

Homework 0

Problem 1

1. Gradient of Lagrangian

$$\nabla_x L(x, \lambda) = (Ax - b)^T A + 2\lambda x^T$$

2. Unconstrained least square

$$x = (A^T A)^{-1} A^T b$$

3.a

$$\begin{aligned} x &= (A^T A + 2\lambda I_{n \times n})^{-1} A^T b \\ \implies h(\lambda) &= (A^T A + 2\lambda I_{n \times n})^{-1} A^T b \end{aligned}$$

3.b We have $h(\lambda)^T h(\lambda) = b^T A (A^T A + 2\lambda I_{n \times n})^{-2} A^T b$, observe that the term $(A^T A + 2\lambda I_{n \times n})^{-1}$ is symmetric. We can write $h(\lambda)^T h(\lambda) = \|(A^T A + 2\lambda I_{n \times n})^{-1} A^T b\|_2$.

We can observe that the 2 norm of a vector will reduce if each component of the vector is reduced. So we will try to show that component of the vector $(A^T A + 2\lambda I_{n \times n})^{-1} A^T b$ reduces for increasing $\lambda \geq 0$.

Observe that diagonal elements of $(A^T A + 2\lambda I_{n \times n})$ will increase if $\lambda \geq 0$ is increased, which implies the diagonal elements of $(A^T A + 2\lambda I_{n \times n})^{-1}$ actually decrease. Consequently, components of the vector $(A^T A + 2\lambda I_{n \times n})^{-1} A^T b$ since each row of the matrix $(A^T A + 2\lambda I_{n \times n})^{-1}$ acts as a set of coefficients for the linear combination of the components of the vector $A^T b$. Hence we observe that since the diagonal elements have reduced, the linear combination from each row have reduced and hence each component of the vector $(A^T A + 2\lambda I_{n \times n})^{-1} A^T b$. Hence the norm decreases as $\lambda \geq 0$ is increased $\implies h(\lambda)^T h(\lambda)$ is monotonically decreasing

Alternatively, observe that $A^T A$ is a positive semidefinite matrix, because for any vector $z \in \mathbb{R}^n$, $z^T (A^T A) z = (Az)^T (Az) = \|Az\|_2^2$, hence all eigenvalues of $A^T A$ are greater than or equal to 0. Now if $\lambda \geq 0$, the matrix $2\lambda I_{n \times n}$ consists of all positive values along the diagonal. Hence the eigenvalues of $A^T A + 2\lambda I_{n \times n}$ increases, but then since eigenvalues of the inverse matrix is the reciprocal of the matrix, we can conclude that the eigen values of $(A^T A + 2\lambda I_{n \times n})^{-1}$ decreases, and hence the components of $(A^T A + 2\lambda I_{n \times n})^{-1} A^T b$ decreases which means the norm decreases. Hence we prove that $h(\lambda)^T h(\lambda)$ decreases.

Hence we have $x = h(\lambda) \implies x^T x - \epsilon = h(\lambda)^T h(\lambda) - \epsilon$, hence our aim is now to find a

value of $\lambda > 0$ which is a root of $h(\lambda)^T h(\lambda) - \epsilon$ which would also minimise the objective function

4. Implement

```
In [2]: import numpy as np
import matplotlib.pyplot as plt
from scipy.misc import derivative
%matplotlib inline
```

In /home/sambarana/anaconda3/envs/spinningup/lib/python3.6/site-packages/matplotlib/mpl-data/stylelib/_classic_test.mplstyle:

The text.latex.preview rcparam was deprecated in Matplotlib 3.3 and will be removed two minor releases later.

In /home/sambarana/anaconda3/envs/spinningup/lib/python3.6/site-packages/matplotlib/mpl-data/stylelib/_classic_test.mplstyle:

The mathtext.fallback_to_cm rcparam was deprecated in Matplotlib 3.3 and will be removed two minor releases later.

In /home/sambarana/anaconda3/envs/spinningup/lib/python3.6/site-packages/matplotlib/mpl-data/stylelib/_classic_test.mplstyle: Support for setting the 'mathtext.fallback_to_cm' rcParam is deprecated since 3.3 and will be removed two minor releases later; use 'mathtext.fallback : 'cm' instead.

