CSCI-4961 - RPI, 05/20 - 08/16/2024

Lecture $|0\rangle$: Syllabus and Logistics for Quantum Computing @ RPI

Tue./Fri. 6:30pm –8:35pm, Location: DCC 337

Contributor: *Lecturer: Yanglet Xiao-Yang Liu*

Figure 1: Pictured: a photo of the utility-scale IBM Quantum System One, which was unveiled on April 5, 2024 in RPI's Voorhees Computing Center, a former cathedral. It is the first IBM Quantum System One to be installed on a university campus. (Credit: IBM)

Class Kickoff

Video [1] (1 min): Quantum Ribbon Cutting Celebration at RPI
Video [2] (1 min): First of its kind quantum computers unveiled at RPI
Video [3] (5 min): What If We Had Working Quantum Computers Today?
Video [4] (10 min): Quantum Computers, Explained With Quantum Physics

[1] https://www.youtube.com/watch?v=-1FEnRAW-A0
[2] https://www.youtube.com/watch?v=hLlwxqRwUrc
[3] https://www.youtube.com/watch?v=fNOJjR1Znds
[4] https://www.youtube.com/watch?v=jHoEjvuPoB8

Course Summary

Enabling a transition from an academic research interest to a nascent industry.

No assumption of background knowledge! But your interest and commitment.

Building on a foundation of computing (CSCI 2200) and algorithms (CSCI 2300), this course delves into such fundamental principles as qubits, superposition, entanglement, quantum gates, and algorithms. Students will engage in hands-on lab sessions using Qiskit/OpenQASM and/or AWS/Azure cloud services, exploring both quantum circuits and algorithms. Students will also explore applications in optimization and machine learning, with discussions on the latest in quantum hardware, including IBM Quantum System One and Google's Sycamore circuits.

We take a mathematical-and-algorithmic approach and also a classical-and-quantum approach, accessible to all STEM students. Two professional levels are expected: 1). Quantum-aware engineers via lectures, and 2). quantum-proficient engineers via labs/projects.

At the *undergraduate* level, we aim to address the educational gap between

- quantum-related graduate programs focused mainly on PhDs, with a few MS programs. E.g., the M.S. in Quantum Science and Technology program at Columbia University will start at September 2024.
- excitement generated both by the popular media and the increased interest in introducing QIST in before-colleague schools;

In particular, students are encouraged to play with the IBM Quantum System One at RPI campus. Please request access via link [5]

Catalog Description: Data structures and algorithms, and the mathematical techniques necessary to design and analyze them. Basic data structures: vector, matrix, associative structures, graphs. Mathematical techniques for designing algorithms, analyzing worst-case and expected-case algorithm efficiency, and characterizing speedups. Algorithm design techniques: Shor's factoring algorithm, Grover's search algorithm, simulated annealing algorithm, etc.. We will also try to cover: QAOA algorithms, quantum suprevacy, Qiskit, etc.

Learning objectives include a solid grasp of quantum computing principles, an understanding of quantum advantages (including the quantum supremacy in the NISQ era), proficiency in reading and analyzing simple quantum algorithms, and insights into practical applications.

[5] `https://itssc.rpi.edu/hc/en-us/articles/17609648152845-Quantum-computing`

Grading and Logistics

There are 13 weeks, 60 students.

Week	Monday	Tuesday	Wednesday	Friday
1 (05/20 - 05/26)		Lecture 1	Lab1	Lecture 2
2 (05/27 - 06/02)	HW1	Lecture 3	Lab2	Lecture 4
3 (06/03 - 06/09)	HW2	Lecture 5	Lab3	Lecture 6
4 (06/10 - 06/16)	HW3	Lecture 7	Lab4	Lecture 8
5 (06/17 - 06/23)		Lecture 9	Review	Lecture 10
6 (06/24 - 06/30)		Review	Exam	Lecture 11
7 (07/01 - 07/07)		Lecture 12	Project 1	Lecture 13
8 (07/08 - 07/14)		Lecture 14		Lecture 15
9 (07/15 - 07/21)	Report-I	Lecture 16	Project 2	Lecture 17
10 (07/22 - 07/28)		Lecture 18		Lecture 19
11 (07/29 - 08/04)	Report-II	Lecture 20	Project 3	Lecture 21
12 (08/05 - 08/11)		Lecture 22		Lecture 23
13 (08/11 - 08/16)	Poster	Presentations (10 min)		Presentations

