$$2x_1 + x_2 = 3$$
$$x_1 + 2x_2 = 3$$

$$Ax = b$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$x_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

If $\vec{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ & $\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ then

we say $\vec{w} = \alpha \vec{u} + \beta \vec{v}$, α, β are Scalars, is a l.c. of $\vec{u}$ & $\vec{v}$

$$\Rightarrow \begin{pmatrix} 3 \\ 3 \end{pmatrix} = x_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

We need to find the scalars x_1 & x_2 s.t

$$x_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

In general

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

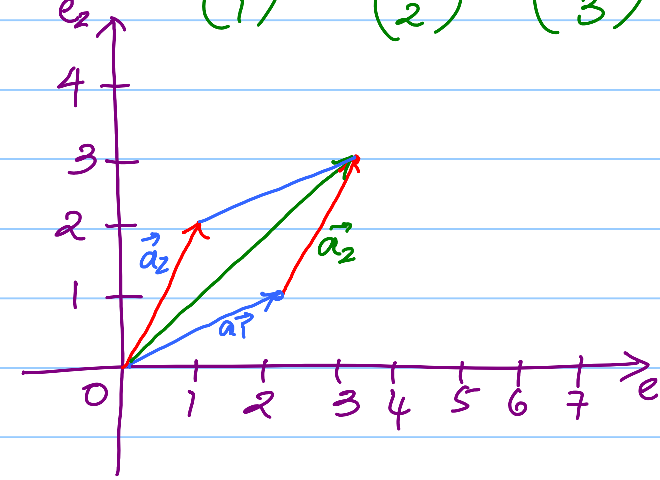
$\vec{a}_1 = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix}$ & $\vec{a}_2 = \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix}$

By $A\vec{x} = \vec{b}$ we mean

$\vec{b} = x_1 \vec{a}_1 + x_2 \vec{a}_2$

$\begin{pmatrix} 3 \\ 3 \end{pmatrix} = x_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

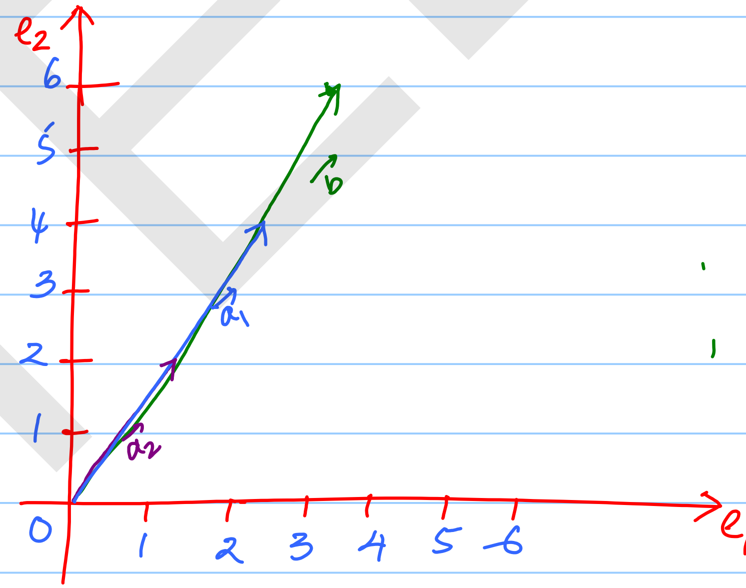
$1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + 1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$



Ex: 2: $\quad 2x_1 + x_2 = 3$

$\qquad\quad 4x_1 + 2x_2 = 6.$

$$x_1 \begin{pmatrix} 2 \\ 4 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$



$\vec{a}_1$, $\vec{a}_2$ & $\vec{b}$ are all along the same direction.

$$\begin{pmatrix}2\\4\end{pmatrix} = 2\begin{pmatrix}1\\2\end{pmatrix}$$

$$\Rightarrow \vec{a_2} = 2(\vec{a_1})$$

$\therefore$ $\vec{a_1}$ & $\vec{a_2}$ are linearly dep. vectors.

$$x_1\begin{pmatrix}2\\4\end{pmatrix} + x_2\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}3\\6\end{pmatrix}$$

$$2x_1\begin{pmatrix}1\\2\end{pmatrix} + x_2\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}3\\6\end{pmatrix}$$

$$(2x_1+x_2)\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}3\\6\end{pmatrix}$$

$x_1 = 1,\ x_2 = 1$

$x_1 = 1.5,\ x_2 = 0$

$x_1 = 0,\ x_2 = 3$

$\vdots$

Infinitely many solutions

Ex: 3: $2x_1 + x_2 = 3$
$2x_1 + x_2 = 4.$

$$x_1 \begin{pmatrix} 2 \\ 2 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

