate optimizer
weights_deform = nn.Parameter(source_tensor)
optimizer = torch.optim.Adam([weights_deform], lr=1e-2)

for it in tqdm(range(iterations)):
    dist1, idx1, dist2, idx2 = Chamfer_distance_torch(weights_deform, target_tensor)
    loss = torch.mean(dist1) + torch.mean(dist2)
    loss.backward()
    optimizer.step()
    optimizer.zero_grad()
    chamfer_losses.append(loss.item())
    if it %100 == 0:
        weights = torch.clone(weights_deform)
        weights =weights.detach().numpy()
        deformed = trimesh.Trimesh(vertices = weights.reshape(-1,3), faces = source.faces)
        plot_mesh(deformed,COLOR_PALETTE[2]/255, f'Achieved mesh using chamfer loss only after {it} iteratio

deformed_chamfer_vertices = weights_deform
```

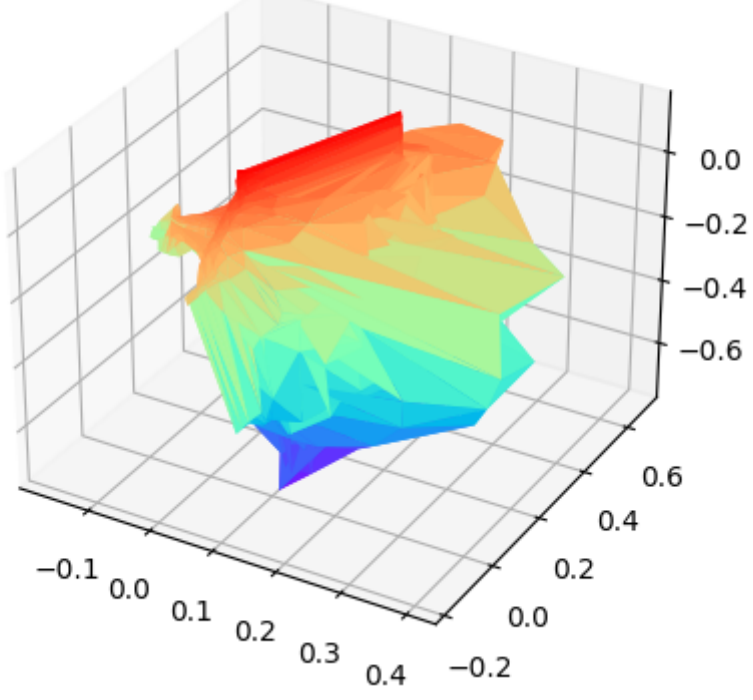
```
  0%|                                                  | 0/1000 [00:00<?, ?it/s]
```

Achieved mesh using chamfer loss only after 0 iterations

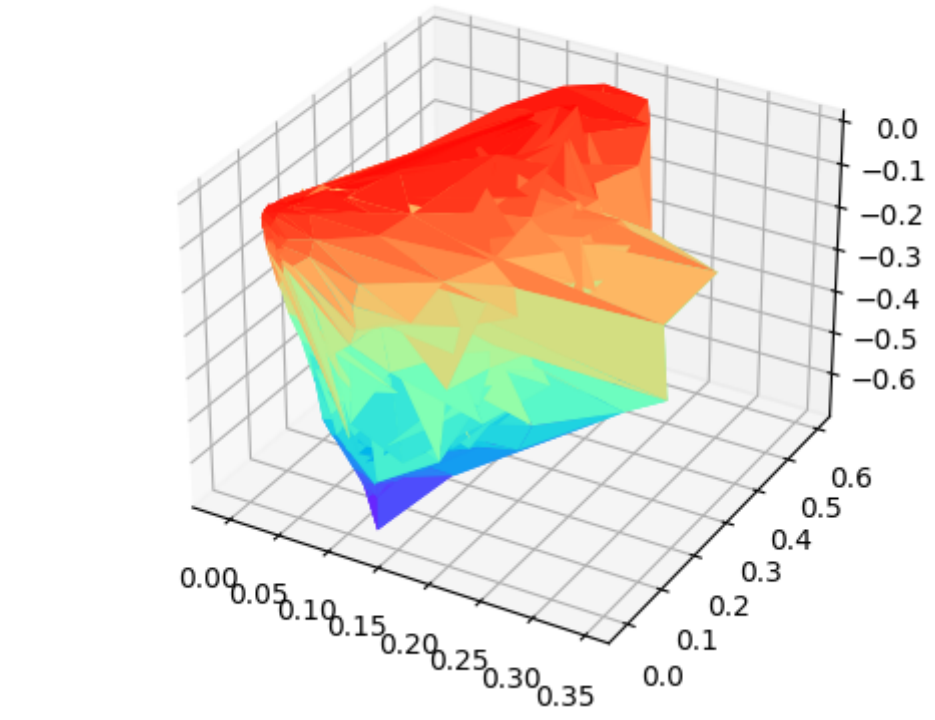



```
10%|███                                               | 98/1000 [00:02<00:17, 52.04it/s]
```

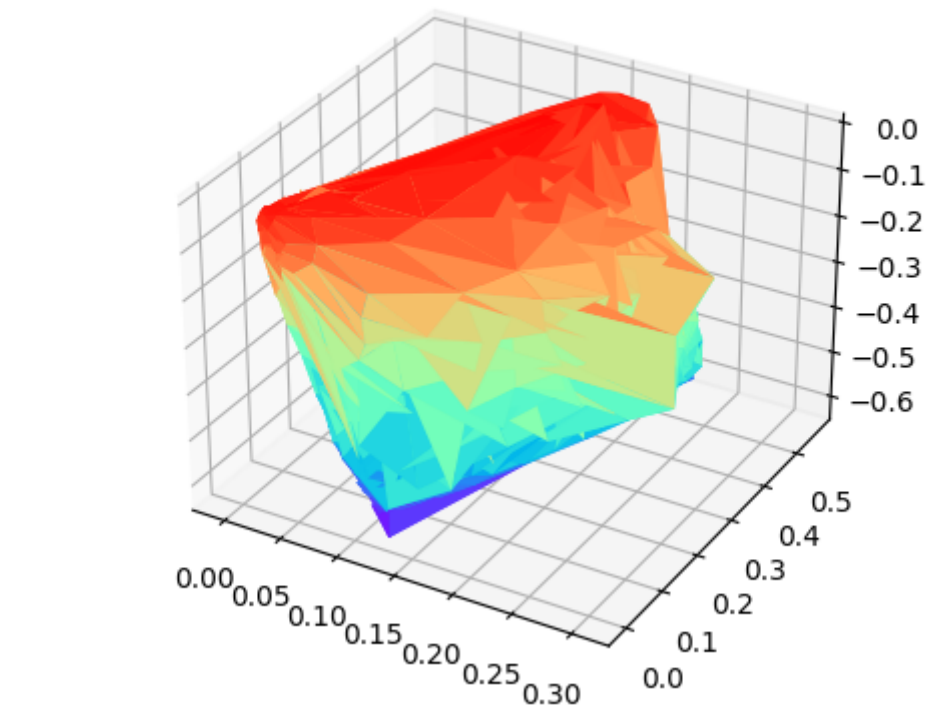
Achieved mesh using chamfer loss only after 100 iterations



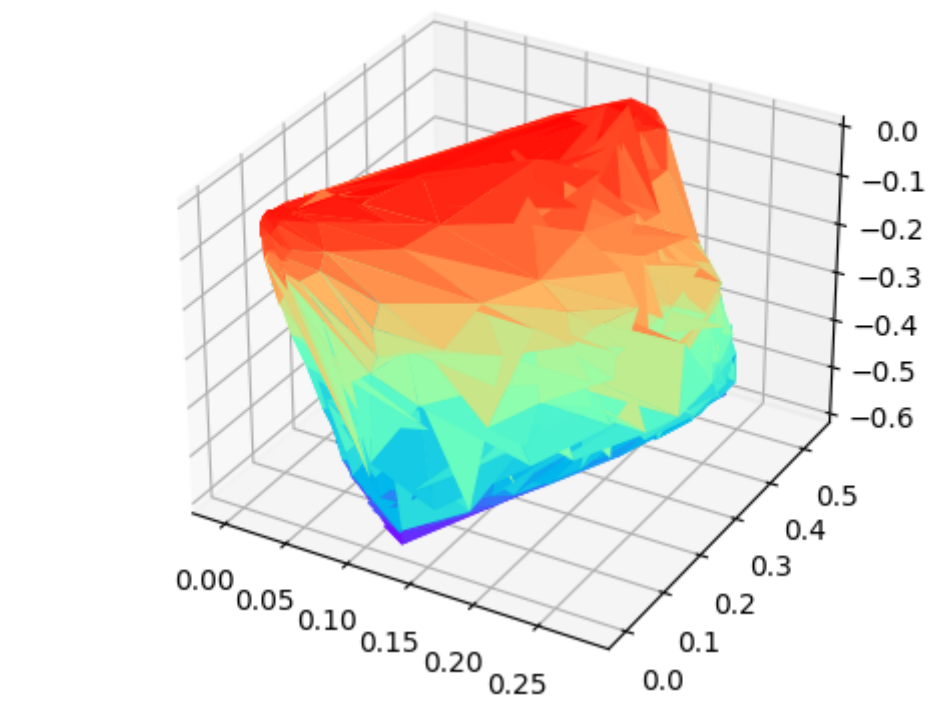