Class participation	quizzes (8%) and classroom sharing (10%)	18%
Homework (3)	$3 \times 5\%$	15%
Labs (4)	$4 \times 2\%$	8%
Exam	15%	15%
Project $1 \sim 3$	$3 \times 3\%$	9%
Report (I, II)	$2 \times 5\%$	10%
Final Poster	5%	5%
Presentation	20%	20%
Total		100%

Scores	90%	86%	82%	78%	74%	70%	66%	62%	58%	54%	$< 54\%$
Grades	A	A -	B+	B	B-	C+	C	C-	D+	D	fail

Logistics:
Course Learning Management System (LMS)
All course materials will be available via Submitty: `https://submitty.cs.rpi.edu/courses/u24/csci4961`
Log in using your RCS ID (e.g., “liux33”)
We will use Submitty’s Discussion Forum for course announcements and for asking questions
Please post questions there; also answer questions...
The course schedule will be posted in Submitty (but will likely change)

Check your RPI email at least once per day

Turn on all email notifications in Submitty — you are responsible for staying current with course announcements, assignment changes, etc.

- Solving problems in group. Each student team has $1 \sim 3$ students (can switch groups for different tasks): HWs, Labs, Projects and Presentations allow collaborations. **But, NO collaboration in exam**!
- **Students can use ChatGPT in this class, including reports and presentations**! Student should state that they use ChatGPT if they do use it. Please avoid fake references/sources (or other misinformation) generated by ChatGPT.

Suggestions: Lab sessions (Wednesday) are regular meetings. For students with a quantum RCOS project, RCOS sessions could also be regular meetings.

Notes: 1). Project $1 \sim 3$ sessions are for attendance purpose; gradings will be done through Report I, II and Final Poster. Poster and Presentation should be be on the same topic and within the same student team.

We will provide templates for Report I, II, Poster, and Presentations.

Bonus points: $1 \sim 5\%$

Project $1 \sim 3$; Presentation, and other projects with professor's approval, or creating course materials.

Each lecture (2 hours):

$10 \sim 15$ minutes for videos of physical experiments;

$70 \sim 80$ minutes' lecture;

$10 \sim 15$ minutes for quizzes;

$15 \sim 30$ minutes for students' sharing.

TA and Undergraduate mentors and Summer URP students:

- Dong Hu (OH: Wednesday 2pm - 4pm): Design, Tutor and Check off Labs, Projects, and Grading Presentations; and join Discussion Forum on Submitty.
- Yue Han (OHs: Monday & Thursday 1pm - 3pm): Design and Grade HWs, Reports, and Presentations; and join Discussion Forum on Submitty.
- Undergraduate mentors: Tutor Labs, Projects; and join Discussion Forum on Submitty.
- Yanglet Xiao-Yang Liu (OH at AE 208: Tuesday 1pm - 3pm): Lectures; and Grading Presentations.

Do NOT email our TAs and mentors...instead, attend OHs and post questions on the Discussion Forum.

OH will primarily be in-person with a few online-only sessions

Always check the posted schedule and watch for announcements in case OH change

Summer URP Students: we have several URP students on quantum computing in Summer 2024, who will provide help with lab tasks and projects.

NO late days! However, school allows Excused Absence.

Textbooks and Materials

Preparation Knowledge

Prerequisites: CSCI 2200 FOCS and CSCI 2300 Algos.

Complex numbers; Trigonometric functions; Sets and functions; Probability

(Complex) linear algebra, matrix operations; tensor product and tensor networks.

- Book 1: Introduction to classical and quantum computing, By Thomas G Wong
- Book 2: Introduction to quantum algorithms via Linear Algebra, Second Edition. Richard J. Lipton and Kenneth W. Regan
- Book 3 (advanced): Quantum computation and quantum information. Cambridge University Press, 2001. Michael A. Nielsen and Isaac L. Chuang
- Lectures [6], Understanding quantum information & computation, by John Watrous (IBM Quantum Education); YouTube [7]
- Lectures, Computing and Quantum Computing, by Malik.
- Lectures [8], Introduction to quantum computing, by Henry Yuen, Columbia University
- Lectures [9], Intro to quantum information science, by Scott Aaronson [10]
- Lectures, The mathematics of quantum mechanics, by Martin Laforest, U. Waterloo, Institute for Quantum Computing.
- Book 4, A Course in Quantum Computing, by Michael Loceff. YouTube [11]
- OptimaLab, Introduction to quantum computing [12]

[6] https://learning.quantum.ibm.com/course/basics-of-quantum-information

[7] https://www.youtube.com/watch?v=tI1SfFuuX-o&list=PLOFEBzvs-VvqoeIypXYLLf0PY-WOQMLR3&index=1

[8] https://www.henryyuen.net/classes/spring2021/

[9] https://scottaaronson.blog/?p=3943

[10] https://www.scottaaronson.com/democritus/

[11] https://www.youtube.com/@michaelloceff2873

[12] https://akyrillidis.github.io/notes/

Statements on the Use of AI

Within this class, you are encouraged to use foundation models (ChatGPT, GPT, DALL-E, Stable Diffusion, Midjourney, GitHub Copilot, and anything after) in a **totally unrestricted fashion, for any purpose, at no penalty**.

