Recall: Elementary Row operations we have

(i) $R_i \longleftrightarrow R_j$ → Row Swap.

(ii) $R_i \leftarrow k R_i$ k: real scalar.

(iii) $R_i \leftarrow R_i + k R_j$

How do we carry out these row operations?

Row Swap operation.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$Ax = b.$$

$$\left[\begin{array}{c|c} A & b \end{array}\right]$$ → Row operations on the augmented matrix

$$\left[\begin{array}{cc|c} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_1} \left[\begin{array}{cc|c} a_{21} & a_{22} & b_2 \\ a_{11} & a_{12} & b_1 \end{array}\right]$$

$$\Rightarrow \mathcal{I}_{2\times 2}\left[A \mid b\right] = \left[A \mid b\right]$$

$$\begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} \longrightarrow \begin{bmatrix} a_{21} \\ a_{11} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} = \begin{bmatrix} a_{21} \\ a_{11} \end{bmatrix}$$

Row Swap:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\left[\begin{array}{cc|c} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \end{array}\right] = \left[\begin{array}{cc|c} a_{21} & a_{22} & b_2 \\ a_{11} & a_{12} & b_1 \end{array}\right]$$

$R_i \leftarrow k R_i$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ k a_{21} & k a_{22} \end{bmatrix}$$

$$\left.\begin{aligned} a(a_{11}) + b(a_{21}) &= a_{11} \\ a(a_{12}) + b(a_{22}) &= a_{12} \end{aligned}\right\} \Rightarrow a = 1 \text{ and } b = 0.$$

$$\left.\begin{aligned} c(a_{11}) + d(a_{21}) &= k a_{21} \\ c(a_{12}) + d(a_{22}) &= k a_{22}. \end{aligned}\right\}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ k a_{21} & k a_{22} \end{bmatrix}$$

$R_i \leftarrow R_i + kR_j$.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21}+ka_{11} & a_{22}+ka_{12} \end{bmatrix}$$

$R_2 \leftarrow R_2 + kR_1$

$$\begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21}+ka_{11} & a_{22}+ka_{22} \end{bmatrix}$$

Every Elementary row oper^n is done using matrix multiplication.

→ Pre multiplication

$$a_{11}x_1 + a_{12}x_2 = b_1 \longrightarrow l_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2 \longrightarrow l_2$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$[A|b] = \left[\begin{array}{cc|c} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \end{array}\right]$$

Row Swap: $\left[\begin{array}{cc|c} a_{21} & a_{22} & b_2 \\ a_{11} & a_{12} & b_1 \end{array}\right]$

$$R_i \leftarrow R_i + kR_j.$$

$$2x_1 + x_2 = 3$$
$$x_1 + 2x_2 = 3.$$

$$\left[\begin{array}{cc|c} 2 & 1 & 3 \\ 1 & 2 & 3 \end{array}\right]$$

$R_2 \leftarrow R_2 + kR_1$

$$= \left[\begin{array}{cc|c} 2 & 1 & 3 \\ 1+2k & 2+k & 3+3k \end{array}\right] \rightarrow$$

$2x_1 + x_2 = 3$

$(1+2k)x_1 + (2+k)x_2 = 3(1+k).$ $\rightarrow \ell_2^{(k)}$.

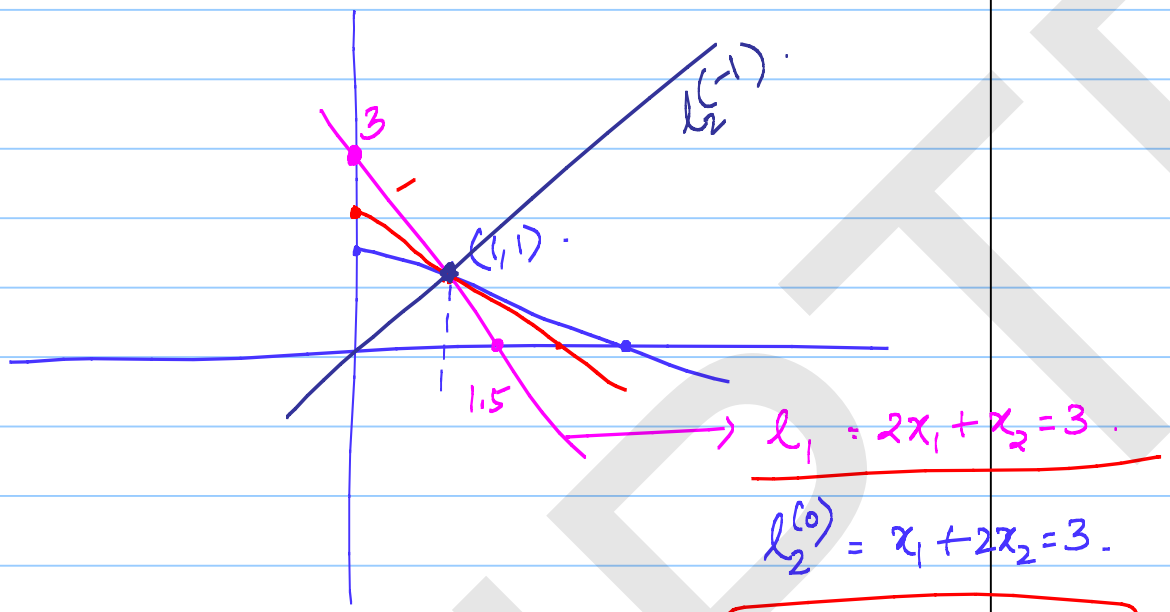
$k = 0, \quad k = 1, \quad k = -1, \quad k = 2.$

$\ell_2^{(0)} = x_1 + 2x_2 = 3$

$\ell_2^{(1)} = 3x_1 + 3x_2 = 6.$

$\ell_2^{(-1)} = -x_1 + x_2 = 0$

$\ell_2^{(2)} = 5x_1 + 4x_2 = 9.$



$l_1 : 2x_1 + x_2 = 3$.

$l_2^{(0)} = x_1 + 2x_2 = 3$.

$l_2^{(1)} = 3x_1 + 3x_2 = 6$.

$l_2^{(-1)} = -x_1 + x_2 = 0$.