Count Of Valleys And Mountains

(n pairs)
- upstroke /
- downstroke \

n	ways
1	1
2	2
3	5

$C_3 = C_0C_2 + C_1C_1 + C_2C_0$

$1 * 2 + 1 * 1 + 2 * 1 = 5$

Count Brackets

1. You are given a number n, representing the number of opening brackets (and closing brackets)
2. You are required to find the number of ways in which you can arrange the brackets if the closing brackets should never exceed opening brackets)

↳ balanced parenthesis expressions

n = 2

n pairs of opening and closing brackets.

Same as previous

/ → (

\ →)

n	count of ways	ways
1	1	( )
2	2	( )( ) , (( ))
3	5	$C_0C_2 + C_1C_1 + C_2C_0$ $1*2 + 1*1 + 2*1 = 5$ ( )( )( ) | (( )) ( ) | (( )( )) ( )(( )) | | (((( ))))
4	14	$C_0C_3 + C_1C_2 + C_2C_1 + C_3C_0$ $1*5 + 1*2 + 2*1 + 5*1 = 14$ ( ) ( ) ( )( ) | (( )) ( )( ) | (( )( )) ( ) | (( )( )( )) ( ) ( ) (( )) | (( )) (( )) | (((( )))) ( ) | (( ) (( ))) ( ) (( )) ( ) | ((( ))) | | ((( ) ( ))) ( ) (( )( )) | | | ((( ) ( ))) ( ) (((( )))) | | | ((( ) (( ))))

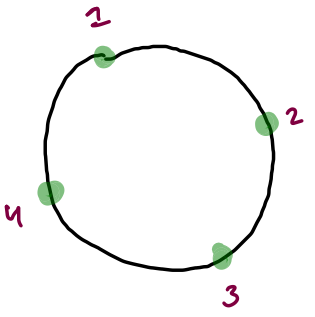
Circle And Chords

1. You are given a number N.
2. There are 2*N points on a circle. You have to draw N non-intersecting chords on a circle.
3. You have to find the number of ways in which these chords can be drawn.

N = 2

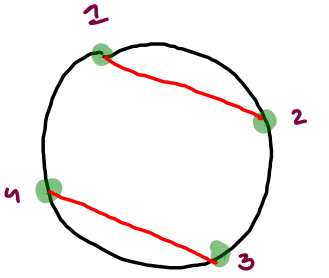
2N points on circle

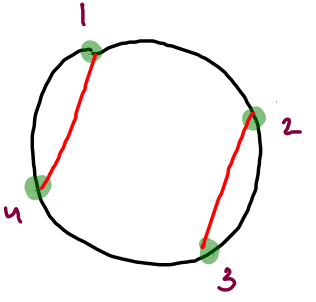
N chords (non-intersecting)