However, you should note that all large language models still have a tendency to make up incorrect facts and fake citations, code generation models have a tendency. You will be responsible for any inaccurate, biased, offensive, or otherwise unethical content you submit regardless of whether it originally comes from you or a foundation model. If you use a foundation model, **its contribution must be acknowledged in the hand in; you will be penalized for using a foundation model without acknowledgement**.

Having said all these disclaimers, the use of foundation models is highly encouraged. I am happy to observe how those advanced AI tools are used to help the young generation to understand quantum.

The Prospects of Quantum Computing

Quantum computing promises several disruptive changes:

- Feymman's argument that the exact simulation of a quantum system is only possible with a quantum computer.
- (End of) Moore's Law. Moore's Law states that the density of transistors on a chip roughly doubles every eighteen months. Current estimates say that in about a decade this should be down to single electron transistors. This is the end of the road for further miniaturization of classical computers based on electronics. Long before that chip designers will have to contend with quantum phenomena. Quantum computation provides a method of bypassing the end of Moore's Law, and also provides a way of utilizing the inevitable appearance of quantum phenomena.
- Cryptography. Quantum computation allows us to do cryptography in a way that doesn't require assumptions about factoring primes, etc. It also allows us to break classical cryptography schemas. Obviously, if we are interested in cryptography, we'll also have to be interested in quantum computation.
- Quantum machine learning....m any speedups are summarized in the Nature 2017 paper: Biamonte, Jacob, Peter Wittek, Nicola Pancotti, Patrick Rebentrost, Nathan Wiebe, and Seth Lloyd. "Quantum machine learning." Nature 549, no. 7671 (2017): 195-202.

Topics

Overview: general info for computing and computational thinking.

- Computing as a discipline: classical and quantum
- A birdview of quantum computing; quantum speedups
- Background on classical and quantum physics

Part $|0\rangle$: Theory of Quantum Computing.
The math.

- Arithmetic-Algebra, Complex Numbers and Trigonometrics
- Quantum Information of Single-Qubit System: Qubit, Bloch Sphere, Quantum Gates
- Quantum Circuits, (Complex) Linear Algebra, and Tensor Networks
- Quantum Information of Multiple-Qubit System: Density Matrices, Block Ball
- Languages and Turing Machine

Part $|1\rangle$: Quantum Algorithms
The advantages.

- P, NP, and Quantum Speedups
- Quantum Supremacy and Empirical Quantum Advantages
- Factoring: Shor's Algorithm
- Search: Grover's Algorithm
- Quantum Approximate Optimization Algorithm (QAOA)
- Quantum Error Correction. NISQ.

Part $|2\rangle$: Quantum Applications
The potentials.

- Optimization + Quantum Computing
- Quantum Machine Learning
- Quantum Simulation for Physical, Chemical, and Biological Systems
- AI + Quantum Computing
- Finance + Quantum Computing

- Material Science + Quantum Computing

Part $|3\rangle$: Lab Sessions and Projects

The hands-on learning.

- Lab 1: IBM Quantum Learning Suite:
- Lab 2: Examples of GHZ Gate and Google's Sycamore Circuits
- Lab 3: Quantum Basic Linear Algebra Subroutines (qBLAS)
- Lab 4: Image Classification (quantum machine learning)
- Project 1: Finding the Ground State of Ising model Using Learning-to-Anneal Algorithm
- Project 2: Quantum approximate optimization algorithm (QAOA) [13]
- Project 3: Challenging the Claim of Quantum Supremacy

Note: Your final presentations and reports can be based on one of Lab 3-4, Project 1-3, or implementation of a quantum algorithm in Chapter 7, or other research ideas (need approval from a professor). Alternatively, the presentation can be about a paper from professor's list, or need approval from a professor.

