Vector Subspaces:

* Consider $W_1 = \left\{ \begin{pmatrix} x_1 \\ x_1 \end{pmatrix}, x_1 \in \mathbb{R} \right\}$.

W_1: Line passing thro' the origin and thro' $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

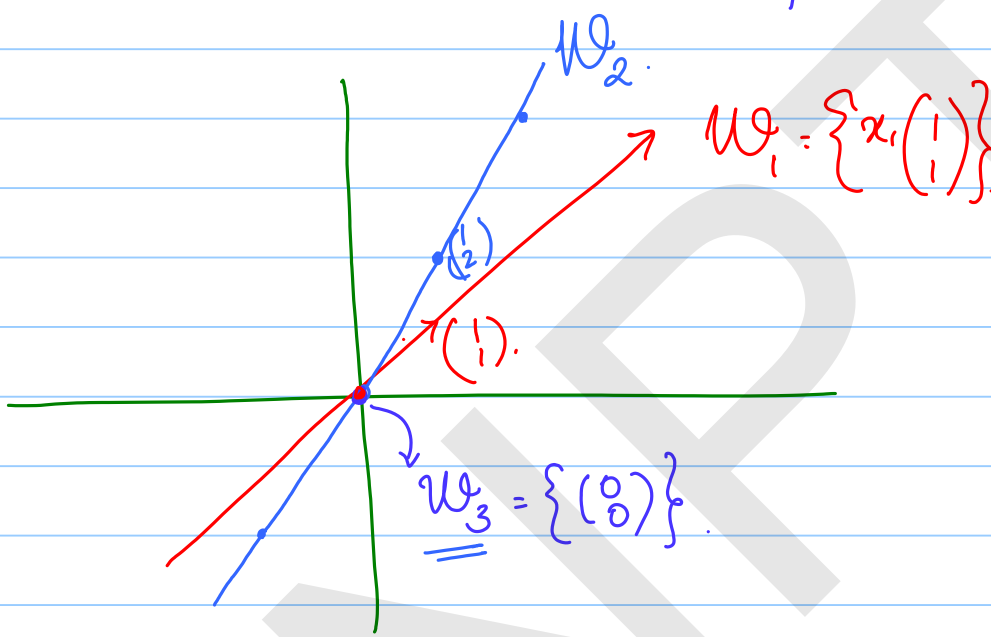
$$W_1 = \left\{ x_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix}, x_1 \in \mathbb{R} \right\}$$

W_1: vector space over $\mathbb{R}$.

* $W_2 = \left\{ \begin{pmatrix} x_1 \\ 2x_1 \end{pmatrix} x_1 \in \mathbb{R} \right\}$.

W_2: Vector space over $\mathbb{R}$
line thro' the origin & $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$W_3 = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$ is a vector space.



<u>Definition</u>:

Any subset W of a vector space V, which by itself is a vector space is called a vector SUBSPACE. (with the same operations as V).

W_1 is a vector Subspace of $V = \mathbb{R}^2$

W_2 is a vector sub space of $\mathbb{R}^2$

$W_3 = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$ is a vector subsp of $\mathbb{R}^2$.

A plane passing through the Origin in $\mathbb{R}^3$ is a Vector Subspace of $\mathbb{R}^3$.

* Any line passing through the Origin in $\mathbb{R}^n$ is a vector subspace of $\mathbb{R}^n$.

* Any plane passing through the Origin in $\mathbb{R}^n$ is Subspace of $\mathbb{R}^n$.

* A vector space V is a subspace of itself

* $\left\{ \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{n\times 1} \right\}$ is a subspace of $\mathbb{R}^n$.

$\Downarrow$

TRIVIAL SUBSPACE of V.

* Every subspace of V other than itself is called a proper subspace of V.

* $W = \{\vec{0}\}$ is called the trivial subspace of V.

Let W_1 & W_2 be two proper subspaces of V.

Questions:

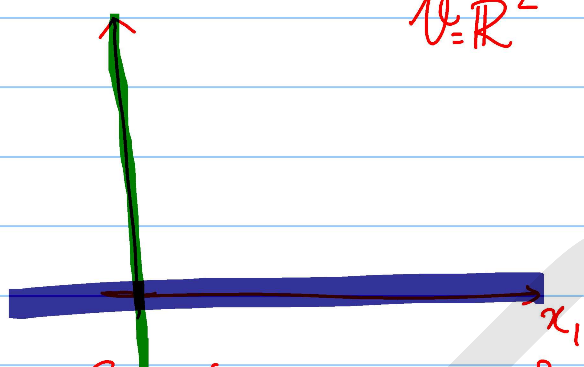
(i) Is $W_1 \cup W_2$ a subspace of V?

(ii) Is $W_1 \cap W_2$ a subspace of V?

(iii) Is W_1^c a subspace of V?

* $W_1 \cup W_2$ $W_1 \subset V$, $W_2 \subset V$.

$V = \mathbb{R}^2$



$W_1 = \left\{ x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ x_1 \in \mathbb{R} \right\}.$

$W_2 = \left\{ x_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \ x_2 \in \mathbb{R} \right\}.$

$W_1 \cup W_2$ = Set of all points along x_1 axis, " " " x_2 axis

$\vec{u} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \in W_1, \in W_1 \cup W_2.$

$\vec{v} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \in W_1 \cup W_2.$

$\vec{u} + \vec{v} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \notin W_1 \cup W_2.$

Union of 2 subspaces NEED not be a subspace.

$W_1 = \mathbb{R}^2.$ $\qquad V = \mathbb{R}^2$

$W_2 = \left\{ x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ x_1 \in \mathbb{R} \right\}.$

$W_1 \cup W_2 = \mathbb{R}^2.$

$W_1 \cup W_2$ is a subspace if one of them is a subset of the other.

(2) Is $W_1 \cap W_2$ a subsp. of V?

$W_1 \cap W_2$ is always a subspace of V as $W_1 \cap W_2$ is atleast $\{\underline{0}\}.$

(3) Is W_1^c a subsp of V?

No.

* Check if the set of vectors $S = \left\{ \begin{pmatrix} x_1 \\ 2 \end{pmatrix}, x_1 \in \mathbb{R} \right\}$ is a subspace of $\mathbb{R}^2$. $\rightarrow$ NO.

* If $A^{2\times2}$ is the set of all invertible 2×2 real matrices, then check if $A^{2\times2}$ is a subspace of $M^{2\times2}$, set of all real matrices. NO.

* Is the set S of all 3 component vectors in $\mathbb{R}^3$, S as given below a subspace of $\mathbb{R}^3$

$$S = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \right\}.$$

NO