CSCI-4961 - RPI, 05/20 - 08/16/2024

Lecture $|5\rangle$: The Matrix (1/3)

May 21, Tue. 6:30pm –8:35pm

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Topics: Complex linear algebra and The Matrix

Quantum gates are linear maps that keep the total probability equal to 1.

$$\boxed{\boldsymbol{U}\,|\psi\rangle = \alpha\boldsymbol{U}\,|0\rangle + \beta\boldsymbol{U}\,|1\rangle}$$

where $\boldsymbol{U}$ is a unitary matrix.

Videos

Video [22] (5 min): The Matrix - Neo sees the Matrix
Video [23] (5 min): What is The Matrix?
Video [24] (10 min): The Story of The Architect | MATRIX EXPLAINED
Video [25] (5 min): The Complete Matrix Timeline
Video [26] (20 min): You are a Simulation & Physics Can Prove It: George Smoot at TEDxSalford

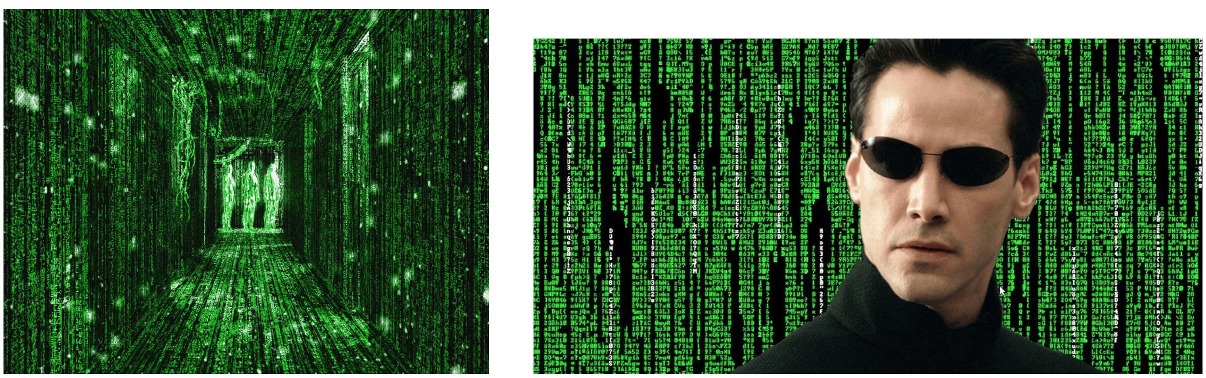

Figure 8: (Credit: The Matrix)

[22] https://www.youtube.com/watch?v=0pYyzolIN3I
[23] https://www.youtube.com/watch?v=O5b0ZxUWNf0
[24] https://www.youtube.com/watch?v=PI4PHI8xLVo
[25] https://www.youtube.com/watch?v=l0N7juslqcc
[26] https://www.youtube.com/watch?v=Chfoo9NBEow

Outline

- (Complex) Linear algebra
- Vectors for Qubit
- Matrices for Quantum Gates

12 Linear Algebra

Chapter 2 Linear Algebra.

13 Vectors for Qubit

Superposition: Qubit

$$\boxed{|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle}$$

where amplitudes $\alpha, \beta \in \mathbb{C}$ satisfy that $|\alpha|^2 + |\beta|^2 = 1$.

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \\ \beta \end{bmatrix} = \alpha \underbrace{\begin{bmatrix} 1 \\ \\ 0 \end{bmatrix}}_{|0\rangle} + \ \beta \underbrace{\begin{bmatrix} 0 \\ \\ 1 \end{bmatrix}}_{|1\rangle} \tag{18}$$

$\boxed{\text{Linearity of function: } f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)}$.

Examples: "plus", "minus", "i", and "minus i", ket and bra representations.

$$\begin{aligned}
|+\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), & \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ \\ 1 \end{bmatrix}, && \frac{1}{\sqrt{2}}[1 \quad 1] \\
|-\rangle &= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle), & \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ \\ -1 \end{bmatrix}, && \frac{1}{\sqrt{2}}[1 \quad -1] \\
|i\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle), & \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ \\ i \end{bmatrix}, && \frac{1}{\sqrt{2}}[1 \quad -i] \\
|-i\rangle &= \frac{1}{\sqrt{2}}(|0\rangle - i\,|1\rangle), & \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ \\ -i \end{bmatrix}, && \frac{1}{\sqrt{2}}[1 \quad i]
\end{aligned} \tag{19}$$

14 One-Qubit Gates

A quantum gate is *linear*, recall the linearity of function such that

$\boxed{\text{Linearity of function: } f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)}$.

We can distribute $\boldsymbol{U}$ across a superposition state (qubit) $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ as follows (pp. 99):

$$\boldsymbol{U}\,|\psi\rangle = \alpha\boldsymbol{U}\,|0\rangle + \beta\boldsymbol{U}\,|1\rangle \tag{20}$$

Probability-preserving linear maps.

$\boxed{\text{Quantum gates are linear maps that keep the total probability equal to } 1.}$

Section 3.3.2 and Lab1: I, X, Y, Z, S, T, H

Gate	Action on Computational Basis	Matrix Representation
Identity	$I\|0\rangle = \|0\rangle$ $I\|1\rangle = \|1\rangle$	$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
Pauli X	$X\|0\rangle = \|1\rangle$ $X\|1\rangle = \|0\rangle$	$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Pauli Y	$Y\|0\rangle = i\|1\rangle$ $Y\|1\rangle = -i\|0\rangle$	$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
Pauli Z	$Z\|0\rangle = \|0\rangle$ $Z\|1\rangle = -\|1\rangle$	$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Phase S	$S\|0\rangle = \|0\rangle$ $S\|1\rangle = i\|1\rangle$	$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$
T	$T\|0\rangle = \|0\rangle$ $T\|1\rangle = e^{i\pi/4}\|1\rangle$	$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$
Hadamard H	$H\|0\rangle = \frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$ $H\|1\rangle = \frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$	$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

Figure 9: Common one-qubit gates as matrices

Scott Aaronson: Quantum mechanics (QM) resembles an opearting system on which the rest of Physics is running its application software.[27]

[27]Except for general relativity "which has not yet been successfully ported to this particular OS"