[13] `https://learning.quantum.ibm.com/tutorial/quantum-approximate-optimization-algorithm`

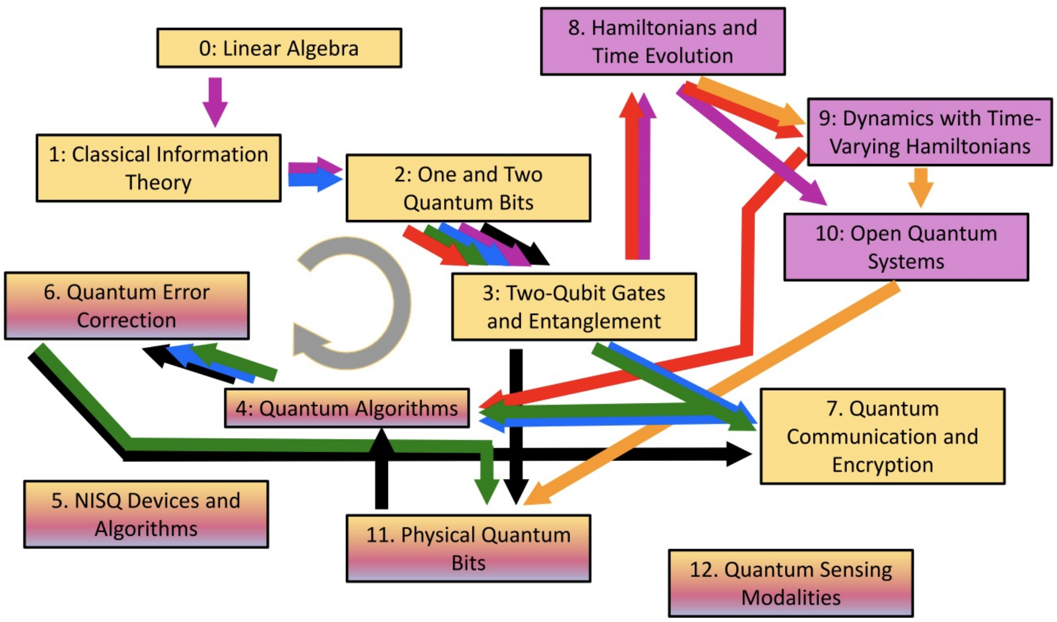


Figure 2: Course modules for STEM students at any level (orange shaded box), all levels plus advanced students (orange faded to purple), and advanced student only (purple).

Course Modules for Quantum Computing

Here, I will describe it based on my survey and resources. The goal is to have a set of modules that can be reorganized for a series of classes at RPI.

Building a Quantum Engineering Undergraduate Program. IEEE TRANSACTIONS ON EDUCATION, 2022.

Module 0 Introductory

To stimulate students' interest through concepts and applications, here we will take a quantum information perspective.

Linear algebra is a strong prerequisite. Either requiring students took it in a separate course (with a refresher lecture), or teaching it as a focused unit (span 2 - 3 lectures).

Concepts-Focused QISE (E)

By learning concepts and application first, (without much math), students may gain appreciation of QIST and intuitions. This would allow the connection between end-use applications and discussion of potential career pathways.

(Complex) Linear Algebra for QISE (E)

Topics: vector spaces (superposition, concept of a basis), linear transformations, matrix multiplication, noncommutativity, diagonalization, inversion, Hermitian and unitary operators, trace and partial traces, outer and tensor products, scaling up to larger matrices numerically.

We will teach those math concepts in the context of single qubit system, i.e., 2×2 matrices, and their tensor products.

Module 1: Classical Information Theory (E)

Topics: basics of bits, gates, communication, randomness and statistics, error correction, parity and data compression.

For the transition from classical bits to qubits, classical bits are represented as vectors and gates as matrices.

Module 2: One and Two Quantum Bits (E)

Topics:

- Qubits, superpositions states, measurements and the Born rule.
- Single-qubit Hilbert space: linear operators, Dirac notation, orthonormal bases and basis changes, qubit rotations, and the Bloch sphere.

Module 3: Two-Qubit Gates and Entanglement (E)

Topics: The CNOT gate and the circuit model of computation. Bell states and nonclassical correlations.

Module 4: Quantum Algorithms (E/A)

Topics:

- Early examples of quantum advantage in computation: the Deutsch, Deutsch-Jozsa, Bernstein-Vazirani, Simon's algorithms.
- Phase kick back from controlled unitaries.
- Oracle algorithms
- Grover's search algorithm, phase estimation.
- Shor's factoring algorithm: QFT and period-finding algorithm.

Module 5: NISQ Devices and Algorithms (E/A)

Topics: NISQ. VQE, QAOA, quantum machine learning. Error mitigation

Module 6: Quantum Error Correction (E/A)

Topics: Error models; Shor code.

