CSCI-4961 - RPI, 05/20 - 08/16/2024

HW $|3\rangle$: Entanglement

Due date: Friday 11:59pm, June 28

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Topics: Product states; maximally entangled states, and partially entangled states; Bell states; GHZ gates.

$$\begin{aligned}|\psi_1\rangle\,|\psi_0\rangle &= (\alpha_1\,|0\rangle + \beta_1\,|1\rangle) \otimes (\alpha_0\,|0\rangle + \beta_0\,|1\rangle) \\ &= \alpha_1\alpha_0\,|00\rangle + \alpha_1\beta_0\,|01\rangle + \beta_1\alpha_0\,|10\rangle + \beta_1\beta_0\,|11\rangle\end{aligned}$$

The mixed product property of tensor product. Given matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, and $\boldsymbol{D}$,

$$(\boldsymbol{A} \otimes \boldsymbol{B})(\boldsymbol{C} \otimes \boldsymbol{D}) = (\boldsymbol{AC}) \otimes (\boldsymbol{BD})$$

where valid dimensions for doing matrix multiplications $\boldsymbol{AC}$ and $\boldsymbol{BD}$.

Overview

- This homework is due by 11:59pm on Friday 11:59pm, June 28
- You may work on this problem set in a group of up to three students; students are encouraged to re-organize teams for different tasks
- Besides the textbook, you may use ChatGPT or any online materials, but **please state clearly the info sources**.
- Please start this homework early and ask questions during Yue's OHs; also ask questions on the Discussion Forum, but be careful not to give any answers away
- Please be concise in your written answers; even if your solution is correct, if it is not well-presented and clear, you may still lose points
- You can type or hand-write (or both) your solutions to the required graded problems below; all work must be organized in one PDF that lists all teammate names
- You are strongly encouraged to use LaTeX, in particular for mathematical symbols; see the corresponding hw1.tex file as a starting point and example

Videos on Entanglement

Video [6] (4 min, TED-Ed): Quantum Explained – Entanglement

Notations

- $|\psi_1\rangle\,|\psi_0\rangle$
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Practice Problems

The problems below are practice problems that will not be reviewed or graded. We encourage you to work on these problems as you study and learn the course material.

Entanglement (page 147 - 149, page 237 - 239)

- **Exercise** 4.10, 4.11
- **Exercise** 6.3

CNOT gates (page 154 - 174)

- **Exercise** 4.16, 4.17

[6] https://www.youtube.com/watch?v=vpxNuicN1rY

Graded Problems

The problems below are required and will be graded.

Q1. Entanglement (Section 2.2.3, pp. 80 - 83)

1). Exercise 4.12 on pp. 150 (also did it in Quiz 4)

2). Exercise 6.2 on pp. 239

Q2. CNOT gates (Section 4.4.2)

CNOT gate can create entanglement.

1). Exercise 4.15 on pp. 158 (also did it in Quiz 4)

2). Exercise 4.18 on pp. 159

2). Exercise 4.23 on pp. 164; it also appears on pp. 273-274
(Notice that please verify by matrix multiplication, NOT by computer).

Q3. GHZ Gates

A quantum gate $\boldsymbol{U}$ is always reversible, and its ivnerse is $\boldsymbol{U}^{\dagger}$.

1). Exercise 6.11 on pp. 254

Q4. Bell States

Bell States/Basis are fundamental for two qubits and discussing the concept of entanglement.

1). Exercise 4.13 on pp. 153 (also did in Quiz 4)

2). Exercise 4.19 on pp. 160

3). Exercise 6.13 on pp. 256