Find Scalars x_1 & x_2 S.t

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} = x_1 \begin{pmatrix} 2 \\ 2 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

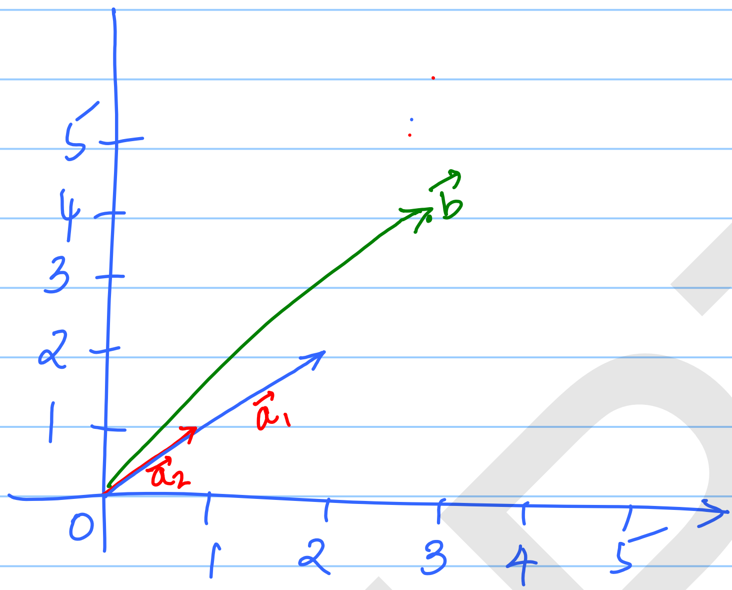
$$\begin{pmatrix} 2 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\therefore \vec{a}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 2\vec{a}_2 = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$\vec{a}_1$ & $\vec{a}_2$ are linearly dependent

$$(2x_1 + x_2)\underline{\underline{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \longrightarrow (*)$$

Inconsistent sys. No x_1, x_2 exist s.t $(*)$ is true.



Inconsistent Sys. of eqns.

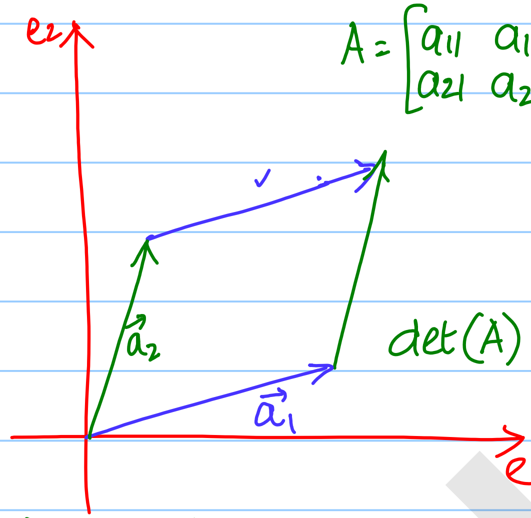
Suppose we have

$$a_{11} x_1 + a_{12} x_2 = b_1$$
$$a_{21} x_1 + a_{22} x_2 = b_2$$

$$x_1 \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Sys. with Unique Soln:

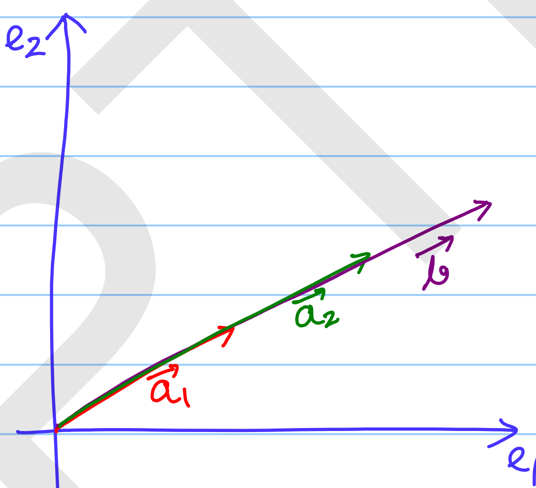
$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$



$\det(A) \neq 0$

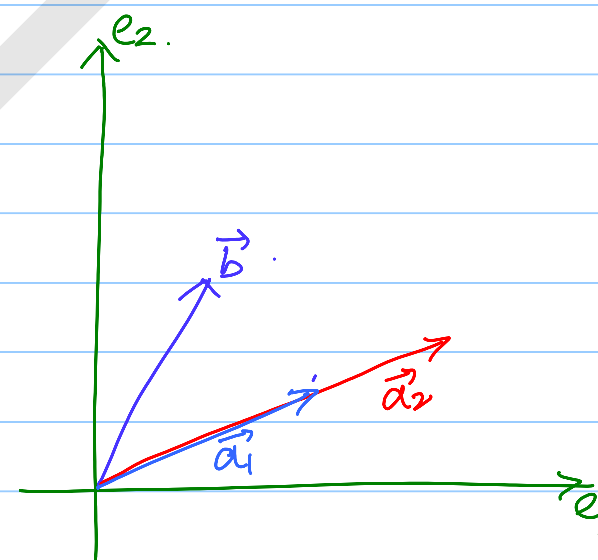
$$\vec{b} = x_1 \vec{a_1} + x_2 \vec{a_2}$$