Module 7: Quantum Communication and Encryption (E/A)

Module 8: Hamiltonian and Time Evolution (E/A)

Module 9: Dynamics with Time-Varying Hamiltonian (A)

Module 10: Open Quantum Systems (A)

Module 11: Physical Quantum Bits (E/A)

Module 12: Quantum Sensing Modalities (E/A)

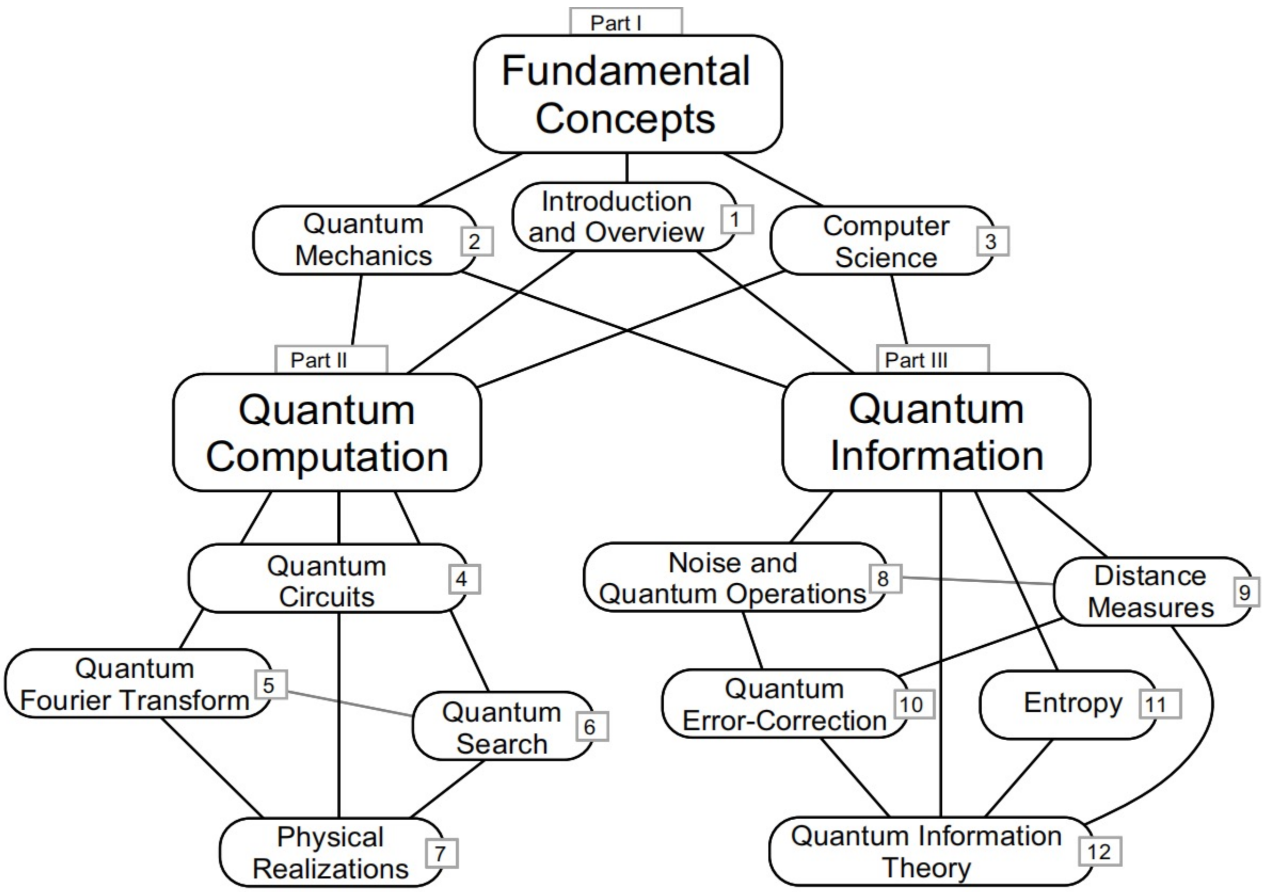


Figure 3: Structure of the Mik-Ike book

Lecture $|1\rangle$: Single Qubit

May 21, Tue. 6:30pm –8:35pm

Contributor: ______________________________ *Lecturer: Yanglet Liu*

Topics: Complex numbers $\mathbb{C}$; Superposition: Qubit, Bloch Sphere, and Single-Qubit Gates.

