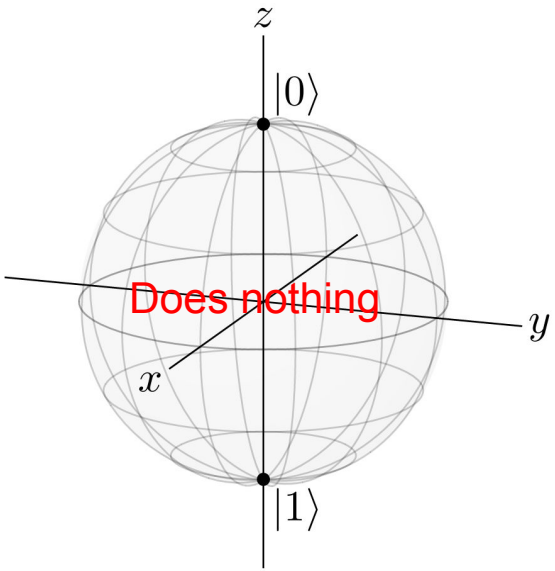
Qubit and Single-qubit Gates

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Gate	Action on Computational Basis	Matrix Representation
Identity	$I\|0\rangle = \|0\rangle$ $I\|1\rangle = \|1\rangle$	$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
Pauli X	$X\|0\rangle = \|1\rangle$ $X\|1\rangle = \|0\rangle$	$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Pauli Y	$Y\|0\rangle = i\|1\rangle$ $Y\|1\rangle = -i\|0\rangle$	$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
Pauli Z	$Z\|0\rangle = \|0\rangle$ $Z\|1\rangle = -\|1\rangle$	$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Phase S	$S\|0\rangle = \|0\rangle$ $S\|1\rangle = i\|1\rangle$	$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$
T	$T\|0\rangle = \|0\rangle$ $T\|1\rangle = e^{i\pi/4}\|1\rangle$	$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$
Hadamard H	$H\|0\rangle = \frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$ $H\|1\rangle = \frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$	$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