Sys. with infinitely Many Solns.



$$\vec{b} = x_1 (\vec{a_1}) + x_2 \vec{a_2}$$

Sys. with no Solution.



$$\vec{b} \neq x_1 (\vec{a_1}) + x_2 (\vec{a_2}).$$

Homogeneous . of eqns.

$$A\vec{x} = \vec{0}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$a_{11} x_1 + a_{12} x_2 = 0 \longrightarrow (1)$$
$$a_{21} x_1 + a_{22} x_2 = 0 . \longrightarrow (2)$$

$$\begin{bmatrix} a_{11} & a_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \leftarrow$$

$\begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix}$ & $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ are Orthogonal

$$\begin{bmatrix} a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$\begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix}$ & $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ are orthogonal.

$\vec{u}$ & $\vec{v}$ are orthogonal if

$$\vec{u} \cdot \vec{v} = 0 \Rightarrow u^T v = v^T u = 0$$

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\vec{x} \cdot \vec{a}_{r_1} = 0$

$\vec{x} \cdot \vec{a}_{r_2} = 0$

If $\vec{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ then

$\vec{x} \cdot \vec{a}_{r_1} = 0$

$\vec{x} \cdot \vec{a}_{r_2} = 0$

If $\vec{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is the only

Vector Orthogonal to $\vec{a}_{r_1}$ & $\vec{a}_{r_2}$

then $A\vec{x} = \vec{0}$ has only trivial

Soln.

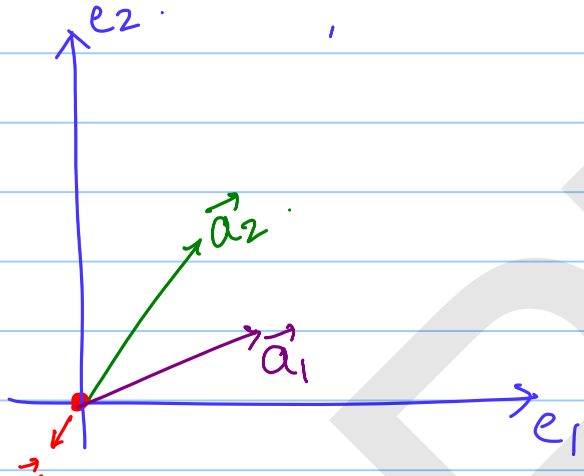
Suppose if $\vec{x} \neq \vec{0}$ is s.t

$\vec{x} \cdot \vec{a}_{r_1} = 0$

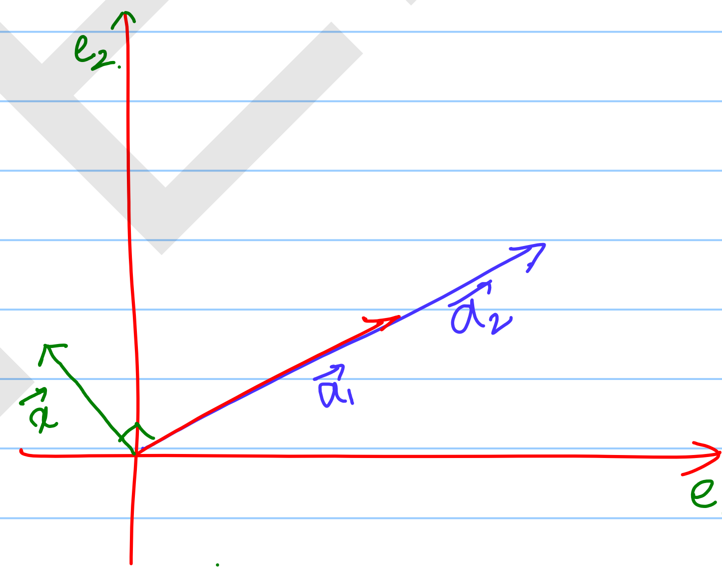
$\vec{x} \cdot \vec{a}_{r_2} = 0$

$Ax = 0$ has non trivial soln.

$A\vec{x} = \vec{0}$



Only trivial Soln Case.



$A\vec{x} = \vec{b}$ $\quad A^{2\times 2} \quad x^{2\times 1} \quad b^{2\times 1}$

Can we have vector $\vec{u}$ s.t $\underline{A\vec{u} = \lambda \vec{u}}$