Suppose A is a transformation

Such that

$$A\vec{x} = \vec{b}$$

where $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Are there vector/vectors

such that $A\underline{\vec{u}} = \lambda \vec{u}$

where λ is a scaling factor?

$$A\vec{u} = \lambda \vec{u}$$

$\Rightarrow$ Since $\vec{u} \neq \vec{0}$, we have

$$\underbrace{(A - \lambda I)\vec{u} = \vec{0}}$$ $I^{3\times 3}$ matrix

Homogeneous sys. of eqns &

we look for non-trivial soln

$\Rightarrow$ $\boxed{\det(A - \lambda I) = 0.}$ Char. eqn of A.

A: 3x3 matrix.

$\det(A-\lambda I)$

$$= \det\begin{bmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{bmatrix} = 0$$

The char. eqn is a cubic equation, that has 3 roots.

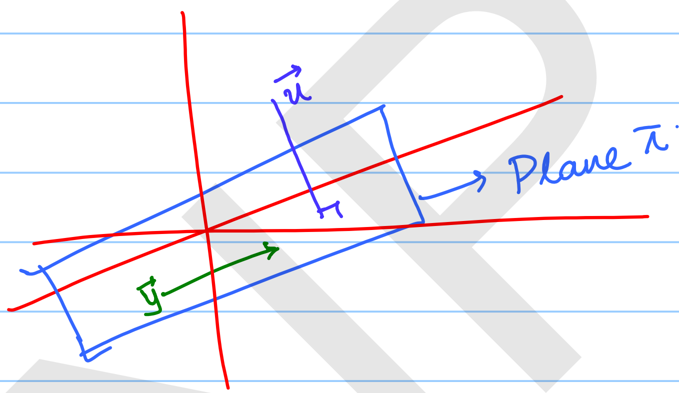
$\Rightarrow$ Atleast one real root for the char. eqn exists

$\Rightarrow$ Atleast one real eigenvector exists for a linear map in 3D.

(a) All three roots are real & distinct
(b) All three roots are real & repeated
(c) One real root and two complex roots (Complex conjugate pairs).

* Consider the reflection of a vector about a plane π passing through the origin & orthogonal to the vector $\vec{u}$.

R_π



Recall:

$R_\pi(\vec{x}) = 2Q(\vec{x}) - \vec{x}$

$Q(\vec{x}) \rightarrow$ Projection of $\vec{x}$ onto π.

Suppose

$R_\pi(\vec{u}) = 2Q(\vec{u}) - \vec{u}$

Since $\vec{u}$ is orthogonal to π

$Q(\vec{u}) = \vec{0}$

$\Rightarrow R_\pi(\vec{u}) = \vec{0} - \vec{u}$

$\Rightarrow R_\pi(\vec{u}) = -1\vec{u}$

$\Rightarrow$ -1 is an eigenvalue of the transf. corresponding to reflection about a plane π Orthogonal to a given vector $\vec{u}$ and π passes thro' the Origin

Let $\vec{y}$ be a vector on the plane π

$$R_\pi(\vec{y}) = 2Q(\vec{y}) - \vec{y}$$

Projection of $\vec{y}$ on to the plane π is $\vec{y}$ itself because $\vec{y}$ is in the plane π

$$R_\pi(\vec{y}) = 2\vec{y} - \vec{y} = \vec{y}$$

$R_\pi(\vec{y}) = 1\,\vec{y}$

For reflection about a plane π thro' the origin, the eigenvalues are ± 1.

Rotation in 3D.

$$(R_\theta)_{e_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$(R_\theta)_{e_2} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$(R_\theta)_{e_3} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[R_\phi]_{e_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{(R_\phi)_{e_1}} \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$$= 3 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Summary:

Linear transformations in 3D

1) Scaling

2) Reflection

3) Shear

4) Rotation

Determinant of $A^{3\times3}$ = Vol. of a ||elopiped