```
20%|██████                                            | 199/1000 [00:04<00:15, 52.21it/s]
```




30%|███████████ | 297/1000 [00:06<00:15, 45.93it/s]

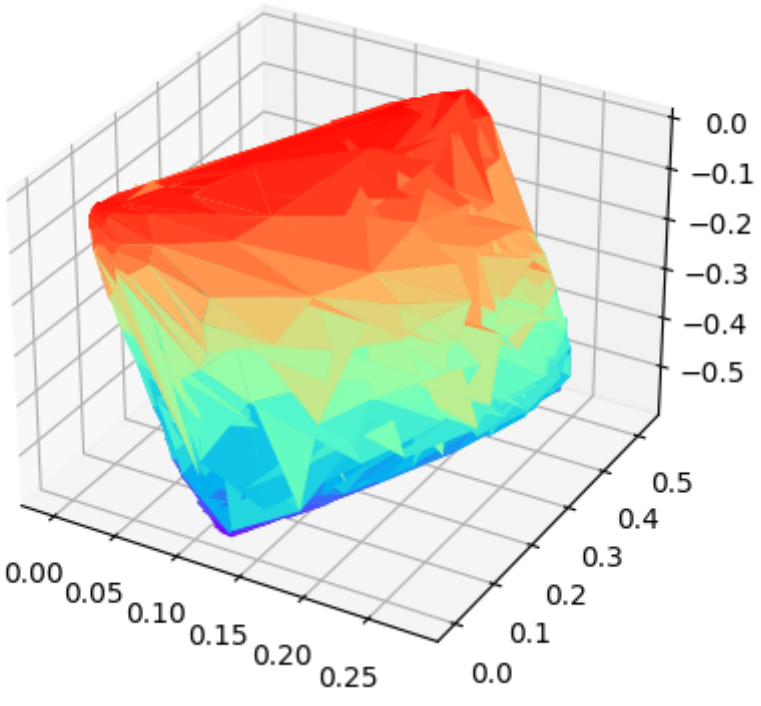



40%|████████████████ | 395/1000 [00:08<00:11, 51.69it/s]



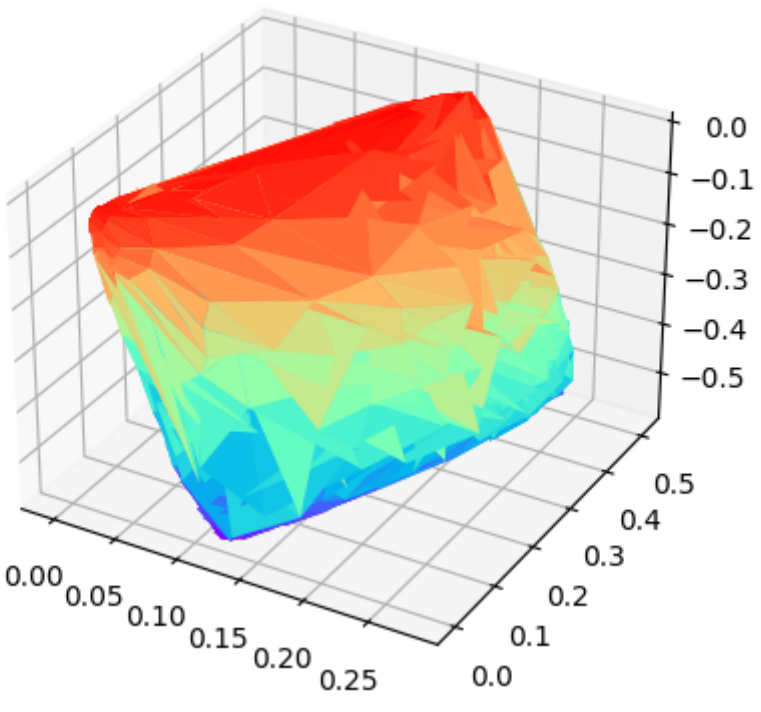
50%|████████████████ | 496/1000 [00:11<00:09, 50.56it/s]

Achieved mesh using chamfer loss only after 500 iterations



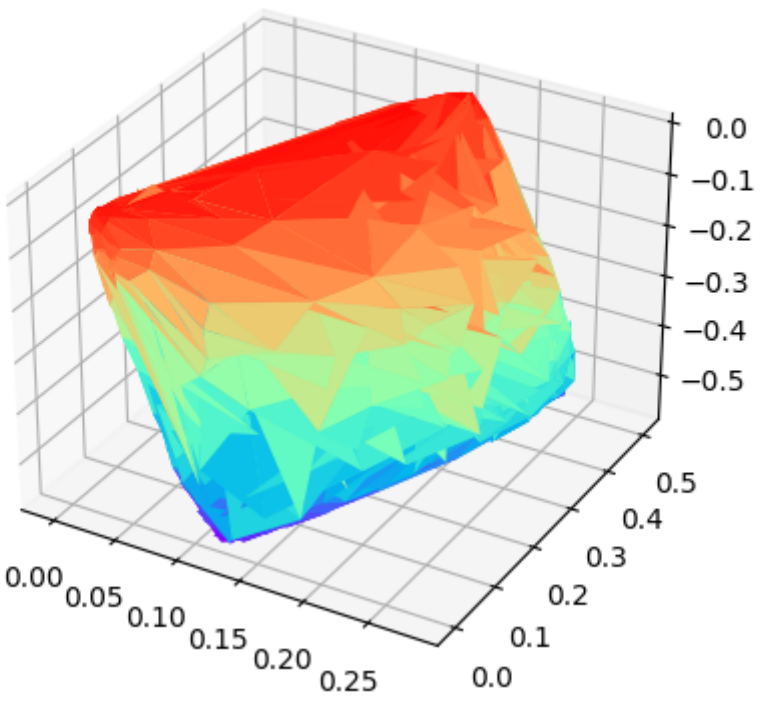

60%|████████████████████ | 600/1000 [00:13<00:07, 51.74it/s]

Achieved mesh using chamfer loss only after 600 iterations



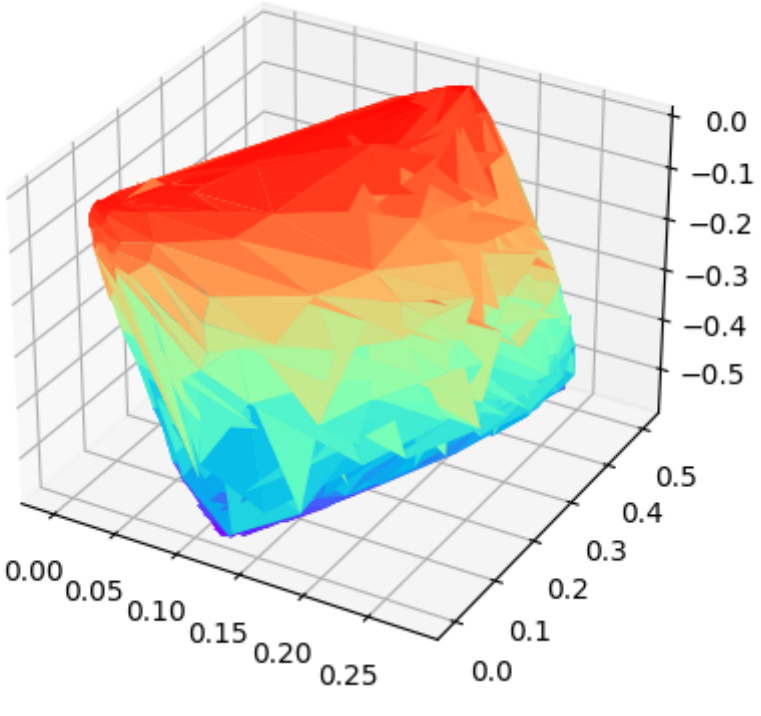

70%|████████████████████████ | 697/1000 [00:15<00:06, 46.18it/s]

Achieved mesh using chamfer loss only after 700 iterations



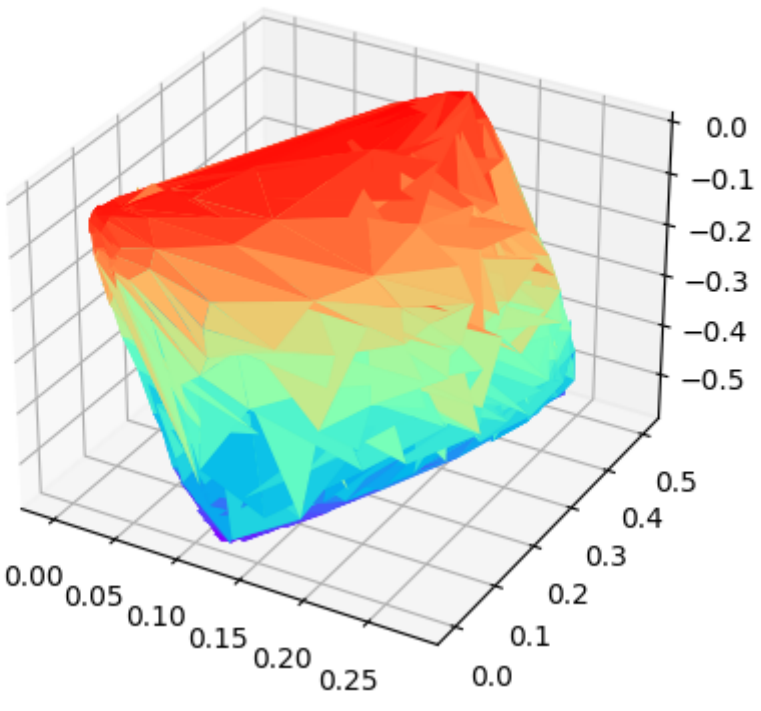
80%|████████████████████████████ | 799/1000 [00:18<00:04, 48.90it/s]

Achieved mesh using chamfer loss only after 800 iterations




90%|████████████████████████████████ | 898/1000 [00:20<00:01, 51.34it/s]

Achieved mesh using chamfer loss only after 900 iterations




100%|████████████████████████████████| 1000/1000 [00:22<00:00, 44.53it/s]

**Using the chamfer loss, the source vertices move closer to the corresponding nearest target vertex. We can see that the corners are not so smooth, are slightly curved whereas the target mesh has sharp corners and overall smooth edges.**

(b) Curvature and normal-based loss

## Using chamfer + curvature normal based loss

using the formula $\Delta p_i = [L_{norm} P]_i$, we nee dto create the adjacency matrix $A$, degree matrix $D$, $L = D - A$ and $L_{norm} = D^{-1} L$

In [9]:
```
def adjacency_matrix(faces, num_pts):
    num_faces = faces.shape[0]
    A = torch.zeros((num_pts, num_pts))