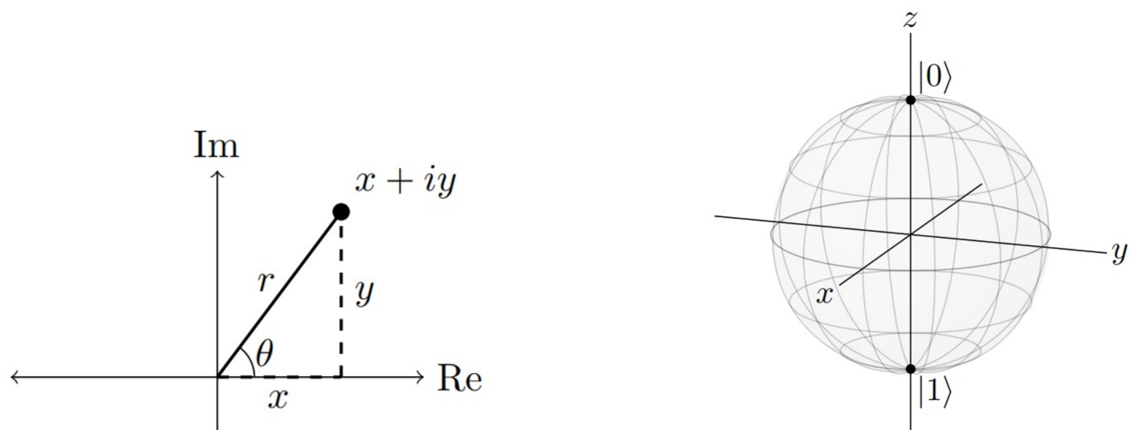


Figure 4: Complex plane (left) and the Bloch sphere (right).

Pop Quizzes

1). Powers of i, where $\boxed{i = \sqrt{-1}}$ (imaginary unit number).
$i^0 =$ _____ $\quad i^1 =$ _____ $\quad i^2 =$ _____ $\quad i^3 =$ _____
$i^4 =$ _____ $\quad i^5 =$ _____ $\quad i^6 =$ _____ $\quad i^7 =$ _____

2). $i^0 =$ _____ $\quad i^{-1} =$ _____ $\quad i^{-2} =$ _____ $\quad i^{-3} =$ _____
$i^{-4} =$ _____ $\quad i^{-5} =$ _____ $\quad i^{-6} =$ _____ $\quad i^{-7} =$ _____

3). Trigonometric functions: SOH-CAH-TOA
A complex number $z = x + iy$, with $r = \sqrt{x^2 + y^2}$, as shown in Fig. 4 (left).
Sine is opposite over hypotenuse (SOH), $\sin\theta =$ _____
Cosine is adjacent over hypotenuse (CAH), $\cos\theta =$ _____
Tangent is opposite over adjacent (TOA), $\tan\theta =$ _____

4). On the Bloch sphere in Fig. 4 (right), mark the four frequent *pure states*:
"plus", "minus", "i", and "minus i".

Paul Dirac: mathematics is the toll specially suited for dealing with abstract concepts of any kind and there is no limit to its power in this field.

Outline

- Light (Photon): Polarization, Superposition, The Born Rule
- System and state
- Complex numbers
- Superposition: qubit
- Bloch sphere
- Quantum gates (q-gates): operating a qubit

We treat *qubits* as *mathematical objects* with certain specific properties.

Notations

- $x, y, a, b \in \mathbb{R}$
- Complex number $z \in \mathbb{C}$: $z = x + iy$ or $z = a + ib$
- Conjugate $z^* = a - ib$, or $\bar{z}$
- *Dirac notation*: $|0\rangle$, $|1\rangle$.
- Quantum bit (qubit) $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$, where $\alpha, \beta \in \mathbb{C}$ and $\alpha^2 + \beta^2 = 1$.

Videos

Video [14] (3 min): Big Bang Theory: "Schrodinger's relationship" [15]
Video [16] (1.5 min): Three polarizing filters: a simple demo of a creepy quantum effect
Video [17] (11.5 min): The principles of quantum mechanics from polarization

[14] https://www.youtube.com/watch?v=oerZnryFxX0
[15] https://www.youtube.com/watch?v=2c66_cha8kg
[16] https://www.youtube.com/watch?v=5SIxEiL8ujA
[17] https://www.youtube.com/watch?v=-ZUw1qJOflU

Short survey of our class:

- Remember trigonometric functions? Complex numbers?
- Familiar with Cartesian coordinates? Spherical coordinates?
- Euler's formula? Euler's identity
- Taken linear algebra? matrix operations? tensor product?
- Anyone did physical experiments?
- Know quantum physics?

