eigenvalues and eigenvectors.

Suppose $A \in \mathbb{R}^{n\times n}$ and, let $u \in \mathbb{R}^n$

s.t

$$Au = \lambda u, \quad \lambda \text{ is a}$$

scalar.

we say that the non-zero vector u is the eigen vector for A associated with the eigenvalue λ.

$$Au = \lambda u$$

$$\Rightarrow Au = \lambda I u \qquad I: n\times n \text{ Identity matrix.}$$

$$\Rightarrow \underbrace{(A - \lambda I)\vec{u} = \vec{0}}$$

Homog. Sys. of eqns for which we are looking for a non-trivial solution.

Since we want $\vec{u} \neq \vec{0}$, we know that the matrix

$(A - \lambda I)$ must be singular

$$\Rightarrow \boxed{\det(A - \lambda I) = 0}$$

Characteristic eqn of A.

$\det(A-\lambda I) \rightarrow$ Char. polynomial of A.

The roots of the characteristic eqn correspond to the eigenvalues of A.

$A^{n \times n}$:

(i) Real distinct roots.

(ii) Real repeated roots

(iii) Roots of the char eqn = 0 (Some of them)

(iv) Real roots & Complex conjugate roots.

* Algebraic Multiplicity of the eigen values:

⇒ No. of times a particular eigenvalue repeats is called the algebraic multiplicity of the eigenvalue

* Geometric Multiplicity

⇒ No. of l.i. eigenvectors associated with a given eigenvalue is called the geometric Multiplicity.

Suppose $\lambda_i = 0$. what does that mean?

$$(A - \lambda I)\vec{u} = \vec{0}$$

Suppose if $\lambda_i = 0$

$$A\vec{u} = \lambda_i \vec{u} \Rightarrow A\vec{u} = \vec{0}$$

eigen vector corresp. to $\lambda = 0$ is the soln to HSE $A\vec{u} = \vec{0}$

Suppose $A^{n \times n}$ has n-eigenvalues and corresponding n eigen vectors

We define a matrix P whose cols are the eigen vectors.

Let $\lambda_1, \lambda_2 \dots \lambda_n$ be the eigenvalues and let $ev_1, ev_2 \dots ev_n$ correspond to the eigenvectors $\lambda_1, \dots \lambda_n$ respectively

$$P = \begin{bmatrix} ev_1 & ev_2 & \cdots & ev_n \end{bmatrix}$$

$$AP = A\begin{bmatrix} ev_1 & ev_2 & \cdots & ev_n \end{bmatrix}$$

$$AP = \begin{bmatrix} Aev_1 & Aev_2 & \cdots & Aev_n \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \lambda_1 ev_1 & \lambda_2 ev_2 & \cdots & \lambda_n ev_n \end{bmatrix}}$$

$$= \underbrace{\begin{bmatrix} ev_1 & ev_2 & \cdots & ev_n \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}}_{D}$$

D: Diagonal matrix containing the

eigenvalues

$AP = PD.$

The eigenvectors corresponding to diff eigenvalues are linearly independent.

Since the eigenvectors form the cols of P and are all li., the matrix P is invertible.

$AP = PD.$

$AP(P^{-1}) = PD(P^{-1})$

$\Rightarrow \boxed{A = PDP^{-1}}$ eigendecomp. of A.

$AP = PD \Rightarrow P^{-1}AP = P^{-1}PD \Rightarrow$

$$\boxed{D = P^{-1} A P.}$$

if and only if the cols of P are the eigenvectors of A and the diagonal elements of D are the eigenvalues that correspond in order to the cols of P.

A square matrix A is diagonalizable if and only if it has n l.i eigen vectors.

Suppose A is diagonalizable then we have

$$\underline{\underline{A = PDP^{-1}}} \quad \& \quad D = P^{-1}AP.$$

$$A = PDP^{-1}$$

$$A^2 = A \cdot A = PD\underbrace{P^{-1} \cdot P}_{I}DP^{-1}$$

$$\Rightarrow A^2 = PD^2P^{-1}$$

$$A^3 = A^2 \cdot A = PD^2\underbrace{P^{-1} \cdot P}_{I}DP^{-1} = PD^3P^{-1}$$

$$\Rightarrow \boxed{A^n = PD^nP^{-1}}$$

$$\boxed{A^{-1} = PD^{-1}P^{-1}}$$

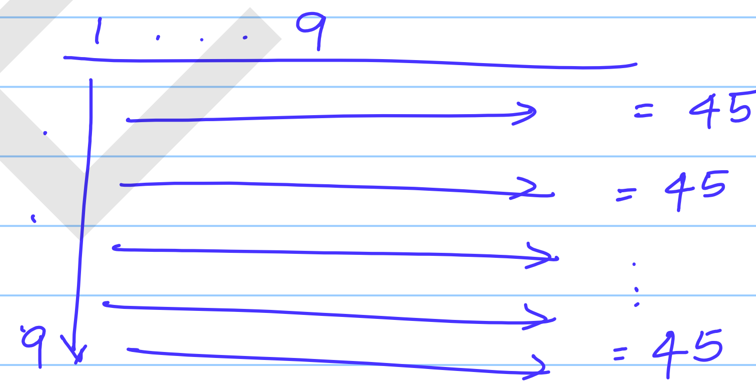
Provided A is invertible.

Sudoku Puzzles :

9x9 table 1 . . . 9.

Every row must contain numbers from 1 to 9, every number only once in a row.

Every col. must contain numbers from 1 to 9, with every number appearing only once in a col.



$S^{9\times 9}$ matrix

$$\underbrace{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}^{9\times 9}}_{A}\underbrace{\begin{bmatrix}1\\ 1\\ \vdots\\ 1\end{bmatrix}^{9\times 1}}_{\vec{u}} = \underbrace{45}_{\lambda}\underbrace{\begin{bmatrix}1\\ 1\\ \vdots\\ 1\end{bmatrix}^{9\times 9}}_{\vec{u}}$$