    for i in range(num_faces):
        vertex = faces[i]
        A[vertex[0], vertex[1]] = 1
        A[vertex[1], vertex[0]] = 1

        A[vertex[0], vertex[2]] = 1
        A[vertex[2], vertex[0]] = 1

        A[vertex[1], vertex[2]] = 1
        A[vertex[2], vertex[1]] = 1

    return A

def Lnorm(A):
    num_pts = A.shape[0]
    D = torch.diag(torch.sum(A, axis=1))
    L = D - A
    Lnorm = torch.inverse(D) @ L

    return Lnorm
```

In [10]:
```
source_A, target_A = adjacency_matrix(source.faces, source_vertex.shape[0]), \
                     adjacency_matrix(target.faces, target_vertex.shape[0])
```

In [11]:
```
Lnorm_source, Lnorm_target = Lnorm(source_A), Lnorm(target_A)
```

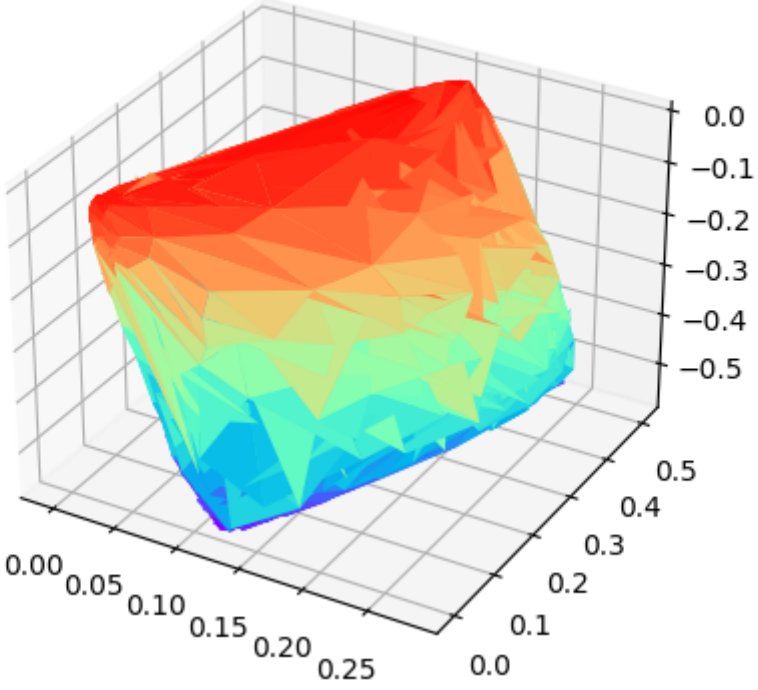
**The $\Delta p_i$ matrix does not change for target since it is not the one undergoing deformation. So it can be computed once and thats it. We need to compute $\Delta p_i$ for source because it is undergoing deformation. But $L_{norm}$ will not change since connectivity does not change.**

In [12]:
```
delta_p_target = Lnorm_target @ target_tensor.reshape(-1,3)
```

In [13]:
```
iterations = 1000
combined_loss = []

weights_deform = nn.Parameter(source_tensor)
optimizer = torch.optim.Adam([weights_deform], lr=1e-3)