In /home/sambarana/anaconda3/envs/spinningup/lib/python3.6/site-packages/matplotlib/mpl-data/stylelib/_classic_test.mplstyle:

The validate_bool_maybe_none function was deprecated in Matplotlib 3.3 and will be removed two minor releases later.

In /home/sambarana/anaconda3/envs/spinningup/lib/python3.6/site-packages/matplotlib/mpl-data/stylelib/_classic_test.mplstyle:

The savefig.jpeg_quality rcparam was deprecated in Matplotlib 3.3 and will be removed two minor releases later.

In /home/sambarana/anaconda3/envs/spinningup/lib/python3.6/site-packages/matplotlib/mpl-data/stylelib/_classic_test.mplstyle:

The keymap.all_axes rcparam was deprecated in Matplotlib 3.3 and will be removed two minor releases later.

In /home/sambarana/anaconda3/envs/spinningup/lib/python3.6/site-packages/matplotlib/mpl-data/stylelib/_classic_test.mplstyle:

The animation.avconv_path rcparam was deprecated in Matplotlib 3.3 and will be removed two minor releases later.

In /home/sambarana/anaconda3/envs/spinningup/lib/python3.6/site-packages/matplotlib/mpl-data/stylelib/_classic_test.mplstyle:

The animation.avconv_args rcparam was deprecated in Matplotlib 3.3 and will be removed two minor releases later.

In [3]: *#Implementing Newtons method to find value of lambda that is root of h^T*

```
def newton(f, x0, eps = 1e-4, iterations = 500):
    """
    : Newtons iterative method to find root of a function
    : Newtons update :  $x \rightarrow x - f(x) / f'(x)$ 
    : input --> function, previous estimate, max iterations, threshold
    : output --> next estimate of x
    """

    x = x0
    for iter in range(iterations):
        df = derivative(f, x, 1e-10)
        x_next = x - f(x)/df
        if np.linalg.norm(x_next - x) < 1e-9:
            print(f'optimal value at {x_next} found in {iter + 1} iterations')
            break
        x = x_next
    return x
```

In [4]: `import numpy as np`
`npz = np.load('HW0_P1.npz')`
`A = npz['A']`
`b = npz['b']`
`eps = npz['eps']`
`A.shape, A.dtype, b.shape, b.dtype`

Out[4]: ((100, 30), dtype('float64'), (100,), dtype('float64'))

In [5]: `def solve(A, b, eps):`
your implementation here
`h = lambda l : np.linalg.inv(A.T @ A + 2 * l * np.eye(A.shape[1])) @ b`
`f = lambda l : h(l).T @ h(l) - eps` *#this is the function we want to minimize*

`l0 = 0` *#starting point value of lambda*

`l = newton(f, l0)`

`return h(l)` *#once we find desired lambda value, x = h(lambda)*

In [6]: *# Evaluation code, you need to run it, but do not modify*
`x = solve(A,b,eps)`
`print('x norm square', x@x)` *# x@x should be close to or less than eps*
`print('optimal value', ((A@x - b)**2).sum())`

optimal value at 0.837045756847062 found in 5 iterations
x norm square 0.4999999999991626
optimal value 17.22012713194599

In [7]: `eps`

Out[7]: array(0.5)

Problem 2

(2.1)

$$A = 0, B = 1 \quad \alpha, \beta \sim \mathcal{U}[(0, 1)]$$

$$\alpha' = \frac{\alpha}{\alpha + \beta} \quad \beta' = \frac{\beta}{\alpha + \beta}$$

$$P = \alpha' A + \beta' B = \beta' = \frac{\beta}{\alpha + \beta}$$

CDF of P

$$\text{Prob}(P \leq t) = \text{Prob}\left(\frac{\beta}{\alpha + \beta} \leq t\right)$$

$$= \text{Prob}(\beta(1 - t) \leq t\alpha) = \text{Prob}\left(\beta \leq \frac{t\alpha}{1-t}\right)$$

Now we see that above cdf depends on the value of random variable α , hence to get the CDF of P, we need to marginalize wrt α

