Let $A\vec{u} = \vec{0}$ be a sys. of 3 linear eqns in 3 unknowns.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\vec{u} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, trivial Soln always exists.

Geometrically, we are looking for that vector $\vec{u}$ Orthogonal to the three vectors, namely

$$\vec{a_1} = \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix} \quad \vec{a_2} = \begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} \quad \vec{a_3} = \begin{bmatrix} a_{31} \\ a_{32} \\ a_{33} \end{bmatrix}$$

$\Rightarrow \vec{a_1}\cdot\vec{u} = 0$; $\vec{a_2}\cdot\vec{u} = 0$, $\vec{a_3}\cdot\vec{u} = 0$

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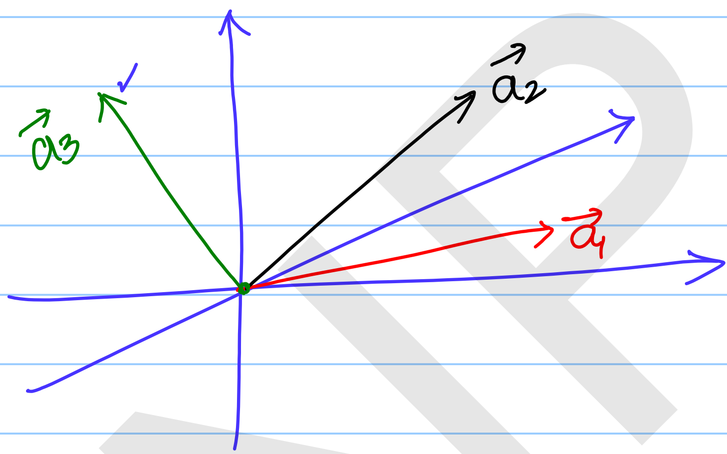
Case - 1:

$\vec{a}_1, \vec{a}_2$ & $\vec{a}_3$ are all linearly indep vectors in 3D-



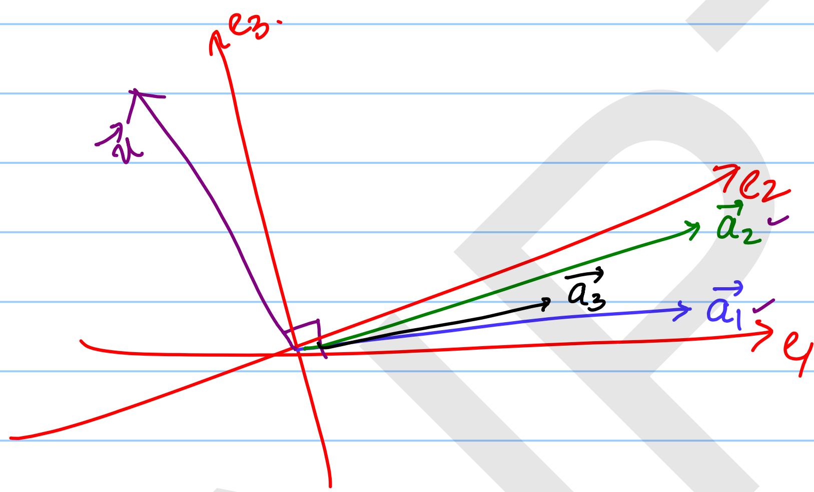
det (A) $\neq 0$ because $\vec{a}_1, \vec{a}_2$ & $\vec{a}_3$ are l.i

$\Rightarrow A\vec{u} = \vec{0} \Rightarrow \vec{u} = \vec{0}$ is the Only Solution to $A\vec{u} = \vec{0}$

Trivial Solution is the only Soln:

Case 2: $\vec{a}_1$ & $\vec{a}_2$ are l.i &

$$\vec{a}_3 = c_1\vec{a}_1 + c_2\vec{a}_2 \qquad c_1, c_2 \text{ Scalars}$$



$\det(A) = 0$

Solution to $A\vec{u} = \vec{0}$

is $\vec{u} = \vec{a}_1 \times \vec{a}_2$

$$\vec{a}_3 = c_1\vec{a}_1 + c_2\vec{a}_2$$

$$\vec{a}_3 \cdot \vec{u} \overset{?}{=} 0 \Rightarrow (c_1\vec{a}_1 + c_2\vec{a}_2)\cdot(\vec{a}_1 \times \vec{a}_2)$$

$$= c_1\vec{a}_1\cdot(\vec{a}_1 \times \vec{a}_2) + c_2\vec{a}_2\cdot(\vec{a}_1 \times \vec{a}_2)$$

$$\Rightarrow \vec{a}_3 \cdot \vec{u} = 0.$$

Case 3: $\vec{a}_2 = c_1\vec{a}_1$ & $\vec{a}_3 = c_2\vec{a}_1$

$$A\vec{u} = \vec{0}$$

$$\begin{bmatrix} \cancel{\vec{a}_1} \\ c_1\vec{a}_1 \\ c_2\vec{a}_1 \end{bmatrix}$$

Ex: $\vec{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\vec{a}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\vec{a}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$A\vec{u} = \vec{0} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Row echelon form of A:

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$u_2 = t \quad ; \quad u_3 = s.$

$u_1 + u_2 + u_3 = 0$

$u_1 + t + s = 0$

$u_1 = -t - s.$

$$\therefore \vec{u} = \begin{bmatrix} -t-s \\ t \\ s \end{bmatrix}$$

$$\Rightarrow \vec{u} = \begin{bmatrix} -t \\ t \\ 0 \end{bmatrix} + \begin{bmatrix} -s \\ 0 \\ s \end{bmatrix}$$

$$= t\underline{\underline{\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}}} + s\underline{\underline{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}}}$$

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$$A^{3\times3} u^{3\times1} = 0^{3\times1}$$

Case 1.

Rows of A are l.i

$\Rightarrow$ $\vec{u} = \vec{0}$ is the only soln

Case 2: Two rows of A are l.i.
(Row vectors)

One of the 3 vectors is l.c
Combinn of the other 2 l.i
vectors

$\vec{u} = \vec{a}_1 \times \vec{a}_2$ is a soln to $A\vec{u}=\vec{0}$

Infinitely many.

Case 3:

$\vec{a}_1$; $\vec{a}_2 = c_1\vec{a}_1$ & $\vec{a}_3 = c_2\vec{a}_1$

$\Rightarrow$ $A\vec{u} = \vec{0}$

is the set of vectors in a plane for which $\vec{a}_1$ is an orthogonal vector.

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