for it in tqdm(range(iterations)):
    delta_p_source = Lnorm_source @ weights_deform.detach().numpy().reshape(-1,3)
    dist1, idx1, dist2, idx2 = Chamfer_distance_torch(weights_deform, target_tensor)
    loss = torch.mean(dist1) + torch.mean(dist2)
    loss += torch.mean(torch.norm(delta_p_source - delta_p_target[idx1], dim = 1))
    loss += torch.mean(torch.norm(delta_p_target - delta_p_source[idx2], dim = 1))
    loss.backward()
    optimizer.step()
    optimizer.zero_grad()
    combined_loss.append(loss.item())
    if it %100 == 0:
        weights = torch.clone(weights_deform)
        weights =weights.detach().numpy()
        deformed = trimesh.Trimesh(vertices = weights.reshape(-1,3), faces = source.faces)
        plot_mesh(deformed,COLOR_PALETTE[2]/255, f'Achieved mesh using chamfer + curvature loss after {it} i
```

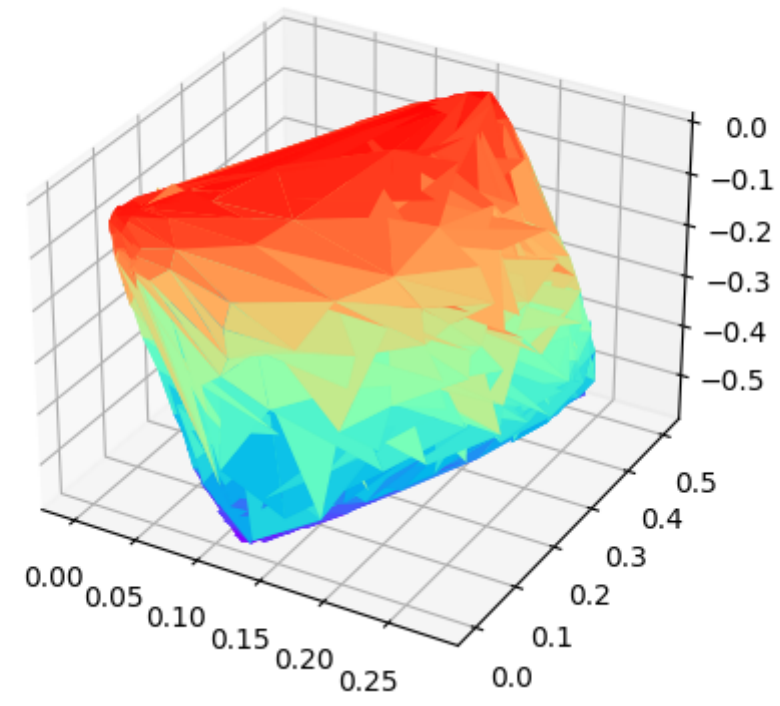
0%| | 0/1000 [00:00<?, ?it/s]

