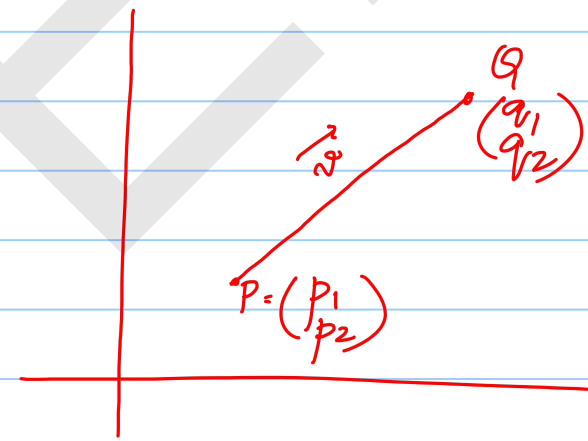
How do we combine vectors?

Linear Combination of vectors.

Recall that adding vectors or scaling vectors was completely different from adding or scaling a point.

Adding points - Coordinate dep operation.



$$P + \vec{v} = Q.$$

$$\Rightarrow Q - P = \vec{v} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} - \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

$$= \begin{pmatrix} q_1 - p_1 \\ q_2 - p_2 \end{pmatrix}$$

We add a vector $\vec{v}$ to the point P to reach Q.

Let R be a new point defined as

$$R = P + \left(\frac{1}{2}\right)\vec{v}$$

$$= P + \frac{1}{2}(Q - P)$$

$$= P - \frac{P}{2} + \frac{Q}{2}$$

$$= \frac{P}{2} + \frac{Q}{2} = \frac{P+Q}{2}$$

$$= \frac{1}{2}(P+Q)$$

$\Rightarrow$ Midpoint of the line joining the points P & Q.

Example:

$$P = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \qquad Q = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$R = \frac{1}{2}[P+Q] = \begin{bmatrix} \frac{1+3}{2} \\ \frac{4+6}{2} \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$R = P + k\vec{v} = P + k(Q-P)$$
$$= (1-k)P + kQ.$$

$R = (1-k)P + kQ$ is ALWAYS a point on the line through P & Q.

k & $(1-k)$ are called the Coeffts.

Defn: A weighted Sum of points where the coefficients sum to 1 - BARYCENTRIC Combination.

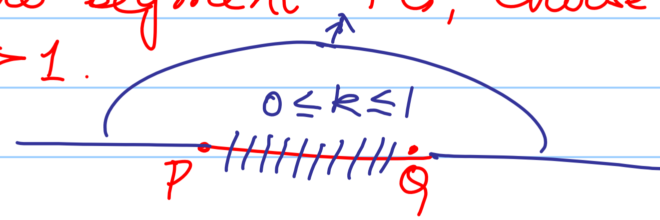
$$R = (1-k)P + kQ$$

- Linear Interpolation.

If $0 \le k \le 1$ we call this combination as the CONVEX Combination.

* Ex: Average of 2 nos → Center of gravity of 2 points.

* To define points outside of the line segment PQ, choose $k < 0$ or $k > 1$.



Combining Vectors:

Let $\vec{u}$ & $\vec{v}$ be a two component vectors

$$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \& \quad \vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$\vec{u} + \vec{v} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$= \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \end{pmatrix} = w = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

where $w_1 = u_1 + v_1$ $\quad w_2 = u_2 + v_2$

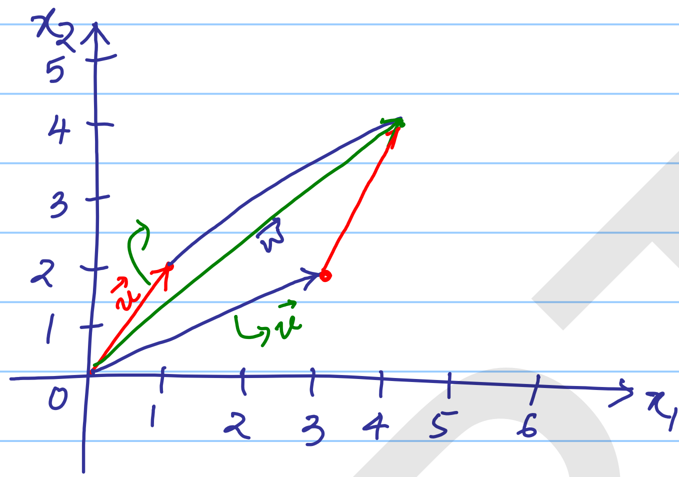
$\vec{w} \rightarrow$ Another vector - Resultant of $\vec{u}$ & $\vec{v}$