Warm-Ups

- Find a solution to polynomial equations

$$x^2 - 1 = 0, \qquad x^2 + 1 = 0.$$

  Does $i = \sqrt{-1}$ bother you?

- **Euler's formula**. Do you see the beauty of $e^{i\pi} + 1 = 0$?
  How about $e^{ix} = \cos x + i \sin x$ for $x \in \mathbb{R}$? How to prove it?

- Are you comfortable with $|0\rangle$? How about $|10\rangle = |1\rangle \otimes |0\rangle$?
  How about $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$ with $\alpha, \beta \in \mathbb{C}$, and $|\alpha|^2 + |\beta|^2 = 1$.

1 Light (Photon): Polarization, Superposition, The Born Rule

Polarization: The polarization of a light wave is the direction in which the electric field of the wave oscillates.

- **Vertical**: up and down, i.e., $|V\rangle$ or $|\uparrow\rangle$
- **Horizontal**: from left to right, $|H\rangle$ or $|\rightarrow\rangle$

The electric field of a light wave has both magnitude and direction, so mathematically, it is a *vector*.

$\boxed{\text{The photon is in a superposition state of vertical and horizontal polarization}}$, using Paul Dirac's notion:

$$|\text{photon}\rangle = |V\rangle + |H\rangle \tag{1}$$

2 System and State

Classical States

A *system*: a collection of objects (or smaller systems) that can be identified. A *state* is a possible configuration of a system.

- in physics: a set of physical objects that are interrelated in complex ways. Ex: solar system; cities; machines. Three types of physical systems: isolated, closed, open.
  - Isolated: a system in which no matter or energy is being exchanged with surroundings.
  - Closed: a system in which only energy is being exchanged with surroundings.
  - Open: a system in which both matter and energy is being exchanged with surroundings.
- in chemistry: A specific portion of matter under study or observation, typically isolated from its surroundings for the purpose of analysis. Ex. Salt Solution, Closed Container Reaction, Biological System (e.g., enzymes in a cell).

Quantum State

- $|0\rangle, |1\rangle$ - Alignment with/Opposition to external field in Z-direction

A quantum state is a mathematical entity that embodies the knowledge of a quantum system. Quantum mechanics specifies the construction, evolution, and measurement of a quantum state. The result is a quantum-mechanical prediction for the system represented by the state. Knowledge of the quantum state, and the quantum mechanical rules for the system's evolution in time, exhausts all that can be known about a quantum system.

Quantum states may be defined differently for different kinds of systems or problems. Two broad categories are wave functions describing quantum systems using position or momentum variables and the more abstract vector quantum states (we focus on the later one in this class).

Quantum bit: qubit

A qubit is a basic unit of quantum information—the quantum version of the classic bit physically realized with a two-state device. A qubit is a two-state (or two-level) quantum-mechanical system, one of the simplest quantum systems displaying the peculiarity of quantum mechanics. Examples include the spin of the electron in which the two levels can be taken as spin up and spin down; or the polarization of a single photon in which the two spin states (left-handed and the right-handed circular polarization) can also be measured as horizontal and vertical linear polarization. In a classical system, a bit would have to be in one state or the other. However, quantum mechanics allows the qubit to be in a coherent superposition of multiple states simultaneously, a property that is fundamental to quantum mechanics and quantum computing.

Bohr's atomic model: for the hydrogen atom ($Z = 1$) or a hydrogen-like ion ($Z > 1$), the negatively charged electron confined to an atomic shell encircles a small, positively charged atomic nucleus and where an electron jumps between orbits, is accompanied by an emitted or absorbed

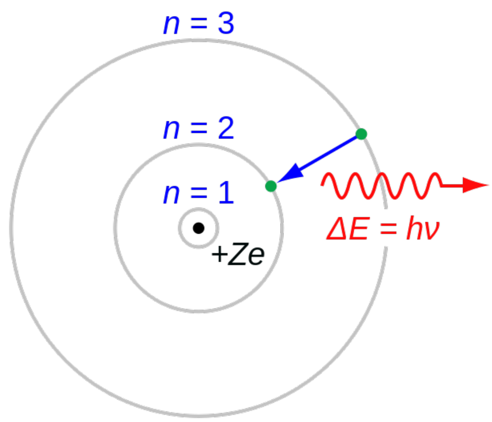


Figure 5: Bohr's atomic model: The electron orbitting the nucleus can only choose from a certain number of allowed orbits. When it jumps from one orbit to another, it emits or absorbs a photon.

amount of electromagnetic energy (hv). The orbits in which the electron may travel are shown as grey circles; their radius increases as $n2$, where n is the principal quantum number. The $3 \to 2$ transition depicted here produces the first line of the Balmer series, and for hydrogen ($Z = 1$) it results in a photon of wavelength 656 nm (red light).