Achieved mesh using chamfer + curvature loss after 0 iterations



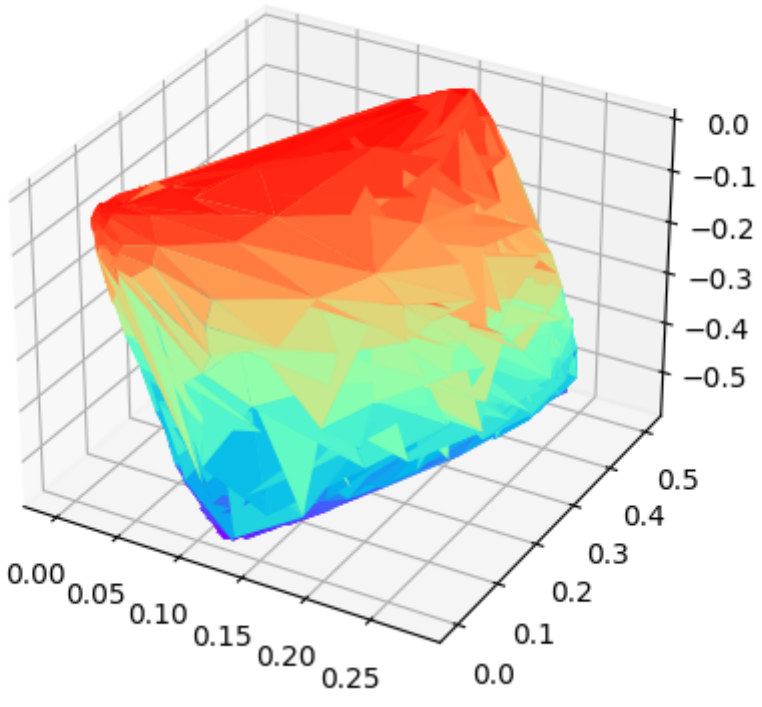

10%|███ | 100/1000 [00:06<00:55, 16.28it/s]

Achieved mesh using chamfer + curvature loss after 100 iterations



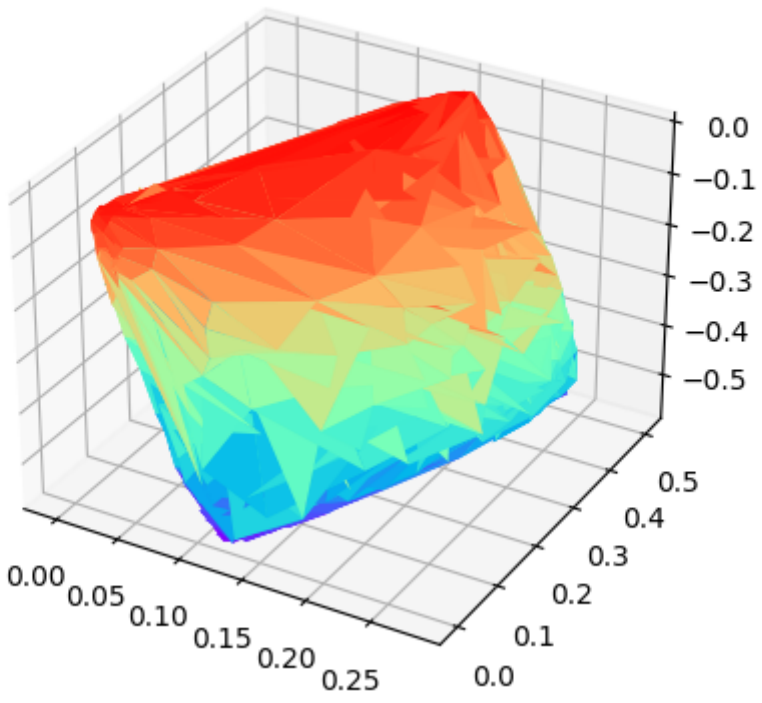

20%|██████ | 200/1000 [00:12<00:53, 14.91it/s]

Achieved mesh using chamfer + curvature loss after 200 iterations



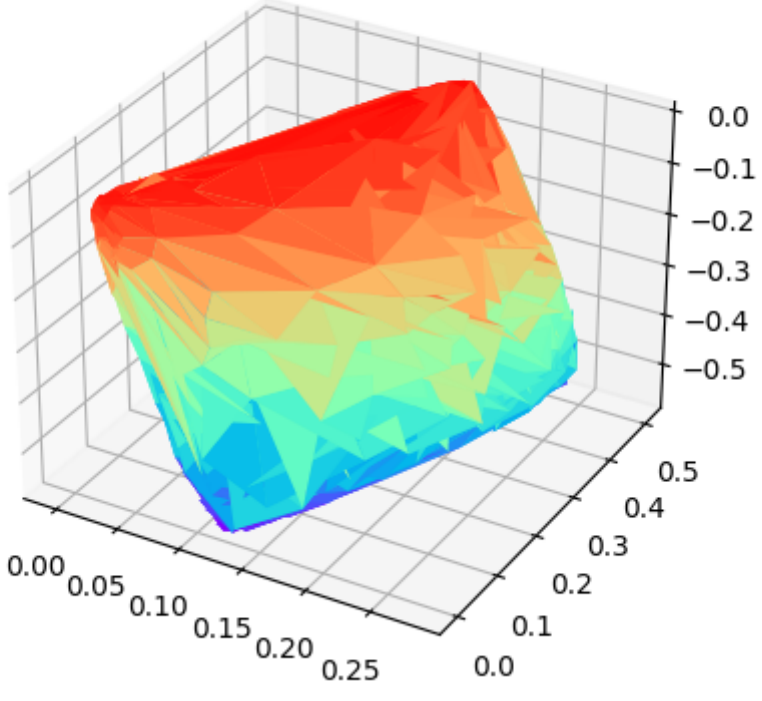

30%|███████████ | 300/1000 [00:17<00:38, 18.18it/s]

Achieved mesh using chamfer + curvature loss after 300 iterations



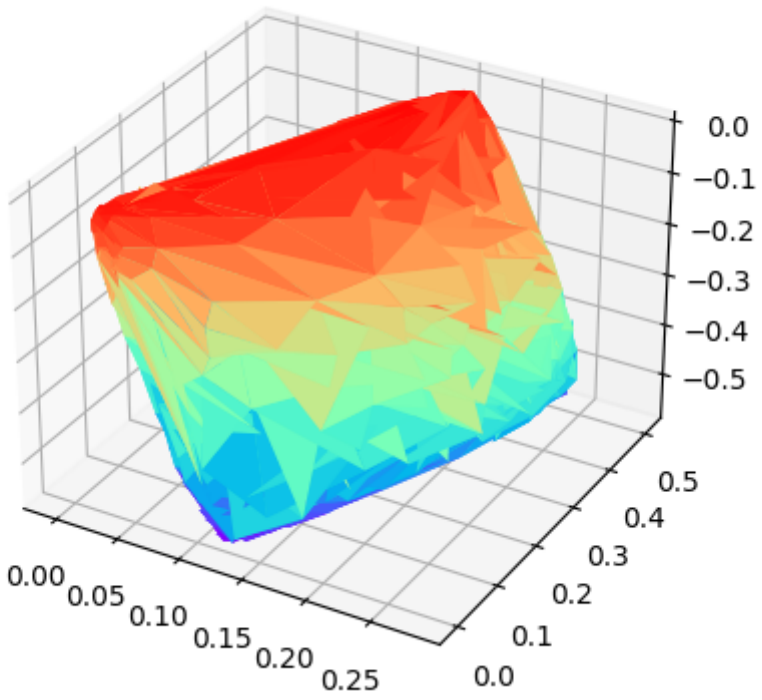

40%|████████████████ | 400/1000 [00:23<00:35, 17.14it/s]

Achieved mesh using chamfer + curvature loss after 400 iterations



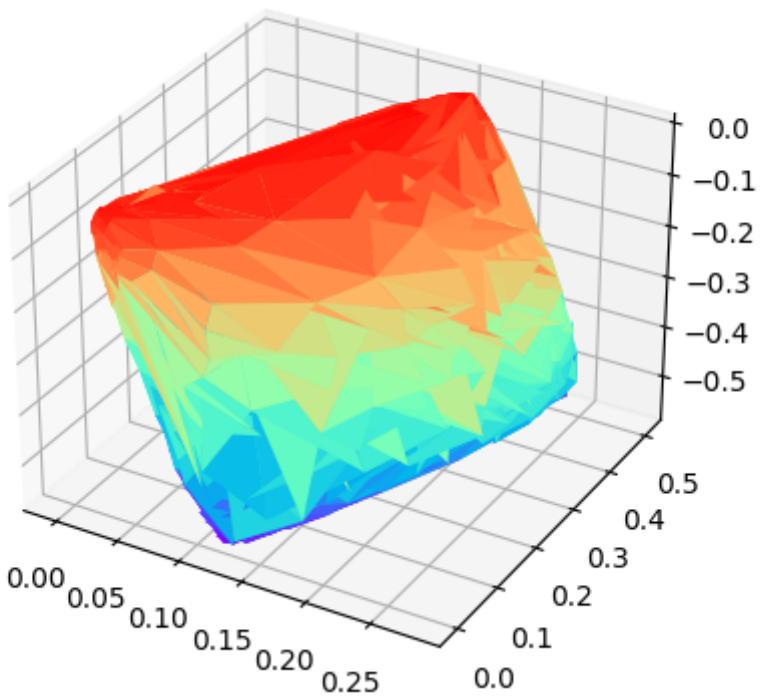

50%|████████████████████ | 498/1000 [00:29<00:25, 19.33it/s]

Achieved mesh using chamfer + curvature loss after 500 iterations