$\int_{\gamma=0}^1 \text{Prob}\left(\beta \leq \frac{t\gamma}{1-t} \mid \alpha = \gamma\right) p_{\alpha}(\alpha = \gamma) d\gamma$ The CDF of $\beta \sim \mathcal{U}[(0, 1)]$ is just x , $0 \leq x \leq 1$ and 0 otherwise, hence we have

$$\int_{\gamma=0}^1 \text{Prob}\left(\beta \leq \frac{t\gamma}{1-t}\right) d\gamma$$

$$\text{Let } u = \frac{t\gamma}{1-t} \implies du = \frac{t}{1-t}$$

$$\int_{u=0}^{\frac{t}{1-t}} \text{Prob}(\beta \leq u) \left(\frac{1-t}{t}\right) du$$

Case 1. $\frac{t}{1-t} \leq 1 \implies 0 \leq t \leq \frac{1}{2}$, then CDF of P

$$= \frac{1-t}{t} \int_{u=0}^{\frac{t}{1-t}} u du = \frac{1-t}{t} \left(\frac{t}{1-t}\right)^2 \frac{1}{2} = \frac{t}{2(1-t)}$$

Case 2. $\frac{t}{1-t} \geq 1 \implies \frac{1}{2} < t \leq 1$, then CDF of P

$$= \frac{1-t}{t} \left(\int_{u=0}^1 u du + \int_{u=1}^{\frac{t}{1-t}} 1 du \right) = \frac{1-t}{t} \left(\frac{1}{2} + \frac{t}{1-t} - 1 \right) = \frac{3t-1}{2t}$$

Therefore, CDF of P is as follows

$$F_P(P \leq t) = \begin{cases} \frac{t}{2(1-t)} & 0 \leq t \leq \frac{1}{2} \\ \frac{3t-1}{2t} & \frac{1}{2} < t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

For the pdf values, first of all we observe that the CDF value at $t = 0$ is continuous, since at $t \rightarrow 0^-$, $F_P(P) = 0$ but for $t \rightarrow 0^+$, $F_P(P) = \frac{1}{2}$. Hence the differential of the CDF does not exist at $t = 0$. SO to get the pdf of P at $t = 0$, we use

$$f_P(P = 0) = \frac{F_P(t \rightarrow 0^+)}{t} = \lim_{t \rightarrow 0^+} \frac{t}{2(1-t)t} = \frac{1}{2}$$

At $t = 0.5$, the CDF is left and right continuous. Hence it is differentiable at $t = 0.5$, and hence pdf of P at $t = 0.5$ is

$$f_P(t = 0.5) = \frac{d}{dt} \left(\frac{3}{2} - \frac{1}{2t} \right) \Big|_{t=0.5} = 2$$

(2.2)

Correct Sampling algorithm

The correct algorithm is as follows :

1. Sample $\alpha \sim \mathcal{U}[0, 1]$, $\beta \sim \mathcal{U}[0, 1]$
2. Let A, B, C, D be the vertices of the parallelogram. Let
 $P' = A + \alpha(B - A) + \beta(C - A)$
3. If P' is inside $\triangle ABC$ then select $P = P'$, else select $P = B + C - P'$

A. To prove : P' has a uniform distribution inside parallelogram ABDC

Proof :

First of all we will show that for $\alpha \sim \mathcal{U}[0, 1]$, $\beta \sim \mathcal{U}[0, 1]$, the point generated P' will always lie inside parallelogram ABDC. For proving this, we will consider the side AB || X axis for simplicity. If we consider values of $\alpha = 1$ and $\beta = 1$, then we have :-

$$\begin{aligned} P'_x &= A_x + \alpha(B_x - A_x) + \beta(C_x - A_x) \\ \implies P'_x &= B_x + C_x - A_x \end{aligned}$$

Now under the assumption stated above of side AB || X axis, we can easily observe that $(B_x - A_x) + C_x = D_x$, hence

$$P'_x = D_x$$

Now doing the same for the Y component of P'

$$\begin{aligned} P'_y &= B_y + C_y - A_y \\ \implies P'_y &= B_y = D_y \end{aligned}$$

since

$$A_y = C_y$$

Similarily we can show for $\alpha, \beta = 0, 0$, generated point $P' = A$.

Hence we show that for the maximum values of α, β the point P' is the point D , and for minimum values of α, β the point P' is A which concludes the fact the the sampling algorithm showed above will always give a point $P' \in ABDC$

Now we try to show that P' follows a uniform distribution inside this region ABDC. We have already concluded from above that any point from inside the parallelogram will satisfy $0 \leq \alpha, \beta \leq 1$.

