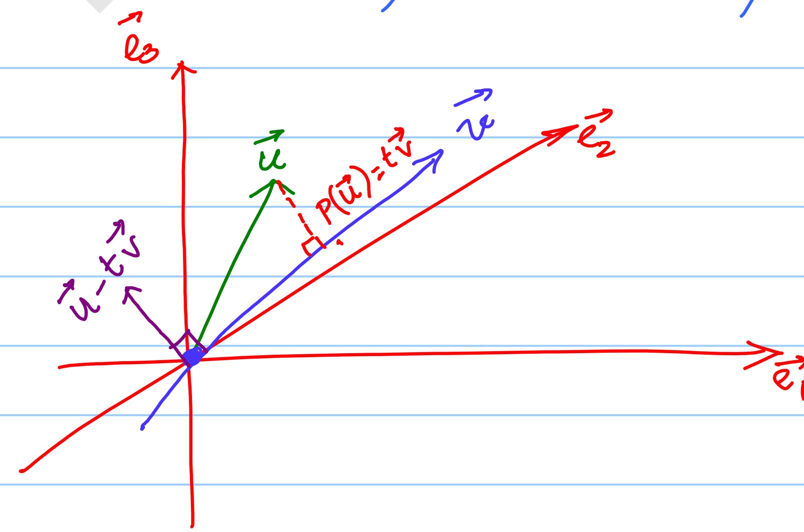
Recall that if $\vec{u} \cdot \vec{v} = 0$, $\vec{u}$ and $\vec{v}$ are orthogonal to each other. $\vec{u} \perp \vec{v}$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \, \|\vec{v}\|}$$

Projection of a vector $\vec{u}$ onto $\vec{v}$

Suppose $\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$ & $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$



Proj of $\vec{u}$ onto $\vec{v}$ is that multiple of $\vec{v}$, $t\vec{v}$ s.t $(\vec{u}-t\vec{v})$ is orthog to $\vec{v}$

$$\Rightarrow \quad (\vec{u}-t\vec{v})\cdot\vec{v} = 0$$

$$\vec{u}\cdot\vec{v} - t(\vec{v}\cdot\vec{v}) = 0$$

$$\Rightarrow \quad t = \frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}$$

Proj of $\vec{u}$ onto $\vec{v}$

$$= P(\vec{u}) = \frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\,\vec{v}$$

This is the multiple of $\vec{v}$ that best approximates $\vec{u}$

Projection of a given vector $\vec{x}$ onto a plane passing through the origin.

Let $\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$ be a non zero vector

and let P_π be the plane passing through the origin & orthogonal to $\vec{u}$

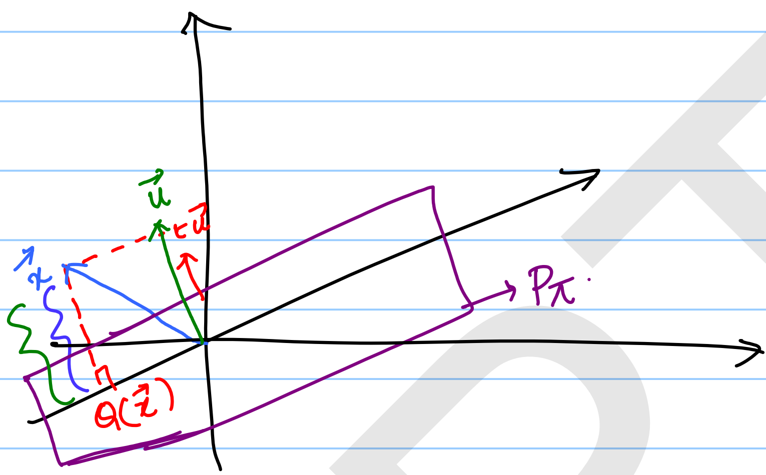
Let $\vec{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$ be a vector in the plane P_π.

Since $\vec{y}$ is in the plane P_π & P_π is orthogonal to $\vec{u}$, we have

$\therefore \vec{u} \cdot \vec{y} = 0$

$$\Rightarrow \boxed{y_1 u_1 + y_2 u_2 + y_3 u_3 = 0}$$

Eqn of the plane P_π.



Let $\vec{x}$ be a vector. Let $Q(\vec{x})$ be the proj of $\vec{x}$ onto P_π.

$\Downarrow$

foot of the $\perp$ar from $\vec{x}$ to P_π

Since the line segment joining $\vec{x}$ and $Q(\vec{x})$ is $\perp$ar to P_π and we know that $\vec{u}$ $\perp$ar to P_π, we have

$$\vec{x} - Q(\vec{x}) \text{ is parallel to } \vec{u}$$

$$\Rightarrow \vec{x} - Q(\vec{x}) = t\vec{u}$$

$$\Rightarrow \vec{x} - t\vec{u} = Q(\vec{x})$$

$Q(\vec{x})$ is in P_π.

