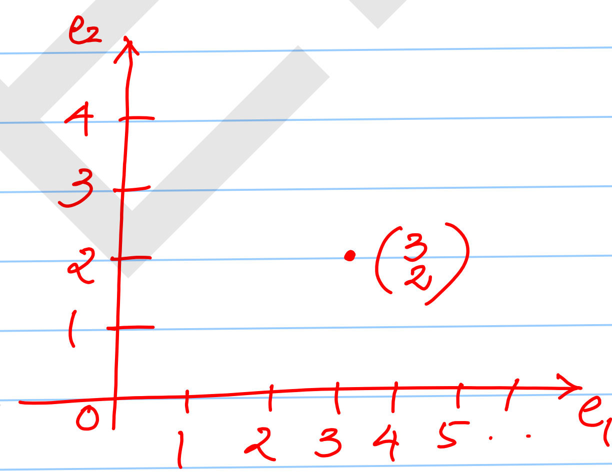
Points:

Point: Most basic geometric entity that indicates a particular location in a coordinate system.

→ Reference to a location

For ex: $A = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ we mean the following by saying $A\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ as



$A\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ → 3 units along the e_1-axis and 2 units along the e_2-axis

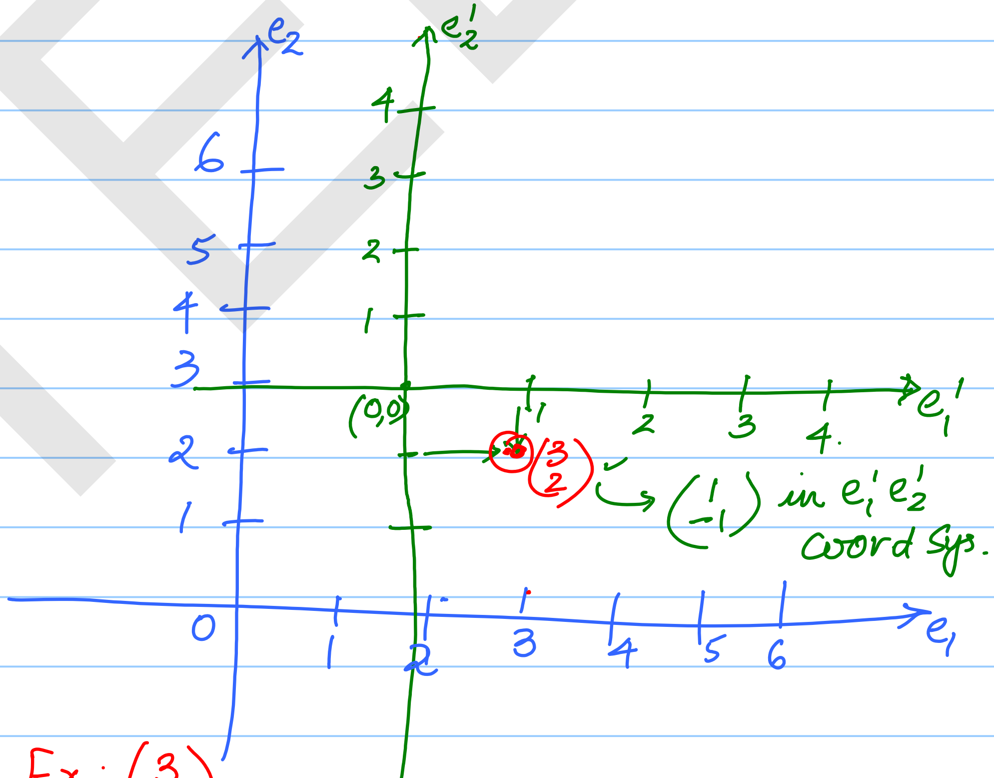
In general, we have

$$P = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

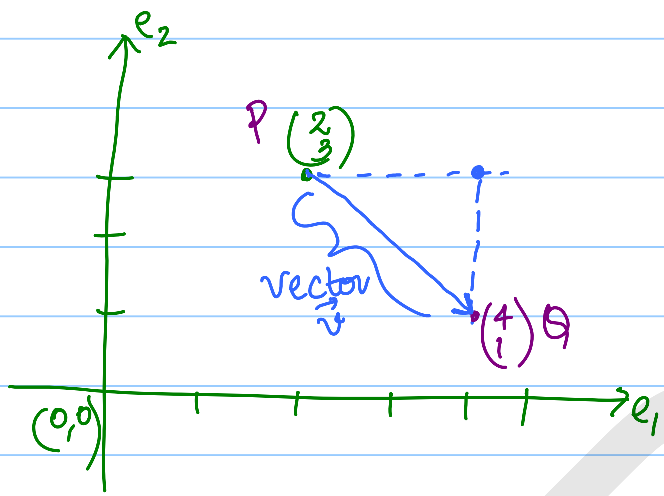
where $p_1 \rightarrow$ Units along e_1 - axis

