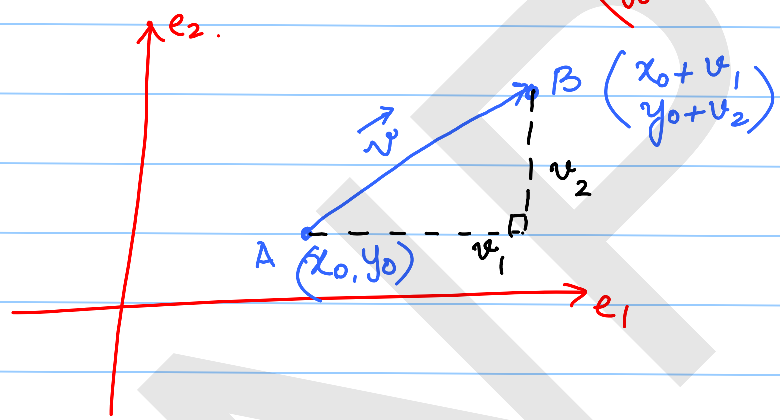
Length of a vector.

Recall if $A(x_0, y_0)$, we want to move from A to $B\begin{pmatrix} x_0+v_1 \\ y_0+v_2 \end{pmatrix}$



$$\vec{v} = \vec{AB} = \begin{pmatrix} x_0+v_1 \\ y_0+v_2 \end{pmatrix} - \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

By Pythagoras Theorem,

$(Hyp)^2 = (Adj)^2 + (opp)^2$ in a Rt. angle triangle

$$\underbrace{\|\vec{v}\|^2}_{\text{Norm of the vector } \vec{v}} = v_1^2 + v_2^2$$

Norm of the vector $\vec{v}$ ⇒ Length of vector $\vec{v}$

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2}$$

$\rightarrow$ Euclidean Norm

Suppose we scale $\vec{v}$ by a factor k

$\Rightarrow k\vec{v} \rightarrow$ Magnitude

$$\|k\vec{v}\| = \sqrt{k^2 v_1^2 + k^2 v_2^2}$$

$$\boxed{\|k\vec{v}\| = |k|\,\|\vec{v}\|}$$

Length is a non neg. quantity

Suppose we want a unit vector along $\vec{v}$

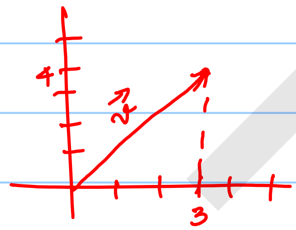
Let $\vec{u}$ be unit vector along $\vec{v}$

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} \rightarrow \text{Normalized Vector}$$

$$\|\vec{u}\| = 1$$

Consider a vector

$$\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$



$$\|\vec{v}\| = \sqrt{3^2+4^2}$$
$$= 5 \text{ units.}$$

Unit vector $\Rightarrow$ $\vec{u} = \dfrac{\vec{v}}{\|\vec{v}\|}$

$$\Rightarrow \frac{\vec{v}}{\|\vec{v}\|} = \begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix} = \vec{u}$$

$$\|\vec{u}\| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{25}{25}} = 1.$$

Since $\|\vec{v}\|$ measures length,

$$\|\vec{v}\| \geq 0$$

When is $\|\vec{v}\| = 0$?

Recall $\|\vec{v}\|^2 = v_1^2 + v_2^2$

$\Rightarrow$ Sum of squares ≥ 0

it is zero if and only if

both $v_1 = 0$; $v_2 = 0$

$\Rightarrow \vec{v} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow$ the zero vector

The length of a vector is 0

if and only if it is the zero vector

$\Rightarrow$ All components of the vector are 0.

Let us look at some examples of unit vectors.

(i) $\vec{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \|\vec{v}\| = 1$

(ii) $\vec{v} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \|\vec{v}\| = 1$

(iii) $\vec{v} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \|\vec{v}\| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$

$= \sqrt{1/2 + 1/2} = 1$

(iv) $\vec{v} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \Rightarrow \|\vec{v}\| = 1$

(v) $\vec{v} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \Rightarrow \|\vec{v}\| = 1$

Look at all the unit vectors that emenate from the Origin in 2D plane

$\Rightarrow$ Geometry we get is the unit circle.

Obtain the length of a vector using the idea of dot product or the inner product.

To conclude:

→ Length of a vector ⇒ $\|\vec{v}\|$

→ Euclidean Norm

→ Pythagoras Theorem.

$\|\vec{v}\| \geq 0$ It is 0 if and only if $\vec{v}$ is the zero vector

Unit vectors → Normalized Vectors.

Geometry of unit vectors emanating from the origin is the unit circle.