For some real valued Scalars α_1 & α_2

$$\alpha_1 \vec{u} + \alpha_2 \vec{v} = \alpha_1 \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \alpha_2 \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$\vec{z} = \begin{pmatrix} \alpha_1 u_1 + \alpha_2 v_1 \\ \alpha_1 u_2 + \alpha_2 v_2 \end{pmatrix}$$

$\vec{z} = \alpha_1 \vec{u} + \alpha_2 \vec{v} \Rightarrow$ Linear Combination of $\vec{u}$ & $\vec{v}$



$\vec{u} = \begin{pmatrix}1\\2\end{pmatrix}$ $\qquad \vec{v} = \begin{pmatrix}3\\2\end{pmatrix}$

$\vec{w} = \vec{u} + \vec{v} = \begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}4\\4\end{pmatrix}$

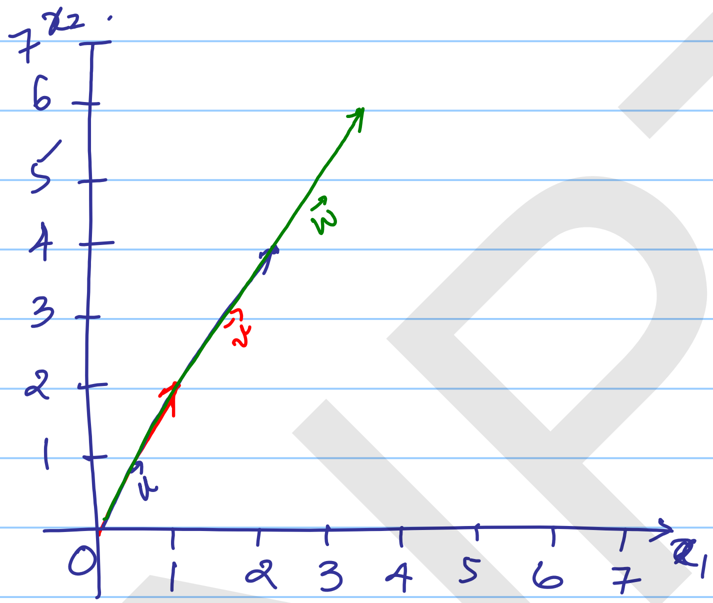
$\alpha_1 = 3 \qquad \alpha_2 = -1$

$$\vec{z} = \alpha_1 \vec{u} + \alpha_2 \vec{v}$$
$$= 3\begin{pmatrix}1\\2\end{pmatrix} + (-1)\begin{pmatrix}3\\2\end{pmatrix}$$
$$= \begin{pmatrix}3-3\\6-2\end{pmatrix} = \begin{pmatrix}0\\4\end{pmatrix}$$

Suppose if $\vec{u} = \begin{pmatrix}1\\2\end{pmatrix}$ and $\vec{v} = \begin{pmatrix}2\\4\end{pmatrix}$

what is $\vec{w} = \vec{u} + \vec{v}$?

$$\vec{w} = \overset{\vec{u}}{\begin{pmatrix}1\\2\end{pmatrix}} + \overset{\vec{v}}{\begin{pmatrix}2\\4\end{pmatrix}} = \overset{\vec{w}}{\begin{pmatrix}3\\6\end{pmatrix}}.$$



$\vec{u} = \begin{pmatrix}1\\2\end{pmatrix}$ & $\vec{v} = \begin{pmatrix}3\\2\end{pmatrix}$ $\vec{w} = \begin{pmatrix}4\\4\end{pmatrix}$

If in 2D, suppose $\vec{u}$ & $\vec{v}$ are 2 vectors that are in the same direction

$\vec{u} = \alpha\vec{v}$ α: Scalar

we call $\vec{u}$ & $\vec{v}$ are linearly dependent vectors.

If $\vec{u} \neq \alpha\vec{v}$ for some scalar α we say that $\vec{u}$ & $\vec{v}$ are linearly indep

If 2 vectors are l.i in 2D, we say that the parallelogram formed encloses an area.

If the vectors are l. dep. then no parallelogram is formed.
Area = 0.

If the vectors are l.i. in 2D, we can reach any destination in 2D ie.,
We can obtain every vector in 2D as linear combin^n of the l.i. vectors.

Any vector
$\vec{w} = \alpha_1 \vec{u} + \alpha_2 \vec{v}$ for
α_1, α_2 scalars &
$\vec{u}$ & $\vec{v}$ l.i. vectors.