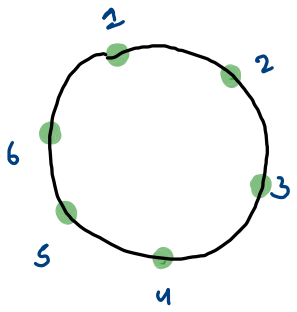


n = 2

2 ways





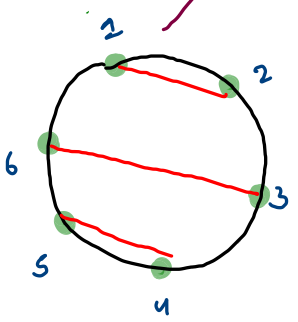


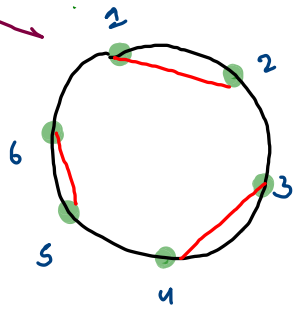
$n = 3$

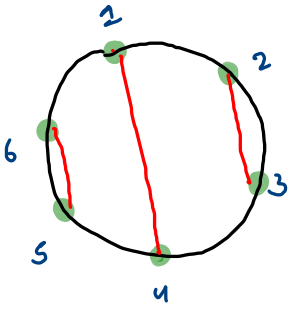
6 points

3 chords

$C_0 C_2$







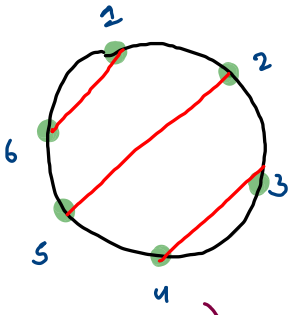
$\rightarrow C_1 C_1$

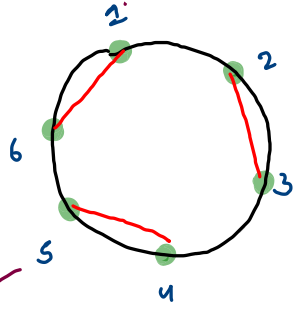
C_3

$= C_0 C_2 + C_1 C_1 + C_2 C_0$

$= 1 * 2 + 1 * 1 + 2 * 1$

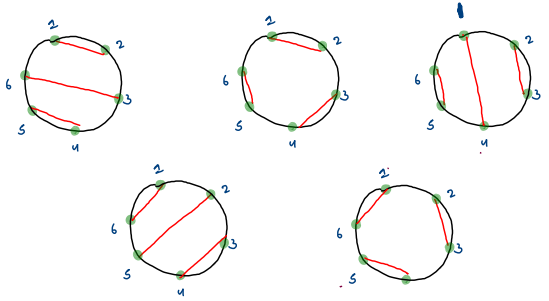
$= 2 + 1 + 2$





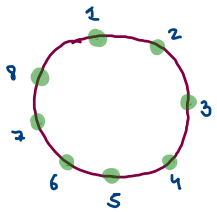
$C_2 C_0$

$n = 3$



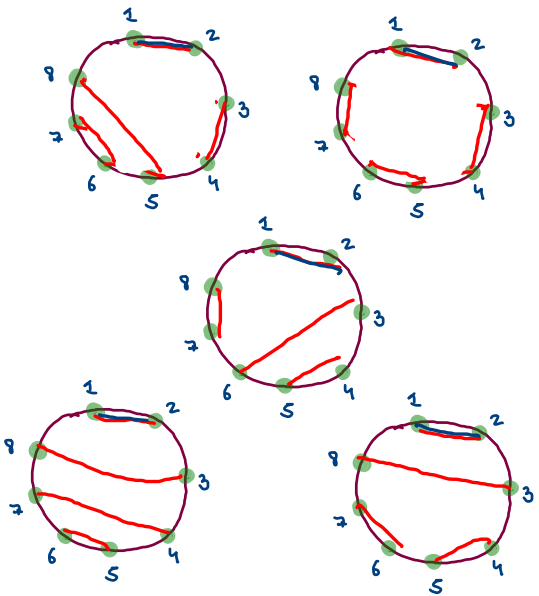
$n = 4$

8 points, 4 chords (non-intersecting)