3 Arithmetic and Algebra

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$

3.1 Arithmetic

Examples

- $+$: recall the recursive definition of $\mathbb{N}$ in CSCI 2200 FOCS;
- $-$: $\mathbb{Z}$
- $\times$:
- $/$: $\mathbb{Q}$

3.2 Algebra

Algebra is the abstract encapsulation of our intuition for composition.

Examples

- The concept of *Unity*. The number 1. As a little baby, I learned to count by starting with 1.

- $\mathbb{N} = \{1, 2, 3, \cdots\}$, equipped with two natural operations $+$ and $\times$.
- $\mathbb{Z} = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$. Additive inverses exist.
- $\mathbb{Q} = \{\frac{a}{b} \mid a \in \mathbb{Z}, b \in \mathbb{N}\}$,

Group and Abelian Group. Let S be a set. A binary operation $* : S \times S \to S$. Then, $(S, *)$ is a *group*, such that the following hold:

- Associativity: $(a * b) * c = a * (b * c)$, $\forall a, b, c \in S$.
- Existence of identity: $\exists e \in S$ such that $e * a = a * e = a$, $\forall a \in S$.
- Existence of inverses: Given $a \in S$, $\exists b \in S$, such that $a * b = b * a = e$.

A group $(S, *)$ is *Abelian* if it also satisfies the commutative property, such that

$$a * b = b * a, \quad \forall a, b \in S. \tag{2}$$

Difference between Arithmetic and Algebra.

S No.	Arithmetic	Algebra
1	It is the branch of mathematics that deals with numbers, their writing systems, and their properties.	It is the branch of mathematics that deals with variables and constants.
2	The operations are carried out with the help of the information provided.	The operations are carried out with the help of standard formulae and expressions.
3	It is generally applicable in real life and associated with elementary education.	Its direct application is not often observed in daily life and is associated with high school education.
4	It has four basic methods of operation (addition, subtraction, multiplication, and division).	It uses numbers, variables, and general rules or formulae to solve problems.
5	It is related to the numbers and number systems.	It is related to equations and formulae.

4 Refresher of Complex Numbers $\mathbb{C}$

Three forms for one number $z = x + iy \in \mathbb{C}$, where $x, y \in \mathbb{R}$.

Complex numbers are absolutely everywhere in quantum mechanics. They also have many applications in physics, chemistry, biology, electrical engineering, statistics, and even finance and economics.

4.1 Cartesian Form

Complex numbers are numbers of the form

$$\boxed{z = x + iy, \quad \text{where } x = \text{Re}(z),\ y = \text{Im}(z)}, \tag{3}$$

where $i = \sqrt{-1}$ or $i^2 = -1$.

The *complex conjugate* z^* of $z = x + iy$ is

$$z = x - iy, \quad \text{replace } i \text{ with } -i, \tag{4}$$

i.e., flipping the sign of the imaginary part.

4.2 Polar Form (and Exponential Form)

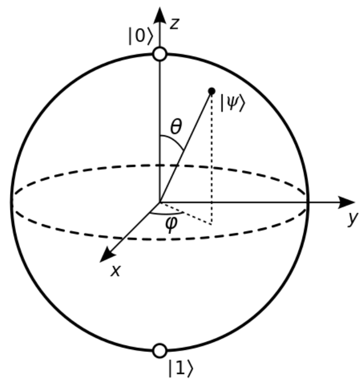
Summary

Complex numbers, $z_1 = a + bi$ and $z_2 = c + di$. Or $z = a + bi$

- Imaginary unit: $\boxed{i = \sqrt{-1}}$
- Addition: $\boxed{z_1 + z_2 = (a + c) + (b + d)i}$
- Multiplication: $\boxed{z_1 z_2 = (ac - bd) + (ad + bc)i}$
- Conjugate: $\boxed{z^* = a - bi}$
- Modulus: $\boxed{|z| = r = \sqrt{zz^*} = \sqrt{z^*z} = \sqrt{a^2 + b^2}}$
- Eler's formular: $\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$
- Exponential form: $\boxed{z = re^{i\theta}}$
- Periodicity: $\boxed{e^{i\theta \pm 2\pi} = e^{i\theta}}$

5 Qubit: Quantum Bit

6 Bloch Sphere



Lab 1

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