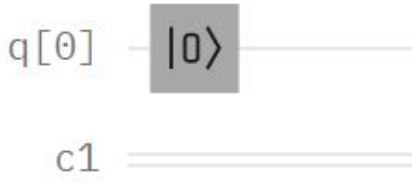
Pauli I Gate

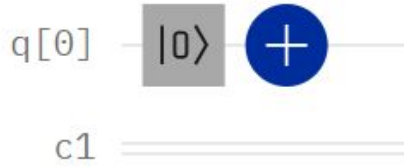


Bloch sphere：I gate

Quantum Composer：I gate

Pauli X Gate

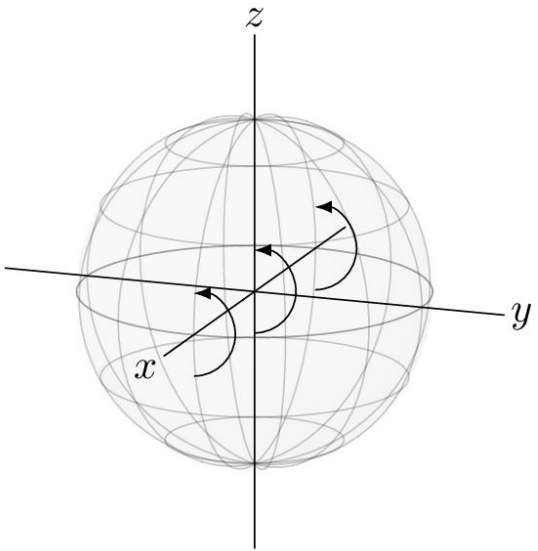




$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

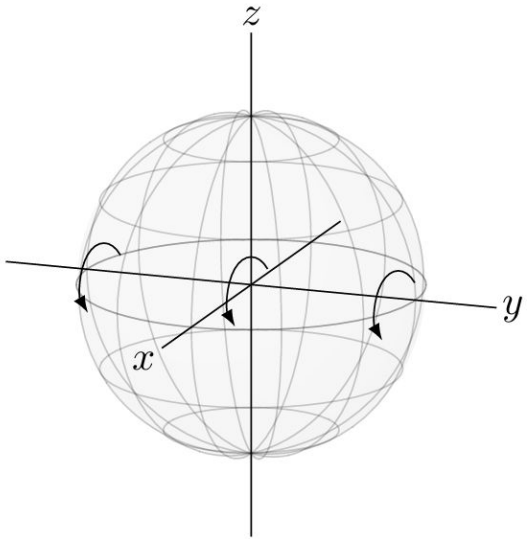
Pauli X Gate



Bloch sphere：X gate

Quantum Composer：X gate

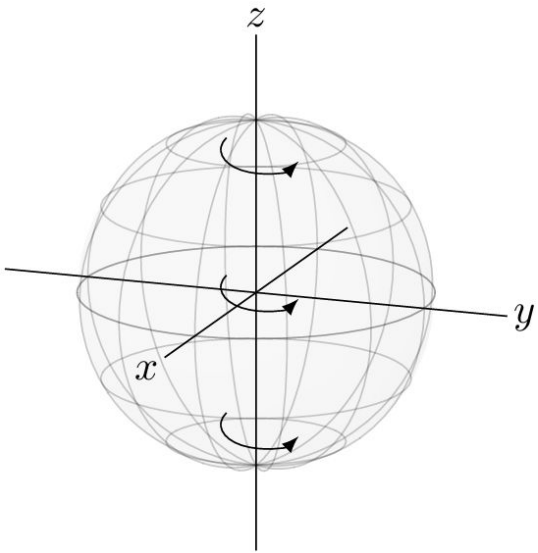
Pauli Y Gate



Bloch sphere：Y gate

Quantum Composer：Y gate

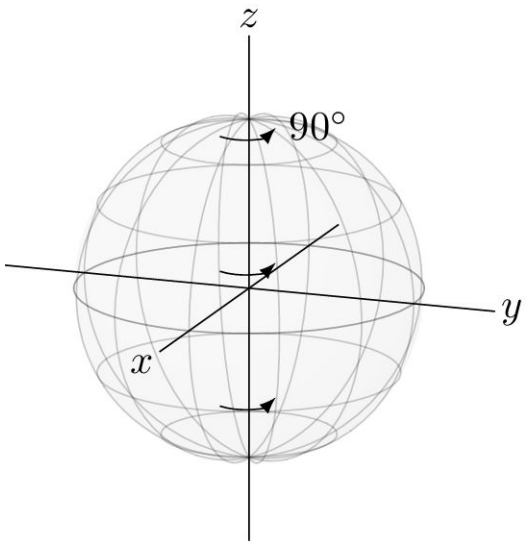
Pauli Z Gate



Bloch sphere：Z gate

Quantum Composer：Z gate

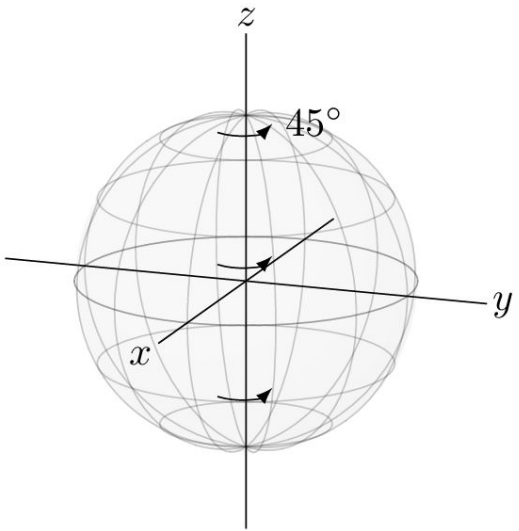
S Gate



Bloch sphere：S gate

Quantum Composer：S gate

T Gate

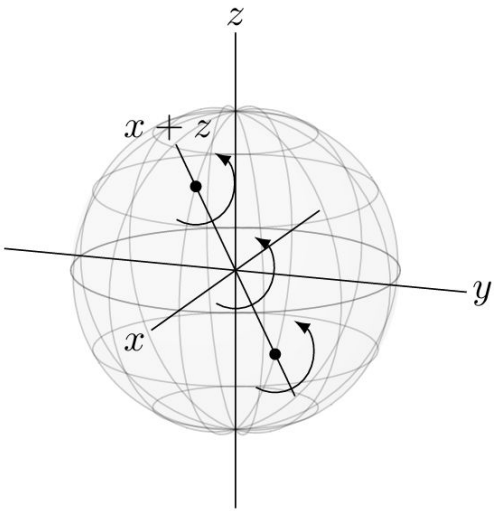


Bloch sphere：T gate

Quantum Composer：T gate

H Gate

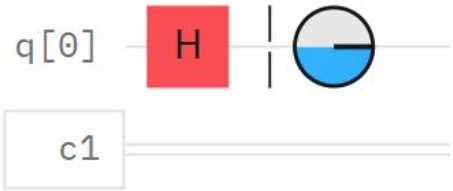
H Gate



Bloch sphere：H gate

Quantum Composer：H gate

Hadmard Gate



$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |-\rangle$$

$$H(|0\rangle) = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix}$$

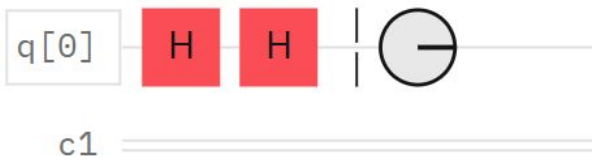
$$= \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\ 0\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}\right)$$

$$= \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

Associative: Hardmard gate

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |-\rangle$$



$$\begin{aligned} H(H(|0\rangle)) &= H(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle) \\ &= \frac{1}{\sqrt{2}}(H(|0\rangle) + H(|1\rangle)) \\ &= \frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \\ &= |0\rangle \end{aligned}$$

$$\begin{aligned} H * H * |0\rangle &= \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} * \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} * \begin{bmatrix}1\\ 0\end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix} \\ &= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} * \begin{bmatrix}1\\ 0\end{bmatrix} \\ &= \begin{bmatrix}1\\ 0\end{bmatrix} \\ &= |0\rangle \end{aligned}$$

Noncomutative: HX != XH

$$HX\ |0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ -1\end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$$XH\ |0\rangle = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} * \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$