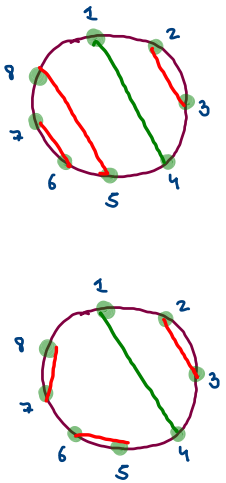


$C_4 = C_0C_3 + C_1C_2 + C_2C_1 + C_3C_0 = 14$

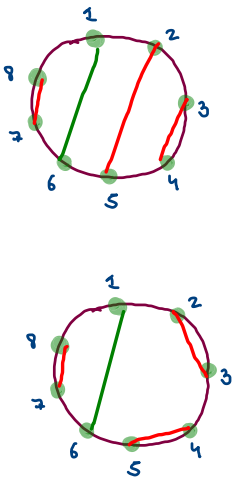
$C_0C_3 = 1 * 5$



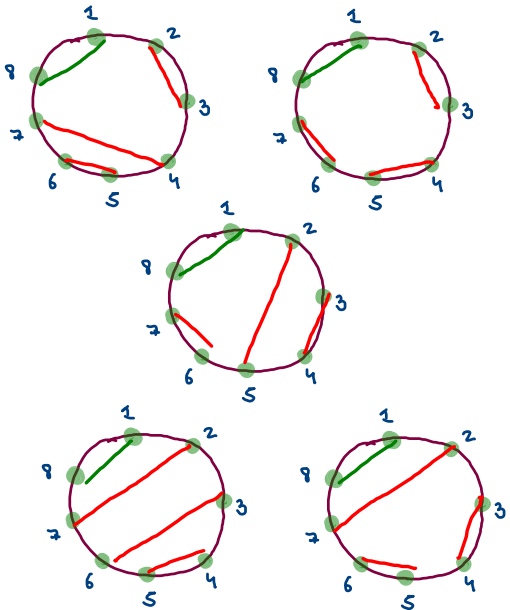
$C_1C_2 = 1 * 2$



$C_2C_1 = 2 * 1$



$C_3 * C_0 : 5 * 1$



Number Of Ways Of Triangulation

1. You are given a number N, which represents the number of sides in a polygon.
2. You have to find the total number of ways in which the given polygon can be triangulated.

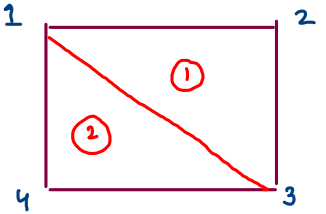
n = 4

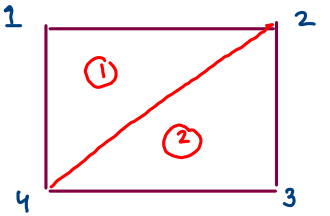
base case

n = 3

(already triangulated)

triangle





$\triangle_n = C_{n-2}$

C_{n-2} → catalan of n-2

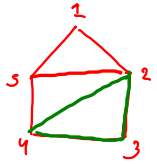
$\triangle_n$ → ways to triangulate a polygon of n side

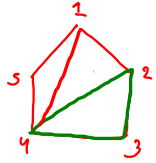
$n = 5$ (pentagon)

$\Delta_5 = C_3 = 5$

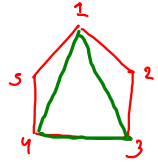
$C_3 = C_0 C_2 + C_1 C_1 + C_2 C_0$

$1 * 2$

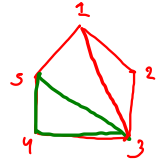


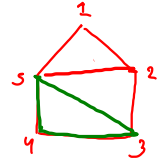


$1 * 1$



$2 * 1$





$\Delta_0 \rightarrow$ X

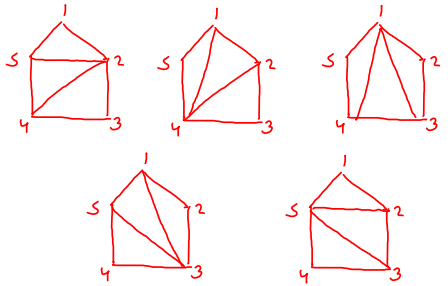
$\Delta_1 \rightarrow$ •

$\Delta_2 \rightarrow$ •—•

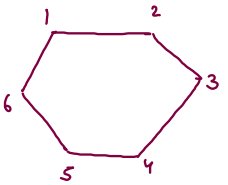
$\Delta_3 \rightarrow$ 

$\Delta_4 \rightarrow$ 

n = 5 (C_3)



n = 6



C_0C_3 (5)	C_1C_2 (2)	C_2C_1 (2)	C_3C_0 (5)