Consider $\mathcal{T} = [\alpha, \beta]$ as a Random Variable. Since α, β are two independent uniform random variables, joint distribution of \mathcal{T} can be described as

$$p_{\mathcal{T}}(\alpha, \beta) = \begin{cases} 1 & 0 \leq \alpha, \beta \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Now

$$\begin{aligned} P'_x &= A_x + \alpha(B_x - A_x) + \beta(C_x - A_x) \\ P'_y &= A_y + \alpha(B_y - A_y) + \beta(C_y - A_y) \end{aligned}$$

which are transformations of the random variables $\mathcal{T} = [\alpha, \beta]$. So $P' = [P'_x, P'_y]$ is the transformed random variable and we are interested in its distribution. We can use the change of density formula to get the pdf of P' . The change of density function for $Y = H(X)$ is given as

$$p_Y(Y = y) = \frac{p_X(H^{-1}(x))}{|det(J)|}$$

where $J = \frac{dH}{dX}$ is the Jacobian matrix of H w.r.t X

$$\begin{aligned} \text{For our transformation, } P' &= \begin{bmatrix} P'_x \\ P'_y \end{bmatrix} = \begin{bmatrix} A_x + \alpha(B_x - A_x) + \beta(C_x - A_x) \\ A_y + \alpha(B_y - A_y) + \beta(C_y - A_y) \end{bmatrix}, \\ J = \frac{dP'}{d\mathcal{T}} &= \begin{bmatrix} \frac{dP'_x}{d\alpha} & \frac{dP'_x}{d\beta} \\ \frac{dP'_y}{d\alpha} & \frac{dP'_y}{d\beta} \end{bmatrix} = \begin{bmatrix} B_x - A_x & C_x - A_x \\ B_y - A_y & C_y - A_y \end{bmatrix} \end{aligned}$$

Hence, for P' inside parallelogram ABDC, we get

$$\begin{aligned} p_{P'}(P'_x, P'_y) &= \frac{p_{\mathcal{T}}(\alpha, \beta)}{|det(J)|} \\ \implies p_{P'}(P'_x, P'_y) &= \frac{1}{|(B_x - A_x)(C_y - A_y) - (B_y - A_y)(C_x - A_x)|} \end{aligned}$$

since $p_{\mathcal{T}}(\alpha, \beta) = 1$ inside parallelogram as we already showed that for points inside ABDC, $0 \leq \alpha, \beta \leq 1$.

Hence we observe that the pdf of P' for points inside the parallelogram is just a constant value that depends on the coordinate points of the parallelogram. Hence using the algorithm shown above, P' follows a uniform distribution inside parallelogram ABDC.

To Prove : $P = B + C - P'$ has uniform distribution inside ΔABC

Proof :

From the above sampling process, we get points P' inside the parallelogram ABDC. For points that lie outside ΔABC , we prove that the transformation $P = B + C - P'$ will bring these points back inside ΔABC .

So if P' lies outside the ΔABC implies P' lies above line BC . The equation of the line BC is given by

$$\begin{aligned}\frac{y - C_y}{x - C_x} &= \frac{B_y - C_y}{B_x - C_x} \\ \Rightarrow y &= C_y + \frac{B_y - C_y}{B_x - C_x}(x - C_x)\end{aligned}$$

For P' to lie outside the line BC , it should satisfy

$$\begin{aligned}P'_y &> C_y + \frac{B_y - C_y}{B_x - C_x}(P'_x - C_x) \\ \Rightarrow (B_y - C_y)(P'_x - C_x) &< (P'_y - C_y)(B_x - C_x) \\ \Rightarrow P'_x(B_y - C_y) &< (P'_y - C_y)(B_x - C_x) + C_x(B_y - C_y)\end{aligned}$$

Now, using the above inequality, we want to prove that $P = B + C - P'$ will result in the point P inside ΔABC i.e. P will lie below the line BC .

To prove this, we need to show that $P_y < f(P_x)$ where $f(x)$ is the equation of line BC .

