CSCI-4961 - RPI, 05/20 - 08/16/2024

Lecture $|2\rangle$: One Qubit (2/3)

May 21, Tue. 6:30pm –8:35pm

Contributor: ______________________________ *Lecturer: Yanglet Liu*

Topics: Bloch Sphere, and One-Qubit Gates.

We treat *qubits* as *mathematical objects* with certain specific properties.

Paul Dirac: mathematics is the toll specially suited for dealing with abstract concepts of any kind and there is no limit to its power in this field.

$$\boxed{|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle}$$

Dirac's notion (bra-ket): $|0\rangle$, $|1\rangle$ (computational basis)

Outline

- Light (Photon): Polarization, Superposition, The Born Rule
- Superposition: qubit
- Bloch sphere
- Quantum gates (q-gates): operating a qubit

Notations

- *Dirac notation*: $|0\rangle$, $|1\rangle$.
- Quantum bit (qubit) $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$, where $\alpha, \beta \in \mathbb{C}$ and $\alpha^2 + \beta^2 = 1$.

Videos

Video [16] (10 min): Quantum Computers, Explained With Quantum Physics

[16] https://www.youtube.com/watch?v=jHoEjvuPoB8

4 Light (Photon): Polarization, Superposition, The Born Rule

Video [17] (1.5 min): Three polarizing filters: a simple demo of a creepy quantum effect
Video [18] (11.5 min): The principles of quantum mechanics from polarization

Polarization of light: The polarization of a light wave is the direction in which the electric field of the wave oscillates.

- **Vertical**: up and down, i.e., $|V\rangle$ or $|\uparrow\rangle$
- **Horizontal**: from left to right, $|H\rangle$ or $|\rightarrow\rangle$

The electric field of a light wave has both magnitude and direction, so mathematically, it is a *vector*.

Photon: light is quantized.

$\boxed{\text{The photon is in a superposition state of }\underline{\text{V}}\text{ertical and }\underline{\text{H}}\text{orizontal polarization}}$,

using Paul Dirac's notion:

$$|\text{photon}\rangle = |V\rangle + |H\rangle \tag{5}$$

Formally, one *qubit* state is a superposition of the basis states $|0\rangle$ and $|1\rangle$,

$$\boxed{|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle}$$

where *probability amplitude* $\alpha, \beta \in \mathbb{C}$ and $\underbrace{|\alpha|^2}_{\text{probability}} + \underbrace{|\beta|^2}_{\text{probability}} = 1$.

$\boxed{\text{We treat } \textit{qubits} \text{ as } \textit{mathematical objects} \text{ with certain specific properties.}}$

Quantum bit: qubit

A qubit is a basic unit of quantum information—the quantum version of the classic bit physically realized with a two-state device. A qubit is a two-state (or two-level) quantum-mechanical system, one of the simplest quantum systems displaying the peculiarity of quantum mechanics. Examples include the spin of the electron in which the two levels can be taken as spin up and spin down; or the polarization of a single photon in which the two spin states (left-handed and the right-handed circular polarization) can also be measured as horizontal and vertical linear polarization. In a classical system, a bit would have to be in one state or the other. However, quantum mechanics allows the qubit to be in a superposition of multiple states simultaneously, a property that is fundamental to quantum mechanics and quantum computing.

Bohr's atomic model: for the hydrogen atom ($Z = 1$) or a hydrogen-like ion ($Z > 1$), the negatively charged electron confined to an atomic shell encircles a small, positively charged atomic

[17] https://www.youtube.com/watch?v=5SIxEiL8ujA
[18] https://www.youtube.com/watch?v=-ZUw1qJOflU

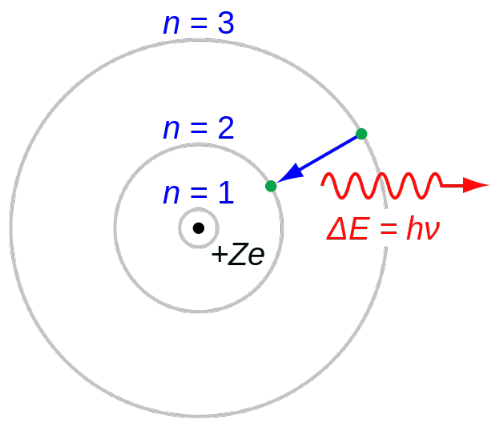


Figure 7: Bohr's atomic model: The electron orbitting the nucleus can only choose from a certain number of allowed orbits. When it jumps from one orbit to another, it emits or absorbs a photon.

nucleus and where an electron jumps between orbits, is accompanied by an emitted or absorbed amount of electromagnetic energy (hv). The orbits in which the electron may travel are shown as grey circles; their radius increases as $n2$, where n is the principal quantum number. The $3 \to 2$ transition depicted here produces the first line of the Balmer series, and for hydrogen ($Z = 1$) it results in a photon of wavelength 656 nm (red light).

4.1 A Puzzle of *Square Amplitudes to Probabilities*

A puzzle of *square amplitudes to probabilities*. Amplitudes are used, which square to probabilities, instead of directly using probabilities, is that it lets you have the same probability values with different phases. Why that is important is because it lets us change the phase without affecting probability.

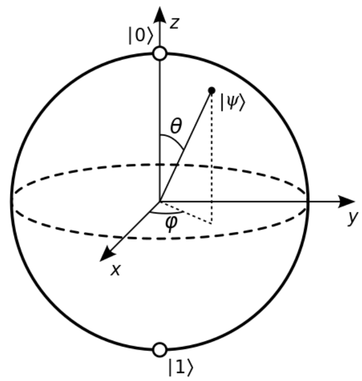
A similar issue in classical physics is we do not know why Newton's law is $F = ma$, instead of $F = ma^2$.

5 Bloch Sphere

$$\boxed{|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle}$$

Spherical Coordinates (Section 2.4.2)

$$\boxed{|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)|1\rangle\,, \;\; 0 \le \theta \le \pi, \;\; 0 \le \phi < 2\pi}$$



6 One-Qubit Gates

Section 3.3.2 Common One-qubit gates (page 127)

Video (14 min) `https://www.youtube.com/watch?v=rD_fH7O-D5Y`

Lab 1: IBM Quantum Learning Suite (Quantum Composer)

See our manual.