3D

A vector $\vec{x}$ in 3D is defined as

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Set of all vectors in 3D space: $\mathbb{R}^3$

2D space: $\mathbb{R}^2$

* Suppose $\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$ & $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

Vector Addition: $\vec{w} = \vec{u} + \vec{v}$

$$\vec{w} = \vec{u} + \vec{v} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$\vec{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} u_1+v_1 \\ u_2+v_2 \\ u_3+v_3 \end{pmatrix}$$

Scalar Multiplication:

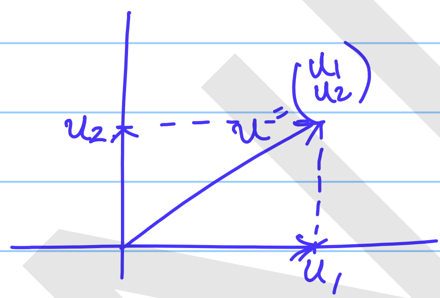
Suppose c is a scalar, then

$$\vec{z} = c\vec{u} = c\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} cu_1 \\ cu_2 \\ cu_3 \end{pmatrix}$$

In 2D we expressed any vector

$$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = u_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + u_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\Rightarrow u_1 \vec{e_1} + u_2 \vec{e_2}$$



In 3D:

$$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = u_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + u_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + u_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= u_1 \vec{e_1} + u_2 \vec{e_2} + u_3 \vec{e_3}$$

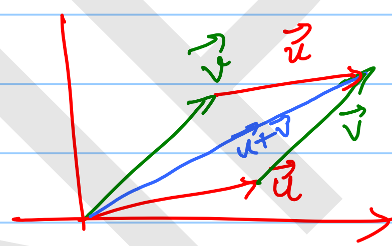
$\vec{e_1} : \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\vec{e_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ & $\vec{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

2D

$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ & $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$$\vec{w} = \vec{u} + \vec{v} = \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \end{pmatrix}$$

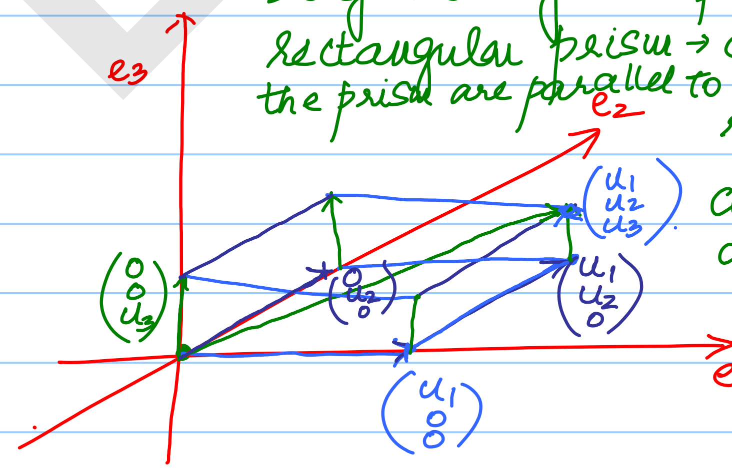
Diagonal of the parallelogram formed by $\vec{u}, \vec{v}$



3D

$$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = u_1\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + u_2\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + u_3\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

→ Diagonal segment of a rectangular prism → Edges of the prism are parallel to the reference or coordinate axes.



Linear Combination of vectors in 3D

$\vec{w} = \alpha_1 \vec{u} + \alpha_2 \vec{v}$ α_1, α_2 are Scalars

$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$ & $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

$\vec{w}$: LC of vectors $\vec{u}$ & $\vec{v}$ for Some real scalars α.

If $\vec{u}$ & $\vec{v}$ are in R^3, then $\vec{u}$ & $\vec{v}$ are linearly dep if $\vec{u} = k\vec{v}$ for Some scalar k.

If $\vec{u}, \vec{v}$ and $\vec{w}$ are vectors in R^3, then they are linearly dep if One of them is a linear combination of the other two.

Ex 1: $\vec{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\vec{v} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$, $\vec{w} = \begin{pmatrix} 3 \\ 6 \\ 9 \end{pmatrix}$

$\vec{v} = 2\vec{u}$, $\vec{w} = 3\vec{u}$

Ex 2: $\vec{u} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ $\vec{w} = \begin{pmatrix} 5 \\ 13 \\ 9 \end{pmatrix}$

$\vec{u} + 3\vec{v} = \vec{w} \Rightarrow \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 12 \\ 6 \end{pmatrix} = \begin{pmatrix} 5 \\ 13 \\ 9 \end{pmatrix}$

$\vec{u}, \vec{v}$ & $\vec{w}$ are linearly dep.

We see that the vectors

$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ & $e_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

are linearly indep. vectors.

For a vector $\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

$$\|\vec{u}\| = \sqrt{u_1^2 + u_2^2}$$

Analogously for a vector $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{v^T v} = \sqrt{v \cdot v}$$

If $\vec{u}$ & $\vec{v}$ are 2 vectors, then

$$\vec{u} \cdot \vec{v} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

$$= \|\vec{u}\| \|\vec{v}\| \cos\theta$$

If $\cos\theta = 0$, $\vec{u}$ and $\vec{v}$ are Orthogonal.

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}.$$