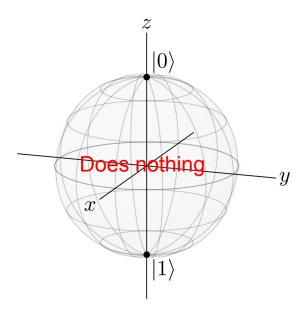
# **Qubit and Single-qubit Gates**

$$|0\rangle = \begin{bmatrix} 1\\0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0\\1 \end{bmatrix}$$

Gate	Action on Computational Basis	Matrix Representation
Identity	$egin{aligned} I 0 angle =  0 angle \ I 1 angle =  1 angle \end{aligned}$	$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
Pauli X	$X 0 angle =  1 angle \ X 1 angle =  0 angle$	$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Pauli Y	$egin{aligned} Y 0 angle &=i 1 angle \ Y 1 angle &=-i 0 angle \end{aligned}$	$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
Pauli $Z$	$Z 0 angle =  0 angle \ Z 1 angle = - 1 angle$	$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Phase S	$S 0 angle =  0 angle \ S 1 angle = i 1 angle$	$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$
T	$T 0 angle =  0 angle \ T 1 angle = e^{i\pi/4} 1 angle$	$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$
Hadamard H	$H 0\rangle = \frac{1}{\sqrt{2}}( 0\rangle +  1\rangle)$ $H 1\rangle = \frac{1}{\sqrt{2}}( 0\rangle -  1\rangle)$	$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

## Pauli I Gate



Bloch sphere: I gate



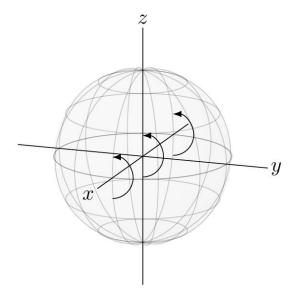
Quantum Composer: I gate

#### Pauli X Gate

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

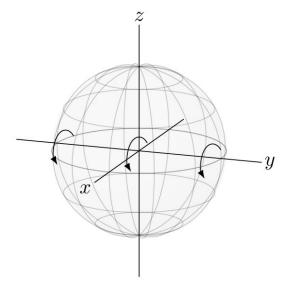
## **Pauli X Gate**



Bloch sphere: X gate

Quantum Composer : X gate

## **Pauli Y Gate**

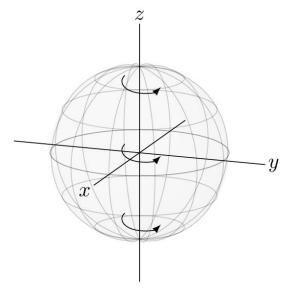


Bloch sphere: Y gate



Quantum Composer : Y gate

# Pauli Z Gate

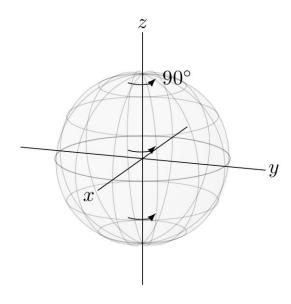


Bloch sphere : Z gate



Quantum Composer : Z gate

# **S** Gate

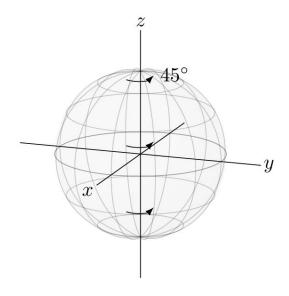


Bloch sphere: S gate



Quantum Composer : S gate

# T Gate



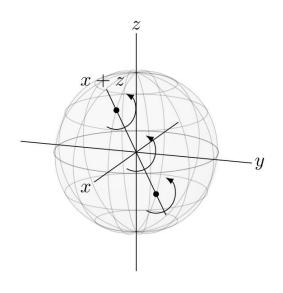
q[0] T

Bloch sphere: T gate

Quantum Composer : T gate

# **H** Gate

## H Gate

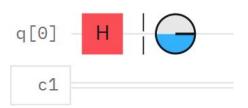




Bloch sphere: H gate

Quantum Composer : H gate

#### **Hadmard Gate**



$$egin{align} H(\ket{0}) &= rac{\ket{0} + \ket{1}}{\sqrt{2}} \ &= rac{1}{\sqrt{2}} \ket{0} + rac{1}{\sqrt{2}} \ket{1} \end{aligned}$$

$$H|0
angle=rac{1}{\sqrt{2}}(|0
angle+|1
angle)=|+
angle$$

$$H|1
angle=rac{1}{\sqrt{2}}(|0
angle-|1
angle)=|-
angle$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$= \frac{1}{\sqrt{2}} (\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix})$$
$$= \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$$

# **Associative: Hardmard gate**

$$H(H(\ket{0})) = H(rac{1}{\sqrt{2}}\ket{0} + rac{1}{\sqrt{2}}\ket{1}) = 1$$

$$egin{align} &=rac{1}{\sqrt{2}}(H(\ket{0})+H(\ket{1}))\ &=rac{1}{\sqrt{2}}(rac{1}{\sqrt{2}}\ket{0}+rac{1}{\sqrt{2}}\ket{1}+rac{1}{\sqrt{2}}\ket{1} \end{aligned}$$

$$=rac{1}{\sqrt{2}}(rac{1}{\sqrt{2}}|0
angle+rac{1}{\sqrt{2}}|1
angle+rac{1}{\sqrt{2}}|0
angle-rac{1}{\sqrt{2}}|1
angle \ =|0
angle \ =$$

$$H|0
angle=rac{1}{\sqrt{2}}(|0
angle+|1
angle)=|+
angle$$

$$H|1
angle=rac{1}{\sqrt{2}}(|0
angle-|1
angle)=|-
angle$$

$$H*H*\ket{0} = rac{1}{\sqrt{2}}egin{bmatrix} 1 & 1 \ 1 & -1 \end{bmatrix}*rac{1}{\sqrt{2}}egin{bmatrix} 1 & 1 \ 1 & -1 \end{bmatrix}*egin{bmatrix} 1 \ 0 \end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$egin{array}{ccc} egin{bmatrix} 0 & 1 \end{bmatrix} \ = egin{bmatrix} 1 \ 0 \end{bmatrix} \ = egin{bmatrix} 1 \ 0 \end{bmatrix}$$

#### Noncomutative: HX != XH

$$HX | 0 \rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}} | 0 \rangle - \frac{1}{\sqrt{2}} | 1 \rangle$$

$$XH \mid 0 \rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} * \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} |0 \rangle + \frac{1}{\sqrt{2}} |1 \rangle$$