Putting $P_x = B_x + C_x - P'_x$ into the equation of the line BC , we have

$$\begin{aligned}f(P_x) &= C_y + \frac{B_y - C_y}{B_x - C_x}(B_x + C_x - P'_x - C_x) \\ &= C_y + \frac{B_y - C_y}{B_x - C_x}(B_x - P'_x) \\ &= \frac{(B_y - C_y)(B_x - P'_x) + C_y(B_x - C_x)}{B_x - C_x} \\ &= \frac{B_y B_x - C_y C_x - P'_x(B_y - C_y)}{B_x - C_x}\end{aligned}$$

Now using the inequality stated above

$$\begin{aligned}P'_x(B_y - C_y) &< (P'_y - C_y)(B_x - C_x) + C_x(B_y - C_y), \text{ we have} \\ f(P_x) &> \frac{B_y B_x - C_y C_x - (P'_y - C_y)(B_x - C_x) + C_x(B_y - C_y)}{B_x - C_x}\end{aligned}$$

(We are now subtracting by a bigger number, so resulting number will be less than the original)

$$\begin{aligned}\Rightarrow f(P_x) &> \frac{B_y B_x - C_y C_x - P'_y(B_x - C_x) + C_y B_x - C_y C_x + C_y C_x - C_x B_y}{B_x - C_x} \\ \Rightarrow f(P_x) &> \frac{B_y(B_x - C_x) + C_y(B_x - C_x) - P'_y(B_x - C_x)}{B_x - C_x} \\ \Rightarrow f(P_x) &> B_y + C_y - P'_y \\ \Rightarrow f(P_x) &> P_y\end{aligned}$$

which proves that P_y lies below the line BC , hence P lies inside ΔABC if P' lies outside it.

This completes our proof that P will lie inside ΔABC if P' lies outside. And since we already prove that P' has uniform distribution in $ABDC$, we arrive at the conclusion that P will have a uniform distribution in ΔABC .

```
In [8]: def incorrect(points, n_samples = 1000):
    incorrect_samples = []
    A,B,C = points
    for n in range(n_samples):
        alpha, beta, gamma = np.random.uniform(0,1), np.random.uniform(0,1), np.random.uniform(0,1)
        total = alpha + beta + gamma
        alpha /= total
        beta /= total
        gamma /= total
        incorrect_samples.append(alpha * A + beta * B + gamma * C)
    return np.array(incorrect_samples)
```

```
In [9]: def area(x1,y1,x2,y2,x3,y3):
    return abs((x1 * (y2 - y3) + x2 * (y3 - y1) + x3 * (y1 - y2)) / 2.0)

def inside_triangle(points,P):
    x1,y1,x2,y2,x3,y3 = points.flatten()
    x_p,y_p = P
    area_ABC = area(x1,y1,x2,y2,x3,y3)
    area_PAB = area(x1,y1,x_p,y_p,x2,y2)
    area_PAC = area(x_p, y_p, x1, y1, x3,y3)
    area_PBC = area(x_p, y_p, x2, y2, x3, y3)
    return area_ABC == area_PAB + area_PAC + area_PBC
```

```
In [10]: def correct(points, n_samples = 1000):
    correct_samples = []
    A,B,C = points
    for n in range(n_samples):
        alpha, beta = np.random.uniform(0,1), np.random.uniform(0,1)
        P_prime = A + alpha * (B - A) + beta * (C - A)
        if inside_triangle(points,P_prime):
            correct_samples.append(P_prime)
        else:
            correct_samples.append(B + C - P_prime)
    return np.array(correct_samples)
```



```

In [11]: import matplotlib.pyplot as plt
from matplotlib.patches import Polygon

pts = np.array([[0,0], [0,1], [1,0]])
def draw_background(index):
    # DRAW THE TRIANGLE AS BACKGROUND
    p = Polygon(pts, closed=True, facecolor=(1,1,1,0), edgecolor=(0, 0,

    plt.subplot(1, 2, index + 1)

    ax = plt.gca()
    ax.set_aspect('equal')
    ax.add_patch(p)
    ax.set_xlim(-0.1,1.1)
    ax.set_ylim(-0.1,1.1)

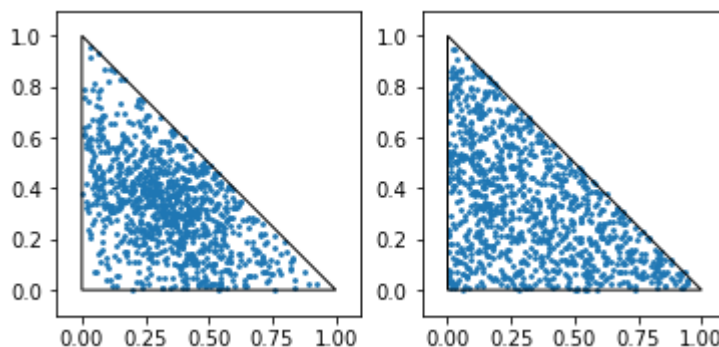
    # YOUR CODE HERE

draw_background(0)
# REPLACE THE FOLLOWING LINE USING YOUR DATA (incorrect method)
incorrect_samples = incorrect(pts)
plt.scatter(incorrect_samples[:,0], incorrect_samples[:,1], s=3)

draw_background(1)
# REPLACE THE FOLLOWING LINE USING YOUR DATA (correct method)
correct_samples = correct(pts)
plt.scatter(correct_samples[:,0], correct_samples[:,1], s=3)

plt.show()

