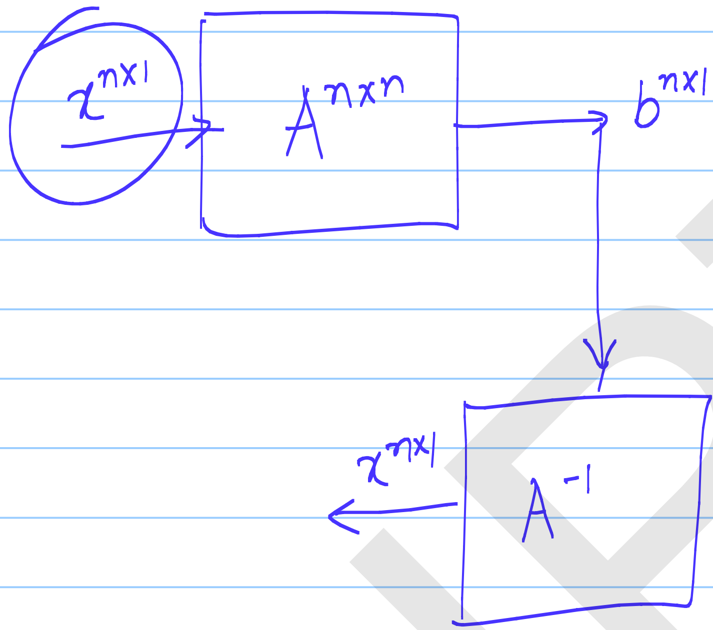
$x^{n\times1}$
$A^{n\times n}$
$b^{n\times1}$
A^{-1}
$x^{n\times1}$

Suppose $Ax = b$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$a_{11} x_1 + a_{12} x_2 = b_1 \longrightarrow 1$$
$$a_{21} x_1 + a_{22} x_2 = b_2 . \longrightarrow 2$$

$1 \times a_{21} - 2 \times a_{11}$

$$\Rightarrow \quad \cancel{a_{11} a_{21} x_1} + a_{12} a_{21} x_2 = a_{21} b_1$$
$$(-) \quad \cancel{a_{11} a_{21} x_1} + a_{22} a_{11} x_2 = a_{11} b_2 .$$

$$x_2 \left(\underbrace{a_{12} a_{21} - a_{22} a_{11}} \right) = \underbrace{a_{21} b_1 - a_{11} b_2} .$$

$$\Rightarrow x_2 = \frac{a_{11} b_2 - a_{21} b_1}{\boxed{a_{11} a_{22} - a_{21} a_{12}}}$$

$$x_1 = \frac{b_1 a_{22} - b_2 a_{12}}{a_{11} a_{22} - a_{21} a_{12}}$$

Denominator of expression for x_1 & x_2 = Det (A).

$$x_1 = \frac{b_1 a_{22} - b_2 a_{12}}{\det(A)}$$

$$x_2 = \frac{a_{11} b_2 - a_{21} b_1}{\det(A)}$$

$$x_1 = \frac{b_1 a_{22} - b_2 a_{12}}{\det(A)}.$$

$$x_1 = \frac{\det\left(\begin{bmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{bmatrix}\right)}{\det(A)}.$$

$$x_2 = \frac{\det\begin{pmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{pmatrix}}{\det(A)}$$

$$a_{11} x_1 + a_{12} x_2 = b_1$$
$$a_{21} x_1 + a_{22} x_2 = b_2.$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$A^{n \times n}$ matrix

$$Ax = b \qquad A^{n\times n} x^{n\times 1} = b^{n\times 1}$$

CRAMER's rule.

$$A^{n\times n} x^{n\times 1} = b^{n\times 1}$$

$$x_i = \frac{\det\left(\left[A_1 \; A_2 \; \cdots \; \overset{i^{th}\, col(A)}{\overset{\downarrow}{b}} \; \cdots \; A_n\right]\right)}{\det\left[\left(A_1 \; A_2 \cdots A_i \cdots A_n\right)\right]}$$

for $i = 1 \ldots n$.

Det $(A^{2\times2})$: Area of a parallelogram whose sides are along the direction of the col. vectors of A.

Suppose we have the homogeneous system of equations

$$Ax = 0$$

$$a_{11} x_1 + a_{12} x_2 = 0$$
$$a_{21} x_1 + a_{22} x_2 = 0.$$

$$x_1 = \frac{\det \begin{bmatrix} 0 & a_{12} \\ 0 & a_{22} \end{bmatrix}}{\det \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}} \Rightarrow 0$$

$\det(A) \neq 0$

$$\underbrace{x_1 = 0; \quad x_2 = 0}_{\text{Only solution to } Ax = 0.}$$ when $\det(A) \neq 0$.

To employ Cramer's rule, the matrix A must be non singular.

$$2x_1 + x_2 = 3$$
$$2x_1 + x_2 = 4.$$

$$\det(A) = \det\left(\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}\right) = 0.$$

$$x_1 = \frac{\det\left(\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix}\right)}{\det(A) = 0.}$$

$$x_1 + x_2 + x_3 = 3$$
$$2x_1 + x_2 - x_3 = 2$$
$$x_1 - x_2 + x_3 = 1$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\det(A) = 1(1-1) - 1(2+1) + 1(-2-1)$$
$$= 0 - 3 + (-3) = -6.$$

$$x_1 = \frac{\det\begin{bmatrix} 3 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}}{\det(A) = -6}$$

$$= \frac{3\cancel{(1-1)}^{0} - 1(2+1) + 1(-2-1)}{-6}$$

$$= \frac{-3-3}{-6} = 1.$$

$$x_2 = \frac{\det\begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix}}{\det(A) = -6.}$$

$$\Rightarrow \frac{1(2+1) - 3(2+1) + 1(2-2)}{-6.}$$

$$= \frac{3 - 9 + 0}{-6} = \frac{-6}{-6} = 1.$$

Similarly x_3 can be obtained by Cramer's rule & $x_3 = 1$.