$p_2 \rightarrow$ Units along e_2 - axis

The coordinates p_1 & p_2 depend on the location of coordinate origin



Ex: $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$



How do we move from $\begin{pmatrix}2\\3\end{pmatrix}$ to $\begin{pmatrix}4\\1\end{pmatrix}$?

$$\begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}2\\-2\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix}.$$

To move from $\begin{pmatrix}2\\3\end{pmatrix}$ to $\begin{pmatrix}4\\1\end{pmatrix}$ we move along the direction of $\begin{pmatrix}2\\-2\end{pmatrix}$

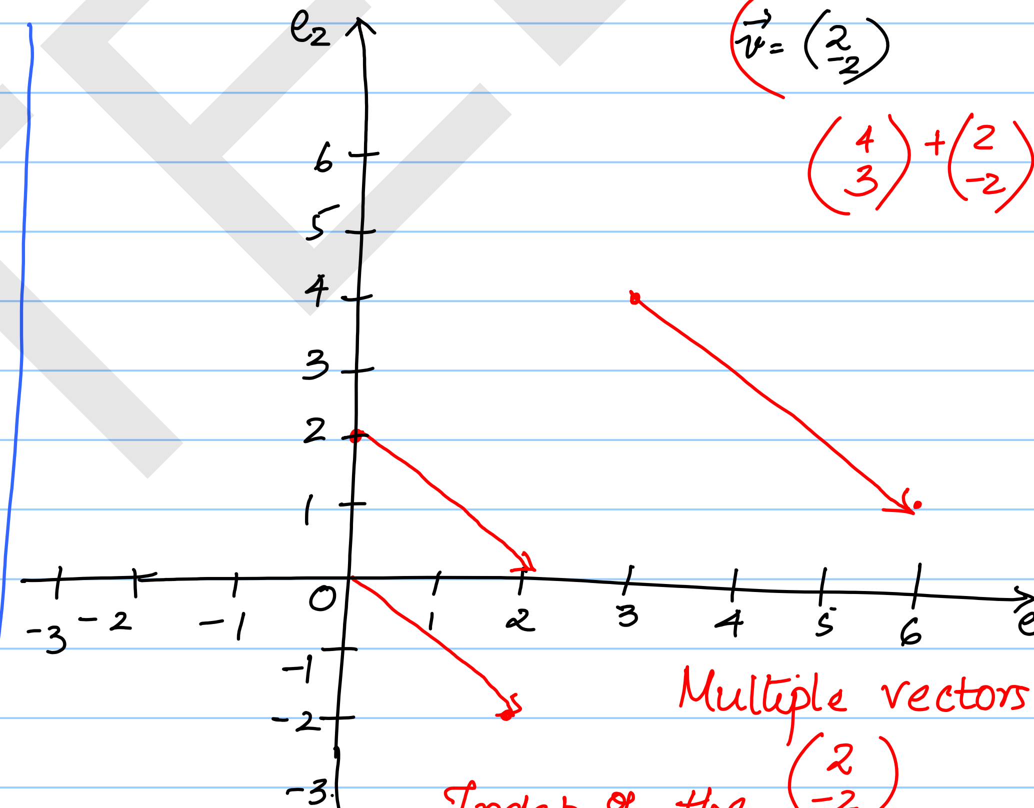
i.e., $P = \begin{pmatrix}p_1\\p_2\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix} \longrightarrow Q = \begin{pmatrix}q_1\\q_2\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix}$

$$P + \vec{v} = Q \Rightarrow \vec{v} = Q - P$$

$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} p_1 + v_1 \\ p_2 + v_2 \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ Difference between points describing a direction & a distance $\rightarrow$ Displacement.

What is it to say the vector $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$?



$\vec{v} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$\begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

Multiple vectors $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$

Indep of the coordinate

The point $P = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ → Location is fixed.

However $\vec{v} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ Can be anywhere in the space

A vector has no fixed position Spatially

How do we differentiate a point & a vector?

Points describe locations & are Coordinate dependent

Vectors are coordinate indep entities.

