Consider

Ex 1: $2x_1 + x_2 = 0$

$x_1 + 2x_2 = 0.$

Solution to the above:

$\underline{x_1 = 0, \quad x_2 = 0}$ is the only

Soln to the above. $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Let N_A be the set of solutions to $A\vec{x} = \vec{0}$ $\quad N_A = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$

Ex 2: $2x_1 + x_2 = 0$

$2x_1 + x_2 = 0.$ $\quad N_A = \left\{ k\begin{pmatrix} 1 \\ -2 \end{pmatrix} k \in \mathbb{R} \right\}$

Solution to the above.

$x_1 = 1, \quad x_2 = -2$

Non trivial Solution to $A\vec{x} = \vec{0}$

x_1:	0	1	-1	2	-2	...
x_2:	0	-2	2	-4	4	...

Any multiple of $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ is a Soln. $\quad N_A = \left\{ k\begin{pmatrix} 1 \\ -2 \end{pmatrix}, k \in \mathbb{R} \right\}.$

NPTEL

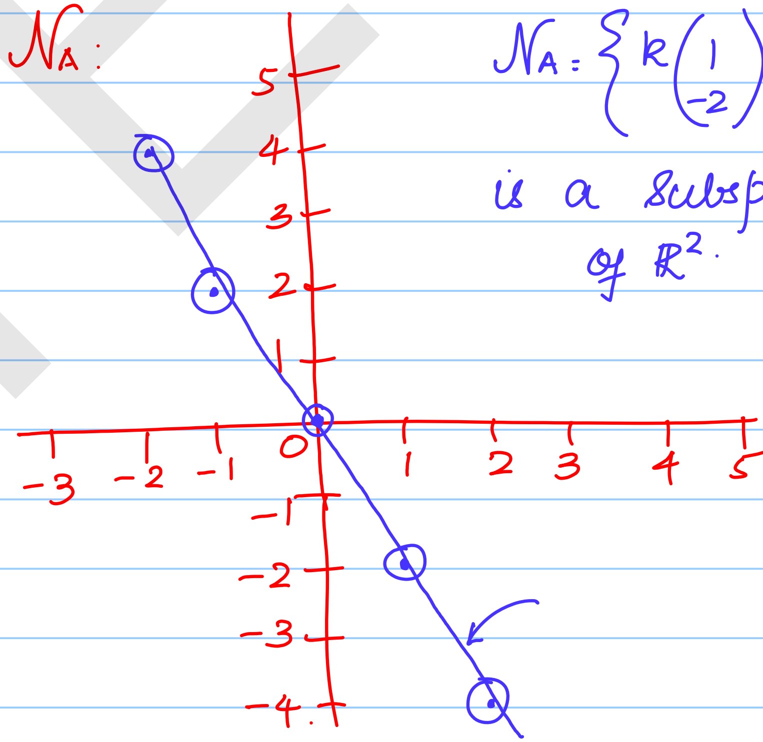
$\mathcal{N}_A = \left\{ \begin{pmatrix}0\\0\end{pmatrix} \right\}$ $\qquad \begin{pmatrix}0\\0\end{pmatrix} \in \mathbb{R}^2$

↳ Subspace of $\mathbb{R}^2$

$\mathcal{N}_A$ in this case Trivial Subspace

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$\mathcal{N}_A$:



$\mathcal{N}_A = \left\{ k\begin{pmatrix}1\\-2\end{pmatrix},\ k \in \mathbb{R} \right\}$

is a subspace of $\mathbb{R}^2$.

NPTEL

Example:

$$x_1 + x_2 + x_3 = 0$$
$$x_1 - x_2 + x_3 = 0$$
$$2x_1 + 2x_3 = 0$$

Solve: $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_2 \leftarrow R_2 - R_1$; $R_3 \leftarrow R_3 - 2R_1$.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 0 \\ 0 & -2 & 0 \end{bmatrix}$$

$R_3 \leftarrow R_3 + R_2$.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

NPTEL

$x_3 = s$

$-2x_2 = 0 \Rightarrow x_2 = 0.$

$x_1 + x_2 + x_3 = 0.$

$x_1 + 0 + s = 0 \Rightarrow x_1 = -s.$

Solution to $A\vec{x} = \vec{0} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -s \\ 0 \\ s \end{bmatrix}$

$= s \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$N_A = \left\{ \underline{s} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \ s \in \mathbb{R} \right\}.$

N_A : Line passing thro' the Origin in $\mathbb{R}^3$.

Ex: $x_1 + x_2 + x_3 = 0$
$$x_1 + x_2 + x_3 = 0$$
$$2x_1 + 2x_2 + 2x_3 = 0.$$

RREF: $R_2 \leftarrow R_2 - R_1$
$R_3 \leftarrow R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_2 = s, \quad x_3 = t.$

$$x_1 + x_2 + x_3 = 0$$
$$\Rightarrow x_1 + t + s = 0$$
$$\Rightarrow x_1 = -t - s$$

Soln: $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -t-s \\ s \\ t \end{pmatrix}$

$$= t \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + s \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$N_A = \left\{ t\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + s\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix},\ t, s \in \mathbb{R} \right\}$$

N_A: Plane passing thro' the origin defined by

$$\left\{ t\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + s\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix},\ s, t \in \mathbb{R} \right\}$$

Ex:
$$x_1 + x_2 + x_3 = 0$$
$$x_1 - x_2 + x_3 = 0$$
$$x_1 + 2x_2 + 3x_3 = 0$$

Soln: $N_A = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

NPTEL

Ex:

$$x_1 = 0$$
$$x_2 = 0$$
$$x_1 + x_2 = 0$$

$$\underline{\underline{A}} = \overset{3\times 2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}} \overset{2\times 1}{\underline{\underline{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}}} = \overset{3\times 1}{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}} \leftarrow$$

Soln: $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$

$$N_A : \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$$

Ex:

$$x_1 + x_2 + x_3 = 0$$
$$x_1 - x_2 - x_3 = 0.$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix} \begin{matrix} \leftarrow \\ \leftarrow \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

RREF:

$$R_2 \leftarrow R_2 - R_1 \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

NPTEL

x_3 is free variable.

$x_3 = t$.

$-2x_2 - 2x_3 = 0$

$\Rightarrow -2x_2 = 2t \quad x_2 = -t$.

$x_1 + x_2 + x_3 = 0$.

$x_1 - t + t = 0 \Rightarrow x_1 = 0$

Soln:

$$N_A = \left\{ \begin{pmatrix} 0 \\ -t \\ t \end{pmatrix} \right\}$$

$$= \left\{ t \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} ; \; t \in \mathbb{R} \right\}.$$

Set of Solutions to $A\vec{x} = \vec{0}$ is a subspace. We call this subspace as the Null space (A).

Let $A^{m\times n}$ be a real matrix

Look at the Homogeneous Sys. of eqns

$$\overset{m\times n}{A}\,\vec{x}^{\,n\times 1} = \vec{0}^{\,m\times 1}$$

Solution to $A\vec{x}=\vec{0}$ ie, set of all $\vec{x}\in\mathbb{R}^n$ s.t $A\vec{x}=\vec{0}$ is a subspace of $\mathbb{R}^n$ and is called as the null space of A, N_A.