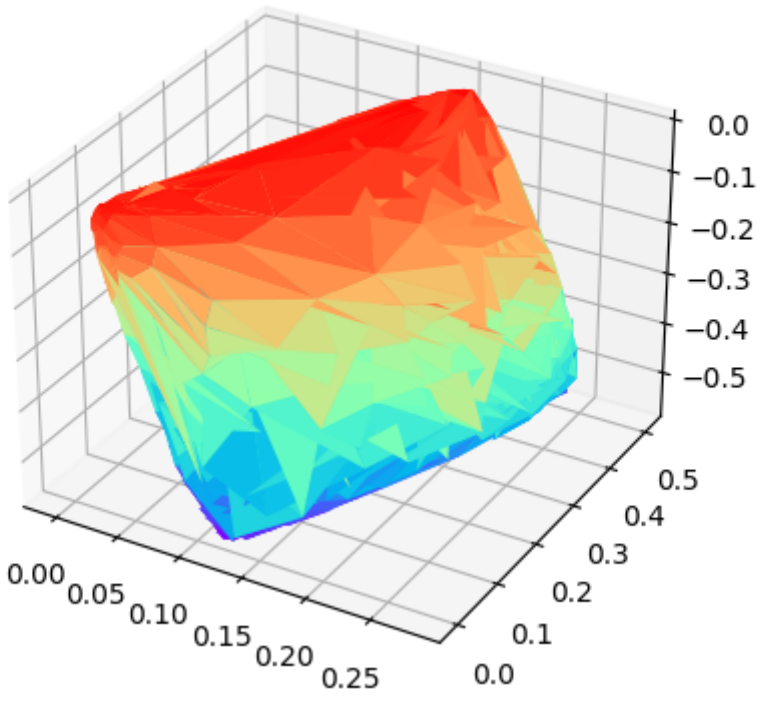

60%|██████████████████ | 600/1000 [00:36<00:23, 16.74it/s]

Achieved mesh using chamfer + curvature loss after 600 iterations

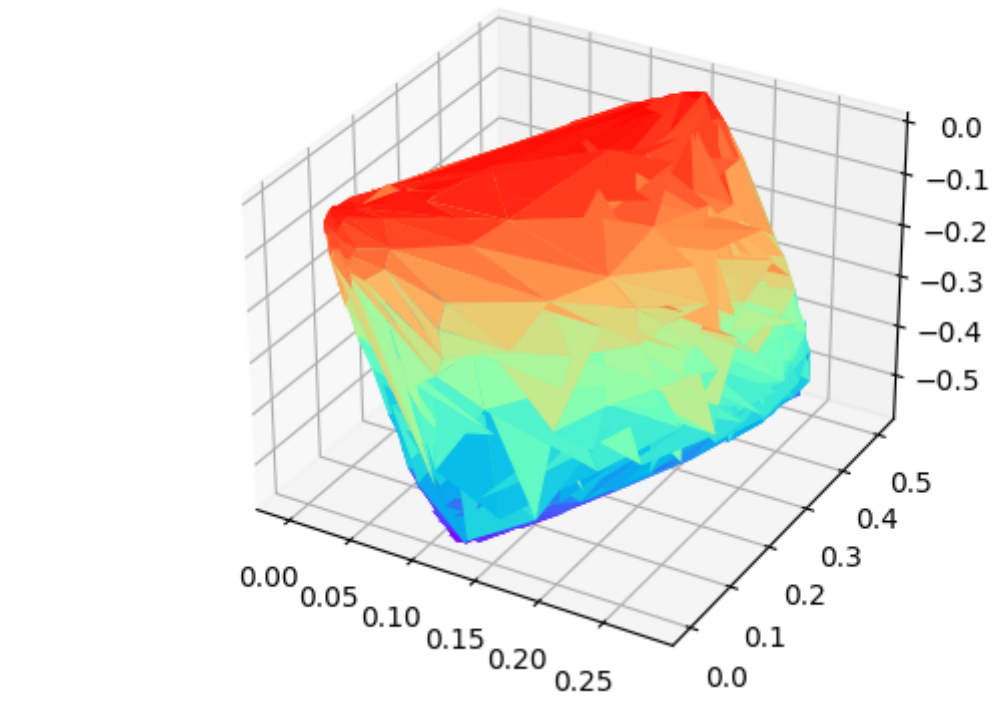



70%|████████████████████ | 699/1000 [00:42<00:16, 18.31it/s]

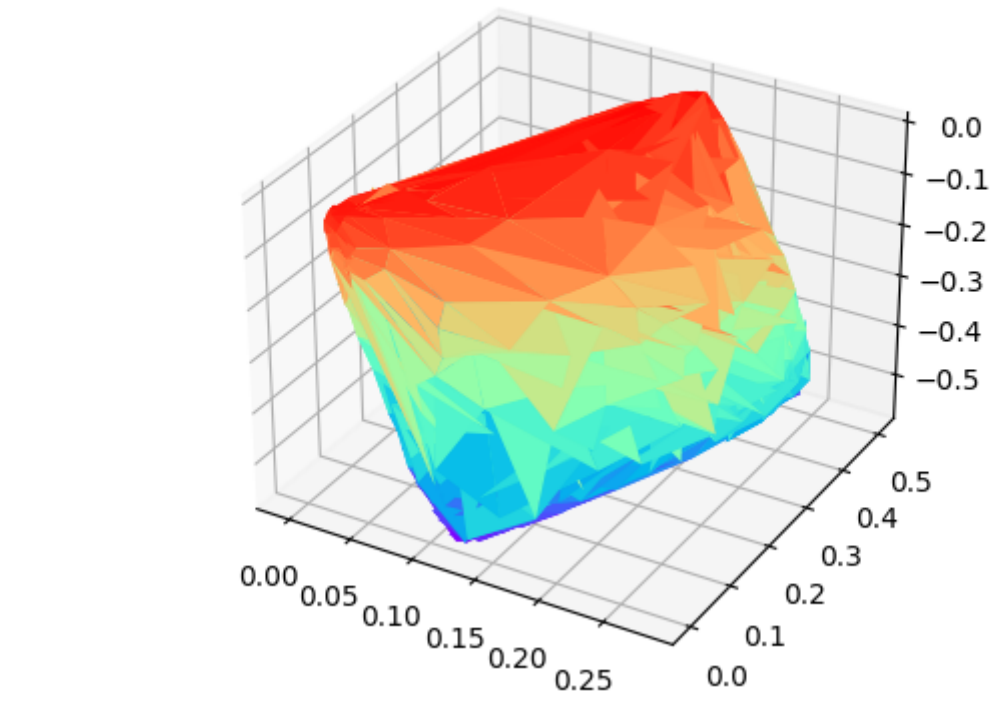
Achieved mesh using chamfer + curvature loss after 700 iterations



80%|████████████████████████ | 800/1000 [00:48<00:09, 20.27it/s]




90%|████████████████████████ | 899/1000 [00:53<00:05, 18.61it/s]




100%|████████████████████████| 1000/1000 [01:00<00:00, 16.66it/s]

**Using the combined loss, we observe that the deformation is a lot smoother than the one using only chamfer loss. Also we observe that the deformed mesh corners are very similar to the target mesh corners whereas the one using only chamfer loss had kind of round corners.**

## Problem 2: ICP

In [14]:
```
"""Visualization utilies."""