Coordinate indep operations

Done on vectors

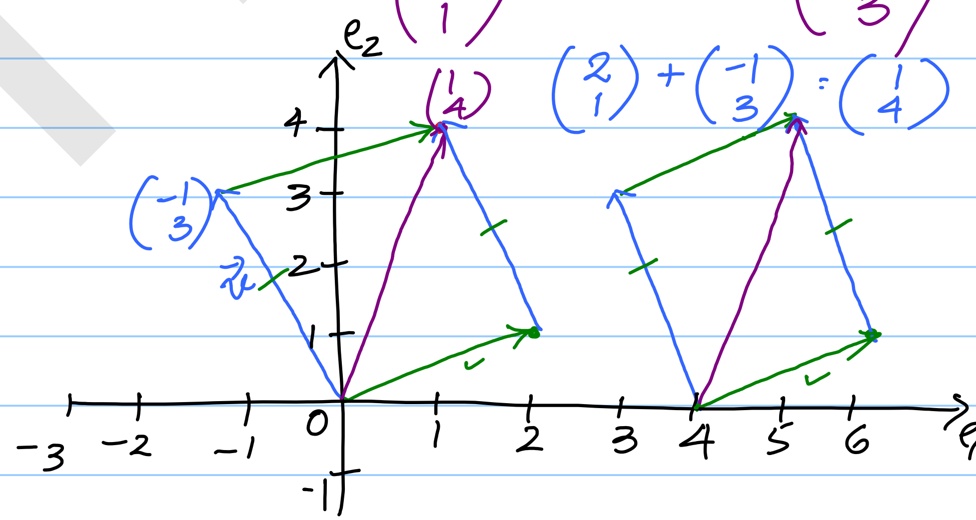
* Point $P\begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$ & Point $Q\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$

$$\vec{v} = Q - P = \begin{pmatrix} q_1 - p_1 \\ q_2 - p_2 \end{pmatrix}$$

* If we add or subtract 2 vectors, we get another vector

Consider $\vec{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$



The resultant of $\vec{u}$ & $\vec{v}$

$\vec{u}+\vec{v} \Rightarrow$ Diagonal of a ||gm formed by the vectors $\vec{u}$ & $\vec{v}$

* Adding a vector to a point results in another point

* Scaling a vector by a const. k.

$$k\vec{v} = k\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} k v_1 \\ k v_2 \end{pmatrix} \rightarrow \text{Another vector}$$

Adjusts the length of a vector

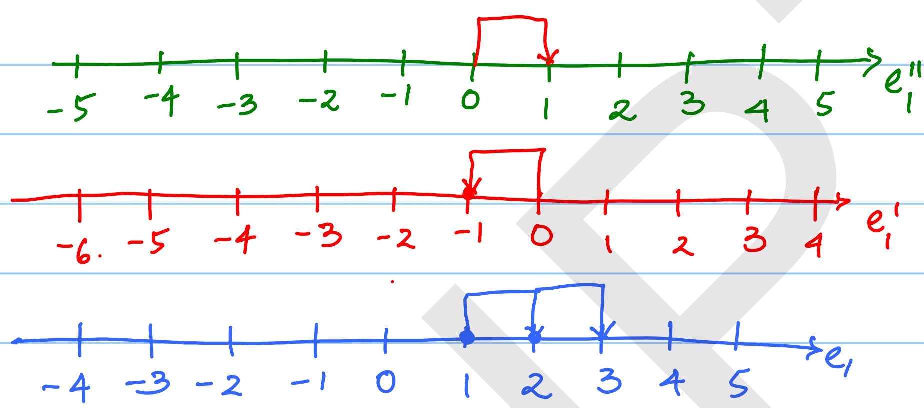
$k > 0 \rightarrow$ Direction remains same

$k < 0 \rightarrow$ Direction is reversed

$k = 0 \Rightarrow$ zero vector $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Coordinate dep. operations.

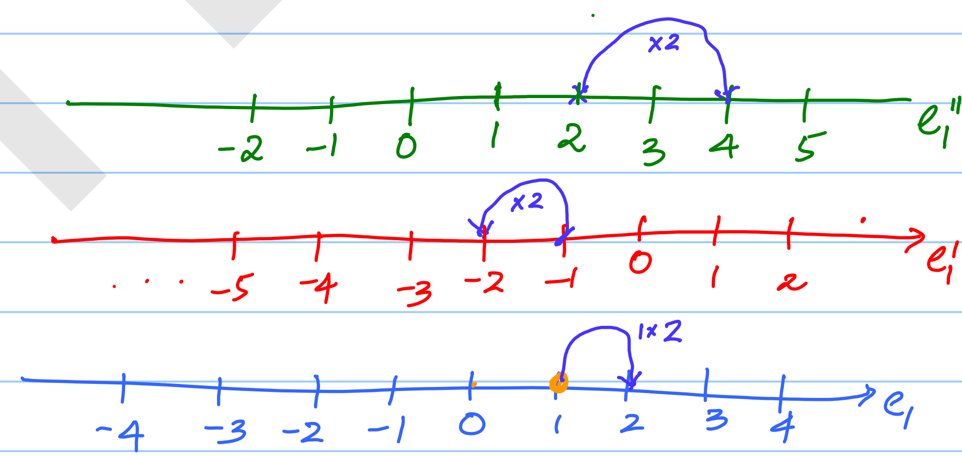
Addition of points - Not well defined



P=1 Q=2 (Reference axis)

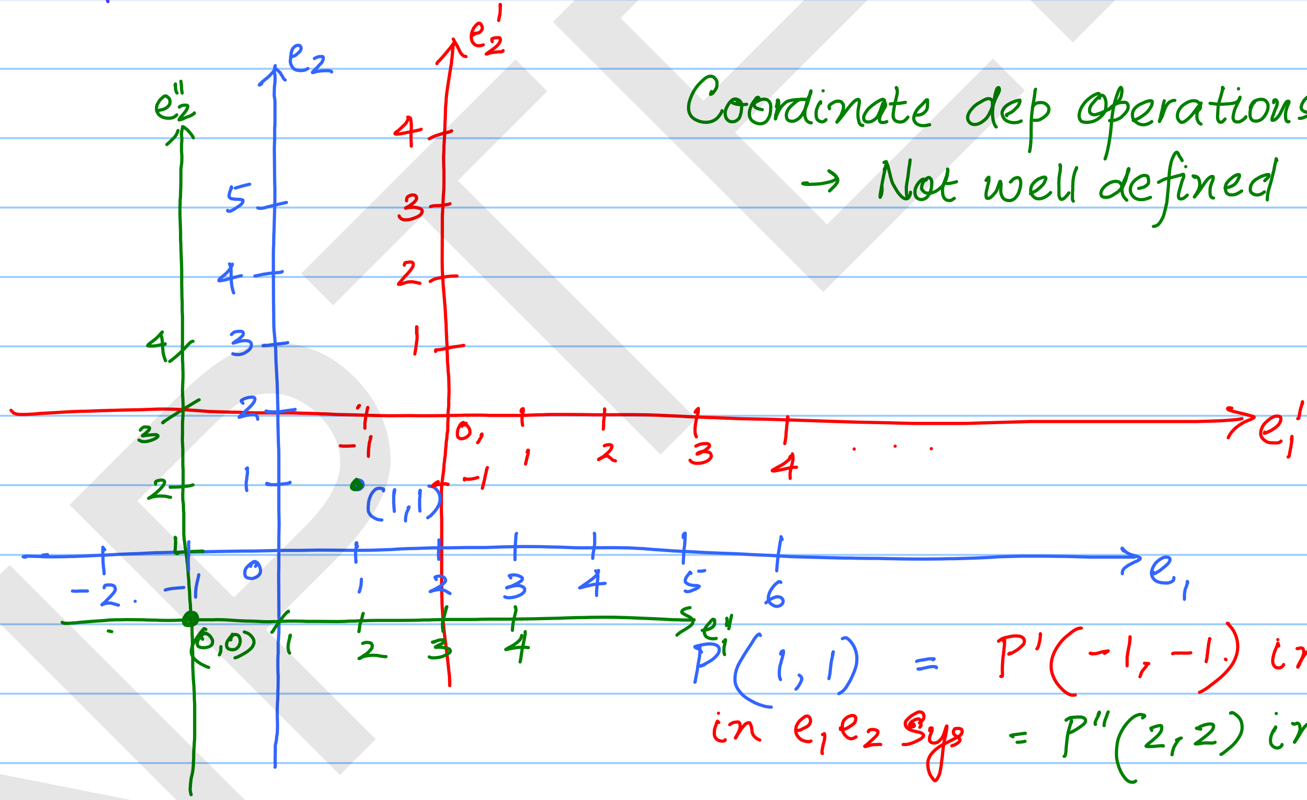
Scaling:

not well-defined



Scaling factor $k = 2$. (Ref axis)

2- axis Sys. e_1 & e_2.



Coordinate dep operations

→ Not well defined

$P(1,1) = P'(-1,-1)$ in $e_1'e_2'$ Sys.

in $e_1 e_2$ Sys $= P''(2,2)$ in $e_1''e_2''$ Sys

To summarize

→ Notion of point → Fixed Location entity → Coordinate dep operns
$\vec{\text{vector}}$ - Not spatially fixed. → Coordinate indep ops
vector add & scaling

⇓

Displacement → Direction & length