$\therefore$ $\vec{x} - t\vec{u}$ is $\perp$ar to $\vec{u}$

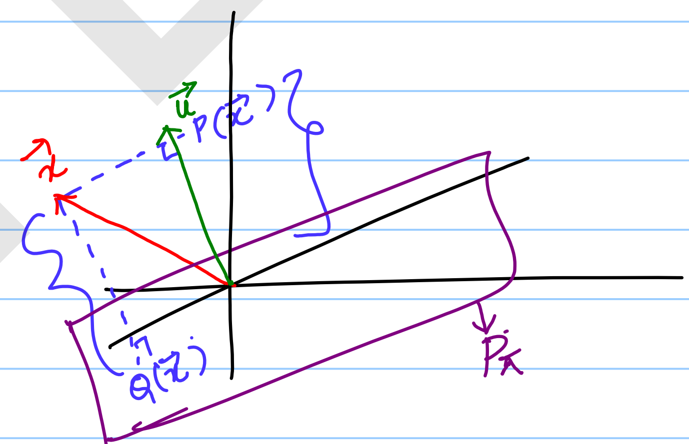
$\Rightarrow t\vec{u}$ is the proj. of $\vec{x}$ onto $\vec{u}$

$\vec{x} - Q(\vec{x}) = P(\vec{x})$, where $P(\vec{x})$ is the proj. of $\vec{x}$ onto $\vec{u}$

Thus we see that

$Q(\vec{x}) = \vec{x} - P(\vec{x})$ is the proj. of $\vec{x}$ onto the plane P_π, passing thro' the origin and orthogonal to $\vec{u}$.

Question: What is the distance from the point $\vec{x}$ to the plane P_π?



The distance b/w $\vec{x}$ and P_π is precisely the length of $P(\vec{x})$

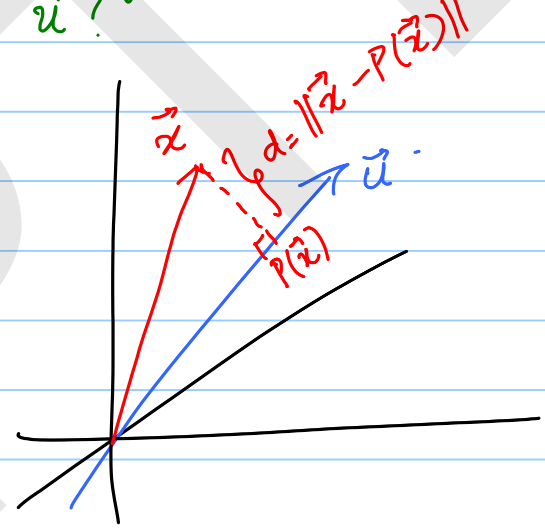
$$\|P(\vec{x})\| = \left|\frac{\vec{x}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\right| \|\vec{u}\|$$

$$= \frac{|\vec{x}\cdot\vec{u}|\,\|\cancel{\vec{u}}\|}{\|\vec{u}\|^{\cancel{2}}}$$

$$= \frac{|\vec{x}\cdot\vec{u}|}{\|\vec{u}\|}$$

$$\boxed{d = \frac{|x_1u_1 + x_2u_2 + x_3u_3|}{\sqrt{u_1^2+u_2^2+u_3^2}}}$$

What is the distance from the point $\vec{x}$ to the line along the nonzero vector $\vec{u}$?



The distance $= \|\vec{x} - P(\vec{x})\|$

$\vec{x} - P(\vec{x})$ is orthogonal to $P(\vec{x})$

$$\|\vec{x} - P(\vec{x})\|^2 = \|\vec{x}\|^2 - \|P(\vec{x})\|^2$$

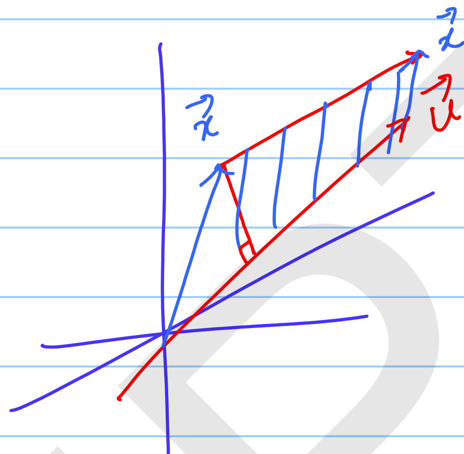
$$= \vec{x}\cdot\vec{x} - \left[\frac{(\vec{x}\cdot\vec{u})\vec{u}}{\vec{u}\cdot\vec{u}}\right]\cdot\left[\frac{\vec{x}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\vec{u}\right]$$

Distance from $\vec{x}$ to line thro $\vec{u}$.

$$= \|\vec{x} - P(\vec{x})\| = \frac{\sqrt{(\vec{x}\cdot\vec{x})(\vec{u}\cdot\vec{u}) - (\vec{x}\cdot\vec{u})^2}}{\|\vec{u}\|}$$

Area of a parallelogram with sides along $\vec{x}$ & $\vec{u}$

$= b \times h.$



$b = \|\vec{u}\|$

$$h = \frac{\sqrt{(\vec{x}\cdot\vec{x})(\vec{u}\cdot\vec{u}) - (\vec{x}\cdot\vec{u})^2}}{\|\vec{u}\|}.$$

$$\therefore \boxed{\text{Area} = \sqrt{(\vec{x}\cdot\vec{x})(\vec{u}\cdot\vec{u}) - (\vec{x}\cdot\vec{u})^2}.}$$