# You can use other visualization from previous homeworks, like Open3D
import numpy as np
import matplotlib.pyplot as plt
from tqdm import tqdm
from mpl_toolkits.mplot3d import Axes3D
import copy

def show_points(points):
    fig = plt.figure()
    ax = fig.add_subplot(1,1,1,projection = '3d')
    ax.set_xlim3d([-2, 2])
    ax.set_ylim3d([-2, 2])
    ax.set_zlim3d([0, 4])
    ax.scatter(points[:, 0], points[:, 2], points[:, 1])


def compare_points(points1, points2):
    fig = plt.figure()
    ax = fig.add_subplot(1,1,1,projection = '3d')
    ax.set_xlim3d([-2, 2])
    ax.set_ylim3d([-2, 2])
    ax.set_zlim3d([0, 4])
    ax.scatter(points1[:, 0], points1[:, 2], points1[:, 1])
    ax.scatter(points2[:, 0], points2[:, 2], points2[:, 1])
```

In [15]:
```
"""Load data."""

import trimesh
import numpy as np

source_pcd = trimesh.load("banana.source.ply").vertices
target_pcd = trimesh.load("banana.target.ply").vertices
gt_T = np.loadtxt("banana.pose.txt")
```

In [16]:
```
source_pcd
```

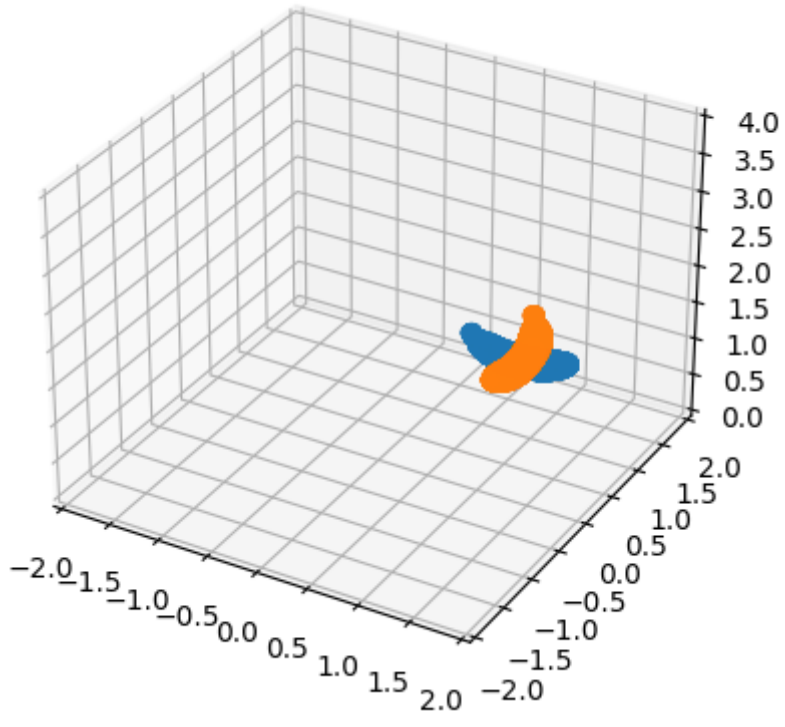
Out[16]:
```
TrackedArray([[-0.23091938, -0.10104883, -0.07753404],
              [ 0.27073169, -0.00146855, -0.0353344 ],
              [-0.39152163, -0.00135914, -0.07481582],
              ...,
              [-0.29234454, -0.14529917, -0.09030698],
              [ 0.19018735, -0.19979134, -0.05973255],
              [-0.09481647, -0.07675853, -0.01570301]])
```

In [17]:
```
target_pcd
```

Out[17]:
```
TrackedArray([[1.07478809, 1.47837925, 0.68953747],
              [1.1249311 , 1.503407  , 0.17942953],
              [1.0583688 , 1.60131693, 0.83220923],
              ...,
              [1.0651176 , 1.44293368, 0.7569496 ],
              [1.12221241, 1.31854594, 0.29003513],
              [1.13867629, 1.48905003, 0.55263776]])
```

In [18]:
```
from sklearn.neighbors import NearestNeighbors

def nearest_neighbor(src, dst):
    '''
    Find the nearest (Euclidean) neighbor in dst for each point in src
    Input:
        src: Nxm array of points
        dst: Nxm array of points
    Output:
        distances: Euclidean distances of the nearest neighbor
        indices: dst indices of the nearest neighbor
    '''

    assert src.shape == dst.shape

    neigh = NearestNeighbors(n_neighbors=1)
    neigh.fit(dst)
    distances, indices = neigh.kneighbors(src, return_distance=True)
#     print("indices from sklearn : {}".format(indices))
    return indices.ravel()
```

In [19]:
```
source = np.copy(source_pcd) + np.mean(target_pcd, axis = 0) - np.mean(source_pcd, axis = 0)
target = np.copy(target_pcd)
compare_points(source, target)
```

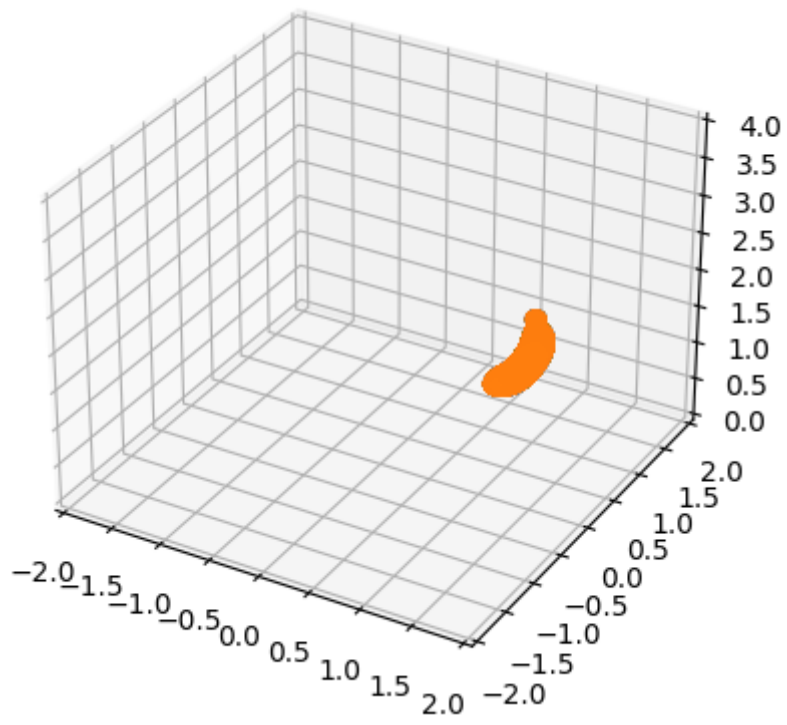



In [20]:
```
"""Implement your own ICP."""

def icp(source_pcd, target_pcd):
    """Iterative closest point.

