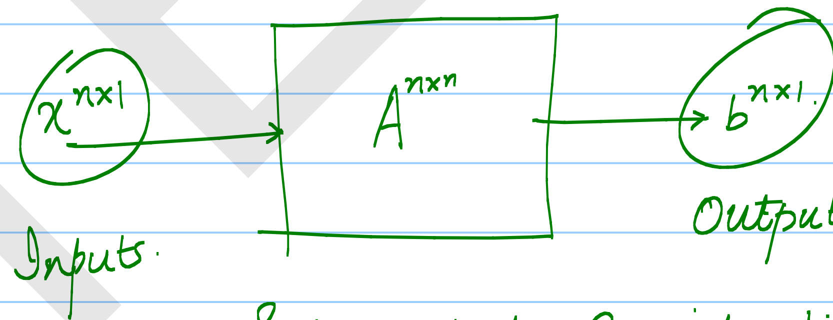
Consider $Ax = b$

where $Ax = b$ is a system of n- linear equations in n- unknowns.

$$A^{n\times n} \, x^{n\times 1} = b^{n\times 1}.$$



System under Consideration

Solve the sys. of l.e. $Ax = b$

Given the sys A & the o/p b, find that input x which made A give this response b.

$$x^{n\times 1} \longrightarrow \boxed{A^{n\times n}} \longrightarrow b^{n\times 1}.$$

Simple example:

$$\boxed{y = 5x} \qquad x \in \mathbb{R}, \; y \in \mathbb{R}.$$

$$x \longrightarrow \boxed{5*} \longrightarrow y = 5x$$

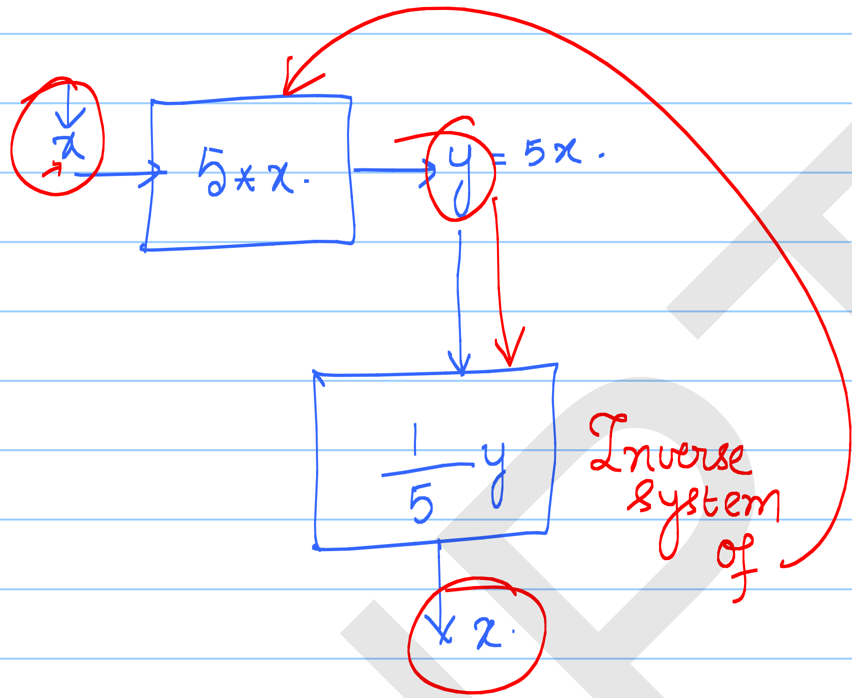
$y = 10$

$$x \overset{?}{\longrightarrow} \boxed{5 * x} \longrightarrow 10 = y = 5x$$

$$5x = 10 \qquad x = \frac{10}{5} = \left(\frac{1}{5}\right)(10) = 2.$$

Given y, we can always find

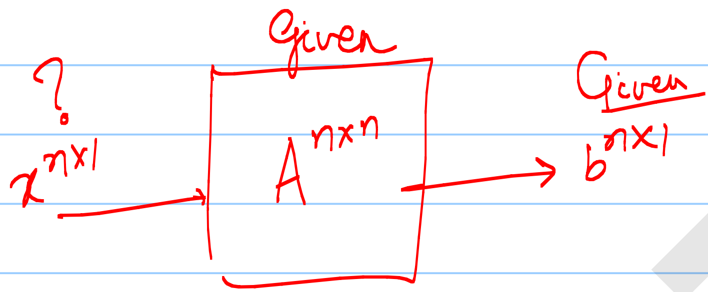
$$\boxed{x = \frac{1}{5}(y).}$$



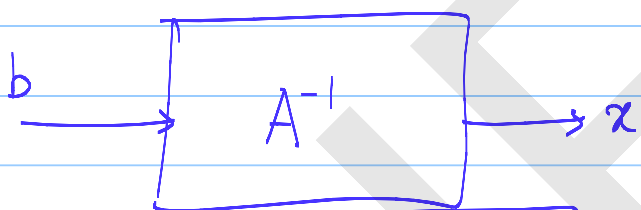
$x = a^{-1} y \quad a \neq 0$

$a \in \mathbb{R}.$

Given a sys. of l.e $Ax = b$, a set of n linear eqns in n-unknowns, can we find x for a given b?



$$\Downarrow$$



$$\boxed{x = A^{-1} b.}$$

Reformulate the problem as find the output of the system A^{-1} Corresponding to the input b.

$$a_{11} x_1 + a_{12} x_2 = b_1$$
$$a_{21} x_1 + a_{22} x_2 = b_2.$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \longrightarrow \boxed{A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}} \longrightarrow \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}.$$

$$a_{11} x_1 + a_{12} x_2 = b_1 \longrightarrow (1)$$
$$a_{21} x_1 + a_{22} x_2 = b_2 \longrightarrow (2)$$

Eliminate x_1

$$-(2) * a_{11} + (1) * a_{21}.$$

$$a_{11} \cancel{a_{21} x_1} + a_{12} a_{21} x_2 = b_1 a_{21}$$
$$a_{11} \cancel{a_{21}} x_1 + a_{22} a_{11} x_2 = b_2 a_{11}$$
$$(-) \qquad (-) \qquad (-)$$

$$x_2 \left(a_{12} a_{21} - a_{22} a_{11} \right) = b_1 a_{21} - b_2 a_{11}$$

$$= x_2 = \frac{b_2 a_{11} - b_1 a_{21}}{\underline{a_{11} a_{22} - a_{21} a_{12}}}.$$

$$x_1 = \frac{b_1 a_{22} - b_2 a_{12}}{a_{11} a_{22} - a_{21} a_{12}}.$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{a_{11} a_{22} - a_{21} a_{12}} \begin{bmatrix} a_{22} b_1 - a_{12} b_2 \\ a_{11} b_2 - a_{21} b_1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \longrightarrow \underbrace{\boxed{\frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}}}_{A^{-1}} \longrightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$x \longrightarrow \boxed{A} \longrightarrow b.$$

$$\longrightarrow \boxed{A^{-1}} \longrightarrow x.$$