Consider $\vec{u}, \vec{v}, \vec{w}$ in 3D

$$\vec{r}_1 = \vec{u}$$

$$\vec{r}_2 = \vec{v} - (\mathrm{Proj}_{r_1} \vec{v})\vec{r}_1$$

$$\vec{r}_3 = \vec{w} - (\mathrm{Proj}_{r_1} \vec{w})\vec{r}_1 - (\mathrm{Proj}_{\vec{r}_2} \vec{w})\vec{r}_2$$

Q: What happens if $\vec{u}, \vec{v}, \vec{w}$ are linearly dep?

$\vec{u}, \quad \vec{v} = \alpha \vec{u}, \quad \vec{w} = \beta \vec{u}$

Verify $\vec{r}_3 = \vec{0}$

$\vec{r}_1 = \vec{u}$

$$\vec{r}_2 = \vec{v} - \mathrm{Proj}_{\vec{r}_1} \vec{v}\, \vec{r}_1$$

$$= \vec{v} - \frac{\vec{v} \cdot \vec{r}_1}{\vec{r}_1 \cdot \vec{r}_1} \vec{r}_1$$

$$= \vec{v} - \frac{\alpha \cancel{\vec{u} \cdot \vec{u}}}{\cancel{\vec{u} \cdot \vec{u}}} \vec{u}$$

$$= \vec{0}$$

$A = QR$

$A^{-1} = (QR)^{-1}$

$= R^{-1} Q^{-1}$

$$\boxed{A^{-1} = R^{-1} Q^{T}}$$

if the matrix A is invertible.

$A^{-1} = \frac{Adj(A)}{det(A)}$.

Cramer's Rule: $A = [A_1 \; A_2 \; A_3]$

$Ax = b \qquad A^{3\times 3}, \; x^{3\times 1}, \; b^{3\times 1}$.

$$x_1 = \frac{\det\begin{bmatrix} b_1 & A_2 & A_3 \\ b_2 & & \\ b_3 & & \end{bmatrix}}{\det\begin{bmatrix} A_1 & A_2 & A_3 \end{bmatrix}}$$

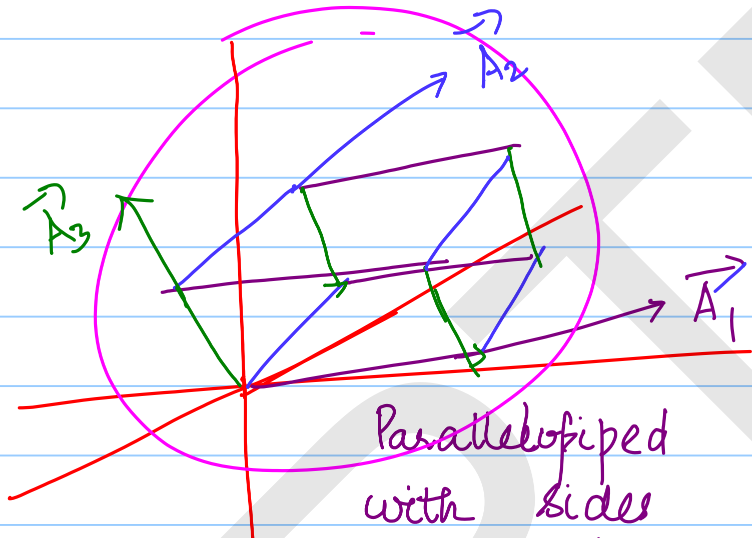
$Ax = b$; $A^{3\times3}$, A invertible

x_1 = Vol. of parallelopiped with the sides along the directions b, A_2, A_3

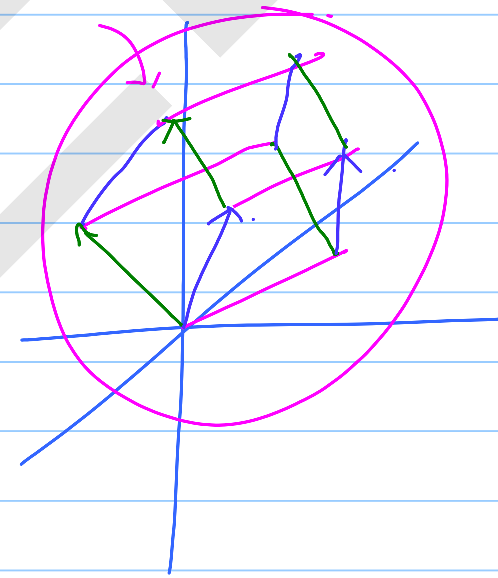
$$\overline{\text{Vol. of parallelopiped with sides along the col. of } A \; A_1, A_2, A_3}$$

$$x_2 = \frac{\det \begin{bmatrix} A_1 & \begin{matrix} b_1 \\ b_2 \\ b_3 \end{matrix} & A_3 \end{bmatrix}}{\det(A)}.$$

$$x_3 = \frac{\det \begin{bmatrix} A_1 & A_2 & \begin{matrix} b_1 \\ b_2 \\ b_3 \end{matrix} \end{bmatrix}}{\det(A)}.$$



Parallelopiped
with sides
along A_1, A_2 & A_3



Eigenvalues and eigenvectors:

Let $A^{3\times3}$ real matrix

$$A\vec{x} = \lambda\vec{x}$$

$\det(A-\lambda I) = 0 \rightarrow$ Cubic equ.

3 roots.

(i) All roots are real & distinct. $\lambda_1, \lambda_2, \lambda_3$ distinct

$\lambda_1 \longrightarrow \vec{u}$

$\lambda_2 \longrightarrow \vec{v}$

$\lambda_3 \longrightarrow \vec{w}$

$$A\vec{u} = \lambda_1\vec{u}$$
$$A\vec{v} = \lambda_2\vec{v}$$
$$A\vec{w} = \lambda_3\vec{w}$$

Case 2. One of the roots is repeated. All 3 roots are real.

$$\lambda_1, \underbrace{\lambda_2, \lambda_2}.$$

We may get 2 eigenvectors corresponding to λ_2.

$$\underbrace{(A-\lambda_2 I)}\vec{x} = 0.$$

$$B\vec{x} = 0.$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad (\lambda-1)^2 = 0$$

$$\lambda = 1,$$

Case 3: One real root & a pair of complex conjugate roots

$\rightarrow$ Only one real eigenvector.