```



Problem 3

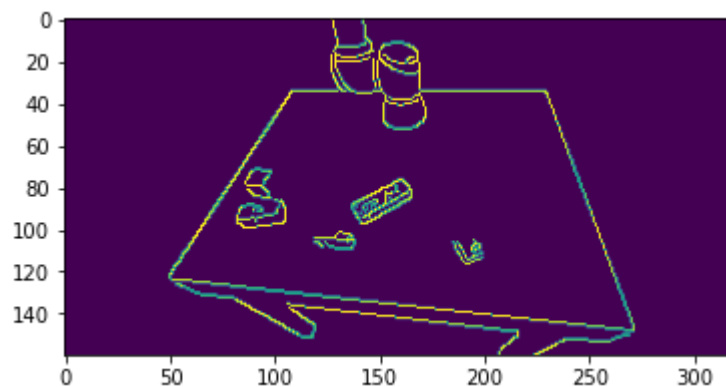
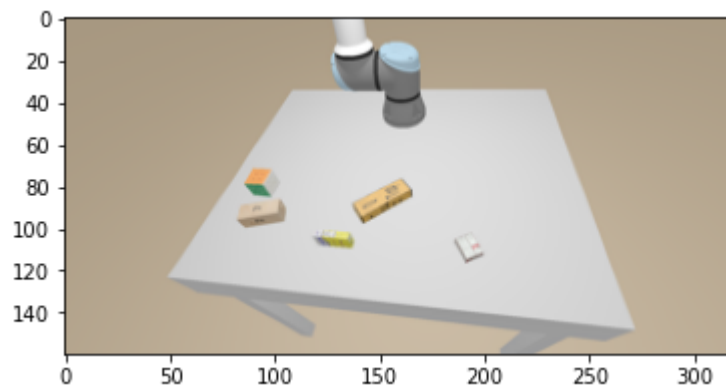
```

In [12]: import numpy as np
npz = np.load("train.npz")
images = npz["images"] # array with shape (N,Width,Height,3)
edges = npz["edges"] # array with shape (N,Width,Height)

```

```
In [13]: plt.figure()  
plt.imshow(images[0])  
plt.figure()  
plt.imshow(edges[0])
```

Out[13]: <matplotlib.image.AxesImage at 0x7ff26428b780>



```
In [14]: images.shape, edges.shape, images.max(), np.unique(edges)
```

Out[14]: ((1000, 160, 320, 3), (1000, 160, 320), 255, array([0, 255], dtype=uint8))

```
In [15]: import torch
```

```
In [16]: def conv_block(in_channels = 3, out_channels = 64):  
          conv1 = torch.nn.Conv2d(in_channels, out_channels, kernel_size = 3,  
          conv2 = torch.nn.Conv2d(out_channels, out_channels, kernel_size = 3,  
          relu = torch.nn.ReLU(inplace=True)  
          conv_down = torch.nn.Sequential(conv1, relu, conv2, relu)  
          return conv_down
```

```

In [17]: #Build the UNet
class UNet(torch.nn.Module):
    def __init__(self, in_channels = 3, out_channels = 1):
        super().__init__()

        #UNet encoder architecture
        self.down_conv1 = conv_block(in_channels = 3, out_channels = 64)
        self.down_conv2 = conv_block(in_channels = 64, out_channels = 128)
        self.down_conv3 = conv_block(in_channels = 128, out_channels = 256)
        self.down_conv4 = conv_block(in_channels = 256, out_channels = 512)
        self.down_conv5 = conv_block(in_channels = 512, out_channels = 1024)
        self.maxpool = torch.nn.MaxPool2d(kernel_size = 2, stride = 2)

        #UNet decoder architecture
        self.upsample4 = torch.nn.ConvTranspose2d(in_channels = 1024, out_channels = 512, kernel_size = 2, stride = 2)
        self.upsample3 = torch.nn.ConvTranspose2d(in_channels = 512, out_channels = 256, kernel_size = 2, stride = 2)
        self.upsample2 = torch.nn.ConvTranspose2d(in_channels = 256, out_channels = 128, kernel_size = 2, stride = 2)
        self.upsample1 = torch.nn.ConvTranspose2d(in_channels = 128, out_channels = 64, kernel_size = 2, stride = 2)

        self.up_conv4 = conv_block(in_channels = 1024, out_channels = 512)
        self.up_conv3 = conv_block(in_channels = 512, out_channels = 256)
        self.up_conv2 = conv_block(in_channels = 256, out_channels = 128)
        self.up_conv1 = conv_block(in_channels = 128, out_channels = 64)

        #Final 1X1 conv layer
        self.final_conv = torch.nn.Conv2d(in_channels = 64, out_channels = 1, kernel_size = 1, stride = 1)

    def forward(self, input):
        x1 = self.down_conv1(input) #Copy + Crop to corresponding upsample
        y1 = self.maxpool(x1) #Input to next down conv

        # print(x1.shape)
        # print(y1.shape)

        x2 = self.down_conv2(y1) #Copy + Crop to corresponding upsample
        y2 = self.maxpool(x2) #Input to next down conv

        # print(x2.shape)
        # print(y2.shape)

        x3 = self.down_conv3(y2) #Copy + Crop to corresponding upsample
        y3 = self.maxpool(x3) #Input to next down conv

        # print(x3.shape)
        # print(y3.shape)

        x4 = self.down_conv4(y3) #Copy + Crop to corresponding upsample
        y4 = self.maxpool(x4) #Input to next down conv

        # print(x4.shape)
        # print(y4.shape)

        x5 = self.down_conv5(y4) #Copy + Crop to corresponding upsample

        # print(x5.shape)

```

```
#x5 is output of encoder network
#Decoder network starts

out4 = self.upsample4(x5)
# print(f'out4 shape : {out4.shape}')

out4 = torch.cat([x4,out4], 1)
# print(f'out4 shape after concatentation : {out4.shape}')

out3 = self.upsample3(self.up_conv4(out4))
# print(f'out3 shape : {out3.shape}')

out3 = torch.cat([x3, out3], 1)
# print(f'out3 shape after concatentation : {out3.shape}')

out2 = self.upsample2(self.up_conv3(out3))
# print(f'out2 shape : {out2.shape}')

out2 = torch.cat([x2, out2], 1)
# print(f'out2 shape after concatentation : {out2.shape}')

out1 = self.upsample1(self.up_conv2(out2))
# print(f'out1 shape : {out1.shape}')

out1 = torch.cat([x1, out1], 1)
# print(f'out1 shape after concatentation : {out1.shape}')

out1 = self.up_conv1(out1)
# print(f'out1 shape after final double conv layer : {out1.shape}')

segmentation = self.final_conv(out1)
# print(f'segmentation output : {segmentation.shape}')

return segmentation
```

In [18]: torch.__version__

Out[18]: '1.10.2'

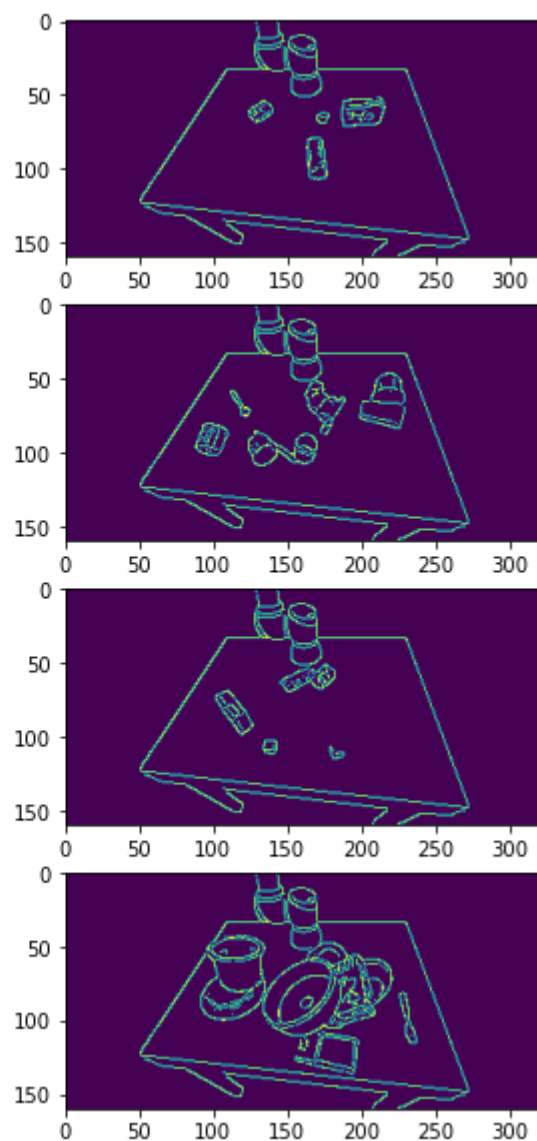
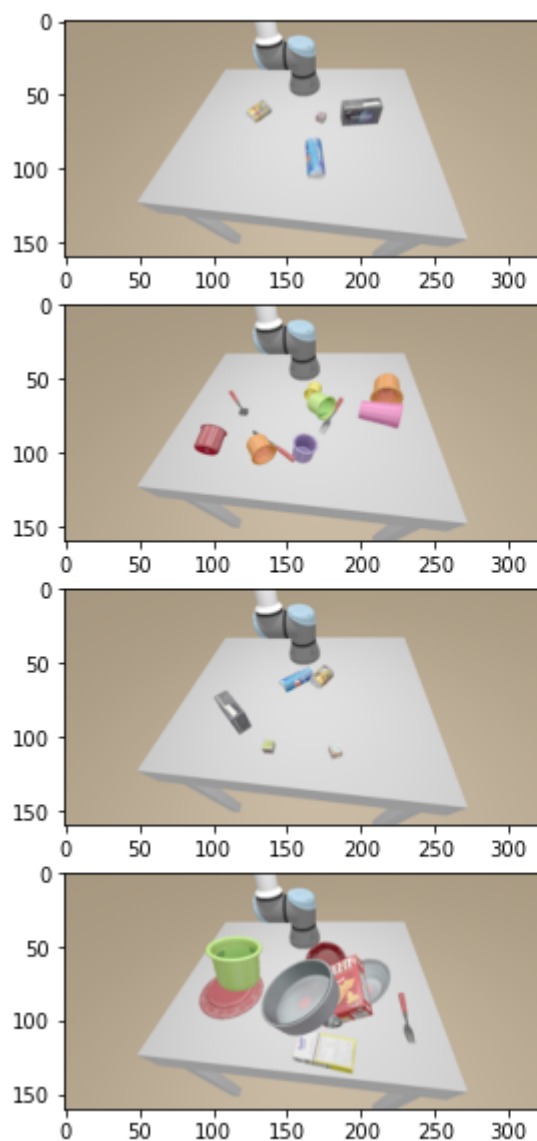
```
In [19]: # Test on the testing set
model = UNet()
checkpoint = torch.load('/home/sambaran/UCSD/CSE291/HW0/HW0/checkpoint-1')
device = ('cuda' if torch.cuda.is_available() else 'cpu')
npz = np.load("test.npz")
test_images = npz["images"]

model.load_state_dict(checkpoint['model_state_dict'])
model.to(device)
model.eval()
plt.figure(figsize=(10, 10))
with torch.no_grad():
    for i, img in enumerate(test_images[:4]):
        plt.subplot(4, 2, i * 2 + 1)
        plt.imshow(img)

        plt.subplot(4, 2, i * 2 + 2)
        # edge = evaluate your model on the test set, replace the following
        img = torch.tensor(img/255, dtype = torch.float32)
        img = img.transpose(0,2).transpose(1,2).to(device).unsqueeze(0)
        print(img.max())
        edge = torch.sigmoid(model(img))
        edge = (edge > 0.5).float()

        plt.imshow(edge.detach().cpu()[0,0])

tensor(1., device='cuda:0')
tensor(1., device='cuda:0')
tensor(1., device='cuda:0')
tensor(1., device='cuda:0')
```



In []: