Recall that if

$$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \& \quad \vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

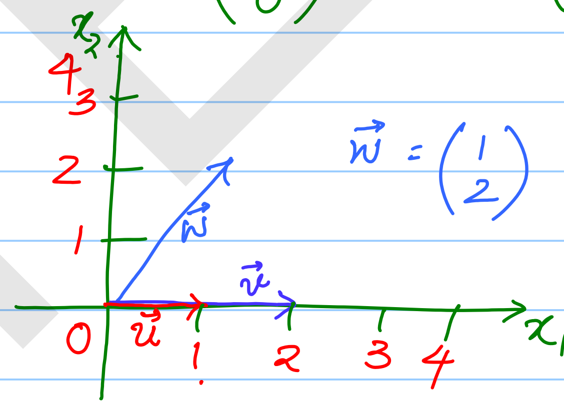
are such that

$$\vec{v} = \alpha \vec{u} \qquad \alpha : \text{Scalar}$$

we called $\vec{u}$ & $\vec{v}$ as linearly dependent vectors.

If not, we call $\vec{u}$ & $\vec{v}$ as l.i vectors.

Ex: $\vec{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ & $\vec{v} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$



$\vec{w} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Can we express $\vec{w}$ as a l.c. of $\vec{u}$ & $\vec{v}$?

$\vec{u}$ & $\vec{v}$ are along the same direction & the vector $\vec{w}$ is in a diff direction

$$\vec{w} \stackrel{?}{=} \alpha_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \stackrel{?}{=} \alpha_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ 0 \end{pmatrix} + \begin{pmatrix} 2\alpha_2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \alpha_1 + 2\alpha_2 \\ \underline{\alpha_1(0) + \alpha_2(0)} \end{pmatrix}$$

No way to get 2^{nd} coordinate of W: $2 = 0\alpha_1 + 0\alpha_2$

If 2 vectors are in 2D l.d then there is no way to get all vectors as l.c. of these 2 vectors.

However if vectors are l.i. in 2D we can get every vector as a unique l.c of these l.i. vectors

Ex: $\vec{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\quad \vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Can $\vec{w} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ be expressed as l.c. of $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ & $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$?

$$\begin{pmatrix} 4 \\ 5 \end{pmatrix} = \alpha_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

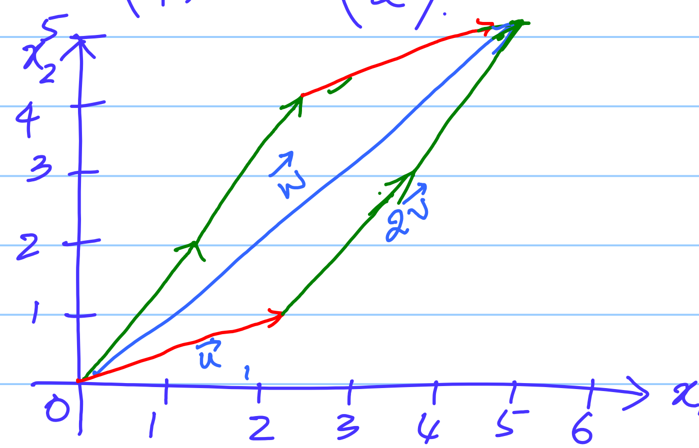
α_1, α_2 are real Scalars

$$4 = 2\alpha_1 + 1\alpha_2$$
$$5 = \alpha_1 + 2\alpha_2$$

$\alpha_1 = 1 \quad \alpha_2 = 2.$

Solving this we get $\alpha_1 = 1, \alpha_2 = 2$

$$\Rightarrow \begin{pmatrix} 4 \\ 5 \end{pmatrix} = 1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$



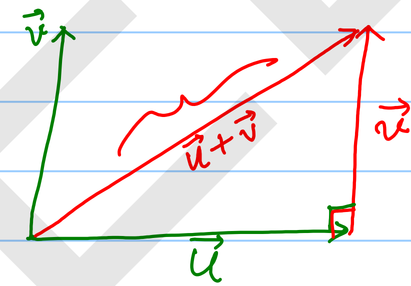
Dot product:

Let $\vec{u}$ & $\vec{v}$ be 2 vectors.

(i) Are $\vec{u}$ & $\vec{v}$ orthogonal/perpendi. to each other?

(ii) What is the angle between $\vec{u}$ & $\vec{v}$?

(iii) Are the 2 vectors, the same vector?



$$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad \vec{u}+\vec{v} = \begin{pmatrix} u_1+v_1 \\ u_2+v_2 \end{pmatrix}$$

$$\|\vec{u}+\vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2$$

$$(u_1+v_1)^2 + (u_2+v_2)^2 = u_1^2 + u_2^2 + v_1^2 + v_2^2$$

$$\cancel{u_1^2} + \cancel{v_1^2} + 2u_1v_1 + \cancel{u_2^2} + \cancel{v_2^2} + 2u_2v_2 = \cancel{u_1^2} + \cancel{u_2^2} + \cancel{v_1^2} + \cancel{v_2^2}$$

$$= 2u_1v_1 + 2u_2v_2 = 0$$

$$\Rightarrow u_1v_1 + u_2v_2 = 0$$

If $\vec{u}$ & $\vec{v}$ are perpendicular then the sum of products of their components is zero

$$\boxed{\vec{u}\cdot\vec{v} = 0}$$

$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$$\vec{u}\cdot\vec{v} = \boxed{u_1 v_1 + u_2 v_2 = \underline{0}}$$

Note: If $\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ is a vector and we want a vector $\vec{v}$ perpend. to $\vec{u}$,

$u_1 v_1 + u_2 v_2 = 0$

$\Rightarrow \vec{v} = \begin{pmatrix} -u_2 \\ u_1 \end{pmatrix}$

$\Rightarrow u_1(-u_2) + u_2(u_1) = 0$

Dot product returns a scalar

↳ Scalar product

Inner product of 2 vectors.

$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \leftrightarrow \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}^{2\times 1}$ matrix

$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ 2x1 matrix

$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2$

$= \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$.

$(\vec{u} \cdot \vec{v}) = u^T v = v^T u = \vec{v} \cdot \vec{u}$

If the vectors are perpendicular then

$\vec{u} \cdot \vec{v} = u^T v = v^T u = \vec{v} \cdot \vec{u} = 0$.

How do we find the angle b/w $\vec{u}$ & $\vec{v}$?