    Args:
        source_pcd (np.ndarray): [N1, 3]
        target_pcd (np.ndarray): [N2, 3]

    Returns:
        np.ndarray: [4, 4] rigid transformation to align source to target.
    """
    T = np.eye(4)
    target_mean = np.mean(target_pcd, axis = 0)
    R = T[:3, :3]
    T[:3,3] = target_mean[:] - np.mean(source_pcd, axis = 0)
    source = np.copy(source_pcd) + target_mean - np.mean(source_pcd, axis = 0)
    target = np.copy(target_pcd)

    # Implement your own algorithm here.
    for it in tqdm(range(50)):
        corr_idx = nearest_neighbor(source, target)
        p = source.T
        q = target_pcd[corr_idx, :].T
        M = (q - np.mean(q,axis = 1, keepdims=True))@ (p - np.mean(p, axis = 1, keepdims=True)).T
        U,S,V = np.linalg.svd(M)
        R = U@V
        if np.linalg.det(R) < 0:
            V[:,-1] = -V[:,-1]
        R = U@V
        t = np.mean(q,axis = 1, keepdims=True) - R@np.mean(p,axis = 1, keepdims=True)
#         print(f"relative translation : {t}")
        source_new = (R@p + t).T
        source = source_new
        T_temp = np.block([[R,t], [np.zeros((1,3)), 1]])
        T = T_temp @ T
        rot_error = np.rad2deg(compute_rre(T[:3, :3], gt_T[:3, :3]))
        rte = compute_rte(T[:3, 3], gt_T[:3, 3])
        print(f"rot error : {rot_error} || translation error : {rte}")
    return T
```

In [21]:
```
"""Metric and visualization."""

def compute_rre(R_est: np.ndarray, R_gt: np.ndarray):
    """Compute the relative rotation error (geodesic distance of rotation)."""
    assert R_est.shape == (3, 3), 'R_est: expected shape (3, 3), received shape {}.'.format(R_est.shape)
    assert R_gt.shape == (3, 3), 'R_gt: expected shape (3, 3), received shape {}.'.format(R_gt.shape)
    # relative rotation error (RRE)
    rre = np.arccos(np.clip(0.5 * (np.trace(R_est.T @ R_gt) - 1), -1.0, 1.0))
    return rre


def compute_rte(t_est: np.ndarray, t_gt: np.ndarray):
    assert t_est.shape == (3,), 't_est: expected shape (3,), received shape {}.'.format(t_est.shape)
    assert t_gt.shape == (3,), 't_gt: expected shape (3,), received shape {}.'.format(t_gt.shape)
    # relative translation error (RTE)
    rte = np.linalg.norm(t_est - t_gt)
    return rte


# Visualization
T = icp(source_pcd, target_pcd)
print(T)
rre = np.rad2deg(compute_rre(T[:3, :3], gt_T[:3, :3]))
rte = compute_rte(T[:3, 3], gt_T[:3, 3])
print(f"rre={rre}, rte={rte}")
compare_points(source_pcd @ T[:3, :3].T + T[:3, 3], target_pcd)
```

```
  8%|███                                       | 4/50 [00:00<00:03, 14.49it/s]
rot error : 86.9389656369664 || translation error : 0.06805395849299617
rot error : 81.75630675910436 || translation error : 0.07271131712948033
rot error : 66.68250314519055 || translation error : 0.07132642738497441
rot error : 37.83088208887614 || translation error : 0.06730364391722787

 16%|██████                                    | 8/50 [00:00<00:02, 16.67it/s]

rot error : 19.15749343913435 || translation error : 0.05435682431670539
rot error : 12.192294583035894 || translation error : 0.041270809782583136
rot error : 8.502858371087228 || translation error : 0.030706188300612954
rot error : 6.144945107571997 || translation error : 0.022948807405673392

 30%|███████████                              | 15/50 [00:00<00:01, 24.56it/s]

rot error : 4.603721323573633 || translation error : 0.017597647737449172
rot error : 3.601915811625485 || translation error : 0.01411664220014118
rot error : 2.926201185042271 || translation error : 0.011923650600620181
rot error : 2.4556035790776765 || translation error : 0.010546637888584977
rot error : 2.087853515398311 || translation error : 0.009634729553363301
rot error : 1.8040245479665649 || translation error : 0.008962465217761758
rot error : 1.5624026077999607 || translation error : 0.008417617450711004

 38%|██████████████                           | 19/50 [00:00<00:01, 27.45it/s]

rot error : 1.3582308483377696 || translation error : 0.00795194333044214
rot error : 1.197291747633845 || translation error : 0.007562006552250469
rot error : 1.0703514222654023 || translation error : 0.007223724460592022
rot error : 0.9724970879253186 || translation error : 0.006911819587216404
rot error : 0.8944459312073222 || translation error : 0.006603292087126549
rot error : 0.82957449358498 || translation error : 0.00632419885598 1207
rot error : 0.7730870919130947 || translation error : 0.006062577240403191

 54%|████████████████████                     | 27/50 [00:01<00:00, 30.53it/s]

rot error : 0.7232375279934821 || translation error : 0.00581022359959737
rot error : 0.6794266686736165 || translation error : 0.005556780191458625
rot error : 0.6386900793785545 || translation error : 0.0053000231284767805
rot error : 0.6022671549025616 || translation error : 0.005037167007824063
rot error : 0.5680862289699448 || translation error : 0.004751109659483922
rot error : 0.5334276490730013 || translation error : 0.004457785228096964
rot error : 0.4974802049341708 || translation error : 0.004124112375002523

 70%|██████████████████████████               | 35/50 [00:01<00:00, 32.60it/s]

rot error : 0.4549059484657815 || translation error : 0.0037319325573458855
rot error : 0.40253700264621073 || translation error : 0.0032507857036372855
rot error : 0.32783487706955144 || translation error : 0.0025978771697115887
rot error : 0.22194712252748913 || translation error : 0.0017044469292163457
rot error : 0.09189078018094955 || translation error : 0.0006621359364264844
rot error : 0.007129130602820963 || translation error : 4.858286288210888e-05
rot error : 6.3890569402603996e-06 || translation error : 4.503890859503922e-08
rot error : 0.0 || translation error : 4.069790255192151e-10

 86%|████████████████████████████████         | 43/50 [00:01<00:00, 35.22it/s]

rot error : 0.0 || translation error : 4.0697916949491695e-10
rot error : 0.0 || translation error : 4.069787321199127e-10
rot error : 0.0 || translation error : 4.069792089503863e-10
rot error : 0.0 || translation error : 4.069789522071813e-10
rot error : 0.0 || translation error : 4.0697895360695704e-10
rot error : 0.0 || translation error : 4.0697881243084497e-10
```

```
rot error : 0.0 || translation error : 4.069790311183021e-10
rot error : 0.0 || translation error : 4.069789944622533e-10

100%|██████████████████████████████████████████████████| 50/50 [00:01<00:00, 28.92it/s]

rot error : 0.0 || translation error : 4.069789944622533e-10
rot error : 0.0 || translation error : 4.069789958620517e-10
rot error : 0.0 || translation error : 4.0697874051864017e-10
rot error : 0.0 || translation error : 4.069790353176969e-10
rot error : 0.0 || translation error : 4.069788180300561e-10
[[ 0.04139069 -0.12505187  0.99128646  1.14856815]
 [-0.15543338  0.9792519   0.13002374  1.55152014]
 [-0.98697886 -0.15946078  0.02109467  0.44714717]
 [ 0.          0.          0.          1.        ]]
rre=0.0, rte=4.069788180300561e-10
```




## 4. Course Feedback

1. I spent around 35-40hrs on this Homework (Most of the time was spent on tuning parameters for Q3)
2. I spent around 10hrs on the course last week because of other assignment deadlines and mid terms.
3. I feel like if given the conda file for the environment that has all packages installed and correct versions, it will help the students a lot.

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]: